Basic Annuities

Financial Mathematics Clinic

SLAS – University of Kent

Introduction

These slides are (mainly) aimed to

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- Postgraduate students doing Financial Mathematics for the first time.

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Objective

- Understand the basic mathematical principles of basic annuities.

- *Discount factor*. Given a rate of interest i, the discount factor is given by $v = (1+i)^{-1}$.
- *Geometric series*. The sum of an infinite number of terms that have a constant ratio r between successive terms. If $|r| < 1$ then the series converges, i.e.

$$\sum_{k=0}^{\infty} r^k = \frac{1}{1-r}$$

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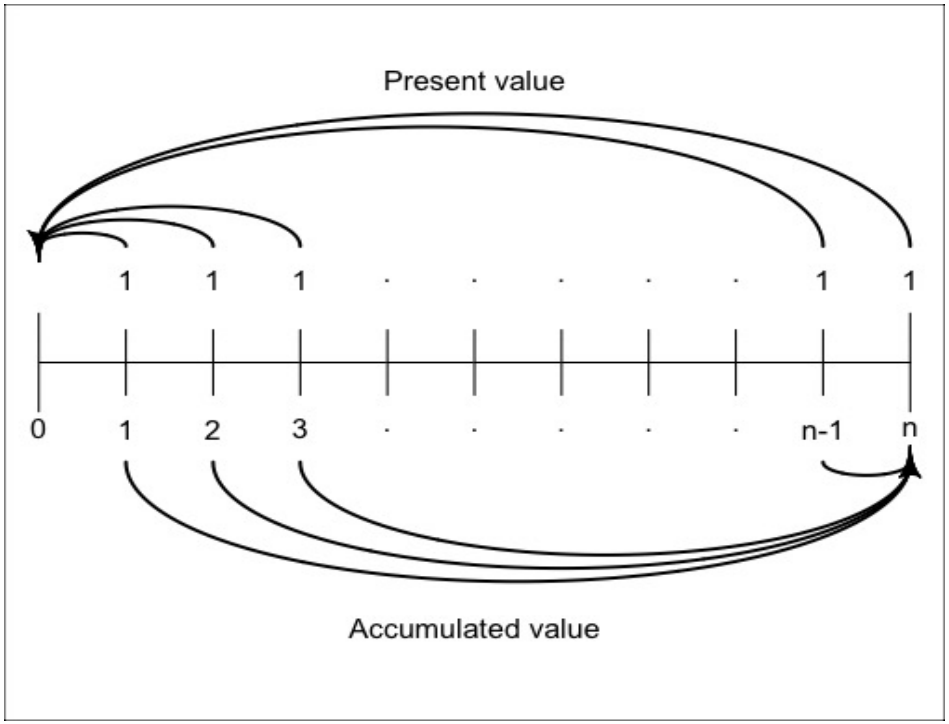
- Annuities are everywhere e.g. house rents, mortgage payments and insurance for retirement.
- Originally it was restricted to annual payments, but is has been extended to other intervals.
- Payments can be "certain" (mortgage and rent) or not (pension plans).

Annuity-Immediate

- An annuity-immediate is the one that pays level payments at the end of n periods, with a constant rate of interest in each of them.



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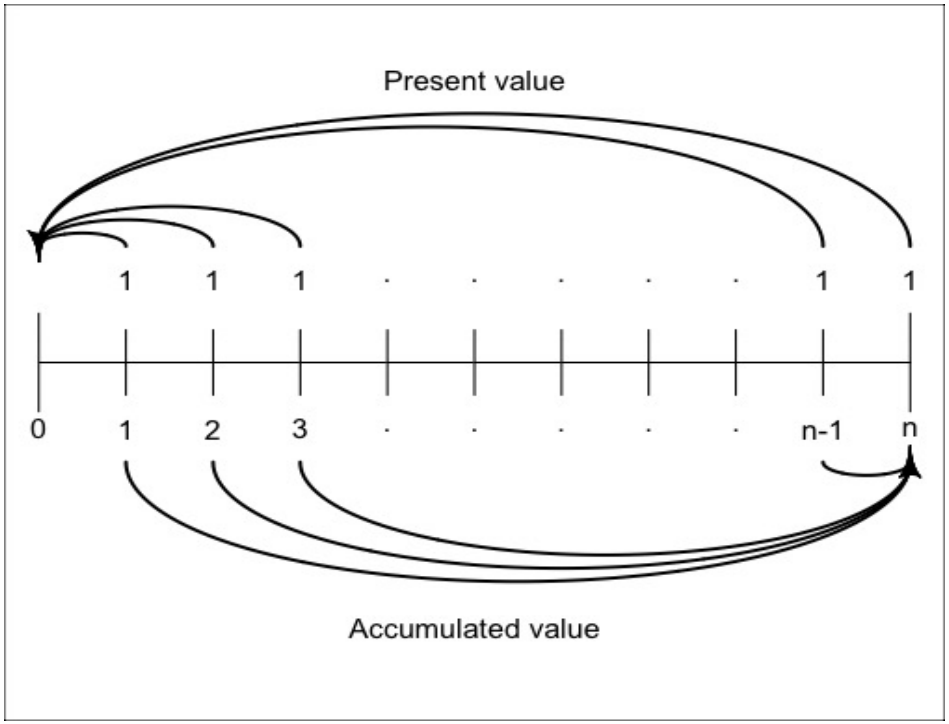

- There are two important equations of value.

Annuity-Immediate (Cont.)

- The present value (using the discount factor v) is

$$\begin{aligned} a_{\overline{n}|} &= v^1 + v^2 + \cdots + v^n && \text{(geometric progression)} \\ &= \frac{v - v^{n+1}}{1 - v} && \text{(some algebra)} \\ &= \frac{1 - v^n}{i}. \end{aligned}$$

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- The accumulated value is

$$\begin{aligned} s_{\overline{n}|} &= a_{\overline{n}|}(1+i)^n \\ &= \frac{(1+i)^n - 1}{i} \end{aligned}$$

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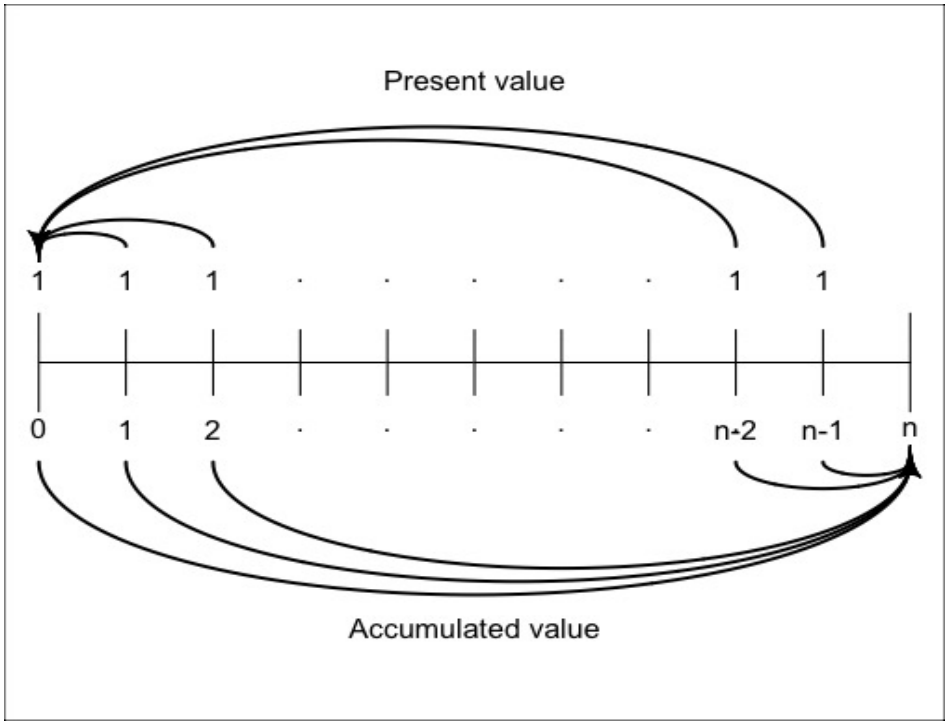

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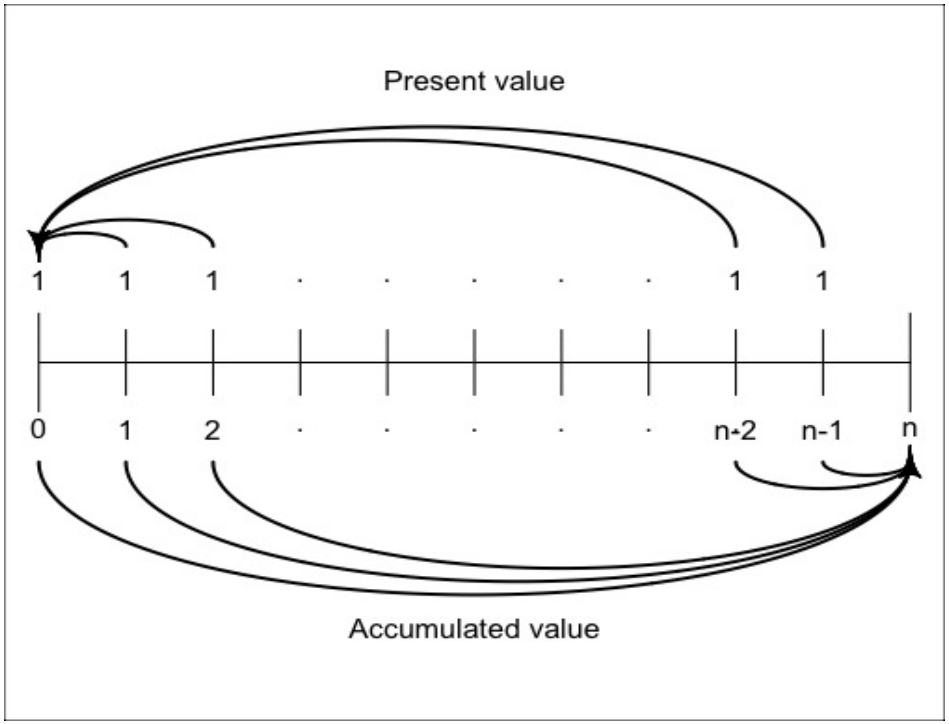

- There are two important equations of value.

Annuity-Due (Cont.)

- The present value (using the discount factor v) is

$$\begin{aligned} \ddot{a}_{\overline{n}|} &= 1 + v^1 + \cdots + v^{n-1} && \text{(geometric progression)} \\ &= \frac{1 - v^n}{1 - v} && \text{(some algebra)} \\ &= \frac{1 - v^n}{iv}. \end{aligned}$$

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- For due perpetuities,

$$\ddot{a}_{\overline{\infty}|} = 1 + v + v^2 \cdots = \frac{1}{v} a_{\overline{\infty}|} = \frac{1}{iv}$$

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- In pratice there are much more general annuities, e.g. *deferred annuities*, annuities with geometrically or arithmetically *increasing (decreasing)* payments, annuities with different payment and interest periods...
- It is impossible to memorise all the formulas!
- But... for each particular case, the basic principles are useful, i.e. draw a timeline, identify the periods and use *equations of value*.

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QUESTIONS?