

RATES OF INTEREST

Financial Mathematics Clinic

SLAS – University of Kent



Student Learning
Advisory Service

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2 GLOSSARY

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4 EFFECTIVE RATE OF INTEREST

5 NOMINAL RATE OF INTEREST

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These slides are (mainly) aimed to

- Undergraduate students.
- Postgraduate students doing Financial Mathematics for the first time.

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Objective

- To understand the difference between effective and nominal rates of interest and their relationship.

① INTRODUCTION

② GLOSSARY

③ MOTIVATION

④ EFFECTIVE RATE OF INTEREST

⑤ NOMINAL RATE OF INTEREST

- *Accumulation function.* An accumulated function $a(t)$ gives the accumulated value at time $t \geq 0$ of an initial investment of 1 monetary unit.

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- Interest is a key concept in Financial Mathematics.

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- Interest measures the cost of borrowing money, the return on any financial instrument and hence, it affects all of our decisions made with respect to our money and investments.
- There is not a unique nor uniform rate of interest and this is why it is so important to have a clear understanding of the mathematical foundations of rates of interest.

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DEFINITION

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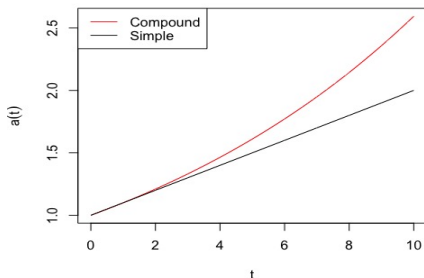
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- Usually denoted in %.
- The principal remains constant.

TYPES OF INTEREST

- *Simple interest.* The amount of interest earned during each period is constant, i.e. a linear accumulation function $a(t) = 1 + it$.

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- *Compound interest.* The earned interest at each period is automatically reinvested, $a(t) = (1 + i)^t$.



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- Some terms used to describe such situations include *payable quarterly*, *compounded semiannually* and *convertible monthly*.
- The frequency in which interest is paid is called the *interest conversion period*.
- $i^{(m)}$ denotes the nominal rate of interest payable m times per period.

EXAMPLE

Bank A offers you an effective rate of interest of 10.5% while Bank B offers you a nominal rate of interest of convertible monthly of 10%. Which offer should you take?

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- We want the one which yields more money but...
- How do we compare effective and nominal rates of interest?
- Solution: consider the equivalent effective rate of interest.

EQUIVALENT RATE OF INTEREST

- We say that two rates of interest are *equivalent* if a given amount of principal invested for the same length of time produces the same accumulated value.

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- In the example, the equivalent rate for the 10% convertible monthly is

$$i' = \left(1 + \frac{.10}{12}\right)^{12} - 1 = .1047.$$

Hence, Bank A should be the chosen one!

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QUESTIONS?