# Controlled and conditioned invariance for polynomial and rational feedback systems

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The investigation of invariant varieties for polynomial control systems leads to several algebraic intersection problems: Let $K$ be a field. Given an ideal $I$ of a commutative polynomial ring $R = K[x_1,\ldots,x_n]$ and polynomials $\alpha, h_1,\ldots,h_p \in R$, we want to find the intersection of the affine ideal $\alpha + I$ with the subalgebra of $R$ generated by the $h_i$'s; shortly, we want to determine the set

$$(\alpha + I) \cap K[h_1,\ldots,h_p].$$

This will be the foundation for the following considerations: If $Q$ is the quotient field of $R$ and $d \in R$ is another polynomial, the set $\frac{1}{d} \cdot I \subseteq Q$ is a fractional ideal. Similar to the above, we are interested in finding the intersection of the affine fractional ideal $\alpha + \frac{1}{d} \cdot I$ with the subfield of $Q$ generated by the $h_i$'s:

$$(\alpha + \frac{1}{d} \cdot I) \cap K(h_1,\ldots,h_p).$$

Using techniques from Gröbner basis theory, we will present methods to compute these intersections. For the second problem, we need to restrict to $p = 1$ and we will point out why the general setting seems more difficult. Further, we give definitions of controlled and conditioned invariant varieties in the control theoretical setting, and show how the methods from above can be applied to this framework.

## References

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