

Binomial Pricing Models

Financial Mathematics Clinic

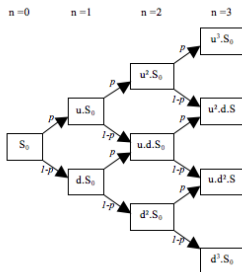
SLAS-University of Kent

Content of study

- ▶ Introduction
- ▶ Replicating portfolio
- ▶ One step binomial pricing model
- ▶ CRR model
- ▶ DerrivaGem
- ▶ Relation with BSM

What is Binomial Model?

- ▶ The binomial option pricing model is an options valuation method proposed by William Sharpe in the 1978 and formalized by Cox, Ross and Rubinstein in 1979.
- ▶ The model assumes that stock price have two possible movement directions at each time point: up or down.
- ▶ It generates different paths of stock price evolution. As a result, we have a distribution of stock prices (as well as option payoff) in the end.



$$p = \frac{e^{rt/n} - d}{u - d}$$

$$u = e^{\sigma \sqrt{t/n}}$$

$$d = e^{-\sigma \sqrt{t/n}}$$

Introduction

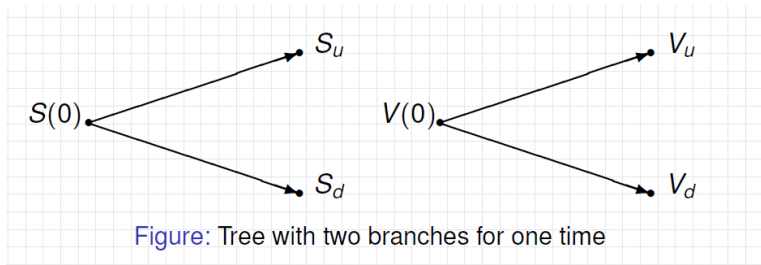


Figure: Tree with two branches for one time

The values of options at maturity are determined by their intrinsic values, which can be discounted to estimate option prices at inception.

One useful tool for pricing options is binomial model. It is a set of all possible stock price evolution.

This methodology assumes that stock price movement follows random walk.

This model is consistent with Black-Scholes model.

Replicating portfolio

Consider a portfolio that consisted of long position in Δ stocks and short position in 1 option contract. Spot price currently is 20 dollars and after 1 period, it can either goes up to 22 dollars or goes down to 16 dollars.

One step binomial pricing model

The example of pricing options can be generalized by

$$V(0) = e^{-r\Delta t} [qV_u + (1 - q)V_d] \quad (1)$$

Where

$$q = \frac{e^{r\Delta t}S(0) - S_d}{S_u - S_d} \quad (2)$$

Here q is often regarded as the risk-neutral probability. It measures how likely asset price may go up during each period in the risk-neutral world.

Now, let's apply this model to price the option in previous example and see what we get.

The Cox-Ross-Rubinstein model

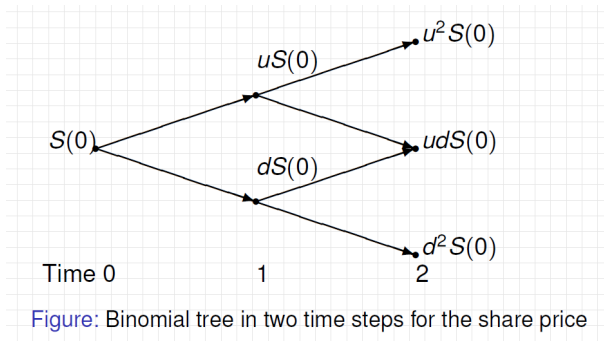


Figure: Binomial tree in two time steps for the share price

For an option that has two periods left before maturity, its price can be calculated by

$$V(0) = e^{-2r\Delta t} [q^2 V_{uu} + 2q(1 - q)V_{ud} + (1 - q)^2 V_{dd}] \quad (3)$$

The Cox-Ross-Rubinstein model

One problem arises as the determination of u and d . As number of steps enlarges, to form a binomial tree with Δt step size, we must select u and d , such that they are consistent with stock return's volatility.

To achieve this, according to Cox, Ross and Rubinstein (1979), we can simply set u and d as

$$\begin{aligned}u &= e^{\sigma\sqrt{\Delta t}} \\d &= e^{-\sigma\sqrt{\Delta t}}\end{aligned}\tag{4}$$

Price options using DerivaGem

DerivaGem is a useful tool for beginners in pricing options and discovering features of options. In the section, let's try to price more complicated options (more steps) using DerivaGem.

It is obvious that, as Δ approaches zero (or number of steps approximates infinity), binomial model prices converges to BSM prices

Pros and Cons

- ▶ Pros: Multi-period view and simplicity.
- ▶ Cons: More time consuming (Consider the number of calculations needed for simulating as step size approximates zero).

To book a maths/stats appointment...

www.kent.ac.uk/learning



University of
Kent

Student Learning
Advisory Service

QUESTIONS?