Computing Liouvillian solutions of linear difference equations

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We present an algorithm to compute Liouvillian solutions of linear difference equations with polynomials coefficients, without the problem of combinatorial search of singularities. The problem of computing Liouvillian solutions comes down to compute first order right factors of a difference operator $L \in \mathbb{Q}[n,\tau]$ but in an extended algebra $\mathscr{A}[\tau]$ where $\mathscr{A}$ is a finite difference ring extension over $\mathbb{Q}[n]$, the interlaced polynomials. This representation allows in particular to compute Liouvillian solutions in the same way as d'Alembertian solutions, the Galois group being by the way always connected over $\mathscr{A}$. A notion of exponential part for such solutions can be defined, and the problem comes down to try all possible exponential parts. Our approach is based on van Hoeijn method for factorizing differential operators: some particular singular solutions are computed and a right factor of L is searched by guessing an annihilating operator. This process, although not sufficient to obtain a complete factorization, is enough to compute all first order right factors. A particular attention will be given to the coefficient field, which can grow in the factorization computation, and in the complexity with respect to the degree in n of L. Several examples which were not accessible by previous methods will be presented.