Equations of Value

Financial Mathematics Clinic

SLAS – University of Kent

These slides are (mainly) aimed to

- Undergraduate students.
- Postgraduate students doing Financial Mathematics for the first time.

Introduction

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Objective

- Understand the basic principles of equations of value and how they can be used to solve some financial problems.

Glossary

- *Present value*. Given a rate of interest i, the term v^t is said to be the present value of 1 to be paid at the end of t periods
- *Root of a function f*. An element x of the domain of the function, such that $f(x) = 0$.

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- However, in certain cases, we need to know how much money do I have to invest to obtain X quantity in the future.
- Other problems of interest include *unknown time* or *unknown rate of interest*.
- To solve these problems an *equation of value* will always be needed.

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- The resulting equation is known as *equation of value*.
- *Time diagrams* are a really useful tool to visualise the problem.
- Under compound interest all equations of value of a series of payments yield the same results.

In order to receive £600 at the end of 8 years, you agree to pay £100 at once, £200 at the end of the fifth year and to make a final payment X at the end of the tenth year. For a nominal rate of interest of 8% convertible semiannually, what is the value of X?.

Example

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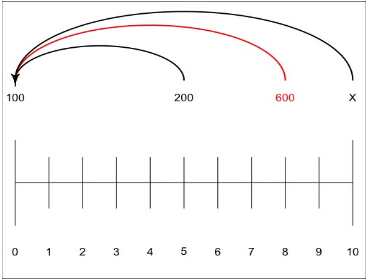
- Draw timeline



Example (continuation)

- Obtain equation of value using the present value of the cashflows

$$100 + 200v^{10} + Xv^{20} = 600v^{16}$$

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- Solve for X

$$X = \frac{600v^{16} - 200v^{10} - 100}{v^{16}} = 186.76$$

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  - Approximations using root finding algorithms (e.g. Newton-Rhapson).

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QUESTIONS?