Correction to Theorem 6.3 of Uniqueness of A_∞-structures and Hochschild cohomology

CONSTANZE ROITZHEIM
SARAH WHITEHOUSE

This is a corrected version of Theorem 6.3 of "Uniqueness of A_∞-structures and Hochschild cohomology". The previous version claims in the proof that the element $D(a_4)$ is trivial for degree reasons, which is simply not true. In order to fix this, a slightly stronger assumption in the statement of the theorem is needed. The corrected statement and proof are below.

Theorem Let A be a dga whose minimal model $H^*(A)$ satisfies $m_i = 0$ for $i \neq 2, 3$. If $F^4\,\mathrm{HH}^2_{alg}(H^*(A), H^*(A)) = 0$, where HH^*_{alg} denotes the Hochschild cohomology of associative algebras, then any A_∞-structure $\bar{m}$ on $H^*(A)$ with $\bar{m}_1 = 0$, $\bar{m}_2 = m_2$ and $\bar{m}_3 = m_3$ is quasi-isomorphic to m.

Proof The proof is extremely similar to the proof of Theorem 5.3.

The differential in the Hochschild complex for $H^*(A)$ is

$$D = D_2 = [m_2, -] : C^{n,k}(H^*(A), H^*(A)) \longrightarrow C^{n+1,k}(H^*(A), H^*(A)).$$

Assume there is an A_∞-structure $\overline{m}$ on $H^*(A)$ with

$$\overline{m} = m_2 + m_3 + a_4 + a_5 + \cdots .$$

Let

$$a = a_4 + a_5 + \cdots .$$

Because $m = m_2 + m_3$ is an A_∞-structure on the minimal model by assumption, we know that a is a twisting cochain, i.e. a satisfies the Maurer-Cartan equation $-D(a) = a \circ a$. Again, for degree reasons $D(a_4) = 0$, so $[a_4]$ is a class in $F^4\,\mathrm{HH}^2_{alg}(H^*(A), H^*(A))$. However, we assumed this to be trivial, thus a_4 must also be a boundary. In other words, there is a p such that $D(p) = a_4$.

By the analogue of Lemma 5.1 for this case, there is a twisting cochain $\overline{a} = \overline{a}_4 + \overline{a}_5 + \cdots$ such that

- $\overline{a}$ is equivalent to a,
- $\overline{a}_k = a_k$ for $k \leq 3$,
- $\overline{a}_4 = a_4 - D(p) = 0$.

The rest of the proof continues inductively following the same steps as the proof of Theorem 5.3. □

Remark Since the Hochschild cohomology of associative algebras is bigraded, we can express the condition $F^4\,\mathrm{HH}^2_{alg}(H^*(A), H^*(A)) = 0$ as $\mathrm{HH}^{n,2-n}_{alg}(H^*(A), H^*(A)) = 0$ for all $n \geq 4$.

C. Roitzheim, University of Kent, School of Mathematics, Statistics and Actuarial Science, Cornwallis, Canterbury, Kent, CT2 7NF, UK

S. Whitehouse, School of Mathematics and Statistics, University of Sheffield, Hicks Building, Hounsfield Road, Sheffield S3 7RH, UK

c.roitzheim@kent.ac.uk, s.whitehouse@sheffield.ac.uk