

Mathematical Analysis (MA552)

Practice Class Test II

Name:

Attempt ALL FOUR questions.

Allowable calculators: Casio fx-83 and Casio fx-85

1. (a) Evaluate the following limit. Justify your answer.

$$\lim_{x \rightarrow 0} \frac{\log(\cos(x))}{x^2}.$$

[12 marks]

- (b) Hence evaluate

$$\lim_{x \rightarrow 0} \cos(x)^{x^{-2}}.$$

Justify your answer.

[6 marks]

2. (a) State the Mean Value Theorem. [5 marks]
- (b) Suppose that $f : \mathbb{R} \rightarrow \mathbb{R}$ is differentiable and satisfies $|f'(x)| \leq 1$ for all $x \in \mathbb{R}$. Show that $|f(b) - f(a)| \leq |b - a|$ for all $a, b \in \mathbb{R}$. [6 marks]

3. (a) Define the *radius of convergence* of a power series. [5 marks]
(b) Determine the radius of convergence for the following series:

$$\sum_{k=1}^{\infty} \frac{x^k}{\sqrt{k}}.$$

[6 marks]

4. On $[0, \pi]$, let $f(x) = \sin(x)$ and let \mathcal{P} be the partition

$$\mathcal{P} : 0 < \frac{\pi}{4} < \frac{\pi}{2} < \frac{3\pi}{4} < \pi.$$

- (a) Sketch the graph of the function f and indicate the rectangles whose areas contribute to the upper and lower Riemann sums. [2 marks]
- (b) Determine the lower Riemann sum $L(f, \mathcal{P})$ and the upper Riemann sum $U(f, \mathcal{P})$. [8 marks]