Basic Annuities

Financial Mathematics Clinic

SLAS – University of Kent
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4 BASIC ANNUITIES
These slides are (mainly) aimed to

- Undergraduate students.
- Postgraduate students doing Financial Mathematics for the first time.
Introduction

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Objective

- Understand the basic mathematical principles of basic annuities.
**Glossary**

- *Discount factor*. Given a rate of interest \( i \), the discount factor is given by \( v = (1 + i)^{-1} \).

- *Geometric series*. The sum of an infinite number of terms that have a constant ratio \( r \) between successive terms. If \(|r| < 1\) then the series converges, i.e.

\[
\sum_{k=0}^{\infty} r^k = \frac{1}{1 - r}
\]
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Motivation

An annuity can be broadly defined as a series of payments made at equal intervals of time.
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- Annuities are everywhere e.g. house rents, mortgage payments and insurance for retirement.
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Motivation

- An *annuity* can be broadly defined as a series of payments made at equal intervals of time.
- Annuities are everywhere e.g. house rents, mortgage payments and insurance for retirement.
- Originally it was restricted to annual payments, but is has been extended to other intervals.
- Payments can be "certain" (mortgage and rent) or not (pension plans).
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An annuity-immediate is the one that pays level payments at the end of \( n \) periods, with a constant rate of interest in each of them.
Annuity-Immediate

- An annuity-immediate is the one that pays level payments at the end of $n$ periods, with a constant rate of interest in each of them.

- There are two important equations of value.
The present value (using the discount factor $v$) is

$$a_{\overline{n}|} = v^1 + v^2 + \cdots + v^n \quad \text{(geometric progression)}$$

$$= \frac{v - v^{n+1}}{1 - v} \quad \text{(some algebra)}$$

$$= \frac{1 - v^n}{i}. $$
Annuity-Immediate (Cont.)

- The present value (using the discount factor \( v \)) is

\[
\bar{a}_n = v^1 + v^2 + \cdots + v^n \quad \text{(geometric progression)}
\]

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= \frac{v - v^{n+1}}{1 - v} \quad \text{(some algebra)}
\]

\[
= \frac{1 - v^n}{i}.
\]

- The accumulated value is

\[
\bar{s}_n = \bar{a}_n (1 + i)^n
\]

\[
= \frac{(1 + i)^n - 1}{i}
\]
An annuity-due is the one that pays level payments at the beginning of \( n \) periods, with a constant rate of interest in each of them.
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There are two important equations of value.
The present value (using the discount factor \( v \)) is

\[
\ddot{a}_{\bar{n}} = 1 + v^1 + \cdots + v^{n-1}
\]

(geometric progression)

\[
= \frac{1 - v^n}{1 - v}
\]

(some algebra)

\[
= \frac{1 - v^n}{iv}.
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The present value (using the discount factor $v$) is

$$\ddot{a}_{\overline{n}|} = 1 + v^1 + \cdots + v^{n-1} \quad \text{(geometric progression)}$$

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The accumulated value is

$$\ddot{s}_{\overline{n}|} = \ddot{a}_{\overline{n}|}(1 + i)^n$$

$$= \frac{(1 + i)^n - 1}{iv}.$$
A perpetuity is an annuity paying an infinite number of level payments, e.g. perpetual bonds.
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- There is no accumulated value.
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- For immediate perpetuities,

\[ a_{\infty} = v + v^2 + v^3 + \cdots \]  

(geometric series)

\[ = \frac{v}{1 - v} \]

\[ = \frac{1}{i} \]
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There is no accumulated value.

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\[ a_\infty = v + v^2 + v^3 + \cdots \quad \text{(geometric series)} \]

\[ = \frac{v}{1 - v} \]

\[ = \frac{1}{i} \]

For due perpetuities,

\[ \ddot{a}_\infty = 1 + v + v^2 \cdots = \frac{1}{v} \ a_\infty = \frac{1}{i}v \]
Immediate and due annuities are just the top of the iceberg.
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In practice there are much more general annuities, e.g. deferred annuities, annuities with geometrically or arithmetically increasing (decreasing) payments, annuities with different payment and interest periods...
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It is impossible to memorise all the formulas!
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But... for each particular case, the basic principles are useful, i.e. draw a timeline, identify the periods and use equations of value.
To book a maths/stats appointment...

www.kent.ac.uk/learning
QUESTIONS?