Symbolic Computation in Studying the Restricted Three-Body Problem with Variable Masses

Alexander N. Prokopenya

Warsaw University of Life Sciences – SGGW, Poland, alexander_prokopenya@sggw.pl

The restricted three-body problem is a well-known model of celestial mechanics (see, for example, [1]). Recall that in the simplest case it describes a motion of the point P_2 of negligible mass in the gravitational field of two massive points P_0 and P_1, moving in Keplerian orbits about their common center of mass. It is assumed that the masses of points P_0 and P_1 are given and their orbits are completely determined by the known solution of the two-body problem. This problem is not integrable, and so the perturbation theory is usually applied to the analysis of the point P_2 motion. As a general solution of the two-body problem is known, one can consider in the first approximation that the point P_2 moves around the point P_0, for example, as a satellite and its Keplerian orbit is disturbed by the gravity of point P_1. Such a model has been used successfully in the study of satellite motion in the system Earth–Moon or Sun–planet [2, 3]. It was shown that doubly averaged equations of motion determining the evolution of satellite orbit may become integrable. The corresponding general solution may be found in analytic form, and it enables investigation of main qualitative features of the orbit parameters (see, for example, [4]).

We consider here a generalized case of a satellite version of the restricted three-body problem when two points P_0 and P_1 form a binary system, losing the mass due to the corpuscular and photon radiation (see [5]). We assume that the points masses vary isotropically with different rates with the only restriction that their total mass reduces according to the joint Meshcherskii law

$$\frac{m_{00}+m_{10}}{m_0(t)+m_1(t)} = \sqrt{At^2+2Bt+1} \equiv \nu(t) \ , \qquad (1)$$

where $m_{00} = m_0(0)$, $m_{10} = m_1(0)$ are initial values of the points P_0, P_1 masses, and parameters A, B are chosen in such a way that $\nu(t)$ is an increasing function for $t > 0$. In this case equations of the points P_0, P_1 relative motion are integrable and their general solution can be written in symbolic form (see [6]).

We assume further that the point P_2 moves around point P_0, being perturbed by the gravity of point P_1. Besides, a distance between points P_0 and P_1 is assumed to be much greater than distance between points P_0 and P_2 and the Hill approximation [7] may be applied. Then the evolutionary equations determining long-term behaviour of the point P_2 orbital parameters become integrable and their solutions may be found in the analytical form (see [8, 9]).

The purpose of this paper is to present the main stages in the investigation of the restricted three-body problem with variable masses which requires tedious symbolic computations. Derivation of the evolutionary equations and determination of a class of functions $m_0(t)$, $m_1(t)$, satisfying equation (1), for which the evolutionary equations are integrable and describe a quasi-elliptic motion of the point P_2, is described in detail. All relevant calculation and visualization of the results are carried out using the Wolfram Mathematica.

References

[1] V. Szebehely. *Theory of orbits. The restricted problem of three bodies*, Academic Press, New York, London (1967).

[2] M.L. Lidov. *The evolution of orbits of artificial satellites of planets under the action of gravitational perturbations of external bodies*, Planetary and Space Science **9**, 10, pp. 719-759 (1962)

[3] M.L. Lidov, M.A. Vashkov'yak. *On quasi-satellite orbits in a restricted elliptic three-body problem*, Astron. Lett. **20**, 5, pp. 676-690 (1994).

[4] M.A. Vashkov'yak. *Evolution of orbits of distant satellites of Uranus*, Astron. Lett. **25**, 7, pp. 476-481 (1999).

[5] A.A. Bekov, T.B. Omarov. *The theory of orbits in non-stationary stellar systems*, Astron. Astrophys. Trans. **22**, pp. 145-153 (2003).

[6] L.M. Berkovič. *Gylden–Meščerski problem*, Cel. Mech. **24**, pp. 407-429 (1981).

[7] G.W. Hill. *Researches in the lunar theory*. Am. J. Math. **1**, pp. 129-147 (1878).

[8] A.N. Prokopenya, M.Zh. Minglibayev, B.A. Beketauov. *On integrability of evolutionary equations in the restricted three-body problem with variable masses*, in *Computer Algebra in Scientific Computing / CASC'2014*, V.P Gerdt, W. Koepf, E.W. Mayr, E.V. Vorozhtsov (ed.), LNCS 8660, Springer-Verlag, Heidelberg, pp. 375-389 (2014).

[9] A.N. Prokopenya, M.Zh. Minglibayev, B.A. Beketauov. *Secular perturbations of quasi-elliptic orbits in the restricted three-body problem with variable masses*. Int. J. Non-Lin. Mech. **73**, pp. 58-63 (2015).