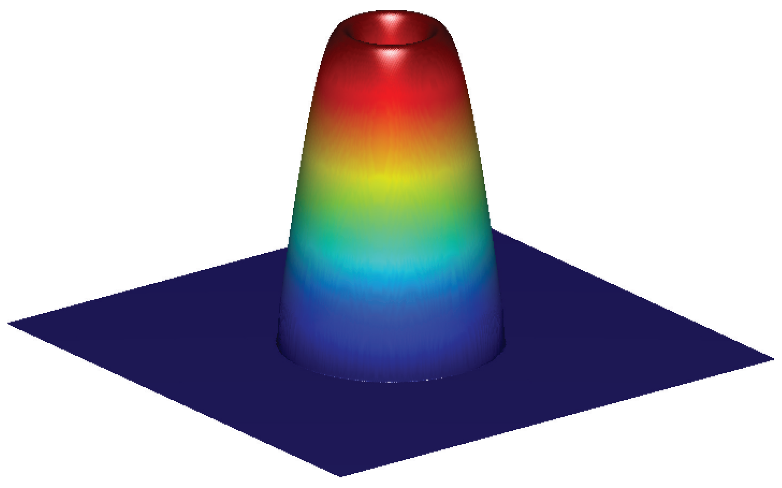

Baby Skyrmions without a potential term

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Motivation

Topological solitons are stable, finite energy solutions of certain partial differential equations which behave like extended particles. In the Skyrme model of nuclear physics, solitons called *Skyrmions* are used to model atomic nuclei. It is computationally intensive to find solutions in the Skyrme model, so interesting processes such as Skyrmion scattering may be first explored in a toy model called the baby Skyrme model. Its solitons are known as *baby Skyrmions*.

There is a key difference in the structure of the two models. To allow stable topological solitons, the baby Skyrme model must contain a potential term. However this is not necessary for the Skyrme model. This provides our motivation: we wish to design a static baby Skyrme model that does not require a potential term to admit topological solitons.

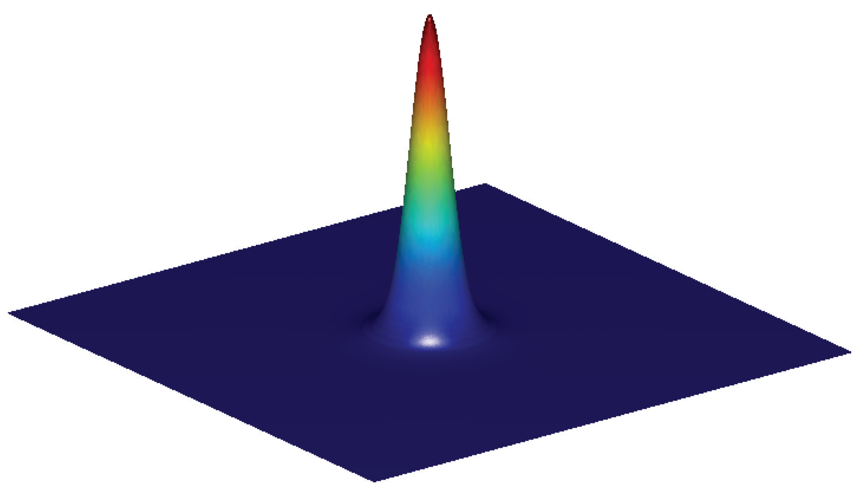

Figure 1: Energy density of a baby Skyrmion

The baby Skyrme model

The static energy of the baby Skyrme model is

$$E_{BS} = \int_{\mathbb{R}^2} \left(\tfrac{1}{2}\partial_i\phi\cdot\partial_i\phi + \tfrac{1}{4}|\partial_i\phi\times\partial_j\phi|^2 + V(\phi) \right) \mathrm{d}^2x, \tag{1}$$

where $i, j = 1, 2$, and $\phi : \mathbb{R}^2 \to S^2$ is a vector $\phi = (\phi_1, \phi_2, \phi_3)$ of unit length. The first term in (1) is called the sigma term, the second is the Skyrme term, and $V(\phi)$ is the potential.

Solutions are labelled by an integer B called the *topological charge* or *baryon number*. It gives the net number of solitons and is written

$$B = -\frac{1}{4\pi}\int_{\mathbb{R}^2} \phi\cdot(\partial_1\phi\times\partial_2\phi)\,\mathrm{d}^2x. \tag{2}$$

Baby Skyrmions are maps ϕ which minimise the energy (1) in a given sector B. A lower bound on the energy of a charge B solution is given by

$$E_{BS} \geq 4\pi|B|. \tag{3}$$

This bound allows us to see how the energy of solutions scales with the number of solitons.

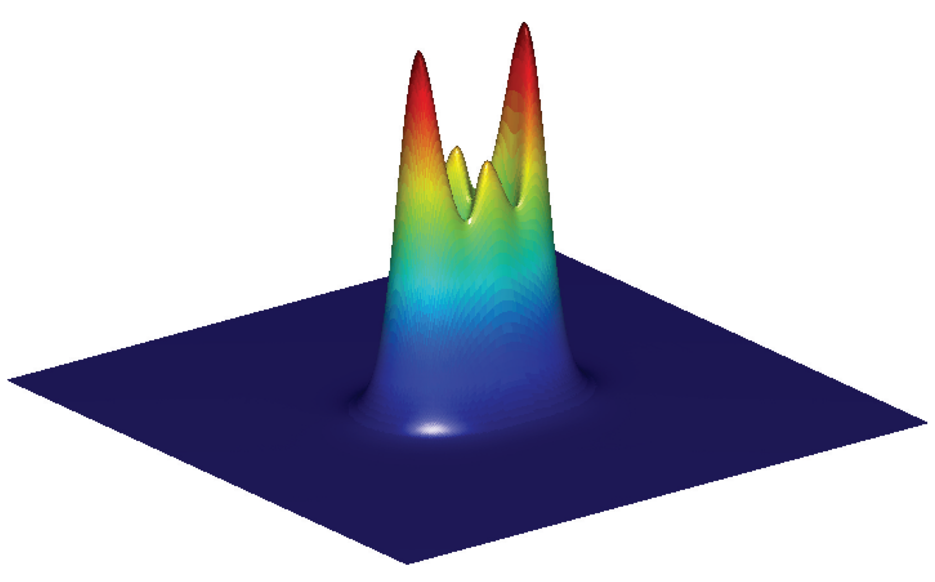

Figure 2: A chain of three baby Skyrmions

Acknowledgements

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Baby Skyrmions of our models

We develop a family of static baby Skyrme models without a potential term by raising the sigma and Skyrme terms to some powers. The static energy of our models is given by $E = 2^{-\frac{3+\alpha}{2}}\int_{\mathbb{R}^2}\mathcal{E}\,\mathrm{d}^2x$, where

$$\mathcal{E} = (\partial_i\phi\cdot\partial_i\phi)^\alpha + 2(|\partial_1\phi\times\partial_2\phi|^2)^{1-\frac{\alpha}{2}}, \tag{4}$$

and $0.5 \leq \alpha \leq 1$. For any α, our models also satisfy the topological energy bound

$$E \geq 4\pi|B|, \tag{5}$$

with equality when $\alpha = 1$. We find the solitons numerically by minimising (4). To the right are contour plots of energy density for charge $B = 1, 2, 3$ solitons in the models with $\alpha = 0.6,\ 0.7,\ 0.8,\ 0.9$. They are circularly symmetric, and the energy density becomes thinner and taller as α increases.

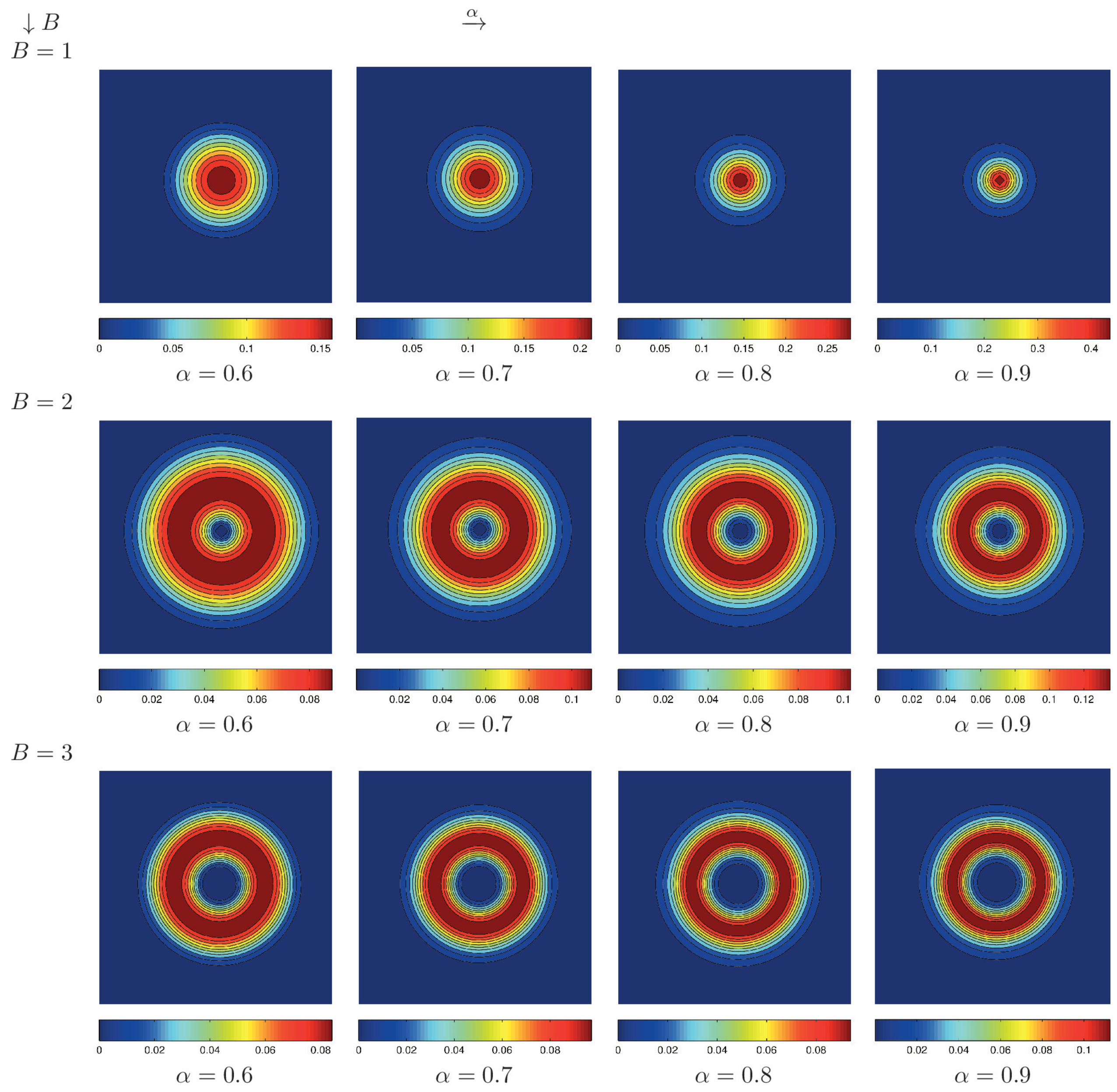
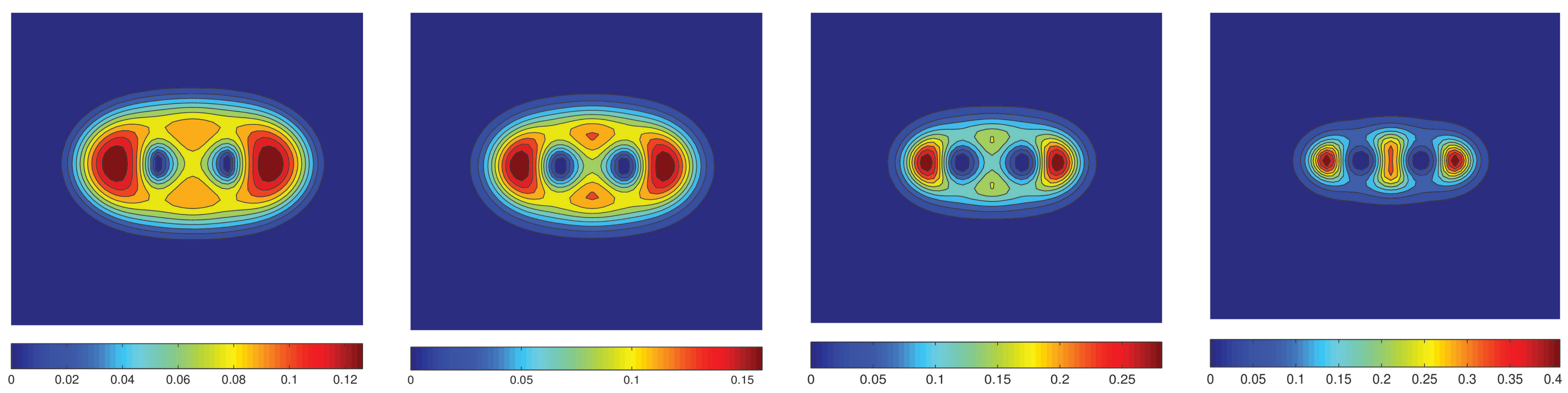


Figure 3: Energy density of $B = 1, 2, 3$ solitons

There are also chain configurations, but these have higher energy than the circularly symmetric ones, and therefore are not the global energy minimisers. Below we display contour plots of energy density for $B = 3$ chains with the same selection of α values as above.



$\alpha = 0.6$ $\alpha = 0.7$ $\alpha = 0.8$ $\alpha = 0.9$

Figure 4: Energy density of $B = 3$ chain configurations

For parameter value $\alpha = 0.5$, the solutions are *compactons*. These are solitons with compact support, having zero energy everywhere outside of a finite region of space.

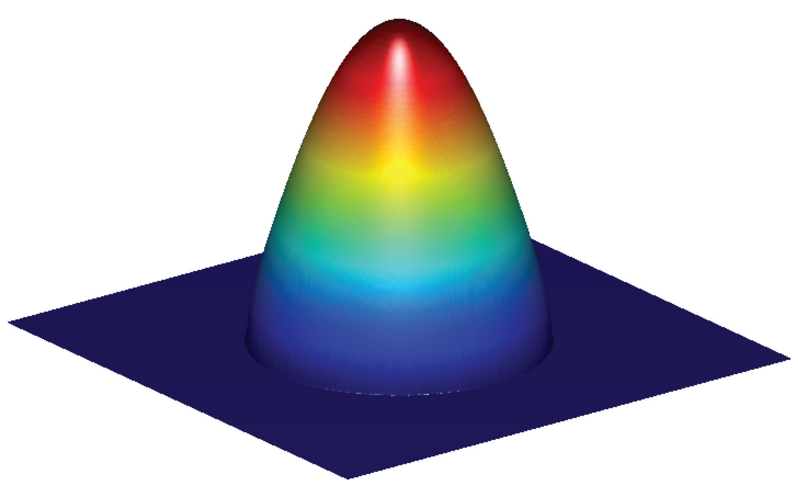


Figure 5: The $B = 1$ compacton

To the left, we present a surface plot of energy density for the $B = 1$ soliton with $\alpha = 0.5$. To the right is energy density for the $B = 2$ soliton. Note the steep approach to zero and extreme localisation of the energy density in both plots.

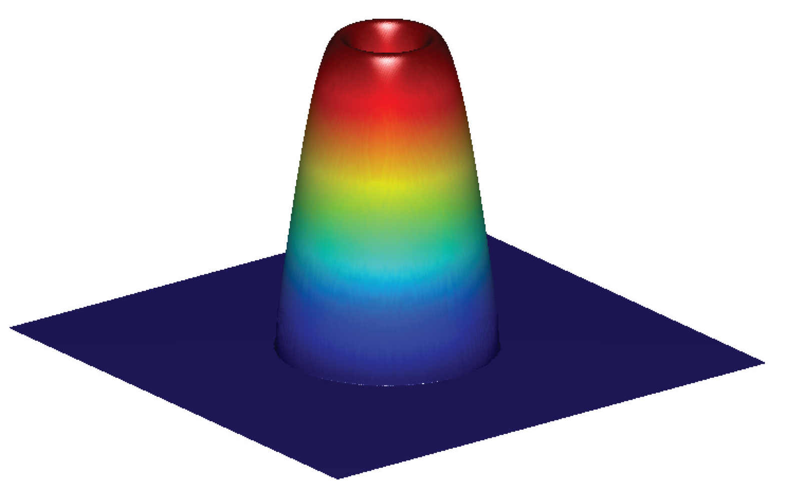


Figure 6: The $B = 2$ compacton

Summary

We developed a family of baby Skyrme models that do not have a potential term, but do allow topological solitons. Our models satisfy the same energy bound as the baby Skyrme model, and it is even saturated for $\alpha = 1$.

To the right, we display the numerically obtained energy as a function of α. Values for the circularly symmetric configurations are given by the solid lines, with black points indicating energy for $\alpha = 0.6,\ 0.7,\ 0.8,\ 0.9$. Energy for the chain configurations is given by the red dashed line. As α increases, the energy of all configurations approaches the black dashed line indicating the energy bound.

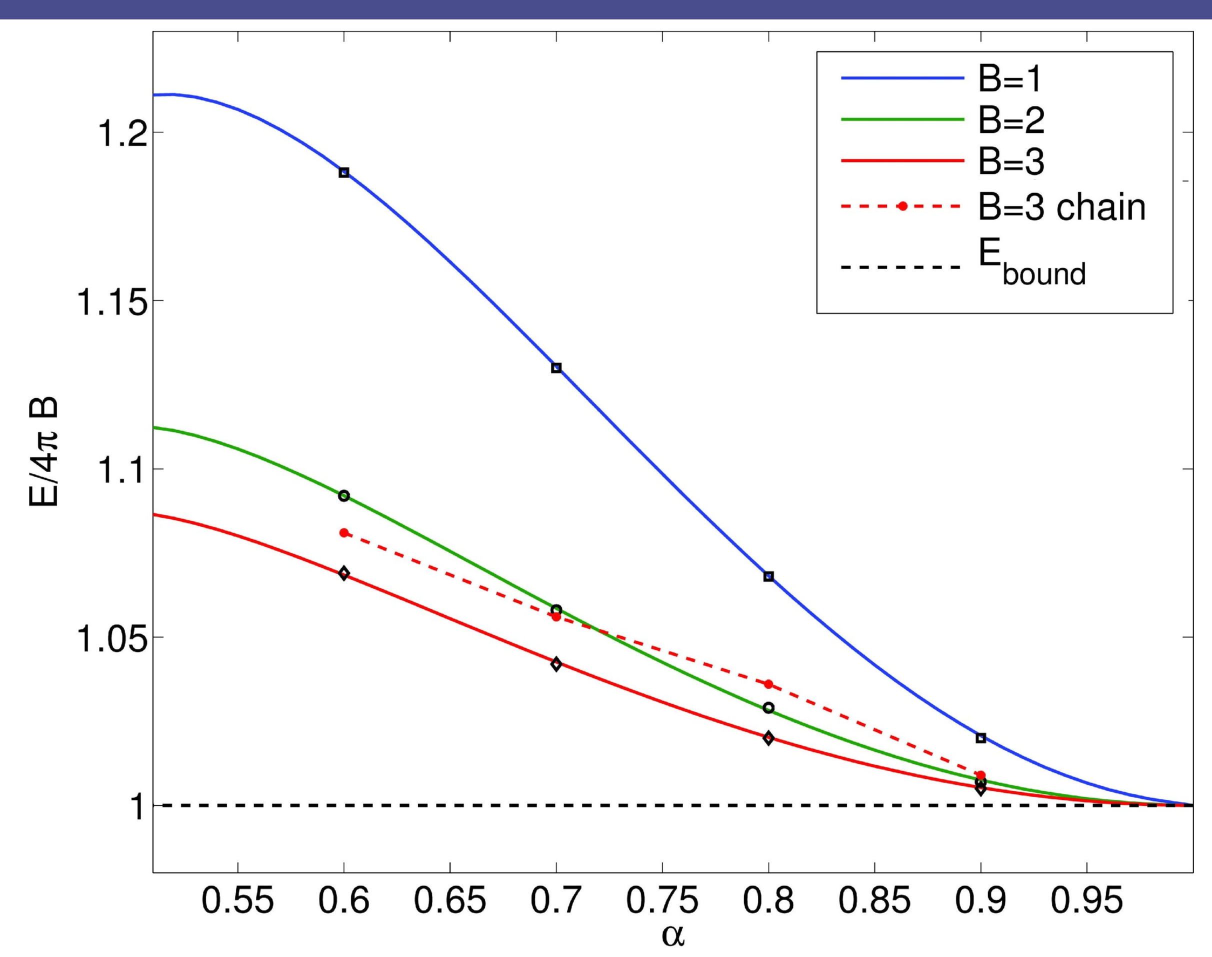


Figure 7: Energy as a function of model parameter α

References

[1] J. Ashcroft, M. Haberichter and S. Krusch, Baby Skyrme models without a potential term, Phys. Rev. D91, 105032, (2015).