MA552 Analysis
Examination Guide 2010

1 General Remarks

This guide is designed to help you study successfully for the MA552 2010 examination. Passing this module requires you to obtain 40% overall; the exam counts towards 80% and coursework 20% of the total final mark.

The format of the examination
The examination will be marked out of 100. There will be a Section A worth 40 marks and a Section B worth 60 marks. There will be six questions in Section A each worth 10 marks; you can do as many as you want, but the total marks available is only 40. Section B has four questions of which marks will be awarded for two. Questions in Section A will be more routine than those in Section B, and will resemble routine problems from the exercise sheets. Questions in Section B will resemble exam questions from before 2005.

Regardless of your coursework mark, you are strongly recommended to attempt four Section A questions followed by the first parts of two Section B questions. *Your first aim is to avoid the resit.* Then complete as far as you can your Section B questions, and then try for another Section A question.

Calculators need to be the approved varieties, and will checked in the exam hall.

2 Exam technique

Any examination is time limited so you need to have relevant basic skills and facts at your fingertips. In particular, if you are in the habit of using a graphical or symbolic calculator to do more than just check your graph or calculation, start weaning yourself off it now.

Memories trigger memories. This means that you should first do those problems where you can make immediate progress. This will calm you, and you will start to remember things relevant to both the question you are working on as well as other questions. If you feel yourself getting lost in a calculation, abandon it. Write down the general principle

or formula, this will attract marks. Leave simplifications involving only arithmetic for the very end, it is better to do additional Section A questions than arithmetic.

Section B questions are structured to lead you through the problem. If you can't do one bit, take it as assumed for the next bit.

Get the easy points: be able to state major definitions and theorems. Know (or be able to quickly derive) standard derivatives, graphs of sin, cos, exp, log functions and so forth. Be certain you can use the chain rule, product rule and quotient rule correctly. Distinguish between the quotient rule and l'Hôpital's rule. Distinguish inverse functions from reciprocal functions. Make sure you can correctly simplify algebraic expressions such as fractions of fractions, difference of square roots, and so forth.

3 Check list

The syllabus is defined by the lectures and exercise sheets.

Routine calculations you should be able to do include

- proofs of convergence and continuity on simple examples with correct setting out (Let $\epsilon > 0$ be given. Choose $0 < \delta < ...$),
- use of limit theorems such as the limit of the sum is the sum of the limit provided both limits exist, the "new limits from old" result, and simple modifications of "famous" limits
- use of convergence tests for series; the comparison test, the ratio test, absolute convergence implies convergence, the alternating series test, comparison with an integral. Sum of a geometric series.
- Taylor series expansions, l'Hôpital's rule,
- use of the ratio test to determine the radius of convergence of a power series

Analytic skills include

- being able to estimate more complicated expressions in convergence and continuity proofs, including using the triangle inequality
- distinguishing between absolute and conditional convergence of series

- being able to set up a lower or upper sum for an integral
- being able to combine calculations, such as l'Hôpital's rule with the use of a continuous function (typically the log function) to simplify a sequence expression
- applications of Mean Value Theorem, Intermediate Value Theorem, Taylor's Theorem, and so forth, such as seen in exercises, assignments and tests
- estimating the error term in Taylor expansions
- being able to calculate derivatives of inverse functions and then use them

Proof skills include being able to use a major theorem in a simple theoretical problem, that is, for an unspecified function or sequence or series. Another type of question is where you must decide if a given statement is true or false. This requires you to think carefully about the precise statement of a theorem and to be able to mentally review the examples you have been given. A third type of question is being able to describe the "heart" of a proof, for example using an illustrative diagram, or giving a brief overview.

Bookwork questions are straightforward. Important definitions and major theorems you should be able to **state precisely** include

- convergence of sequences and series
- lower and upper bounds and the supremum and infimum of a set of real numbers
- continuity, differentiabilty and integrability of a function
- tests for convergent series as above
- convergence results for "famous" series such as $\sum c^n$, $\sum 1/n^k$, binomial expansions, Taylor series for exp, cos
- Intermediate Value Theorem
- Extreme Values Theorem
- Mean Value Theorem and Fixed Point Theorem
- Taylor's Theorem
- radius of convergence of a power series
- l'Hôpital's rule

- lower and upper sums, Riemann integrability
- Fundamental Theorem of Calculus.

Proofs of the "short and cute" variety are viewed as examinable. For example,

- the Sandwich Theorem, sum of limit is the limit of sum, a convergent sequence is bounded,
- "famous" limits such as $c^n \to 0$ if $|c| < 1$, or $\sin(x)/x \to 1$ as $x \to 0$,
- a differentiable function is continuous
- proofs of "famous" derivatives such as those for sin, log, arccos
- Rolle's Theorem
- The simplest l'Hôpital's rule

You are *not* required to reproduce long proofs such as those for the Boundedness Theorem, Comparison or Alternating Tests for series, and so forth.

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