Theory and Decision (2006) 61: 159–190
DOI 10.1007/s11238-006-0007-y

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RETHINKING RISK ATTITUDE: ASPIRATION AS PURE RISK

ABSTRACT. There exists no completely satisfactory theory of risk attitude in current normative decision theories. Existing notions confound attitudes to pure risk with unrelated psychological factors such as strength of preference for certain outcomes, and probability weighting. In addition traditional measures of risk attitude frequently cannot be applied to non-numerical consequences, and are not psychologically intuitive. I develop Pure Risk theory which resolves these problems – it is consistent with existing normative theories, and both internalises and generalises the intuitive notion of risk being related to the probability of not achieving one's aspirations. Existing models which ignore pure risk attitudes may be misspecified, and effects hitherto modelled as loss aversion or utility curvature may be due instead to Pure Risk attitudes.

KEY WORDS: risk, pure risk attitude, aspiration levels, Subjective Expected Utility Theory, Prospect Theory, Pure Risk Prospect Theory, strength of preference

1. INTRODUCTION

Faced with a choice between acts in an uncertain world, which goals should rationally dictate one's choice of act? One intuitive answer is that, since the decision maker wishes to achieve the best *ex post* result, this requires a trade-off between some measure of the "anticipated benefit" of choosing each act, and some measure of the "risk" that the actual outcome will turn out to be worse than this anticipation. However, precisely what is meant by "risk" and how this trade-off should work is not immediately apparent. Certainly we have intuitive notions of risk, but these often seem widely divergent of traditional risk measures used in decision science and economics.

The rather surprising answer given by Expected Utility Theory (EUT) (von Neumann and Morgenstern, 1947) and Subjective Expected Utility (SEU) (Savage, 1954) is that, given any preference ordering of acts that meets the rather minimal requirements of the axioms, a utility value may be ascribed to each possible consequence in such a way that preferences are fully represented by the expected utility ordering of these acts. Since this representation is complete these utilities must reflect all attitudes to risk held by the decision maker, and no further trade-off is required. However, the von Neumann-Morgenstern utilities also represent all the other relevant attitudes of the decision maker to the available acts, which may include psychophysical responses to the potential outcomes, strength of preference for these outcomes, and social preferences, amongst others. Thus, although SEU enables us to represent preferences without explicit reference to "risk", the representation confounds risk attitude with other factors influencing the decision, making it impossible to understand the role that the introduction of uncertainty plays in the ordering of preferences. Where our objective is solely to arrive at a rank ordering of preferences over acts this confound is unimportant, but where we wish to understand in more depth *why* preferences are as they are, it becomes very useful to understand precisely the degree to which the von Neumann-Morgenstern utilities are attributable to the introduction of risk.

We are missing a theory of risk that is both psychologically intuitive, and that provides an explanation for the way in which risk attitudes affect preference orderings. In this paper I develop a normative theory of *pure risk attitude* which is grounded in our psychological intuitions of what is meant by the "risk" of an act, and which enables us to overcome the confound with other non-risk concerns in decision making. I shall build on ideas introduced by Dyer and Sarin (1982). These are introduced in the next section, along with a discussion of how traditional risk measures have ignored the confound between pure risk attitude and other factors.

Section 3 extends Dyer and Sarin's basic ideas, grounding pure risk attitude as a primitive concept in the intuitive notion that risk is related to the chance of not achieving levels of aspiration.

Section 4 discusses the implications of pure risk theory, and as an example shows how pure risk theory may be combined with Cumulative Prospect Theory (CPT) (Tversky and Kahneman, 1992), one of the leading current descriptive models of decision making under uncertainty. Section 5 concludes.

2. UNCONFOUNDING RISK ATTITUDES

There have been a number of characterisations of risk attitude within the frameworks of EUT, SEU and their later variants. In more sophisticated versions risk attitude may arise from three or more distinct components (Davies and Satchell, 2004), all of which affect the risk premium, that is, the amount by which the expected utility of the option exceeds its expected value. In CPT for example, risk attitude may be affected by the concavity or convexity of the value function, by loss aversion, or by the way decision makers distort probabilities by giving greater or less weight to extreme outcomes in a lottery.

However, a number of problems arise when we *identify* risk attitudes with the conjunction of the multiple factors which influence the risk premium. First, none of the contributing factors seem to offer an intuitive psychological interpretation of what we mean by "risk". Fishburn (1982) refers to "the conventional notion that risk is a chance of something bad happening". Identifying risk attitude with the overall effect that arises from unrelated psychological effects that do not mention risk, does not at all reflect this intuitive notion. Introspecting on what the "risk" of a decision intuitively means, reveals that it is not an ambivalent concept which one may plausibly seek to maximise or minimise depending on how our value functions and probability distortions combine, but rather that risk is, in a deep sense, a negative concept, something that a coherent concept of rationality should mandate that we minimise. As Lopes (1987) says of EUT, "after all the study and all the clever theorizing, we are left with a theory of risk taking that fails to mention risk."

A second problem is that comparisons to expected value can only be performed if the outcomes themselves can be completely described on a single numerical scale. In any decision problem where the outcomes have some non-numerical descriptive component we cannot presume the risk premium completely reflects the degree to which the von Neumann-Morgenstern utilities are affected by risk attitudes. A related problem arises in multi-attribute decision making in which there might be expected values on each attribute, and therefore a risk premium pertaining to each, without any way of arriving at a single measure of risk aversion. These concerns considerably limit the range of decision problems to which traditional measures of risk attitude can be applied.

It is important to note that a single numerical scale is *not* required for risk attitude to be an intelligible concept within SEU. We just cannot use the risk premium to determine the effects of risk attitude without such a scale. Yaari's (1969) definition of risk attitude in terms of acceptance sets shows that we may speak of decision makers having different degrees of risk aversion even if the outcomes are completely general. Specifically, one decision maker T^1 is more risk averse than the another T^2 if T^2 is willing to take a gamble rather than a certain outcome α (his acceptance set) in every circumstance when T^1 is. Yaari's definition may be stated without reference to any particular theory of decision making, and may be applied to acts without any restrictions on outcome descriptions. For EUT with a single numerical scale T^1 is more risk averse than T^2 in Yaari's sense iff the von Neumann-Morgenstern utility function of T^1 is a concave transformation of that of T^2. Peters and Wakker (1987) show that this result still holds for von Neumann-Morgenstern utility functions on non-convex domains with non-numerical outcomes, or with finitely many outcomes. It is thus certainly meaningful to speak of and compare risk attitudes in decision problems with general outcomes, but we do not have a measure of risk aversion like that provided by the risk premium in situations where the expected value may be computed.

For many decisions this issue may not appear important because, it may be argued, monetary value provides a clear numerical scale on which to describe outcomes. However, it is not technically possible to calculate risk premia unless consequences are *completely* described numerically. It is doubtful that this can ever be the case. Consider that a complete description of a future outcome must necessarily incorporate the decision itself. To describe a particular outcome as, for example, "the decision maker receives £X", is incomplete. Completeness requires at a minimum a description such as: "The decision maker receives £X having chosen act A". To receive £X after having chosen some other act is a different outcome, and may well be assigned a different utility, even though the monetary outcome is identical in both cases. Mental accounting (Thaler, 1999) and behavioural game theory (see Camerer, 2003) provide other reasons to doubt the possibility of purely numerical outcome descriptions. In both, monetary amounts do not exhaustively describe the outcomes, which have in addition "mental labels" reflecting their source (regular income, bonus, windfall, etc.), earmarked uses (housing, leisure, etc.), as well as aspects of "fairness", reciprocity, and social preferences.

It is the third problem, however, that has the most serious consequences: the expected utility of the gamble confounds pure risk attitude with all other aspects of the decision. Even where it exists, the risk premium cannot be taken to reflect only risk attitudes. Because perceptual effects, such as diminishing sensitivity to value away from the reference point, may be given interpretations in terms of risk attitudes, they have been confused with attitudes to risk itself. Even the characterisations of risk attitude with regard to general outcomes (i.e., of Yaari, and Peters and Wakker) are just as susceptible to the problem of confounding pure risk attitudes with other factors influencing preferences.

Dyer and Sarin (1982) assume that two factors influence preferences for risky alternatives, *strength of preference* for certain outcomes, and attitudes to *pure risk*. As they point out, if you have a positive risk premium when faced with a gamble with a 50% chance of winning £8 and a 50% of winning £0, then you would be regarded as risk averse in Pratt's (1964) sense. However, lets say that your risk premium is £1, meaning the you are indifferent between the gamble and a sure outcome of £3. If your strength of preference for gaining £3 for sure when you have £0 is equal to your strength of preference for gaining £5 for sure when you already have £3, then your preferences can be entirely explained by your diminishing marginal utility for monetary value, without any reference at all to the introduction of risk. In other words, your "risk aversion" has nothing to do with any attitude to pure risk at all, but is derived entirely from your strength of preference for certain monetary outcomes. You may be said to be pure risk neutral, but still display risk averse behaviour when choosing between gambles.[1]

Dyer and Sarin take strength of preference to be a primitive concept and show how differences in strength of preference may be ordered using a *measurable value function* $v(x)$, which encodes only strength of preference. On this function they define a measure analogous to the Pratt-Arrow measure of risk attitude on the von Neumann-Morgenstern utility function, $r(x) = -u''(x)/u'(x)$. Their *measure of value satiation*, $m(x) = -v''(x)/v'(x)$ is a local measure of strength of preference. Pure risk attitude is defined as the gap between the measurable value function that represents strength of preferences and the von Neumann-Morgenstern utilities that represent overall preferences. In particular they take an individual to be locally averse to pure risk if $m(x) < r(x)$ and to be locally pure risk seeking if $m(x) > r(x)$. If $0 < m(x) < r(x)$, then both v and u are locally concave, but u is relatively more concave than v, meaning that strength of preference alone is insufficient to account for overall risk aversion.

An alternative way of examining this is to define a function $u_v[v(x)] = u(x)$ which transforms the outputs from the measurable value function into the final von Neumann-Morgenstern utilities. Dyer and Sarin show that $u_v[v(x)]$ is concave with regard to $v(x)$ if the agent is averse to pure risk. They have formally separated the effects on the risk premium due to strength of preference (through the shape of $v(x)$) from that due to introduction of risk (through the shape of $u_v[v(x)]$). It is now possible for an individual to be globally averse to pure risk whilst being risk seeking in Pratt's sense, i.e., with a negative risk premium. In particular, evidence from choices using reference dependent theories such as CPT often show risk seeking behaviour in the domain of losses. This need not be because people seek pure risk itself. Instead, the convexity of $v(x)$ in the loss domain, due to diminishing sensitivity to value away from the reference point, may locally outweigh the concavity of $u_v[v(x)]$. Risk averse behaviour is thus induced by psychophysical responses that have nothing to do with attitudes to risk itself.

Despite the clarity of Dyer and Sarin's exposition there are some difficulties with this approach. Their theoretical development relies on the existence of both $r(x)$ and $m(x)$, which requires that both $u(x)$ and $v(x)$ are continuously twice differentiable. Their version of pure risk attitude is once again only operational for single attribute, completely numerical consequences.

Furthermore, in treating strength of preference as primitive, they have defined pure risk attitudes not by what they are, but by what they are not. Pure risk attitudes on this view consist precisely in the gap between the measurable value function and the von Neumann-Morgenstern utility function. This is acceptable only if these are the only two factors affecting the preference ordering. We have strong reasons to doubt that this is the case. It is true that the measurable value function can cope with reference dependence and with different responses to losses and gains. However, since the measurable value function deals only with certain outcomes, it will not reflect the distortions to probability through decision weights that are central to many current theories of decision making. These weights can be taken to reflect attitudes to hope and fear in the way that these psychological notions direct attention to particularly good or bad outcomes (Diecidue and Wakker, 2001) and, whilst they are only operational when risk is introduced, are conceptually distinct notions from attitudes to pure risk. I see no good reason to subsume by definition the effects on overall risk attitude from nonlinear decision weighting into pure risk attitudes. The effects of decision weights will be reflected in the von Neumann-Morgenstern utilities, but not in $v(x)$. Thus by Dyer and Sarin's account they will be automatically included in the definition of pure risk attitudes, confounding the two notions. Other contenders for confounding factors that remain when using their approach are social preferences and context effects—both may affect preferences, but neither will affect Dyer and Sarin's measurable value function.

A positive definition of pure risk attitude should allow it to be separated from *all* other influences on preferences, in the same way that the measurable value function separates strength of preference. In contrast to Dyer and Sarin then, I will treat pure risk attitude as primitive in choice under uncertainty and build on a concept that encapsulates intuitive notions of what "risk" means to individuals: risk is related the chance of something bad happening. An added advantage of this approach will be that it leads to a normative theory of pure risk attitudes which may be used both to enhance our understanding of actual behaviour, as well as to understand how rational individuals *should* react to the introduction of risk.

3. PURE RISK ATTITUDES

To approach pure risk attitude as a primitive of choice I proceed by examining what it means to be unaffected by an attitude to pure risk, or to be pure risk neutral. Pure risk attitude thus resides in the difference between the preferences of an individual who is not pure risk neutral, and the preferences that the same individual would have were we to excise all the effects of pure risk attitude on the original preference ordering.

If the original preferences of an individual are represented by the preference ordering $\succeq$, removing all the effects of pure risk attitude would result in a different, but related hypothetical preference ordering, $\succsim^N$, that is pure risk neutral. Thus $\succsim$ is separated into a pure risk neutral component and a second component that embodies only the individual's attitudes to pure risk. This second component may be seen as an ordering $\succsim^{PR}$ over acts, where the preferences are informed solely by attitudes to pure risk. I assume that $\succeq$ obeys the axioms required by SEU and thus develop a version of pure risk theory consistent with that base theory. This assumption is for simplicity, although other representations (e.g., CPT) could equally be used as a foundation for pure risk theory.

This approach is analogous to that of Dyer and Sarin, except that they treat strength of preference as the primitive component of overall preferences and define pure risk attitude to be everything accounted for by the gap between the overall preference ordering and the preference ordering due only to strength of preferences. The current decomposition treats pure risk attitude as primary, thus isolating it from other components of preferences. Having removed pure risk attitudes, the pure risk neutral preferences then account for all other aspects of the decision, including Dyer and Sarin's strength of preference, psychophysical attitudes to gains and losses, and decision weights. Note that no assumptions have been made limiting the nature of the consequences—both $\succsim$ and $\succsim^N$ may be applied to acts with completely general consequences. It will be necessary for the development of the theory that $\succsim^N$ admits an expected utility representation, so $\succsim^N$ must obey the same axioms as $\succsim$.

Given this assumption, there exists an attribution of utilities to consequences that is unique up to an affine transformation that preserves this pure risk neutral preference ordering. Denoting the pure risk free utilities as u^N, for any gambles $\mathbf{f}$ and $\mathbf{g}$ it is the case that $\mathbf{f} \succsim^N \mathbf{g}$ if $E\left[u^N(\mathbf{f})\right] \geq E\left[u^N(\mathbf{g})\right]$. This contrasts with the original utilities, u, which reflect the full preference order such that $\mathbf{f} \succsim \mathbf{g}$ if $E[u(\mathbf{f})] \geq E[u(\mathbf{g})]$. Unless the decision maker is originally pure risk neutral, the ordering $\succsim^N$ is likely to be different from $\succsim$. It is in the gap between these two preference orderings that we find pure risk attitude.

Pure risk theory must explain in a plausible way, both theoretically and intuitively, how $\succsim^N$ is derived from $\succsim$, or equivalently, how the utilities u are related to the pure risk neutral utilities u^N.

For clarity I summarise the assumptions that are required to arrive at the framework in which pure risk theory will be developed.

ASSUMPTION 1. *Pure risk attitude is a primitive of human choice and is distinct from strength of preference for certain outcomes, from psychophysical probability distortions and from reference dependence, amongst others.*

ASSUMPTION 2. *Given a complete preference ordering between acts, $\succsim$, that admits an expected utility representation,*[2] $\succsim^N$ *represents the related preference ordering where the effect of pure risk attitudes have been eliminated.* $\succsim^N$ *embodies all other aspects of the rational preference ordering and is pure risk neutral.*

ASSUMPTION 3. $\succsim^N$ *admits an expected utility representation—it thus permits an allocation of pure risk neutral utilities* u^N *to all consequences, which may be distinct from the overall utilities* u *that represent* $\succsim$.

3.1. *Aspirations as pure risk*

CLAIM 4. *Pure risk is related to the concept of aspiration levels. That is, the probability of attaining an outcome above some aspiration level. Since pure risk is primitive, the aspiration level must reflect all non pure risk aspects of the decision and must therefore be evaluated on pure risk neutral utility levels* u^N.

Lopes (1987) posited a dual criterion theory of decision making under risk, SP/A theory. It was not intended as a normative theory of rational decision making, but rather a descriptive theory incorporating a psychological perspective on how individuals assess risk. However, being concerned with the psychology of risk it contains insights and supporting data for an approach to risk that is intuitively appealing. The first criterion is *Security versus Potential* which is concerned with how people focus differentially on extremely good, or extremely bad outcomes depending on the degree to which they are motivated by achieving security (fear) or achieving potential (hope). This component is essentially a form of nonlinear probability distortion which involves a linear value function and a decision weighting function that gives rise to an inverse-S shape (Lopes and Oden, 1999). It is interesting that in discussing the psychology of risk, Lopes feels no need to discuss the forms of risk that derive from diminishing marginal returns as reflected in the value function, except to argue that these do not seem to be adequate descriptions of risk as a psychological notion.

It is the second criterion of *Aspiration*, however, that contains the kernel of a rational theory of pure risk. Lopes postulates that individuals have an aspiration level and that, in addition to maximising the SP criterion, they also wish to maximise the probability of achieving this level. An intuition for the second criterion is that even a risk-averse person may be inclined to take large risks if playing it safe in a particular context fails to provide a high enough outcome to ensure survival. For example, an impoverished farmer unable to meet subsistence levels by planting entirely low-risk subsistence crops, may quite rationally choose to take the risky alternative of planting cash crops, which at least provide a possibility of achieving the minimum survival level (Lopes, 1987; Shefrin and Statman, 2000).

Letting α represent the aspiration level, this criterion means maximising $A = \Pr(x \geq \alpha)$. This intuitive conception of risk, which is absent from traditional measures based on value functions, will form the cornerstone of pure risk theory.[3] Lopes' Security and Potential form no part of pure risk attitude, although the intuitions they reflect may still affect traditional risk attitudes through the risk premium insofar as they underpin distortions of probabilities.

That the probability of achieving some aspiration level is a psychologically plausible consideration when choosing among risky options is supported by Lopes' 1987 protocol analyses, as well as descriptive data (Lopes, 1987, Lopes and Oden, 1999, Payne et al., 1980, 1981; Payne, 2004) and simple introspection. This notion has been widely discussed, but rarely formalised as a theoretical basis for risk attitude (Dubins and Savage, 1976; March and Shapira, 1992; Payne, 2004; Roy, 1952; Sokolowska and Pohorille, 2000; Sokolowska, 2003). A recent exception is Diecidue and van de Ven (2004) who build a single aspiration level into the value function to provide a descriptive model of choice where a single aspiration level is particularly salient.[4]

The first way in which Lopes' aspiration level criterion does not meet our requirements is easily amended. Her theory applies only to initial consequences described in monetary values. I wish, however, to ask how this criterion would be applied by a hypothetical individual whose preference ordering, $\succsim^{PR}$, embodies only pure risk attitude. By assuming that this hypothetical individual is making a decision between acts where the consequences already encompass all choice preferences but pure risk, we can change Lopes' criterion to $A = \Pr\left(u^N \geq \alpha\right)$, where α is a pure risk neutral utility level. Pure risk attitude is concerned solely with the probability of achieving a pure risk neutral utility that is greater than some level utility, α.

This method enables us to work directly with preferences rather than with consequences; second, by construction it eliminates all other aspects of the decision that might influence the preference ordering in ways that give the appearance of relating to risk attitude but are, in fact, unrelated.

This version of pure risk attitudes is simplistic, but nonetheless expresses a notion of risk that is congruous with the intuitive idea that avoiding risk involves avoiding the worst that can happen. The fact that the aspiration criterion is similar to Value at Risk (VaR) models, which are widely used in practical finance, is indicative of the degree to which the folk psychology of risk is captured by examining the probability of failure.

The major problem with the aspiration criterion is its essential arbitrariness: how precisely do we choose the aspiration point α? And how do we defend this choice against other possible contenders? A case can be made for many possible levels with special significance: survival; the status quo reference level; a peer group benchmark; a regret based reference level from the outcomes of options not taken, etc. No doubt plausible justifications can be found for numerous other possible aspiration points in specific contexts. In addition, since the aspiration level is a hypothetical pure risk neutral utility, it is not clear how specific aspiration points might be identified.

One option is to utilise multiple utility aspiration points. However, whilst a step forward, this does not go far enough, and results in an unspecified number of pure risk minimisation criteria, each of which is still arbitrary in nature. In addition, combining these multiple criteria presents a problem. For instance, it seems intuitively reasonable that the aspiration points should take a lower weighting as the threshold associated with each increases: surviving is more important than not taking a loss, which is in turn more important than reaching a positive benchmark.

3.2. *Aspiration weighting function*

The intuition of risk as aspiration may instead be generalised beyond a single level by making *every* possible pure risk neutral utility point an aspiration point. This enables us to arrive at a preference condition for pure risk attitude. Since the hypothetical pure risk neutral utility allocations u^N reflect all aspects of the preference ordering $\succsim$ except pure risk attitude it must be the case that, if one act $\mathbf{f}$ has a higher probability of satisfying an aspiration level than another act $\mathbf{g}$, for every possible aspiration level, then we must have $\mathbf{f} \succsim^{PR} \mathbf{g}$.

CLAIM 5. *(Pure Risk Neutral Dominance) If, for any two acts* $\mathbf{f}$ *and* $\mathbf{g}$ *we have* $\Pr_{\mathbf{f}}\left(u^N \geq \alpha\right) \geq \Pr_{\mathbf{g}}\left(u^N \geq \alpha\right)$ *for all* α*, then* $\mathbf{f} \succsim^{PR} \mathbf{g}$.

That is, if we equate pure risk with the chance of not achieving levels of pure risk neutral utility, we should never choose a gamble that is dominated for all possible aspiration levels.

At first sight this proposition is not much use as it deals only with pairs of acts, where one is dominated by the other at every possible aspiration level. The criterion also requires that we compare any two acts for an infinity of aspiration levels which are themselves on hypothetical pure risk neutral utilities. However, notice that the criterion given in the proposition is precisely that of first-order stochastic dominance applied to pure risk neutral utility.

Applying standard stochastic dominance results (Rothschild and Stiglitz, 1970) this means that there exists a function of u^N, which I term the *Aspiration Weighting Function* $\alpha\left(u^N\right)$ such that:

PROPOSITION 6. *If, for any two acts* $\mathbf{f}$ *and* $\mathbf{g}$ *we have* $F\left(u^N\right) \leq G\left(u^N\right)$ *for all* u^N *(so* $\mathbf{f} \succsim^{PR} \mathbf{g}$*) then*

$$\int \alpha\left(u^N\right) dF\left(u^N\right) \geq \int \alpha\left(u^N\right) dG\left(u^N\right) \tag{1}$$

if and only if $\alpha\left(u^N\right)$ *is a nondecreasing function.*

Bawa (1975) provides a proof of this for the case where the function is bounded from below. This use of dominance enables us to represent pure risk attitude using a single transformation of pure risk neutral utilities, and the expectation of the aspiration weighting function represents pure risk preferences.

CONCLUSION 7. *Pure risk attitude is related to the probability of not achieving aspiration levels of pure risk neutral utility (in which all aspects of choice except pure risk attitude are accounted for). All possible aspiration levels (and thus the entire distribution of pure risk neutral utility) are important for pure risk attitude. An act that is pure risk neutral dominant will be as least as good as the alternative act. The decision maker will therefore choose the act that maximises*

$$U = E\left[\alpha\left(u^N\right)\right] \tag{2}$$

where $\alpha\left(u^N\right)$ *is a nondecreasing aspiration weighting function that transforms pure risk neutral utility,* U^N*, to final utility* U.

The $\alpha\left(u^N\right)$ is a consequence of adapting and extending Lopes' Aspiration criterion to an infinite number of aspiration levels over the whole space of possible pure risk neutral utility outcomes. There is an obvious comparison between $\alpha\left(u^N\right)$ and Dyer and Sarin's function $u_v\left[v\left(x\right)\right]$. In both cases the transformation is intended to represent pure risk attitudes. However, the aspiration weighting function excludes everything but pure risk attitude by definition, which cannot be assumed for $u_v\left[v\left(x\right)\right]$. In addition, since u^N can represent any pure risk neutral preference ordering that obeys the axioms, the aspiration weighting function is defined for decision problems with general outcomes, which, as we have seen, is not the case for $u_v\left[v\left(x\right)\right]$.

A further assumption can put additional structure on the shape of the function. The $\alpha\left(u^N\right)$ governs the degree to which the decision maker is more, or less, concerned with achieving lower values of u^N when faced with a choice. The slope governs the importance given to each increment of the cumulative distribution of u^N. By following through on our intuition that fulfilling a given aspiration point should become less important the higher the level of utility attached to that point, it must be the case that the aspiration weighting function is concave as well as nondecreasing. In this case the choice between distributions over pure risk neutral utilities satisfies second-order stochastic dominance, as well as our intuition that pure risk is inherently negative.

Furthermore, the concavity of the aspiration weighting function may be derived if we apply to pure risk theory Yaari's (1969) general definition of risk aversion. That one individual's acceptance set is contained in another's implies that the first decision maker is more risk-averse. For pure risk, as for SEU, Yaari's definition characterises a concave function. Like the value function, the aspiration weighting function will be unique only up to an affine transformation.

PROPOSITION 8. *If the importance attached to achieving any given aspiration level of u^N is greater the lower the level, then the aspiration function is concave as well as nondecreasing, and pure risk attitude satisfies second-order stochastic dominance. This also precisely characterises pure risk aversion.*

The value of $U = E\left[\alpha\left(u^N\right)\right]$ given to each act is necessarily equal to the final expected utility of that act when the original consequences are evaluated by the total preference function $\succsim$. Thus we have:

$$\int \alpha\left(u^N\right) dF\left(u^N\right) = \int u dF(u) \tag{3}$$

Pure risk attitude is thus all that is required to apply to U^N in order to transform the pure risk neutral utility to the final utility U.

3.3. *An alternative derivation of pure risk theory*

A striking aspect of this criterion is its similarity to SEU which is given by

$$U = \int u(x) dF(x) \tag{4}$$

There is, however, a crucial difference: expected utility maximisation is the expectation of the utility of outcomes, whereas pure risk is an expectation on a transformation of pure risk neutral utility values themselves.

Nonetheless, because (2) is an SEU representation over acts with pure risk neutral outcomes, it must be the case that $\succsim^{PR}$ is a rational preference ordering that obeys some set of axioms that permit such a representation. This also suggests a possible way of axiomatising the theory of pure risk attitude. If we are prepared to assume in advance that $\succsim^{PR}$ obeys the axioms of SEU when applied to u^N, then any axiomatisation of SEU can be adapted to produce a criterion of pure risk attitude. So, constraining $\succsim^{PR}$ to satisfy continuity and independence, and taking the probabilities as objectively given,[5] it must be the case that there exists a function $\alpha\left(u^N\right)$, such that

$$\mathbf{f} \succsim^{PR} \mathbf{g} \Longleftrightarrow \int \alpha\left(u^N\right) dF\left(u^N\right) \geq \int \alpha\left(u^N\right) dG\left(u^N\right) \tag{5}$$

In addition, given the monotonicity of u^N (in the absence of uncertainty the individual will always prefer greater certain pure risk neutral utility to less) we can say that $\alpha\left(u^N\right)$ is increasing. However, the fact that $\succsim^{PR}$ obeys the axioms of SEU is not sufficient to guarantee the concavity of $\alpha\left(u^N\right)$ which, as before, must come from additional assumptions about the declining importance of aspiration as the level increases.

3.4. *Restrictions on the aspiration weighting function*

For two prospects with identical valuations from a pure risk neutral perspective (so $E\left[u^N\right]$ is the same for both), the prospect that is less purely risky will necessarily have the higher utility U. We may thus use $-E\left[\alpha\left(u^N\right)\right]$ as a measure of pure risk for a distribution over u^N for a given decision maker:

$$PR = -E\left[\alpha\left(u^N\right)\right] \tag{6}$$

The fact that pure risk is always concerned with minimising cumulative probabilities (i.e., is inherently concerned with the downside), means that, given a consistent set of pure risk neutral utilities, in maximising $E\left[\alpha\left(u^N\right)\right]$ we minimise pure risk.

To give further structure to the aspiration function we need to delve deeper into the existing literature on risk measures. This is surprisingly sparse, and attempts either to add a risk measure as a secondary variable to be considered in addition to SEU (Coombs, 1975), to derive measures of perceived risk that are incidental to the actual preference structure over outcomes (Luce, 1980, 1981; Pollatsek and Tversky, 1970; Sarin, 1987), or to examine risk measures for practical application in finance with no necessary link to the normative decision theories (Artzner et al., 1999; Szegö, 2002).

All of these measures have been concerned primarily with the measurement of risk inherent in monetary outcomes and not with separating a concept of pure risk from risk effects derived from other sources. However, the axiomatic approaches to risk measurement used by Pollatsek and Tversky, Luce, and Sarin may be easily applied to gambles over pure risk neutral utilities and, since they intended to axiomatise a measurement of risk alone, some of the axioms may have better traction when applied to that component of choice which, by definition, focusses solely on pure risk. Sarin (1987) uses two assumptions to derive a model of risk, both of which may be placed within the context of pure risk. Our risk measure for a density function f on u^N is the value of $PR(f) = -\int \alpha\left(u^N\right) dF\left(u^N\right)$. Define the density of the modified gamble, where a constant amount β is added to every pure risk neutral utility, as f^{β}. Sarin's first assumption is that $PR\left(f^{\beta}\right)$ is a multiplicative function of $PR(f)$ and β. This assumption is justified by both intuition and evidence that the pure risk of an option should decrease when a constant is added to all outcomes of the gamble (Coombs and Lehner, 1981; Jia et al., 1999; Keller et al., 1986; Pollatsek and Tversky 1970).

ASSUMPTION 9. (*Risk Multiplicitivity*) *There is a strictly monotonic function S such that for all density functions f and all real* $\beta > 0$.

$$PR\left(f^{\beta}\right) = PR(f)\, S(\beta) \tag{7}$$

$S(\beta)$ *is strictly decreasing if* $PR : f \to \mathbb{R}^{+}$ *and strictly increasing if* $PR : f \to \mathbb{R}^{-}$.

The second assumption used by Sarin (and by Luce 1980) is that the densities can be aggregated into a single number using a form of expectation.

ASSUMPTION 10. *There is a function T such that for all densities f*

$$PR(f) = \int T\left(u^N\right) dF\left(u^N\right) = E\left[T\left(u^N\right)\right] \tag{8}$$

This assumption is already satisfied given the structure required for pure risk theory.

THEOREM 11. *Pure Risk Theory and Risk Multiplicitivity together ensure that the aspiration weighting function takes the form*

$$\alpha\left(u^N\right) = -Ke^{-\rho u^N} \tag{9}$$

with $\rho > 0$.

Proof. Sarin (1987) proves that given these two assumptions, (which amount to the sole additional assumption of multiplicitivity for pure risk theory), that, for some constants K and ρ,

$$PR(f) = \int Ke^{-\rho u^N} dF\left(u^N\right) \tag{10}$$

where $K > 0$, $\rho > 0$, or $K < 0$, $\rho < 0$.

Given our definition of pure risk, this implies

$$\alpha\left(u^N\right) = -Ke^{-\rho u^N} \tag{11}$$

and since $\alpha\left(u^N\right)$ is required to be nondecreasing and concave, we can restrict the constants to $K > 0$, $\rho > 0$. □

In addition, given that $\alpha\left(u^N\right)$ is unique only to an affine transformation we can, without changing the resulting preference ordering, define $K = \frac{1}{\rho}$, and add the constant $\frac{1}{\rho}$, to obtain the familiar negative exponential

$$\alpha\left(u^N\right) = \frac{1 - e^{-\rho u^N}}{\rho} \tag{12}$$

with $\rho > 0$ as a single parameter that governs the curvature of the aspiration weighting function, and thereby the degree of pure risk aversion. This implies that pure risk aversion should be constant with respect to u^N. It is interesting to note that Bell and Raiffa (1988) have argued that pure risk aversion *should* be constant in risky situations.

4. IMPLICATIONS OF PURE RISK THEORY

4.1. *Unconfounding utility functions*

Extending Dyer and Sarin's conception to decision problems with completely general outcomes is theoretically and conceptually useful. However, since the pure risk neutral utilities u^N

are hypothetical and unobserved, it will be additionally useful to examine the implications of pure risk attitude for the restricted class of decisions with real valued outcomes. In these cases we can examine choice using a continuous von Neumann-Morgenstern function on outcome values and ask what part is played by the pure risk attitude component.

Previous attempts to fit value functions to observed choice data have generally taken as their forms value functions that represent psychological or economic concepts affecting strength of preference, such as diminishing marginal returns (or diminishing sensitivity from a reference point in reference dependent utilities (RDU) theories (Quiggin, 1982; Schmeidler, 1989)). This selection has not considered that both strength of preference and pure risk attitudes may affect preferences. Thus, existing empirical fitting may have been mis-specified: by ignoring pure risk attitude we may have forced loss aversion, value function curvature and probability distortions to take on values that do not reflect their actual role in decision making.

Let us assume that empirical data suggested that a negative exponential von Neumann-Morgenstern function could be used to fit preferences for money gambles exactly.[6] The implications of this are either that (a) the individual is pure risk neutral but faces diminishing marginal utility with respect to money such that the measure of value satiation $m(x)$ is constant (b) the individual shows a completely rational aversion to pure risk, and no diminishing marginal utility with respect to money, or (c) that the individual shows some rational aversion to pure risk, but that this does not completely account for the shape of the von Neumann-Morgenstern function. In this case the influence of strength of preference may be to either increase or decrease the observed risk aversion. If pure risk aversion is weaker than the Pratt-Arrow risk aversion then the measurable value function must display diminishing marginal utility and therefore add to the overall (Pratt-Arrow) risk aversion. However, if pure risk aversion is actually stronger than Pratt-Arrow risk aversion, then it must be the case that strength of preferences result from increasing marginal utility to certain monetary amounts. When pure risk attitude is ignored as an influencing factor, it will be concluded that the Dyer and Sarin measurable value function is concave, whereas in reality the two effects need to be separated in order to say anything about marginal utility of certain monetary outcomes.

Dyer and Sarin show that the combination of constant pure risk aversion and constant value satiation must produce *decreasing* Pratt-Arrow risk aversion, commenting that "This combination may help to explain the appeal of decreasing Pratt-Arrow risk aversion as an appropriate description of a risk attitude". Certainly, decreasing Pratt-Arrow risk aversion is the standard belief in classical economics (Gollier, 2001), and in order to maintain constant Pratt-Arrow risk aversion in the presence of pure risk aversion, it would be necessary for value satiation to be *increasing* in wealth to counterbalance pure risk attitude. Similarly risk seeking behaviour occurs only if the strength of preference (value) function is sufficiently convex to overcome pure risk aversion, which for EUT implies increasing marginal returns. Since risk seeking behaviour is observed, this must sometimes be the case.

A psychologically plausible account for when we might expect convex value functions is provided by reference dependent theories: if sensitivity to value is diminishing away from the reference point, then the value function will be convex in the domain of losses. The next section explores the implications of combining pure risk attitudes with reference dependent theories, adding pure risk attitudes to CPT to develop Pure Risk Prospect Theory (PRPT).

4.2. *Pure risk prospect theory*

Pure risk theory may be applied to any preference structure that admits an expected utility representation (including those where the expectation incorporates decision weights), and can thus be made compatible with SEU, RDU, or their variations. Without specifying fully the remaining factors that influence choice through the hypothetical pure risk neutral utilities u^N, however, the resulting framework is highly theoretical and not particularly useful. Using the theory requires applying it to actual choices, not to hypothetical transformations of these choices.

Arriving at u^N requires specification of the content of the pure risk neutral preference ordering $\succsim^N$. If Dyer and Sarin are correct that strength of preference and pure risk attitude are the only two factors in decision making, then the measurable value function $v(x)$ completely represents $\succsim^N$.[7] Using CPT as a basis for $v(x)$ allows a particularly rich set of effects to be incorporated into strength of preference: reference dependence, differential attitudes to gains and losses, and loss aversion.

A further component of $\succsim^N$ is the existence of non-linear decision weights. Since probability distortions arise from rank-dependence rather than an attitude to pure risk, these must be accounted for in $\succsim^N$. The expectation in (2), which arrives at the final evaluation of the act, takes the subjective decision weights from $\succsim^N$ as objectively given—there is no second probability distortion caused by pure risk neutral preferences. Also, since pure risk attitudes are primitive one can postulate any desired form of decision weighting in the pure risk neutral stage, without influencing pure risk attitudes, though such a change would alter the u^N values to which pure risk attitudes are applied.

Of course, there may be many psychological effects that influence our strength of preference with regard to certain monetary amounts of which we are currently unaware. In addition, choices may be influenced by unrelated factors such as social preferences or context effects. In these cases $v(x)$ will at best be a good proxy for the translation of monetary values to pure risk neutral utilities.

Until now CPT has used the combination of decision weights and a value function to go directly from monetary outcomes to final utility values. This has been based on the assumption that a combination of rank dependence, reference-dependence and loss aversion are the sum total of effects that influence decision making under risk. Thus, the values

$v(x)$ have been used as proxies for the final utilities u that are required for their expectations to actually preserve preferences. The existence of pure risk attitudes means that the value function has been stretched too far—we have tried to incorporate effects due to pure risk attitudes into parameterisations of value functions that ignore such attitudes. In addition, because decision weights influence overall (Pratt-Arrow) risk attitudes, estimates of the weighting function parameters may also have been registering some of the effects of pure risk attitude. However, if these components do cover the majority of perceptual effects for monetary outcomes and probability, then the CPT value function may be a good proxy for u^N. Using this insight we can adapt CPT to form PRPT.

4.3. *The structure of PRPT*

Using the pure risk formulations of (2) and (12) we have:

$$U = \int \alpha\left(u^N\right) dF\left(u^N\right) = E\left[\alpha\left(u^N\right)\right] = E\left[\frac{1 - e^{-\rho u^N}}{\rho}\right] \tag{13}$$

Given the assumption that the CPT reference-dependent and loss aversion value function encompasses completely the strength of preferences for certain monetary outcomes, we have in addition

$$u^N(x) = \begin{cases} v_+(x) & \text{if } x > 0 \\ 0 & \text{if } x = 0 \\ \lambda v_-(x) & \text{if } x < 0 \end{cases} \tag{14}$$

where the monetary outcomes are measured relative to the reference point, λ is an index of loss aversion, and $v_+(x)$ and $v_-(x)$ are the basic strength of preference functions over gains and losses respectively, which exclude the effects of loss aversion (Köbberling and Wakker, 2005). $v'(x) \geq 0$ and the assumption of diminishing sensitivity, which is supported by much of the empirical data,[8] particularly for median or representative individuals, implies that the value function is convex for losses ($v''_-(x) \geq 0$) and concave for gains ($v''_+(x) \leq 0$).

Combining these two gives PRPT:

$$U = E\left[\frac{1-e^{-\rho\lambda v_{-}(x)}}{\rho}\,|x<0\right] + E\left[\frac{1-e^{-\rho v_{+}(x)}}{\rho}\,|x\geq 0\right] \qquad (15)$$

Köbberling and Wakker (2005) propose an exponential basic value function which, given their definition of the loss aversion index, exactly separates loss aversion from diminishing sensitivity, whilst retaining an index of loss aversion that is invariant to the unit of payment. Employing the same function here, the overall transformation from monetary outcomes to utility requires a double exponential (expo-expo) transformation: once of the reference-dependent monetary values through an exponential that is concave above $x=0$ and convex below, and the second time of the resulting u^N through a globally concave exponential that reflects pure risk attitude. With curvature of gains governed by $g>0$, losses by $l>0$ and loss aversion by $\lambda>1$, pure risk neutral utilities are:

$$u^N(x) = \begin{cases} \frac{1-e^{-gx}}{g} & \text{if } x\geq 0 \\ \lambda\left(\frac{e^{lx}-1}{l}\right) & \text{if } x<0 \end{cases} \qquad (16)$$

and the final utility allocations with $\rho>1$ are:

$$u(x) = \begin{cases} \frac{1-e^{\frac{\rho}{g}\left(e^{-gx}-1\right)}}{\rho} & \text{if } x\geq 0 \\ \frac{1-e^{\frac{\rho\lambda}{l}\left(1-e^{lx}\right)}}{\rho} & \text{if } x<0 \end{cases} \qquad (17)$$

The double transformation permits a far richer set of behaviour than standard CPT and, although an additional parameter has been introduced, this pure risk attitude parameter ρ plays a normative role and may, in fact explain some or all of the effects currently described by parameters of CPT. Indeed, examining closely the behaviour of the expo-expo function over both gains and losses reveals some very interesting results. Empirical data show that, for the CPT framework, the most common pattern for individual choices displays loss aversion ($\lambda>1$), and a value function that is concave for gains, convex for losses, and more linear for losses than for gains ($g>l>0$).

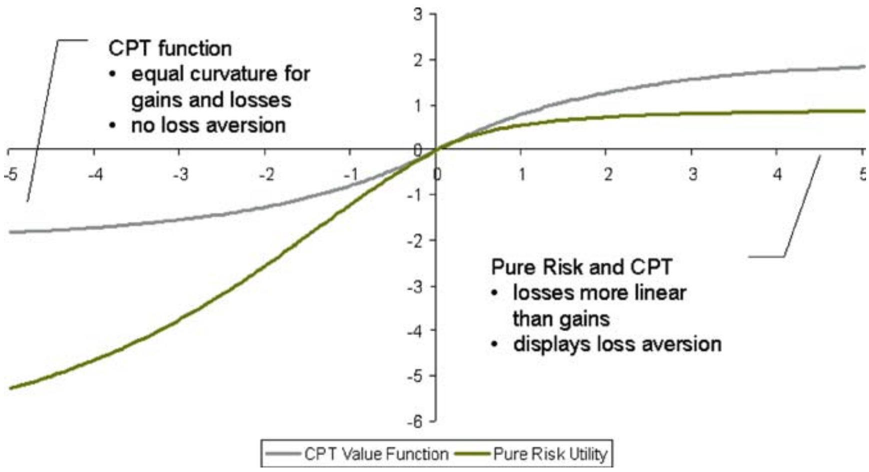

Figure 1. Transformations of monetary gains and losses for CPT and PRPT (dotted line). CPT is exponential and restricted to two parameters and thus does not display loss aversion ($\lambda = 1$) or differential values of gain and loss curvature ($g = l = 0.5$). The Aspiration Weighting Function is governed by $\rho = 1$.

The upper line in Figure 1 shows a two parameter exponential function that cannot fit these patterns: $\lambda = 1$, thereby removing all loss aversion, the curvature for gains and losses is equal and governed by a single parameter ($g = l = 0.5$), and $\rho = 1$ governs pure risk attitude. The lower line shows the effect of using these values as the CPT input into PRPT. The values have been chosen for illustrative purposes only. The resulting function is steeper for losses than for gains in a manner that is consistent with many definitions of loss aversion, but does not require the slopes to be different at the reference point from above and below. It is also more linear for losses than for gains. *Neither effect has been introduced as an assumption of the model*. PRPT can reproduce the effects of CPT with fewer parameters, and with greater normative justification. This is not to say that these psychophysical effects do not exist in reality and that a model with all four parameters might not do a significantly better descriptive job. However, we can now provide a normative basis for some effects that could hitherto be explained solely through descriptive patches to the model.

The expo-expo utility function permits Pratt-Arrow risk aversion to vary for gambles of different stakes in more complex ways than traditional CPT, and thus better fit empirical choice data for real payoffs. For example, it captures the dual effects of increasing relative risk aversion (which occurs as individuals become more risk-averse as the stakes rise), but decreasing absolute risk aversion for large stakes, which may help to explain why the curvature of the value function required to adequately express reasonable risk aversion over small gains may imply absurd risk aversion over larger gains (Rabin, 2000). The use of expo-expo functions has also been previously suggested by Luce (2000) for RDU where gambles are more complex than binary alternatives, and by Holt and Laury (2002), although in neither case motivated by pure risk attitudes.

Much further work is required to test how much of the pattern of choices currently attributed to loss aversion, curved utility, or non-linear decision weights is actually a reflection of a rational attitude to pure risk. All these concepts may have a place in describing the overall Pratt-Arrow risk attitude to uncertainty over monetary outcomes, but pure risk theory adds both normative and descriptive power to the model. The expo-expo function has the additional property that it is concave for small losses before turning convex for larger losses, whilst being everywhere concave for gains. Although this pattern can also be produced through decision weights (which might also imply such a reversal for gains), it could be an interesting non-parametric test of PRPT to examine whether this asymmetry could be produced after accounting for probability distortions.

4.4. *Implications for multi-attribute or general outcomes*

Having explored some implications of pure risk theory in a restricted domain, it remains to comment on a few implications in more complex decisions where consequence descriptions require more than a single numerical value. Since pure risk is primitive, it may be conceptually isolated from other factors in all such decisions. This is particularly useful in the case of outcomes with multiple attributes. Without a concept of pure risk attitude there is no single risk premium from comparing expected utility to expected value, but rather multiple premia arising from each numerical dimension of the decision outcomes. Compared with the single attribute case it is a lot less credible to argue that total risk attitude should be identified with a number of such premia simultaneously. Pure risk attitudes resolve this problem. The decision maker in multi-attribute cases has distinct strength of preference functions for each attribute, but only a single attitude to pure risk. This may go some way to explaining why it has frequently been observed that measured risk attitudes of a single individual appear to differ widely across different outcome domains (e.g., money, health, time) (Slovic, 1972). These measures may be confounding variable strength of preference for different outcomes with a single stable attitude to pure risk. It also raises the intriguing prospect that if a stable pure risk preference could be measured for an individual in domains that are more easily explored experimentally, this knowledge could be used to remove the pure risk component of attitudes in other decision making domains and thus reveal strength of preference for general outcome descriptions that are more difficult to test empirically.

Pure risk theory also holds implications for prescriptive approaches to risk. In many cases it may be argued that decisions *should* be made using linear value functions—diminishing marginal utility or sensitivity may be seen in certain contexts as psychological effects that a rational decision maker would want to avoid. In particular, in an institutional context, or when making decisions on behalf of some non-human legal entity diminishing strength of preference for certain outcomes could be seen as irrational and should thus play no part in a rational decision. Without recourse to pure risk theory, however, a linear value function necessarily implies that the decision maker is completely risk neutral. Pure risk theory permits an entity to have linear reactions to certain outcomes, whilst remaining risk averse in any decision that involves uncertainty. In addition, the theory stipulates the form that this attitude to pure risk should have, namely constant (i.e., exponential) pure risk aversion.

5. CONCLUSION

Conventional theories of risk used within the tradition of expected utility maximisation do not characterise attitudes to pure risk. Traditional approaches to risk attitude confound pure risk attitudes with the way in which utility is attached to monetary outcomes, or with the way in which decision makers distort probabilities. Dyer and Sarin's work drew attention to these confounds, but did not completely isolate pure risk attitude. In addition their theory applied only to decisions where outcomes can be completely described on a single numerical scale. The traditional model of rational choice in a risky environment has hitherto lacked a theory to explain precisely that element which makes the environment complex: it lacks a theory of how preferences are affected by the introduction of risk itself to the decision.

Pure risk theory fills this gap. It arises naturally from the extension of an intuitive psychological notion of risk as the chance of something bad happening—we should rationally wish to minimise the probability of not achieving our goals. It is instructive that this aspiration based approach to risk is commonplace in practical measures used by the finance industry, even though it is at odds with existing theoretical risk measures. I extend the aspiration concept to a continuum of aspiration levels and show that rational choice requires minimising the probability of not achieving the aspiration level for all possible choices of aspiration level simultaneously, where the importance of reaching each particular level of utility is expressed through a nondecreasing weighting function. By assuming the decomposition of a rational preference ordering into a component embodying only pure risk attitudes, and one which is pure risk neutral we may employ pure risk neutral utilities u^N as the substrate for the theory, which obviates the need for numerical outcomes. Final utilities in this view are obtained from pure risk neutral utilities by the application of the aspiration weighting function.

In addition, the assumption that the probability of not achieving each aspiration level decreases in importance as the level increases allows us to stipulate that the aspiration weighting function must be concave, and a further assumption that the risk of a prospect must increase if a constant positive value is added to each outcome leads to an exponential function.

Pure risk theory arises from both an axiomatic derivation and the generalisation of an intuitive psychological notion of risk. Furthermore pure risk theory may be applied to choices with completely general outcomes, and thus promises greater universality than most previous notions of risk attitude. An implication is that existing models ignore a fundamental component of risk attitude. However, pure risk may be adapted to fit into any extant rational theory of choice, and I show that by combining it with CPT, much of the empirical explanation that has been achieved through descriptive patches to SEU in the past may now have a normative explanation in terms of pure risk attitudes.

ACKNOWLEDGEMENT

I would like to thank the ESRC for funding (Ref: PTA–030-2002-00325).

NOTES

1. Dyer and Sarin call this *relative risk aversion*, a terminology I avoid for two reasons. The first is that I will develop pure risk attitude as a normative concept which is somewhat more general than Dyer and Sarin's concept. The second is that, subsequent to their paper, the term relative risk aversion has come to be associated strongly with an unrelated concept, namely the degree of local risk attitude relative to wealth levels.
2. With the possible addition of decision weights.
3. Note that I will not use SP/A theory as the foundation for pure risk theory, just Lopes' insight that aspiration is very closely related to intuitive notions of risk. Pure risk attitude is a component of von Neumann-Morgenstern preferences so decision making is governed the single decision criterion of expected utility maximisation. A dual criterion model

does not make normative sense as the expected utility criterion already fully represents the preferences $\succsim$.

4. Diecidue and van de Ven's approach may be seen as using an aspiration level to introduce a discrete jump into the strength of preference function when applied to numerical values. This may add descriptive power to the model in cases when one level of monetary outcome is particularly salient, but does not address the question of pure risk attitude.
5. The probabilities over states are constrained to match those of the overall preference ordering, $\succsim$, and so may be taken as given when applied to $\succsim^{PR}$.
6. Though in practice we could never be sure of this except where choices always involved only two options as in all other cases observed choice only ever reveals the top ranked choice of a preference ordering.
7. Assuming as before that consequences are numerical.
8. Although this interpretation of these data confounds diminishing sensitivity to certain outcomes with pure risk attitudes.

REFERENCES

Artzner, P., Delbaen, F., Eber, J.-M. and Heath, D. (1999), Coherent measures of risk, *Mathematical Finance* 9, 203–228.

Bawa, V.S. (1975), Optimal rules for ordering uncertain prospects, *Journal of Financial Economics* 2, 95–121.

Bell, D.E. and Raiffa H. (1988), Marginal value and intrinsic risk aversion, in Bell, D.E. Raiffa, H. and Tversky, A. (eds.), *Decision Making: Descriptive, Normative, and Prescriptive Interactions*, Cambridge University Press, Cambridge, pp. 384–397.

Camerer, C. F. (2003), *Behavioral Game Theory: Experiments in Strategic Interaction*. Russel Sage Foundation, New York.

Coombs, C.H. (1975), Portfolio theory and the measurement of risk, in Kaplan, M.F. and Schwartz, S. (eds.), *Human Judgement and Decision Processes*, Academic Press, New York, pp. 63–86.

Coombs, C.H. and Lehner, P.E. (1981), An evaluation of two alternative models for a theory of risk: part I, *Journal of Experimental Psychology: Human Perception and Performance* 7, 1110–1123.

Davies, G.B. and Satchell, S.E. (2004), The behavioural components of risk aversion, *Cambridge Working Papers in Economics*. Faculty of Economics, University of Cambridge.

Diecidue, E. and van de Ven, J. (2004), Probability of success and aspiration level: a descriptive extension of expected utility, *INSEAD Working Paper* 2004/99/DS.

Diecidue, E. and Wakker, P.P. (2001), On the intuition of rank-dependent utility, *Journal of Risk and Uncertainty* 23(3), 281–298.

Dubins, L.E. and Savage, L.J. (1976), *Inequalities for Stochastic Processes: How to Gamble if You Must*. Dover, New York.

Dyer, J.S. and Sarin, R. (1982), Relative risk aversion, *Management Science*. 28(8), 875–886.

Fishburn, P.C. (1982), Foundations of risk measurement. II. Effects of gains on risk, *Journal of Mathematical Psychology* 25, 226–242.

Gollier, C. (2001), *The Economics of Risk and Time*. The MIT Press, Cambridge, Massachusetts.

Holt, C.A. and Laury, S.K. (2002), Risk aversion and incentive effects, *American Economic Review* 92(5), 1644–1655.

Jia, J., Dyer, J.S. and Butler, J.C. (1999), Measures of perceived risk, *Management Science* 45(4), 519–532.

Keller, L.R., Sarin, R. and Weber, M. (1986), Empirical investigation of some properties of the perceived riskiness of gambles, *Organizational Behavior and Human Decision Processes* 38(1), 114–130.

Köbberling, V. and Wakker, P.P. (2005), An index of loss aversion, *Journal of Economic Theory* 122(1), 119–131.

Lopes, L.L. (1987), Between hope and fear: the psychology of risk, *Advances in Experimental Social Psychology* 20, 255–295.

Lopes, L.L. and Oden, G.C. (1999), The role of aspiration level in risky choice: a comparison of cumulative prospect theory and SP/A theory, *Journal of Mathematical Psychology* 43, 286–313.

Luce, R.D. (1980), Several possible measures of risk, *Theory and Decision* 12, 217–228.

Luce, R.D. (1981), Correction to 'Several possible measures of risk', *Theory and Decision* 13, 381.

Luce, R.D. (2000), *Utility of Gains and Losses: Measurement-Theoretical and Experimental Approaches*. Lawrence Erlbaum Associates, Mahwah, New Jersey.

March, J.G. and Shapira, Z. (1992), Variable risk preferences and the focus of attention, *Psychological Review* 99(1), 172–183.

Payne, J.W. (2006), It is whether you win or lose: The importance of the overall probabilities of winning or losing in risky choice, *Journal of Risk and Uncertainty* 30(1), 5–19.

Payne, J.W., Laughhunn, D.J. and Crum, R. (1980), Translation of gambles and aspiration level effects in risky choice behavior, *Management Science* 26, 1039–1060.

Payne, J.W., Laughhunn, D.J. and Crum, R. (1981), Aspiration level effects in risky behavior, *Management Science* 27, 953–958.

Peters, H.J.M. and Wakker, P.P. (1987), Convex functions on non-convex domains, *Economics Letters* 22, 251–255.

Pollatsek, A. and Tversky, A. (1970), A theory of risk, *Journal of Mathematical Psychology* 7, 540–553.

Pratt, J.W. (1964), Risk aversion in the small and in the large, *Econometrica* 32, 122–136.

Quiggin, J. (1982), A theory of anticipated utility, *Journal of Economic Behavior and Organization* 3, 323–343.

Rabin, M. (2000), Risk aversion and expected-utility theory: a calibration theorem, *Econometrica* 68(5), 1281–1292.

Rothschild, M. and Stiglitz, J. (1970), Increasing risk: I. A definition, *Journal of Economic Theory* 2, 225–243.

Roy, A.D. (1952), Safety first and the holding of assets, *Econometrica* 20(3), 431–439.

Sarin, R.K. (1987), Some extensions of Luce's measures of risk, *Theory and Decision* 22, 125–141.

Savage, L.J.: (1954), *The Foundations of Statistics*. Dover, New York.

Schmeidler, D. (1989), Subjective probability and expected utility without additivity, *Econometrica* 57(3), 571–589.

Shefrin, H. and Statman, M. (2000), Behavioral portfolio theory, *Journal of Financial and Quantitative Analysis* 35(2), 127–151.

Slovic, P. (1972), Psychological study of human judgment: implications for investment decision making, *Journal of Finance* 27(4), 779–799.

Sokolowska, J. (2003), Risk perception and acceptance—one process or two? The impact of aspirations on perceived risk and preferences, *Working Paper*, Institute of Psychology, Polish Academy of Sciences.

Sokolowska, J. and Pohorille, A. (2000), Models of risk and choice: Challenge or danger, *Acta Psychologica* 104, 339–369.

Szegö, G. (2002), Measures of risk, *Journal of Banking and Finance* 26, 1253–1272.

Thaler, R.H. (1999), Mental accounting matters, *Journal of Behavioral Decision Making* 12, 183–206.

Tversky, A. and Kahneman, D. (1992), Advances in prospect theory: Cumulative representation of uncertainty, *Journal of Risk and Uncertainty* 5, 297–323.

von Neumann, J. and Morgenstern, O. (1947), *The Theory of Games and Economic Behaviour*. Princeton University Press, Princeton.

Yaari, M.E. (1969), Some remarks on measures of risk aversion and on their uses, *Journal of Economic Theory* 1, 315–329.

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