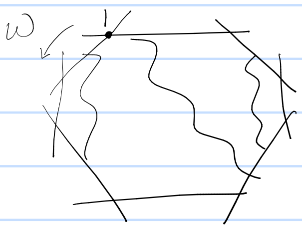


$\overset{8}{\overset{\shortparallel}{n}}$-particle interaction.
$\underset{\underset{3}{\shortparallel}}{K}$ - MHV interaction.

$([8], \{(1,3),(5,8),(5,7)\})$

- joint w/ Marin-Amat
- 1509.06150

Def| Wilson loop diagram: $([n], \mathcal{P})$
$\mathcal{P} = K$ pairs in $[n]$

Thm| If $\nexists$ m vertices adjacent upon both endpoints of m-2 propagators $\Rightarrow$ W defines representable matroid (a.k.a. elem of $Gr(K, n+1)$)

Dfn|
- $\underbrace{Z_* \underbrace{Z_1 \ldots Z_n}_{\text{physical}}}_{\text{given}} \in \mathbb{R}^4$ $\quad \mathcal{Z}_* = \begin{bmatrix} -Z_*- \\ -Z_1- \\ \vdots \\ -Z_n- \end{bmatrix} \in M_{n+1 \times 4}$, $\quad \mathcal{Z} = \begin{bmatrix} Z_1 \\ \vdots \\ Z_n \end{bmatrix} \in M_{n \times 4}$ $\Big\}$ no zero max minor. (a.k.a. in general position in $\mathbb{R}^4$)
- $\sim\sim \longrightarrow Z_* + c_{pi} Z_i + c_{p\,i+1} Z_{i+1} + c_{pj} Z_j + c_{p\,j+1} Z_{j+1} = (1, c_p) \cdot \mathcal{Z}_*$
- $([n], \mathcal{P}) \longrightarrow \left[\mathbb{1} \,\middle|\, C(W(Z)) \right]$ rows given by $(1, c_p)$

$$I(W) = \int_{(\mathbb{CP}^1)^{\times n}} \prod_{p=1}^{K} \frac{d\hat{c}_{p0}\, d\hat{c}_{pi} \cdots d\hat{c}_{pj+1}}{\hat{c}_{p0} \cdots \hat{c}_{pj+1}} f(z)\, \delta^4_p\big((1, c_p) \cdot \mathcal{Z}_*\big)$$

$\hat{c}_{p,i}$ shifted coordinates such that variables take positive values.

$C_p(W)$ above
$$\begin{bmatrix} \hat{c}_{11} & \hat{c}_{12} & \hat{c}_{13} & \hat{c}_{14} & 0 & 0 & 0 & 0 \\ -\hat{c}_{21} & 0 & 0 & \hat{c}_{25} & \hat{c}_{26} & 0 & -\hat{c}_{28} \\ 0 & 0 & 0 & 0 & \underbrace{\hat{c}_{25}\hat{c}_{35}}_{c_{35}} & \underbrace{\hat{c}_{26}\hat{c}_{35} + \hat{c}_{36}}_{c_{36}} & \hat{c}_{37} & \hat{c}_{38} \end{bmatrix}$$

Note!| Delta function in $I(W)$ gives values for $\frac{c_{pi}}{c_{p0}}$
$(1, c_p) \in \operatorname{Ker} \mathcal{Z}_*^T$

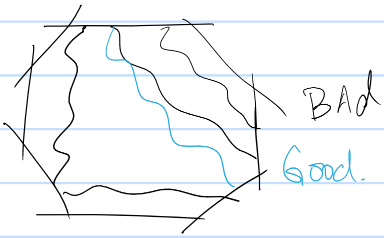
Better with dfn:
$q \in \mathcal{P}$; $V(q) \subset [n]$ subset of 4 verts defining endpts of q
$P \subset \mathcal{P}$, $V(P) = \bigcup_{q \in P} V(q)$

$\Rightarrow (1, C_q) \in \operatorname{Ker} Z^T_{*}\big|_{V(q)}$ selecting colums.

$\{(1, C_q) \mid q \in P\} \in \operatorname{Ker} Z^T_{*}\big|_{V(P)}$

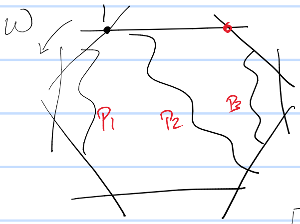
Namely: If $|V(P)| = m$ $\quad |P| > m-3 \Rightarrow (1, C_q)$ not all indep.

Call these graphs overdefined. IGNORE.

Eg 

Cell structure of $([n], P)$:

Dfn | Prop (V) = set of Propagator adjascent upon elems in V.

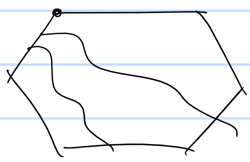


$\text{Prop}(8) = \{P_2, P_3\}$

$\text{Prop}(6,7) = P_3$

$\boxed{V(\text{Prop}(\omega)) \supset \omega}$

Prop(V) can be empty.



Prop $5 = \phi$

$\{5\} = V(\phi)$

Thm | $V \subset \binom{[n]}{|P|}$ a (representable) cell $\Leftrightarrow \nexists\, U \subseteq V$ s.t. $|U| > |\text{Prop}(U)|$

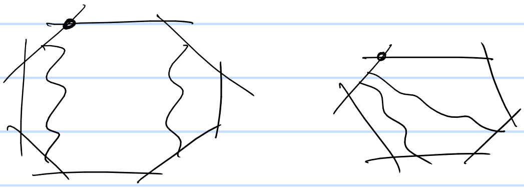
dim $= 3|P|$

When is this positive?

Step 1 | When is ω connected?

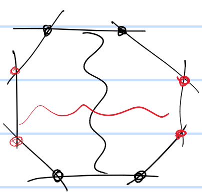
ω disconnected $\Leftrightarrow \exists\, P_1, P_2$ (possibly empty) portioning P s.t.

$V(P_1), V(P_2)$ portion $[n]$, not empty.



disconnected.

Step 2 | if Connected components of ω positive, don't cross $\Rightarrow \omega$ positive.

Eg 

not positive.

Step 3 | Assume ① W connected ② For all $P \subseteq \mathcal{P}$, $|V(P)| > |P| + 3$

No crossing propagators $\Rightarrow$ Positive.

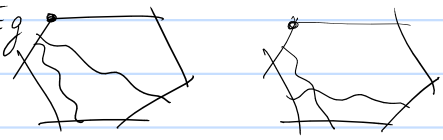
Conj | Each such diagram defines a unique cell.

Step 4 | If W ① Connected, ② $\exists P \subset V(\mathcal{P})$ s.t. $|V(P)| = |P| + 3$

$\mathcal{P} \setminus P$ don't cross $\Rightarrow$ positive.

In particular: If $V(\mathcal{P}) = |\mathcal{P}| + 3$

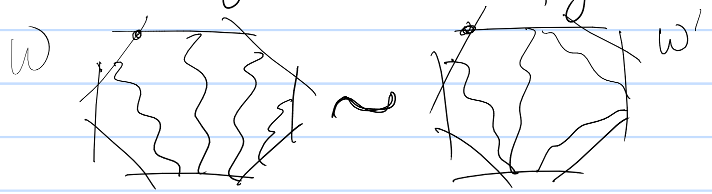
Eg.



totally positive Grassmannian.

More: $\{(1, C_q)\}_{q \in P} \in \operatorname{Ker} Z^T_* \big|_{V(P)}$ $\dim = |V(P)| - 3$

Any such configuration gives same grassmannian.



Notes, Problems, future directions:

$$A_{n,k}(Z_1 \ldots Z_n) \text{ "=" volume form on a geometric object defined by } Z_i$$

$$\wr$$

$$\sum_{\substack{\text{positive,} \\ \text{non crossing } \mathcal{P}}} I(([n], \mathcal{P})) \rightarrow \text{also gives a volume form, collection of cells.}$$

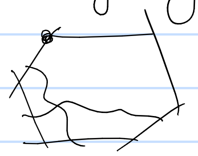
- WLD more than just a matroid.

① $C(W) = C(W')$ but $I(W) \neq I(W')$

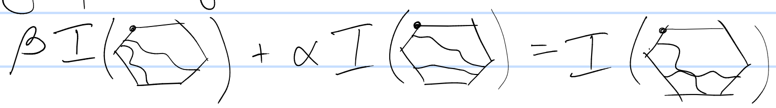
$\longrightarrow$

①! If the goal is to attach a volume form to each diagram that defines a volume on the (dual) amplituhedron → What is going on?

② Wilson loop diagrams know to ignore crossing propagator.



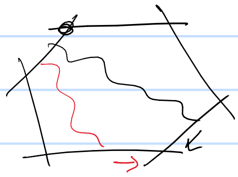
Fact



③ $\overline{\text{MHV}}$ diagrams. $\underbrace{A_{n,n-3}}_{\overline{\text{MHV}}}(Z_1 \cdots Z_n)$ canonically defined.

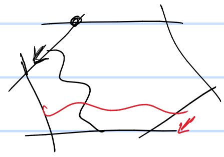
We see that $\overline{\text{MHV}}$ sub diagrams special.

• What are the boundaries of these cells? $[\,\vdots\,|\,C(\omega)]$ good bdndary ✓ others must cancel.



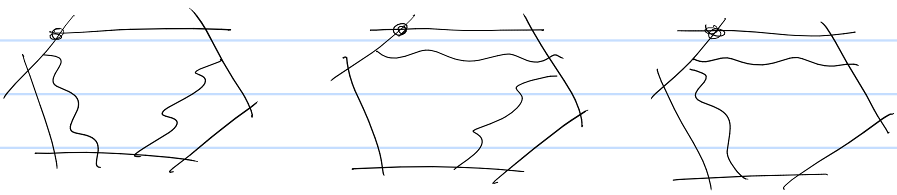
$[a\ b\ 0\ c\ d\ 0]$

$[a\ b\ c\ d\ 0\ 0]$



$[a\ b\ c\ d\ 0\ 0]$

$[0\ a\ b\ c\ d\ 0]$

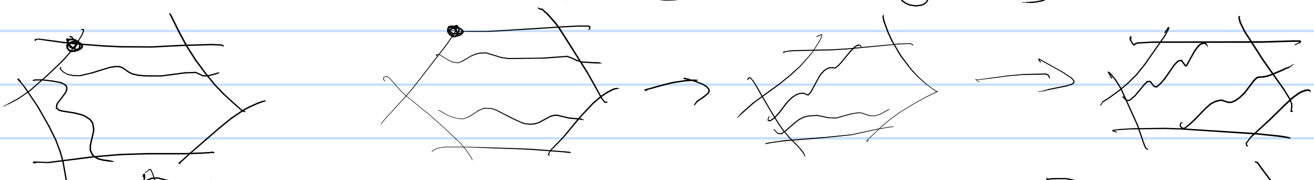


all share codim 2 bndary.

What about codim 1?

④ Ignored first col. $[\,1\,|\,C(\omega)] \in Gr(|P|, n)$

$$\begin{bmatrix} a & b & 0 & 0 & -c & -d \\ 0 & e & f & g & h & 0 \end{bmatrix} \to \begin{bmatrix} d & a & b & 0 & 0 & -c \\ 0 & e & f & g & h & 0 \end{bmatrix}$$



$$\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

$1 \to -1$

Not some elem in $Gr(k, n+1)$!