

Conservation Laws and the Chazy Equation

T. M. N. Gonçalves ¹, I. L. Freire ²

¹ Universidade Federal de Goiás, Catalao, Brazil, tmng@kentforlife.net

² Universidade Federal do ABC, Sao Paulo, Brazil, igor.freire@ufabc.edu.br

Noether's theorem yields conservation laws for systems derived from a variational principle. As not all systems can be derived from a variational principle, the applicability of Noether's theorem is substantially reduced. Ibragimov in [1, 2] provided a solution to this problem through the construction of a Lagrangian for a system which is composed of the equation of interest and its adjoint equation.

Using these developments by Ibragimov and the new format for Noether's conservation laws (see [3, 4]) which is

$$d(\mathcal{A}d(\rho)^{-1}(v_1, \dots, v_p) M_{\mathcal{J}} dx) = 0,$$

where $\mathcal{A}d(\rho)^{-1}$ is a moving frame, v_i are vectors of invariants and $M_{\mathcal{J}}$ comes from the action of the variational symmetry group on the volume form, can lead to the simplification of integration problems. In particular, here we will show how such conservation laws can be used to solve the Chazy equation,

$$y_{xxx} - 2yy_{xx} + 3y_x^2 = 0.$$

References

- [1] N. H. Ibragimov, *Integrating factors, adjoint equations and Lagrangians*, J. Math. Anal. Appl. **318**, pp. 742–757 (2005).
- [2] N. H. Ibragimov, *A new conservation theorem*, J. Math. Anal. Appl. **333**, pp. 311–328 (2006).
- [3] T. M. N. Gonçalves and E. L. Mansfield, *On moving frames and Noether's conservation laws*, Stud. Appl. Math. **128**, pp. 1–29 (2011).
- [4] T. M. N. Gonçalves and E. L. Mansfield, *Moving frames and Noether's conservation laws – the general case*, submitted to Forum Math. Sigma (2015).