BMC Operator Theory Workshop

University of Kent, Canterbury

April 17 -18, 2012

Organizers: Bas Lemmens and Ian Wood

1 Programme

Tuesday		
14.00-14.30	Marco Marletta	Approximating the spectra of self-adjoint and dissipative operators
14:40-15.10	Matthias Langer	Triple variational principles for self-adjoint operator functions
15.20-15.50	Brian Winn	Quantum ergodicity for expander graphs with large girth
16.00-16.30	David Rule	An end-point result for bilinear Fourier integral operators
Wednesday		
14.00-14.30	Jens Bolte	Many-particle quantum graphs
14:40-15.10	Malcolm Brown	A uniqueness result for one-dimensional inverse scattering
15.20-15.50	Simon Eveson	Some unexpected rank 1 asymptotics
16.00-16.30	Charles Batty	Quasi-hyperbolic operators and semigroups

Room: KS13 (Keynes College)

2 Titles & Abstracts

Quasi-hyperbolic operators and semigroups

Charles Batty
(Oxford)

Abstract: An operator on a Banach space is hyperbolic (aka exponentially dichotomous) if it is a direct sum of two operators, the powers of one summand decays exponentially and the powers of the inverse of the other summand decay exponentially. Quasi-hyperbolic operators are not necessarily hyperbolic but they behave similarly. They arise naturally in pure operator theory, PDEs and differential geometry. The theory for single operators is quite complete. For one-parameter semigroups of operators the failure of spectral mapping theorems prevents a simple characterisation of quasi-hyperbolicity in terms of the generator, and the picture is incomplete.

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Many-particle quantum graphs

Jens Bolte
(Royal Holloway)

Abstract: Quantum graphs are defined in terms of self-adjoint realisations of a Laplacian on a metric graph. I will introduce many-particle quantum systems on graphs with singular two-particle interactions that are either localised in the vertices, or along the edges. The latter case includes a Lieb model on a graph. The self-adjoint many-particle Laplacians are constructed via suitable quadratic forms, and a Weyl-type asymptotic eigenvalue count is proved. We also prove H^2-regularity in certain cases.

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A uniqueness result for one-dimensional inverse scattering

Malcolm Brown
(Cardiff)

Abstract: We consider the whole-line inverse scattering problem for Sturm-Liouville equations which have constant coefficients on a half-line. Since in this case the reflection coefficient determines a Weyl-Titchmarsh m-function, it determines the coefficients up to some simple Liouville transformations. Given inverse spectral theory this provide extensions of known results as we require less smoothness and less decay than is customary.

Joint work with Christer Bennewitz (Lund) and Rudi Weikard (Birmingham AL).

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Some unexpected rank 1 asymptotics

Simon Eveson
(York)

Abstract: I consider three situations in which iterates of linear mapping have an asymptotic rank 1 nature.

The simplest example is just the power method: if a bounded operator T has a dominant, simple eigenvalue λ, then $T^n \sim \lambda^n P$, where P is a rank 1 spectral projection.

Some quasi-nilpotent operators behave in a similar way. For example, for many Volterra convolution operators

$$(V_k f)(t) = \int_0^t k(t-s)f(s)\mathrm{d}s$$

acting on, say, $L^p(0,1)$, we can construct sequences (S_n) of rank 1 operators such that $V_k^n \sim S_n$ as $n \to \infty$.

As a final example, take the polylaplacian Δ^n in the unit ball B_m in $\mathbb{R}^m$, with Dirichlet boundary conditions, and let $\mathcal{G}_n$ be the solution operator (Green's operator), acting on $L^2(B_m)$. Then, as $n \to \infty$, $\mathcal{G}_n \sim c_n P_n$, where c_n is a known sequence of scalars and P_n is a rank 1 orthogonal projection.

This all invites the question (to which I have no answer!): is there an underlying reason why these rather different situations all give rise to rank-1 asymptotics?

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Triple variational principles for self-adjoint operator functions

Matthias Langer
(Strathclyde)

Abstract: Many eigenvalues problems that depend nonlinearly on the eigenvalue parameter can be studied using functions whose values are operators in a Hilbert space. In this talk I present an inequality between eigenvalues of such operator functions and a triple variation involving a generalised Rayleigh functional. Moreover, a method is considered that can be used to show that certain sets are contained in the resolvent set. The results are applied to self-adjoint operators and block operator matrices where eigenvalues in gaps of essential spectrum can be studied.

The talk is based on joint work with Michael Strauss.

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Approximating the spectra of self-adjoint and dissipative operators

Marco Marletta
(Cardiff)

Abstract: We discuss nesting results for Dirichlet to Neumann maps, and dissipative perturbation tricks, for finding the spectra of differential operators on singular domains. Parts of this work are joint with R. Scheichl (Bath).

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An end-point result for bilinear Fourier integral operators

David Rule
(HeriotWatt)

Abstract: I will describe an extension of a theorem of R. Coifman and Y. Meyer regarding bilinear pseudo-differential operators to bilinear Fourier integral operators. More precisely, we prove the global $L^2 \times L^2 \to L^1$ boundedness of bilinear Fourier integral operators with amplitudes in the class $S^0_{1,0}(n, 2)$. The proof makes use of a quadratic $T(1)$-theorem and commutator estimates. This is joint work with Wolfgang Staubach and Salvador Rodriguez-Lopez.

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Quantum ergodicity for expander graphs with large girth

Brian Winn
(Loughborough)

Abstract: Differential operators on graph-like structures have been much-studied in recent years, both as interesting mathematical objects in their own right, and as models of complex quantum systems. In the latter context, it is perhaps surprising that a completely rigorous proof of the quantum ergodicity property is currently lacking, as this is known to hold in very many other situations. We prove a partial result in this direction, restricting attention to expander graphs with large girth (such as, for example, families of Ramanujan graphs due to Lubotzky, Phillips and Sarnak, and others) quantised with an operator that dis-allows back-scattering at vertices of the graph. This is joint work with Matthew Brammall.

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