

# MA552 Analysis Assignment 1

This assignment counts for 5% of the final mark. Attempt all **THREE** questions. You may use any theorems proved in lectures or in the exercise sheets, but you must tell me which theorems you are using, and why the hypotheses they require hold. Question 1 is worth 7 marks, Question 2 is worth 5 and Question 3 is worth 8.

**DUE DATE Noon, 29rd JANUARY, 2010**

**Q1.** State whether the following series converge or diverge. Justify your reasoning.

$$(i) \quad \sum (-1)^n \frac{2\sqrt{n}}{3n^2 + 5} \quad (ii) \quad \sum \frac{2n}{3n + 4}$$

$$(iii) \quad \sum \frac{2^n n^n}{3(n!) + 4}$$

**Q2.** By comparing

$$\sum_{m=2}^N \frac{1}{m (\log m)^2}$$

with the integral

$$\int_2^N \frac{dx}{x (\log x)^2}$$

determine whether the series converges or diverges. Justify your reasoning.

**Q3.** Suppose  $\sum a_n$  and  $\sum b_n$  are infinite series with positive terms, and that

$$\frac{a_{n+1}}{a_n} \geq \frac{b_{n+1}}{b_n}$$

for each  $n$ . Prove that

$$a_n \geq \frac{a_1}{b_1} b_n$$

for each  $n$ . Deduce that  $\sum a_n$  diverges if  $\sum b_n$  diverges.

When  $a_n = \left\{ \frac{1.3.5 \dots (2n-1)}{2^n n!} \right\}^2$  show that

1.  $\frac{a_{n+1}}{a_n} = \left( \frac{2n+1}{2n+2} \right)^2$
2. the ratio test can not be used to decide whether  $\sum a_n$  converges or diverges
3.  $\frac{a_{n+1}}{a_n} \geq \frac{n}{n+1}$  for each  $n$
4.  $\sum a_n$  diverges.

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