Anglo-Franco-German Network in Representation Theory and its applications

Stéphane Launois (PI) and Iain Gordon (CoI)

Part 1: Previous research track record

The Principal Investigator (PI). Principal Investigator Stéphane Launois was appointed to a lectureship in Pure Mathematics at the University of Kent in 2007 and was promoted to Professor in October 2016. He has management experience through his role of Director of Research and REF coordinator for the School of Mathematics, Statistics and Actuarial Science at the University of Kent (2013–2014), and is currently Head of Mathematics at the University of Kent. The PI's research has been supported by a Marie Curie Fellowship (2005–2007), a Marie Curie Reintegration Grant (2007–2010), an EPSRC First Grant (2011–2013), a Marie Curie Fellowship to host Dr. Karel Casteels (2013–2015) and he was awarded a 3-year Standard EPSRC grant in June 2016. His recent research is mainly concerned with quantum groups/algebras and their classical counterparts: Poisson algebras. In particular, he is interested in the prime and primitive spectra of noncommutative deformations of coordinate rings of affine or projective spaces such as flag varieties and their Schubert cells. The ultimate aim here is to understand the representation theory of these algebras (and their Poisson counterparts). One of the features of his research is the interplay between representation theory, (Poisson and algebraic) geometry and combinatorics. More precisely, in joint works with Goodearl, Lenagan and Rigal, they uncovered a deep link between the representation theory of quantum grassmannianns (and their Schubert cells) and the totally nonnegative grassmannians which have recently appeared in different areas of mathematics, e.g. Integrable Systems (KP equation), Theoretical Physics (scattering amplitudes).

More recently, he used differential algebraic geometry/model theory to solve problems in the representation theory of Poisson algebras and noncommutative algebras. The PI is/has supervised 5 PhD students and 3 postdocs on these topics.

The PI has been involved in many UK-EU research activities. In particular, he founded the Kent Algebra Days series. These workshops ran in 2008, 2010, 2011, 2013 and 2014 at the University of Kent. Each workshop served as a link meeting between representation theorists from France, Germany and the UK. In addition, he organised conferences in Luminy and Oberwolfach, and together with Rowena Paget a conference on the applications of representation theory to integrable systems, theoretical physics and total positivity in January 2016. This well-attended workshop encompassed lecture series by Nima Arkani-Hamed (Princeton) and Lauren Williams (Berkeley).

The Co-Investigator (CoI). Iain Gordon is the Professor of Mathematics at the University of Edinburgh, and also the Head of the School of Mathematics. He won the London Mathematical Society Berwick Prize in 2005, held an EPSRC Leadership Fellowship (2008–2014), was an invited speaker at the International Congress of Mathematicians in 2010, and served on the 2014 REF subpanel for Mathematical Sciences. He is a representation theorist, proving results in a wide range of related fields, including Lie theory, combinatorics, and algebraic geometry. He established Macdonald positivity via the Harish-Chandra D-module; he uncovered many noncommutative algebraic geometric connections between rational Cherednik algebras and Hilbert schemes. He has been recognised for his work that introduced baby Verma modules for rational Cherednik algebras. In one of his early papers, he proved conjectures of Haiman on the quotient ring by diagonal invariants, generalising Haiman's work on the $n!$ theorem.

Gordon has experience managing large grants and networks; he serves on the ICMS board, and is the initiator and coI of the Anglo-Franco-German Representation Theory Network. He founded the LMS Algebra and Representation Theory in the North (ARTIN) network in 2003, has founded and grown the Hodge Institute in Edinburgh to its current world-leading status, serving as Head of Research Groups for 5 years, and has further management experience as Head of School. He has organised 12 conferences across the spectrum of representation theory and noncommutative

geometry. He has successfully supervised 9 postdocs and 8 PhD students, with usually between 2–3 PhD students and 2–3 postdocs at any one time.

Network Partners. At UK level, the proposed network will involve research teams from major Mathematics Departments, including Aberdeen, Bath, Birmingham, Bristol, Cambridge, Durham, East Anglia, Edinburgh, Glasgow, Heriot-Watt, Kent, Lancaster, Leeds, Leicester, Liverpool, Imperial College London, King's College London, City University London, Queen Mary University of London, Manchester, Newcastle, Nottingham, Oxford, Southampton, Sheffield, Swansea, Warwick, York. It will engage leading researchers from a wide spectrum of topics related to representation theory and its applications. In particular, it will support applications of representation theory to its traditional neighbours (e.g. algebraic topology, combinatorics, geometry, number theory) but also newer connections (e.g. computer science, theoretical physics, probability, theoretical physics).

For several years there has been a successful international cooperation between France, Germany and the United Kingdom in Mathematics around the theme of "Representation Theory". Initially, this was launched in 2007 between CNRS and EPSRC, for the mutual benefit of mathematicians in both countries, especially for young researchers. Later Germany joined in, through the "DFG Priority Programme SPP 1388". In the UK, this cooperation has been supported by two EPSRC Network Grants:

• EPSRC Network Grant EP/F029381/1 "Representation Theory Across The Channel", which was coordinated by Meinolf Geck (University of Aberdeen) and the CoI from 2008 until the end of 2011;

• EPSRC Network Grant EP/K016326/1 "Anglo-Franco-German Representation Theory" which was coordinated by Radha Kessar (City University) and the CoI from 2012 until February 2016.

The initial network was set up following discussions between EPSRC and CNRS, mediated through the British Embassy in Paris in 2006. It was "twinned" with two French partner networks to form a European Research Partnership" (GDRE 571). In 2009, CNRS and EPSRC took initial steps towards enhancing the original GDRE agreement by approaching the DFG priority programme SPP 1388 Representation Theory in Germany. A memorandum for the creation of an international scientific coordination network (GDRI) between the three countries was set up. This memorandum has now come to an end, but a new one is currently being drafted. Moreover, CNRS has already approved the funding of the GDRI on the French side until 2020 (coordinator: Professor Nicolas Jacon). On the German side, the DFG Schwerpunktprogramm came to an end recently, and a new DFG programme has just been approved for the next four years (coordinator: Günter Malle).

References

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Part 2: Case for support

Representation Theory is one of the most vibrant fields of mathematics today. It was introduced at the end of the nineteenth century thanks to the work of Frobenius, Burnside and Schur. It was then concerned with the study of properties of abstract groups via their representations as linear transformations of vector spaces. The idea was extended to other mathematical structures such as associative algebras, Hopf algebras or Lie algebras. Since its introduction, it is a story of rich innovations and applications to other topics in mathematics and to all sciences. Originally, representations were studied over the field of complex numbers. However, the idea was soon to extend the definition to other fields of positive characteristic or over p-adic fields. New difficulties have then appeared even for basic problems and the introduction of geometric methods to solve them has given a new impulse to the field.

Representation theory is at the heart of an intense research activity over the world, and examples of driving questions/problems are/have been: Questions and problems around the Langlands program; Lusztig's conjecture on irreducible representations of simple Lie algebra in positive characteristic; James' conjecture concerning the representations of symmetric group in positive characteristic; Broué abelian defect group conjectures concerning representations of finite groups in positive characteristic; Kac conjecture; minimal model programme.

Examples of recent spectacular advances (most partly supported by the previous EPSRC grants) include, but are not limited to, works of:

- Kessar-Malle on Donovan conjecture and Brauer's height zero conjecture;
- Elias, Fiebig, Juteau, Williamson and Webster around Soergel bimodules which have led to a proof of a positivity result of the Kazhdan-Lusztig polynomials, and to counter-examples of both Lusztig and James conjecture;
- Teleman on TQFTs (classification of 2-D semisimple field theories);
- Hausel-Letellier-Rodriguez-Villegas on geometric representation theory, and in particular, their proof of Kac conjecture;
- Ardakov-Wadsley on representation theory of p-adic analytic groups and Iwasawa algebras;
- Fordy-Marsh, Hernandez-Leclerc, Geiss-Leclerc-Schröer on cluster algebras and their catergorifications;
- Iyama-Wemyss on connections between algebraic geometry (minimal model), representation theory and categories;
- Premet et al. on the representation theory of W-algebras;
- Chardin-Symonds who gave counterexamples to Derksen's conjecture regarding degree bounds for the syzygies of rings of polynomial invariants in the non-modular case;
- Ciubotaru, Sécherre-Stevens, etc. on the representation theory of p-adic groups, and in particular on the smooth representations of $GL_m(D)$;
- Benson, Broomhead-Pauksztello-Ploog, Chuang-Lazarev, Joyce, Miemietz, etc. on higher categories and connections to derived geometry.

There are also interactions between Representation Theory and other areas of mathematics that are studied. For example, during the three last decades, Representation Theory has become a classical tool for studying problems of probabilistic nature:

- conjecture of Bálint Tóth (Berestycki et al.)
- conditioning of random walks and Brownian motions can be obtained from the Littelmann path model (Biane et al.),

- analysis of Whittaker functions is related to the geometric Robinson-Schensted-Knuth correspondance (O'Connell et al.)
- very recent results by Breuillard, Green, Guralnick and Tao use the representation theory of simple finite groups of Lie type as a important ingredient in the study of their expansion.

Moreover, while applications of representation theory to its traditional neighbours (e.g. algebraic topology, combinatorics, geometry, number theory) continue to flourish, new connections have been established with:

- theoretical physics (work of Arkani-Hamed and co-authors on Scattering Amplitudes);
- noncommutative projective geometry (work of Sierra and Walton on the representation theory of Virasoro Algebra);
- combinatorics (work of Kang, Kashiwara, Kim and Oh on Lusztig's dual canonical basis);
- model theory of differential fields (work of PI and co-authors on the representation theory of Poisson algebras).

The UK is a major international player in Representation Theory with leading practitioners throughout the country, with several members of the previous network speaking at the last two International Congress of Mathematicians. To maintain and enhance this strength, international collaboration on a whole range of levels is vital. This applies, in particular, to the training of PhD students and early-career researchers. Representation Theory is characterised by its use of and connection to many diverse areas of mathematics: it is crucial for the next generation of researchers that we provide this breadth, both to allow young representation theorists to gain experience of other areas of mathematics, and to allow talents from other topics to see representation theory in action. The training and opportunities that we intend the network to offer will be broad, but high level, at a scope far beyond anything that can be offered for instance by the Taught Course Centres in the UK.

Our previous grant and the corresponding GDRI agreement came to an end in February 2016. We are proposing an essentially new network, still partnered with France through the CNRS and Germany through DFG. The aim of the new network is to facilitate the study of these new links between Representation Theory and other fields within the UK, France and Germany. The support of EPSRC is crucial for this as a single team of researchers would not have the expertise in all these areas and so we need support to keep developing tools in representation theory and keep abreast of the latest developments—as mentioned above, several excellent results have been obtained during the period of the previous network by researchers in France, Germany and the UK—and also to develop and create new interactions with other fields of mathematics/sciences. This strongly differs from the previous network which was centred around Representation Theory rather than its applications. This new, pan-european network will have significant scientific benefits for the UK. Representation theoretic interests in the three countries overlap significantly but there are clear complementary strengths. For instance, the UK and France have considerable expertise in group theory and in the interactions between representation theory and probability, while France and Germany are particularly strong in Lie theory and the link between representation theory and number theory, with the UK and Germany being masters in representations of algebras and the interactions between representation theory and mathematical physics. Of course, the research communities in these three countries are already linked through a number of individual contacts and collaborations. But we strongly believe that, given the eclectic nature of representation theory and its wide-ranging applications, coordinated cooperation in the framework of a network with the appropriate financial support from each partner, will lead to a significantly higher synergy. The resulting breadth and medium-term activity will be of significant benefit to UK representation theorists at all levels, and for all mathematicians/scientists whose research interacts with representation theory.

Initial Membership: The main feature of this new network will be a close cooperation and coordination of activities with partner networks and institutions in both France and Germany. At UK

level, the proposed network will involve research teams from all major Mathematics Departments. Administratively, this will be done through a GDRI agreement with the following initial members:

- **France:** CNRS (contact: Gabriel Boutang); University Paris 6 and Paris 7 (contact: Emmanuel Letellier); University of Montpellier (contact: Cédric Bonnafé); University of Picardie (contact: Ivan Marin); University of Reims (contact: Nicolas Jacon)
- **Germany:** University of Kaiserslautern (contact: Günter Malle)
- **UK:** University of Kent (contact: Stéphane Launois)

The PI will be responsible for the overall management of the network working closely with the CoI. The duties of the PI and CoI include:

- coordinating the exchanges between the three partner countries, in close consultation with the representatives of the French and German networks;
- encourage network partners to initiate activities in targeted areas;
- maintaining a webpage containing lists of members with interests, information about forthcoming meetings, links to earlier meetings with active hyperlinks and notes, etc.
- organising conferences as necessary (the conference on Representation Theory and Art will take place at the University of Kent);
- ensuring equality and diversity in selection of speakers, participants, etc.

Matters of a more administrative nature (e.g., refunding of expenses for travel and subsistence) will be handled through the administration of the University of Kent. A steering committee will oversee the scientific content and balance of the network's activities. The membership will initially be as folllows: PI, CoI, representative of our network partners in France and Germany, representatives of smaller groups in the UK (ARTIN, BLOC, etc.), and a few carefully selected researchers who have interest in applying Representation Theory techniques to other fields. Regarding the later, we have already the secured the agreement of the following researchers:

- Ivan Fesenko, University of Nottingham: interactions with Number Theory;
- Paul Martin, University of Leeds: interaction with Mathematical Physics;
- Lionel Mason, Oxford University: interactions with Integrable Systems and Physics;
- Neil O'Connell, University of Warwick: interactions with Probability;
- Konstanze Rietsch, King's College London: interactions with Geometry and liaison with the London School of Geometry and Number Theory (EPSRC CDT).

Objectives: While the internet and electronic communication play an increasingly important role in the exchange of research and ideas, the possibility to meet and discuss individually or in small groups and workshops remains both a key source of information about what is happening at the international level and a key ingredient in the discovery and development of new routes of research; given the interdisciplinary nature of the proposed network, these interactions will be crucial in order to make sure that researchers from other areas of Mathematics/sciences learn about representation-theoretic methods, and also that representation theorists learn interesting open problems from them. In a somewhat different direction, one of the high-level recommendations of the Panel of the International Review of Mathematics 2010 concerns "continued action and attention directed toward improving the quality of PhD training". Thus, the central objectives of the proposed network are:

- Provide workshop programs and training opportunities for early-career researchers at the crossroad between representation theory and its applications;

- Facilitate and encourage contacts and exchanges between researchers in France, Germany and the UK;
- Facilitate interactions between representation theory and other research areas.

By providing the opportunity for frequent contact and collaboration between researchers at different career levels and from institutions across the three countries, the network will be a key facilitator in shaping the direction of a central area of mathematics. Pursuing these objectives leads to a significant increase in international research collaborations and all the benefits that come with these: short time visits of PhD students, invited lecture series, enhanced scope of mini-events and so on. Numerous examples can be found on the summary page of the GDRI at http://njacon.perso.math.cnrs.fr/gdri.html. The proposed new network offers a strong european effort in Representation Theory and its applications at a most needed time.

Activities:

1. Support travel of early-career and established researchers to work on specific projects with colleagues overseas, or to attend workshops. This includes, in particular, participation in the relevant annual GDR workshops at the CIRM in France. (The most recent appropriate meeting would have been in August 2016; see http://scientific-events.weebly.com/1490.html. The next one has just been approved by CIRM and will take place late 2018. It will be dedicated to Representation in Lie Theory and Interactions.) As a general rule, the proposed network will provide support for subsistence for visitors to the UK and travel support for visits out of UK with the explicit understanding that the French and German partner networks reciprocate, applying analogous rules.

2. Enhance the scope (audience, speakers, range of topics) and international standing of regional and interregional meetings (e.g., ARTIN, BLOC, Kent Algebra Days), with the added benefit to encourage and initiate new collaborations. To gain support from the network, organisers will need to demonstrate a willingness to cross boundaries and invite not only representation theorists but also (potential) users of representation theory.

3. Run a biannual representation theory seminar focussing on enhancing the research collaborations between representation theory and other topics both within mathematics (e.g. number theory, algebraic geometry) and beyond mathematics (e.g. quantum computing, art). This will meet on a rotating principle in universities in France, Germany and the UK.

4. Develop bespoke courses for CDT centres and/or Taught course centres. For instance, we have already discussed the possibility to organise a Summer School in collaboration with the London School Geometry & Number Theory (EPSRC CDT) around Serre's modularity conjecture (Galois representations) or geometric representation theory. The proposed network will bring an international dimension to these courses. In particular, it will allow to get more of the CDT's expertise spread to others in the rest of the UK, France and Germany.

5. Two large conferences focussing on the applications of Representation Theory. These large meetings would have the following structure.

    - First, 3-4 days dedicated to mini-courses (e.g. 2 in Representation Theory and 2 in the chosen field of application).
    - Next, 2-3 days dedicated to advanced talks in both areas (and most importantly at their interface).
    - Finally, 2-3 days dedicated to focus group(s) with a few participants working on a specific problem at the interface of the two disciplines. The idea would be to have two leaders per focus group (on from Representation Theory, one from the chosen field of application) and participants from both areas.

Possible topics include Representation Theory and Probability, Representation Theory and Theoretical Computer Science, Representation Theory and Model Theory, etc. The proposed structure for these conferences has been chosen to ensure that researchers from each area (Representation Theory and the chosen field of application) first develop their knowledge of the other area, before working collaboratively on problems at the meeting place between these two areas. While 1-week conferences work well for most meetings centred around only one area/discipline, our conferences aim to be genuinely at the crossroad between two different areas, and we believe a few extra days are needed for researchers in both disciplines to create a common world/language.

6. A more experimental conference on the interactions of Representation Theory in Art (through symmetry), Magic (shuffling), Juggling (geometry) etc. For instance, Diaconis for his work at the crossroad between card shuffling and representation theory, and Knutson for his recent work on juggling and total positivity/representation theory of Poisson algebras, would be two potential speakers. This conference would take place at the University of Kent towards the end of the grant period, and would include a poster competition, a picture competition and a competition for the best art creation inspired by Representation Theory and its applications. This conference has been designed for Representation Theory to reach out a wider audience, and is part of our impact strategy.

Plans for dissemination: We will have a network webpage with an RSS feed which will be the main gateway to information about the network. It will contain lists of members with interests, forthcoming meetings, links to earlier meetings with active hyperlinks, notes and videos, etc. Dissemination will also take place in the proposed conferences, workshops, seminars and research visits. Whenever possible, videos of talks will be recorded to maximise the visibility of the network. Finally we plan several public interest lectures, especially during the experimental conference on the interactions of Representation Theory in Art.

Potential for collaboration: The key aspect of network is collaboration. The network is designed to facilitate a variety of different formats for collaboration, some of which are novel and others tried and tested, but throughout a crucial ingredient is that the intellectual content will be formulated by the members of the network themselves. Thus topics will emerge in response to new developments in the field as well as the fundamental intellectual needs of the UK community, as understood by the mathematicians working at the international forefront of representation theory and its applications. The role of the PI and CoI, in collaboration with the network partners in France and Germany, will be to make sure that over the term of the grant these topics cohere into a broad and balanced portfolio. Given the interdisciplinary nature of the proposed network, potential for collaborations both within and outwith mathematics is huge and access to resources in both France and Germany will increase the possibility for cross-fertilisation with subjects beyond the typical boundaries of representation theory. For instance, it is commonly the case that workshops in Luminy address topics that straddle more than one field, with a recent relevant example being ”Algebraic Combinatorics in Representation Theory” where interactions between representation theory and probability, mathematical physics and dynamical systems where all represented.

Two recent successful conferences supported by the previous network grant are: “Probability and Representation Theory in Edinburgh” (February 2014) which brought together experts in Representation Theory and Probability; and “Total Positivity: A bridge between Representation Theory and Physics” that took place at Kent in January 2016 and which brought together experts in representation theory, integrable systems and theoretical physics. The PI was one of the organiser of the later event, while the CoI was an organiser of the Edinburgh event. The network will aim to extend this type of activities. More precisely, we aim to facilitate new interactions and continue to support traditional interactions.

It is necessary to provide researchers with the chance to develop a broader vision of the fundamental underpinnings and future directions in representation theory, to learn about neighbouring disciplines, and to train researchers in neighbouring disciplines in representation theory. It is also

crucial for the researchers to have the occasion to meet to begin and develop the collaborations. That is why the support will be used for the following actions:

- Support the students for the participations of conferences, summer schools etc.
- Support for the short terms visit between a researcher of one country of the GDRI to a researcher of another country of the GDRI.
- Support for the regular meetings and workshops which are organized around representation theory mainly in the three countries of the GDRI (such as ARTIN and BLOC meetings, the DFG meetings in Germany or the annual conferences of the GDR Théorie de Lie algébrique et géométrique)

The decision on the support will be given following advice of the steering-committee. In addition, the GDRI plans to organize one conference per year concerning the interactions between the representation theory with other areas (such as Probability, Number Theory, Theoretical Physics, Noncommutative Geometry, etc.). Such conferences will include several mini-courses as well as more advanced talks and will take place in the UK in 2017/18, in France in 2018/19, in Germany in 2019/20 and in the UK in 2020/21 (see above for more details on the format of these conferences).

Building on recently discovered links with other fields will ensure that Representation Theory not only continues to flourish but also strengthen its impact on other areas of Mathematics/Sciences and beyond. The european dimension of the network will guarantee the UK strengthen its research collaboration in the future.

National Importance: Mathematical Sciences underpin our 21st century technology, economy and society, and serve as one of the pillars of education at all levels. Major contributions to the health and prosperity of society arise from insights, results and algorithms created by the entire sweep of the mathematical sciences, ranging across the purest of the pure, theory inspired by applications and hands-on applications. This has been the case already with the Ancient Greeks, and it is as true as ever now.

Representation Theory, understood in the broad sense of this proposal, is a key area of mathematical sciences, with multiple and complex connections to neighbouring sciences like Physics (e.g., elementary particle theory, quantum computing, random matrices, string theory), Biology and Chemistry (e.g., symmetry in molecular structures). It is a rich area of research in its own right, touching upon fundamental and deep mathematical problems, and, through its wide range of connections, it is a key to maintaining the health of other research disciplines.

The 2010 International Review of Mathematical Sciences by EPSRC said that “the work in representation theory is world-leading” and that “a challenge for mathematicians working in algebra is to realise the opportunities created by several outstanding young appointments, to establish new vital research areas in the associated sub-disciplines and pursue synergies between them and beyond”. The proposed network will not only help maintain the international standing of representation theory in the UK, but will also act as a bridge between representation theory and other areas of mathematics/sciences.

Another aspect of national importance is the training of PDRAs and PhD students as their contribution is extremely important for the health and vitality of UK mathematics. The proposed network will contribute to this objective by organising relevant activities (Summer Schools, conferences, visits) that will ensure the training of the next generation of mathematicians in a highly interdisciplinary context.

Summary: The proposed new network will allow to increase the reach and influence of the world-class UK research in Representation Theory and its applications, to keep up-to-date and bring complementary expertise, and to keep the UK at the forefront of international visibility. It will also give access to dedicated funds from our European partners for UK researchers. Thus, we are now in a unique position to develop a world-leading cooperation over the next 4 years around the theme of “representation theory and its application”, between France, Germany and the UK.