Measures of Investment Risk

Financial Mathematics Clinic

SLAS – University of Kent

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Introduction

These slides are (mainly) aimed to

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- Postgraduate students doing Financial Mathematics for the first time.

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Objective

- Learn some ways to measure the risk of different investments when we don't know the whole distribution of the returns.

Glossary

- *Return*. The return of an asset or investment is the percentage increase of its market value over particular period of time.
- *Random variable*. Let $(\Omega, \mathscr{F}, \mathbb{P})$ be a probability space and consider the measurable space $(\mathbb{R}, \mathscr{B}(\mathbb{R}))$ and $X : \Omega \to \mathbb{R}$ be a measurable function. Then X is called a random variable.
- *Downside risk*. It can be thought as the potential loss derived from a fall in the price of an asset (or the value of a security).

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  - One could choose the safest one.
- One way to do it is through the underlying risk of each investment.
- Hence we need methods to measure it.
- Hereafter, we let X be the random variable associated to the return.

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- It measures the variability.
- It is also common to use the standard deviation

$$\text{SD}(X) = \sqrt{\text{Var}(X)}.$$

- The semi-variance of return (SV) is defined as:

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- We can estimate it using simulation techniques or historical data.

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- It quantifies the expected value of the loss given that it has fallen below the VaR.

Exercise

Suppose that the annual return X for a particular stock has the following pdf

$$f(x) = .00075(100 - (x-5)^2) \qquad -5 \leq x \leq 15$$

Obtain

- Expected return μ (ans. 5).
- Variance of return (ans. 20).
- Semi-variance of return (ans. 10).
- VaR over 1 year with 95% confidence for a portfolio consisting of £100 millions invested in the stock (ans. 2.293).
- TVaR with 95% confidence (ans. 2.3392).

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QUESTIONS?