Newton-Rhapson Method

Financial Mathematics Clinic

SLAS – University of Kent

University of Kent | Student Learning Advisory Service

Introduction

These slides are (mainly) aimed to

- Undergraduate students.
- Postgraduate students doing Financial Mathematics for the first time.

These slides are (mainly) aimed to

- Undergraduate students.
- Postgraduate students doing Financial Mathematics for the first time.

Objective

- Learn the Newton-Rhapson method and some of its applications in Financial Mathematics.

- *Taylor series*. An infinite sum expansion of a function about a point.
- *Root of a function f*. An element x of the domain of the function, such that $f(x) = 0$.
- *Continuous function*. A function f is continuous at some point a if

$$\lim_{x \to a} f(x) = f(a).$$

- In numerical analysis, the Newton-Rhapson method is a root finding algorithm.

Motivation

- In numerical analysis, the Newton-Rhapson method is a root finding algorithm.
- Extremely useful when $f(x) = 0$ cannot be solved analytically.

Motivation

- In numerical analysis, the Newton-Rhapson method is a root finding algorithm.
- Extremely useful when $f(x) = 0$ cannot be solved analytically.
- Widely used in many fields of Applied Mathematics.

Algorithm

The Newton-Rhapson method is a recursive algorithm that makes use of the Taylor series in order to approximate the root.

The Newton-Rhapson method is a recursive algorithm that makes use of the Taylor series in order to approximate the root.

Algorithm

- Start with an initial guess x_0.

The Newton-Rhapson method is a recursive algorithm that makes use of the Taylor series in order to approximate the root.

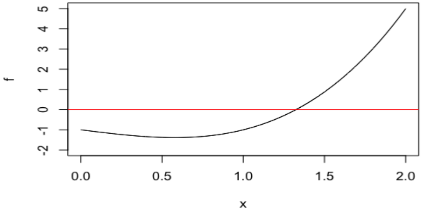
Algorithm

- Start with an initial guess x_0.
- At step n

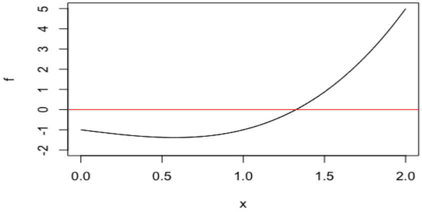
$$x_n = x_{n-1} - \frac{f(x_{n-1})}{f'(x_{n-1})}$$

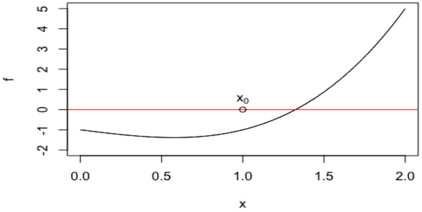
.

Graphical example

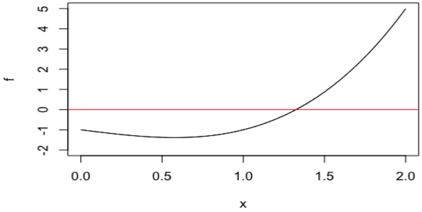


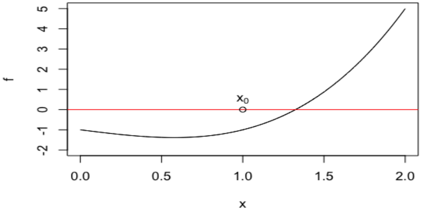
Graphical example

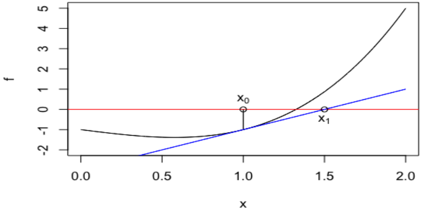




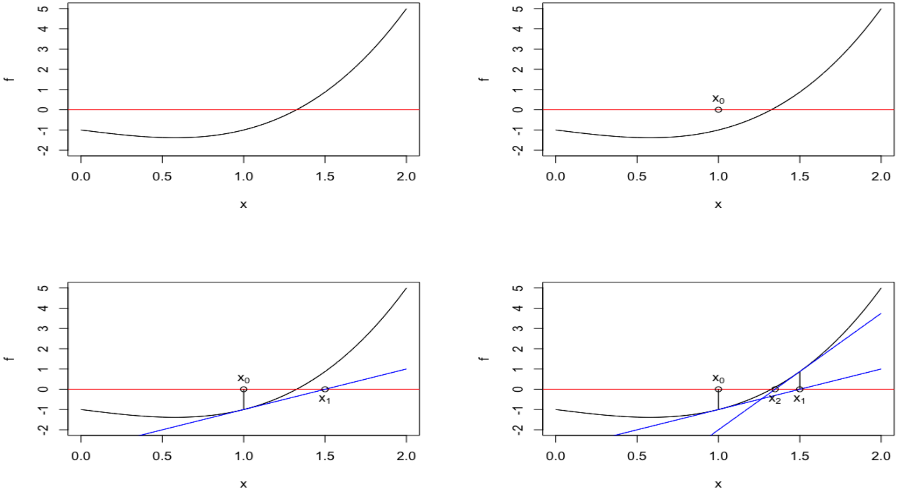
Graphical example







Graphical example



In Financial Mathematics the N-R technique is useful for problems involving

In Financial Mathematics the N-R technique is useful for problems involving

- Unknown rate of interest, i,

In Financial Mathematics the N-R technique is useful for problems involving

- Unknown rate of interest, i,
- Implied volatility, σ in the Black-Scholes-Merton model.

Newton-Rhapson in Finances

At what rate of interest is £16,000 the present value of £1000 paid at the end of every year for 20 years?

Newton-Rhapson in Finances

At what rate of interest is £16,000 the present value of £1000 paid at the end of every year for 20 years?

- We want to solve

$$
\begin{aligned}
16000 &= 1000(1+i)^{-1} + \cdots + 1000(1+i)^{-20} \\
&= 1000 \times \left(\frac{1-(1+i)^{-20}}{i} \right)
\end{aligned}
$$

At what rate of interest is £16,000 the present value of £1000 paid at the end of every year for 20 years?

- We want to solve

$$\begin{aligned} 16000 &= 1000(1+i)^{-1} + \cdots + 1000(1+i)^{-20} \\ &= 1000 \times \left(\frac{1-(1+i)^{-20}}{i} \right) \end{aligned}$$

- We define

$$f(i) = (1+i)^{-20} + 16i - 1$$

- We obtain the derivative

$$f'(i) = -20(1+i)^{-21} + 16$$

Newton-Rhapson in Finances

- We obtain the derivative

$$f'(i) = -20(1+i)^{-21} + 16$$

- We start with $i_0 = .5$ and we start the algorithm.

  1. $i_1 = .5 - \frac{f(.5)}{f'(.5)} = .0623$
  2. $i_2 = .0623 - \frac{f(.0623)}{f'(.0623)} = .0246$
  3. $i_3 = .0246 - \frac{f(.0246)}{f'(.0246)} = .0224$
  4. $i_4 = .0224 - \frac{f(.0224)}{f'(.0224)} = .0222$ (Convergence achieved!)

To book a maths/stats appointment...

www.kent.ac.uk/learning

University of Kent

Student Learning Advisory Service

QUESTIONS?