On the Formal Reduction of Singularly-Perturbed Linear Differential Systems.

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Abstract
We consider the singularly-perturbed linear differential system of the form
\[ \epsilon \frac{dY}{dx} = A(x, \epsilon) Y = \epsilon^{-h} x^{-p} \sum_{k=0}^{\infty} A_k(x) \epsilon^k Y. \] (1)

where \( x \) is a complex variable, \( \epsilon \) is a small parameter, \( h, p \) are integers, and the entries of \( A(x, \epsilon) \) lie in \( \mathbb{C}[[x, \epsilon]] \), the ring of formal power series in \( x \) and \( \epsilon \) over the field of complex numbers.

Such systems have countless applications which are traced back to the year 1817 and their study encompasses a vast body of literature (see, e.g., [8, 15, 10, 9] and references therein). However, their symbolic resolution is still open to investigation.

Clearly, system (1) is a singular perturbation of the widely studied linear singular system of differential equations (see, e.g., [2, 13]) given by
\[ x \frac{dY}{dx} = A(x) Y = x^{-p} \sum_{k=0}^{\infty} A_k x^k Y. \] (2)

The methods proposed in the literature of system (1) either exclude its turning points or are not algorithmic throughout. Moreover, they make an essential use of the so-called Arnold-Wasow form. On the other hand, for system (2), the research advanced profoundly in the last two decades making use of methods of modern algebra. The former classical approach is substituted by efficient algorithms [3, 5, 7, 4] giving rise to the Maple package ISOLDE [6].

It was the hope of Wasow, in his 1985 treatise summing up contemporary research directions and results on system (1), that techniques of system (2) be generalized to tackle the problems of system (1). This generalization is the interest of this talk.

In [1], we set a first step towards the formal reduction of system (1) by giving algorithms to eliminate turning points and reduce the \( \epsilon \)-rank \( h \) to its minimal integer value using a Moser-based algorithm. In this talk, we continue this work to achieve full formal reduction by defining and computing an \( \epsilon \)-Katz invariant, in analogy to that defined for system (2) (see, e.g., [3]).

Keywords
Moser-based reduction, Newton Polygon, Perturbed linear differential systems, Turning points, Computer algebra.

References


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