Group 3: Investigating Identifiability Using Maple

This worksheet is about investigating identifiability using the Symbolic method and Hybrid Symbolic-Numeric Method, which involves using Maple.

The following files are needed:

- Symbolicmethod.mw
- Eg2Maple.mw
- Eg3Maple.mw

A. Symbolic Method and B. Hybrid Method

Open Maple then go to File > Open to open Symbolicmethod.mw . This will execute the Symbolic and Hybrid method using the introductory Lapwing example with parameters ϕ_1, ϕ_a, ρ.

Most of the instructions are given in the comments, but note the following:

- Code is executed by pressing enter whilst on the specific line of code.
- The procedure Dmat finds the derivative matrix, the intrinsic procedure Rank can be used to find the rank (procedure from in Cole et al, 2010).
- The procedure Estpar finds the estimable parameter combination (based on a procedure in Cole *et al*, 2010).
- The procedure Formnum can be used to execute the Hybrid method (based on procedure in Choquet and Cole, 2012).
- The two exhaustive summaries for state-space models can be found using the procedures Expan and Ztransformys (procedures from Cole and McCrea, 2016)

Further Examples

Now look at the two further examples below.

Example 2: Immigration Model

Abadi *et al* (2010) examine an integrated population model for a population of Little Owls, where the population is affected by immigration. The integrated population model consists of census data and capture-recapture data (there is also productivity data which is not considered here). The model for the census data is:

$$\begin{bmatrix} x_{1,t} \\ x_{2,t} \end{bmatrix} = \begin{bmatrix} 0 & 0.5\rho\phi_1 \\ \phi_a + imm & \phi_a + imm \end{bmatrix} \begin{bmatrix} x_{1,t-1} \\ x_{2,t-1} \end{bmatrix} + \begin{bmatrix} \epsilon_{1,t} \\ \epsilon_{a,t} \end{bmatrix}$$

$$y_t = [1 \quad 1] \begin{bmatrix} x_{1,t} \\ x_{2,t} \end{bmatrix} + \eta_t$$

where $x_{1,t}$ is the number of 1-year old females, $x_{2,t}$ is the number of females older than a year, y_t is the total number of females. The parameters are: ϕ_j juvenile survival probability, ϕ_a adult survival, ρ productivity and imm the immigration rate. (η_t and ϵ_i are error processes.) The capture-recapture data consists of separate data for juveniles and adults. A CJS model is used with parameters ϕ_j, ϕ_a and p, where p is the recapture probability. (Note here we ignore the data on males, and only include the female data – this does not change the identifiability results).

The procedures are given in eg2Maple.mw along with extra code on how to create an exhaustive summary for the integrated model. This makes use of a simpler exhaustive summary for capture-recapture models from Hubbard *et al* (2014).

Use both the symbolic and the hybrid method to answers the following questions:
Is the model with census data alone identifiable?
Is the integrated model with census and capture-recapture data identifiable?
Which parameters can be estimated in both cases?
If parameter redundant what are the estimable parameter combinations? (symbolic method only)

Example 3: Salmon State-Space Model

This example is from Newman *et al* (2014). The number of juvenile salmons alive in year *t* in a certain river can be modelled as

$$N_t|N_{t-1} \sim Po(\alpha N_{t-1} \exp(-\beta N_{t-1})), \qquad \alpha > 0, \beta > 0.$$

The parameter α models survival and fecundity, the parameter β models density dependence and the parameter $n_0 = N_0$. The observation process is

$$y_t|N_t \sim lognormal\left(\log(N_t) - \frac{\sigma^2}{2}, \sigma^2\right).$$

σ^2 is assumed to be a fixed known value.

The Maple code to find the exhaustive summary is given in eg3maple.mw.

Is the model identifiable? What if the parameter α was time dependent? (Hint change alpha to alpha[t] in the code for the exhaustive summary.)

References

Abadi, F., Gimenez, O., Arlettaz, R. and Schaub, M. (2010). An assessment of integrated population models: bias, accuracy, and violation of the assumption of independence. *Ecology*, **91**, 7-14.

Choquet, R. and Cole, D.J. (2012) A Hybrid Symbolic-Numerical Method for Determining Model Structure. *Mathematical Biosciences*, **236**, 117-125

Cole, D. J., Morgan, B. J. T and Titterington, D. M. (2010) Determining the Parametric Structure of Non-Linear Models. *Mathematical Biosciences*, **228**, 16-30.

Hubbard, B. A., Cole, D.J. and Morgan, B.J.T. (2014) Parameter Redundancy in Capture-Recapture-Recover Models. *Statistical Methodology*, **17**, 17-29.

Newman, K.B., Buckland, S.T., Morgan, B.J.T., King, R., Borchers, D.L. and Cole, D.J. and Besbeas, P. and Gimenez, O. and Thomas, L. (2014) Modelling Population Dynamics: model formulation, fitting and assessment using state-space methods. Springer.