Tips for the May 2007 MA552 Analysis examination

There was evidence that people did all 6 Section A questions before starting on Section B, and lost marks as a consequence.

Question 1. The answer requires you to follow the format, "Let ϵ be given. Let $N = N(\epsilon) >$ ...." This is because proofs from the definition have to follow the format of the definition. Using the Algebra of Limits gets no marks because the question requires you to use the definition of convergence. If you didn't "rationalise the numerator" (mentioned many times in lectures), you had no chance.

Question 2. There are several theorems given in lectures from which convergence of a sequence can be proved. You are allowed to use "famous limits" without proof and to use theorems proved in lectures without proof. You do, however, need to state what those theorems are, and check that the conditions they require do, in fact, hold. For example, the Algebra of Limits theorem requires denominators to be non-zero, including in the limit.

Question 3. This is "Spot the Test" time for series. Note that for the series $\sum a_n$ it is necessary but NOT sufficient that $a_n \to 0$. So, showing $a_n \to 0$ is not enough. (If a_n does not converge to zero, then the series diverges). If you use the ratio test, you need to be able to correctly simplify the ratio. If you use comparison test, you need to get the inequalities the right way round. If you use limit form of the comparison test, you need to calculate the limit correctly. Check your work using Maple.

Question 4. The answer requires you to follow the format, " Let ϵ be given. Let $\delta = \delta(\epsilon) <$ ...." This is because proofs from the definition have to follow the format of the definition. If you do not simplify $f(x) - f(1)$ correctly, you will get lost. If f is a polynomial, then $(x - 1)$ must be a factor of $(f(x) - f(1))$.

Question 5. The first part is bookwork. For the second part, you did a similar question in the class test so if you went to the lecture in which I went through the solutions, you know what to do. Some students thought that $10\frac{1}{10}$ simplified to 1!! If you saw $1\frac{1}{2}$ you wouldn't think it was just $\frac{1}{2}$ would you? If you saw $5\frac{1}{2}$ you'd know it was 5 and a $\frac{1}{2}$ right? Yes, the "plus sign" is implicit. Hints for the left hand side of the inequality: $\sqrt{121} = 11$ and $102 < 121$.

Question 6. You need to be able to correctly differentiate $\log(2^x + 1)$ three times and evaluate one of these at $x = 0$. (Check your answer using Maple. If it's wrong, figure out why, learn from any errors you make). You also need to be able to state correctly one form of the error term. We did only one form in lectures but any correct form will get the marks.

Question 7. Since all the easy questions on sequences and series appear in Section A, you can expect that the Section B question on this topic will require a degree of

confidence and flair.
(a) Bookwork. Hint for the first part: Think about an ϵ-interval about ℓ if $\ell < 0$ and ϵ is small enough that $\ell + \epsilon < 0$. Can all a_n for large n be in it? The second part follows from the first by considering the sequence $b_n - a_n$.
(b) The Taylor series for both $\exp(x)$ and $1/(1-x)$ are famous. You should know them. The big problem here was that everyone who got this question wrong *either*

1. derived the wrong Taylor series for $1/(1-x)$. Hint: there are NO negative coefficients.

   and/or

2. completely misunderstood the phrase "the Taylor series about $x = 0$". The "Taylor series of $f(x)$ about $x = a$" means the series

$$f(x) = f(a) + f'(a)x + \frac{1}{2}f''(a)x^2 + \ldots + \frac{1}{n!}f^{(n)}(a)x^n + \ldots.$$

   If you set $x = 0$ into this expression, then you don't understand that a is fixed/given and x is a dummy variable. If you had coefficients depending on x, then you don't understand that a Taylor series is a function of the form $x \mapsto \sum_n a_n x^n$ where the a_n are constants.

Once you have the correct series, then you have a chance to see how to apply part (a) of this question to get the desired result.

Question 8. (a) Bookwork, followed by a routine calculation.
(b) If you draw the graph of the function, then you have a chance of seeing....that you need to apply the IVT twice. As for the last part, it has nothing to do with physics, and everything with seeing that a function on a circle is like a function on an interval with the same value at the endpoints.

Question 9. (a) bookwork.
(b) Most people understood that l'Hôpital's rule was the key to the calculations here. The main problem was that people confused $k'(0)$ with $\lim_{x\to 0} k'(x)$. How are they different? What does it mean if the two are equal? You also have to understand the difference between the quotient rule and the calculation for l'Hôpital's rule.

Question 10. (a) is bookwork. For (b), it is a matter of applying

$$\lim_n \mathcal{L}_{\mathcal{P}_n}(f) \to \int f$$

and getting the calulations right. (Use Maple to check your answer, see where you went wrong etc).