1. **Title of the module**

MAST6022 (MA6522) - Integrable Systems

MAST7022 (MA7522) – Integrable Systems

1. **School or partner institution which will be responsible for management of the module**

School of Mathematics, Statistics and Actuarial Science

1. **The level of the module (e.g. Level 4, Level 5, Level 6 or Level 7)**

MAST6022: Level 6 ; MAST7022: Level 7

1. **The number of credits and the ECTS value which the module represents**

15 credits (7.5 ECTS)

1. **Which term(s) the module is to be taught in (or other teaching pattern)**

Autumn or Spring

1. **Prerequisite and co-requisite modules**

**Level 6:**

For delivery to students completing Stage 1 before September 2016:

Pre-requisite: MA552 (Analysis), MA588 (Mathematical Techniques and Differential Equations);

Co-requisite: None

For delivery to students completing Stage 1 after September 2016:

Pre-requisite: MAST5013 (Real Analysis 2); MAST5005 (Linear Partial Differential Equations), MAST5012 (Ordinary Differential Equations)

Recommended: MAST5004 (Lagrangian and Hamiltonian Dynamics); MAST6002 (Linear and Nonlinear Waves).

Co-requisite: None

**Level 7:**

Pre-requisite: Students are expected to have studied material equivalent to that covered in the modules above.

Co-requisite: None

1. **The programmes of study to which the module contributes**

For the level 6 module, BSc Mathematics (including programme with a Year in Industry), BSc Mathematics with a Foundation Year, MMath Mathematics, Graduate Diploma in Mathematics, International MSc Mathematics and its Applications (including programme with an Industrial Placement).

For the level 7 module, MMath Mathematics, International MSc Mathematics and its Applications, MSc Mathematics and its Applications (including programmes with an Industrial Placement).

1. **The intended subject specific learning outcomes.**

**On successfully completing the level 6 module students will be able to:**

1. demonstrate systematic understanding of key aspects of integrable systems;
2. demonstrate the capability to deploy established approaches accurately to analyse and solve problems using a reasonable level of skill in calculation and manipulation of the material in the following areas: nonlinear differential equations, Hamiltonian systems, nonlinear difference equations;
3. apply key aspects of integrable systems in well-defined contexts, showing judgement in the selection and application of tools and techniques.

**On successfully completing the level 7 module students will be able to:**

1. demonstrate systematic understanding of integrable systems;
2. demonstrate the capability to solve complex problems using a very good level of skill in calculation and manipulation of the material in the following areas: nonlinear differential equations, Hamiltonian systems, nonlinear difference equations;
3. apply a range of concepts and principles in integrable systems in various different contexts, showing good judgment in the selection and application of tools and techniques.
4. **The intended generic learning outcomes.**

**On successfully completing the level 6 module students will be able to:**

1. manage their own learning and make use of appropriate resources;
2. understand logical arguments, identifying the assumptions made and the conclusions drawn;
3. communicate straightforward arguments and conclusions reasonably accurately and clearly;
4. manage their time and use their organisational skills to plan and implement efficient and effective modes of working;
5. solve problems relating to qualitative and quantitative information;
6. make competent use of information technology skills such as online resources (Moodle) and internet communication;
7. communicate technical material competently;
8. demonstrate an increased level of skill in numeracy and computation;
9. demonstrate the acquisition of the study skills needed for continuing professional development.

**On successfully completing the level 7 module students will be able to:**

1. work competently and independently, be aware of their own strengths and understand when help is needed;
2. demonstrate a high level of capability in developing and evaluating logical arguments;
3. communicate arguments confidently with the effective and accurate conveyance of conclusions;
4. manage their time and use their organisational skills to plan and implement efficient and effective modes of working;
5. solve problems relating to qualitative and quantitative information;
6. make effective use of information technology skills such as online resources (Moodle) and internet communication;
7. communicate technical material effectively;
8. demonstrate an increased level of skill in numeracy and computation;
9. demonstrate the acquisition of the study skills needed for continuing professional development.
10. **A synopsis of the curriculum**

Integrable systems are special dynamical systems which can be solved exactly in some sense. They arise in a variety of settings, ranging from Hamiltonian systems and nonlinear wave equations to difference equations. This module covers the origins of the subject as well as modern topics like integrable maps and lattice equations.

* **Liouville integrability in classical mechanics.** Hamiltonian mechanics. Canonical symplectic form and Poisson brackets. Liouville's theorem (statement and examples). Lax pairs for finite-dimensional systems.
* **Soliton equations.** History and physical origins (e.g. Korteweg-de Vries and/or sine-Gordon). Conservation laws. Hamiltonian formalism.  Lax pairs.
* **Construction of solitons**. Introduction to inverse scattering. Darboux-Bäcklund transformations. Hirota’s method.
* **Discrete integrability.** Symplectic maps. Liouville's theorem (discrete version). Integrable lattice equations. Discrete Lax pairs with examples.

At level 7, topics will be studied and assessed to greater depth.

1. **Reading List (Indicative list, current at time of publication. Reading lists will be published annually)**

O. Babelon, D. Bernard and M. Talon, Introduction to Classical Integrable Systems, Cambridge Monographs on Mathematical Physics, Cambridge University Press, 2003.

M.J. Ablowitz and P.A. Clarkson, Solitons, Nonlinear Evolution Equations and Inverse Scattering, London Mathematical Society Lecture Note Series 149, Cambridge University Press, 1992.

P.G. Drazin and R.S. Johnson, Solitons: an introduction, Cambridge Texts in Applied Mathematics 2, Cambridge University Press, 1989.

**J. Hietarinta, N. Joshi and F. W. Nijhoff,** Discrete Systems and Integrability, Cambridge Texts in Applied Mathematics, Cambridge University Press, 2016.

1. **Learning and Teaching methods**Total contact hours: 42

Private study hours: 108

Total study hours: 150

1. **Assessment methods**

13.1 Main assessment methods

**Level 6:**

Assessment 1 Exercises, requiring on average between 10 and 15 hours to complete 10%

Assessment 2 Exercises, requiring on average between 10 and 15 hours to complete 10%

Examination 3 hours 80%

The coursework mark alone will not be sufficient to demonstrate the student’s level of achievement on the module.

**Level 7:**

Assessment 1 Exercises, requiring on average between 10 and 15 hours to complete 10%

Assessment 2 Exercises, requiring on average between 10 and 15 hours to complete 10%

Examination 3 hours 80%

The coursework mark alone will not be sufficient to demonstrate the student’s level of achievement on the module.

13.2 Reassessment methods

Like-for-like

1. **Map of Module Learning Outcomes (sections 8 & 9) to Learning and Teaching Methods (section12) and methods of Assessment (section 13)**

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Level 6 Module learning outcome** | 8.1 | 8.2 | 8.3 | 9.1 | 9.2 | 9.3 | 9.4 | 9.5 | 9.6 | 9.7 | 9.8 | 9.9 |
| **Learning/ teaching method** |  |  |  |  |  |  |  |  |  |  |  |  |
| Private Study  | **X** | **X** | **X** | **X** | **X** | **X** | **X** | **X** | **X** | **X** | **X** | **X** |
| Lectures/Exercise classes | **X** | **X** | **X** |  | **X** | **X** |  | **X** |  | **X** | **X** |  |
| Revision classes | **X** | **X** | **X** |  | **X** | **X** |  | **X** |  | **X** | **X** |  |
| **Assessment method** |  |  |  |  |  |  |  |  |  |  |  |  |
| Examination | **X** | **X** | **X** | **X** | **X** | **X** | **X** | **X** |  | **X** | **X** | **X** |
| Coursework | **X** | **X** | **X** | **X** | **X** | **X** | **X** | **X** | **X** | **X** | **X** | **X** |

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Level 7 Module learning outcome** | 8.4 | 8.5 | 8.6 | 9.10 | 9.11 | 9.12 | 9.13 | 9.14 | 9.15 | 9.16 | 9.17 | 9.18 |
| **Learning/ teaching method** |  |  |  |  |  |  |  |  |  |  |  |  |
| Private Study  | **X** | **X** | **X** | **X** | **X** | **X** | **X** | **X** | **X** | **X** | **X** | **X** |
| Lectures/Exercise classes | **X** | **X** | **X** |  | **X** | **X** |  | **X** |  | **X** | **X** |  |
| Revision classes | **X** | **X** | **X** |  | **X** | **X** |  | **X** |  | **X** | **X** |  |
| **Assessment method** |  |  |  |  |  |  |  |  |  |  |  |  |
| Examination | **X** | **X** | **X** | **X** | **X** | **X** | **X** | **X** |  | **X** | **X** | **X** |
| Coursework | **X** | **X** | **X** | **X** | **X** | **X** | **X** | **X** | **X** | **X** | **X** | **X** |

1. **Inclusive module design**

The School recognises and has embedded the expectations of current equality legislation, by ensuring that the module is as accessible as possible by design. Additional alternative arrangements for students with Inclusive Learning Plans (ILPs)/declared disabilities will be made on an individual basis, in consultation with the relevant policies and support services.

The inclusive practices in the guidance (see Annex B Appendix A) have been considered in order to support all students in the following areas:

a) Accessible resources and curriculum

b) Learning, teaching and assessment methods

1. **Campus(es) or centre(s) where module will be delivered:** Canterbury
2. **Internationalisation**

Mathematics is an international language with techniques developed and refined by mathematicians across the globe. Mastery of the subject-specific learning outcomes, 8.1 to 8.6, will equip students to apply the theories and techniques of this module in a wide range of international contexts. The module team is drawn from the School of Mathematics, Statistics and Actuarial Science, which includes many members of staff with international experience of teaching and research collaboration.

In compiling the reading list, consideration has been given to the range of texts that are available internationally and a selection of texts has been identified to complement the delivery of the material.

Examples with an international dimension are included in the module where appropriate.

The support SMSAS provides to its students is also internationally attuned given our international student body.

**FACULTIES SUPPORT OFFICE USE ONLY**

**Revision record – all revisions must be recorded in the grid and full details of the change retained in the appropriate committee records.**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Date approved | Major/minor revision | Start date of the delivery of revised version | Section revised | Impacts PLOs( Q6&7 cover sheet) |
| 16/12/2019 | Major | September 2020 | 6, 10, 12 | No |
|  |  |  |  |  |