Measures of Investment Risk

Financial Mathematics Clinic

SLAS – University of Kent
1 Introduction

2 Glossary

3 Motivation

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Introduction

These slides are (mainly) aimed to

- Undergraduate students.
- Postgraduate students doing Financial Mathematics for the first time.
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Objective

- Learn some ways to measure the risk of different investments when we don’t know the whole distribution of the returns.
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• **Return.** The return of an asset or investment is the percentage increase of its market value over a particular period of time.

• **Random variable.** Let \((\Omega, \mathcal{F}, \mathbb{P})\) be a probability space and consider the measurable space \((\mathbb{R}, \mathcal{B}(\mathbb{R}))\) and \(X : \Omega \rightarrow \mathbb{R}\) be a measurable function. Then \(X\) is called a random variable.

• **Downside risk.** It can be thought as the potential loss derived from a fall in the price of an asset (or the value of a security).
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4. Measures of investment risk
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- Hence we need methods to measure it.

- Hereafter, we let $X$ be the random variable associated to the return.
4 Measures of investment risk
One of the most commonly used measurements of risk, is given by the variance of $X$, i.e.,

$$\text{Var}(X) = \begin{cases} \int_{-\infty}^{\infty} (x - \mu)^2 f(x) \, dx & \text{if } X \text{ is continuous} \\ \sum_{i} (x_i - \mu)^2 p_i & \text{if } X \text{ is discrete} \end{cases}$$

It measures the variability. It is also common to use the standard deviation

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Value at Risk (VaR)

- It estimates the potential loss on a portfolio over a given future time period \( T \) i.e., \((\text{in a bad period, how much could we loose?})\).

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\text{VaR}(X; q) = -L \quad \text{where} \quad P(X < L) = q
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In other words, with probability \( q \) we will not lose more than \( L \) in time \( T \).

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- It quantifies the expected value of the loss given that it has fallen below the VaR.
Exercise

Suppose that the annual return $X$ for a particular stock has the following pdf

$$f(x) = 0.00075(100 - (x - 5)^2) \quad -5 \leq x \leq 15$$

Obtain

- Expected return $\mu$ (ans. 5).
- Variance of return (ans. 20).
- Semi-variance of return (ans. 10).
- VaR over 1 year with 95% confidence for a portfolio consisting of £100 millions invested in the stock (ans. 2.293).
- TVaR with 95% confidence (ans. 2.3392).
To book a maths/stats appointment...

www.kent.ac.uk/learning
QUESTIONS?