Targeted fiscal policy to increase employment and wages of unskilled workers*

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Abstract

We extend the canonical model of search and matching frictions by including capital-skill complementarity in production, labour markets with skilled and unskilled workers and on-the-job-learning (OJL) within and across skill types. These extensions capture key characteristics of skilled and unskilled labour markets in the data. We find that increases in public spending to enhance unskilled productivity via OJL are beneficial to employed unskilled workers and reduce earnings inequality between employed skilled and unskilled labour. However, unskilled unemployment and labour income inequality within the group of unskilled labour rises. We next find that vacancy subsidies work to increase employment and returns to unskilled workers. However, unemployment for skilled workers rises and skilled wages and labour income fall in the short-run. We finally show that it is possible to increase skilled vacancy subsidies to nullify the negative effects on skilled employment following an increase in unskilled vacancy subsidies.

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Non-technical summary

The evolution of inequality has been well documented in the data. Inequality in earnings has increased in recent decades and, in particular, wage inequality has increased dramatically since the beginning of the 20th century. As a result of this rise and its deleterious implications for the welfare of a large part of the population, societies and policymakers at large are paying increasing attention to better understanding causes and consequences of inequality.

This paper aims to contribute to the inequality literature by studying the difference in employment opportunities and labour productivities for workers with (skilled) and without (unskilled) college education which is a main contributor to wage and earnings inequality. The paper, in particular, focuses on the U.S. economy. The literature on the skill premium has demonstrated that there are significant differences in the wages between skilled and unskilled labour, and that the skill premium has increased in recent decades to its highest levels in a century. We investigate the so-called "college premium" or "skill premium" in the U.S. and its relationship with basic earnings inequality.

To this end, we model inequality in wages jointly with differences in employment opportunities between skilled and unskilled workers over the business cycle, with the aim of evaluating the effects of supply side fiscal interventions which intend to increase labour productivity and employment for the unskilled and to reduce inequality. We employ a standard approach to modeling unemployment using a setup with search and matching frictions that belongs to the Mortensen-Pissarides (MP) family, and extend this by allowing for *ex ante* heterogeneous workers who are employed in skilled or unskilled jobs and produce output under capital-skill complementarity. While their skill type is given, workers' productivity benefits from lifelong learning associated with working experience and on-the-job learning (OJL), so that workers' productivity is endogenous and a positive function of employment. As a result, differences in employment opportunities and inequality in wages are closely linked. This paper also allows for capital-skill complementarity in production. These extensions capture key characteristics of skilled and unskilled labour markets in the data.

We find that increases in public spending to enhance unskilled productivity via OJL are beneficial to employed unskilled workers and reduce earnings inequality between employed skilled and unskilled labour. However, unskilled unemployment and labour income inequality within the group of unskilled labour rises. We next find that vacancy subsidies work to increase employment and returns to unskilled workers. However, unemployment for skilled workers rises and skilled wages and labour income fall in the short-run. We finally show that it is possible to increase skilled vacancy subsidies to nullify the negative effects on skilled employment following an increase in unskilled vacancy subsidies.

1 Introduction

Inequality in earnings has increased in recent decades and, in particular, wage inequality has increased dramatically since the beginning of the 20th century. The difference in employment opportunities and labour productivities for workers with and without college education, broadly capturing different returns to skilled and unskilled labour, is a main contributor to wage and earnings inequality. Among others in a big literature, Goldin and Katz (2008) and Heathcote *et al.* (2010) document and evaluate the evolution of the so-called "college premium" or "skill premium" in the U.S. and its relationship with basic earnings inequality. This increase is wage inequality has been accompanied by a stagnation in the real incomes of the lower quintals over recent decades. Research also shows that changes in earnings inequality are closely linked to changes in employment (see e.g. Heathcote *et al.* (2010)). Moreover, differences in the employment experience of separate income groups (see also the discussion below using data from the Current Population Survey (CPS)) suggest that unskilled workers face a consistently higher unemployment rate than skilled workers.

The connection between earnings inequality and unemployment is especially relevant in business cycle frequencies (see e.g. Quadrini and Rios-Rull (2015)). The cyclical properties of unemployment differ between skilled and unskilled labour (see e.g. Hagedorn *et al.* (2016)). For example, the CPS data suggest that unemployment is more volatile for unskilled compared to skilled labour. Hence, inequality in returns to work and differences in unemployment spells jointly determine earnings inequality and the distinction between skilled and unskilled labour is important in this nexus.

The combined effects of rising wage inequality, stagnation in real income growth for a large proportion of lower income, unskilled workers, and permanently higher unemployment for the unskilled has created a disadvantaged environment for a large part of the population, which, commentators often link to the rise of populism. It is widely acknowledged that intervention to reverse this situation for lower income unskilled workers is required. Targeted interventions to increase the skills and productivity of unskilled workers (see e.g. Goldin and Katz (2008) on the role of learning, skills and education) are naturally considered as a means to support the incomes and employability of workers with lower skills.[1] While maintaining visibility, and thus being attractive to elected policymakers, importantly such policies are expected to have real effects by directly addressing the underlying causes of the increased

[1]For example, see the subsidized jobs program run by the Office of National Assistance (Temporary Assistance for Needy Families) for policies aiming to offset hiring costs, www.acf.hhs.gov/ofa/programs/tanf.

inequality and social tensions.

Motivated by these observations, we model inequality in wages jointly with differences in employment opportunities between skilled and unskilled workers over the business cycle, with the aim of evaluating the effects of policies which intend to increase labour productivity and employment for the unskilled and to reduce inequality. We employ a standard approach to modeling unemployment using a setup with search and matching frictions that belongs to the Mortensen-Pissarides (MP) family, and extend this by allowing for *ex ante* heterogeneous workers who are employed in skilled or unskilled jobs and produce output under capital-skill complementarity. While their skill type is given, workers' productivity benefits from lifelong learning associated with working experience and on-the-job learning, so that workers' productivity is endogenous and a positive function of employment. As a result, differences in employment opportunities and inequality in wages are closely linked.

Equilibrium unemployment models with search and matching frictions have been extensively used in macroeconomic analyses of unemployment (see e.g. Shimer (2010) and Rogerson and Shimer (2011) for an analytical overview of this research). Among other extensions, this literature has considered the importance of both differences in workers' skills and the potential for skill erosion due to unemployment (see e.g. Cahuc *et al.* (2006), Krause and Lubik (2006 and 2010), Dolado *et al.* (2009), Laureys (2014), Hagedorn *et al.* (2016) and Doppelt (2016)).[2]

Given that in business cycle frequencies there is not much labour movement between the skilled and unskilled sectors[3], we assume that unskilled workers cannot become skilled. Instead, skilled workers work in skilled jobs and, if unemployed, search for employment in the skilled sector. Similarly, unskilled workers work in unskilled jobs, and if unemployed, search for employment in the unskilled sector. In addition, we assume that the production structure allows for skill-biased technical change and, in particular, is characterised by capital-skill complementarity. This setup has been shown to explain key characteristics of the skill premium in the data, both in terms of its evolution over the past several decades (see e.g. Katz and Murphy (1992), Krusell *et al.* (2000) and He (2012)) as well as over the business cycle (Lindquist (2004) and Pourpourides (2011)). The search and matching mechanism for employment creation that we employ follows the benchmark MP framework with the wage being determined via Nash bargaining. Moreover,

[2] Hagedorn *et al.* (2016) also extend a version of the MP search and matching model to allow for skilled and unskilled workers. Their aim, however, is to evaluate the effect of technology and tax shocks on sectoral unemployment.

[3] This is consistent with time series data on relative skill supply, which we discuss below.

our setup allows for differentiation between the two labour markets across a number of dimensions, such as differences in relative bargaining power, job separation rates and job posting costs to reflect the relevant empirical observations.

Given the potential for lifelong learning and knowledge spillovers associated with learning by doing in the work place, we allow skilled and unskilled workers' productivities to be positive functions of skilled and unskilled employment. We model this as on-the-job learning (OJL). Alternatively, since the sectoral productivities are decreasing functions of unemployment, this can equivalently capture skill erosion due to not working (see e.g. Laureys (2014) and Doppelt (2016)). We allow the workers of each type to internalise the effect of their own employment on their labour productivity via OJL, but assume that the employment of the other worker type is taken as given, thus working as a positive externality. Including OJL in the model also allows us to evaluate the effect of policies aimed at increasing worker's labour productivity, since the latter is now endogenous.

The model is calibrated to match the steady-state of aggregate and sectoral labour market data in the U.S., following the calibration strategy in Shimer (2010). We find that the calibration does a good job at matching the second moments in the labour market data for the skilled and unskilled (the cyclical properties in the data are analysed in Section 2). In particular, the model predicts more volatile employment for skilled versus unskilled workers, while unemployment is more volatile for unskilled workers. At the same time, employment for both types is less volatile than output[4], whereas unemployment for both types is more volatile than output and wages for both skilled and unskilled are at least as volatile as employment. The mechanism introduced by OJL also helps to bring the model's predictions for the output correlations of employment/unemployment and wages closer to the data. In particular, OJL contributes to increasing the output correlations of employment and unemployment and to decreasing the output correlations of wages, which in its absence are too low and too high, respectively, compared with the data.

We then evaluate the productivity, employment and inequality effects of government policies that are designed to stimulate labour productivity and employment. A natural policy that can increase the productivity and wages of low skilled workers and reduce the skill premium is to improve their com-

[4]Consistent with the results in Shimer (2005 and 2010), the model under predicts quantitatively the volatility of employment, but the gap is not very big. In particular, the model variants considered predict an employment volatility of about 65% of the volatility of employment in the data, whereas in the canonical models, e.g. Shimer (2010), this ratio is typically about 25%.

petence set by offering educational and continuing professional development programmes. These can increase labour productivity and enhance the ability of the worker for OJL. For instance, government-financed programmes to train workers to use new software or improve their generic skills not only increase workers' productivity but also their capacity to interact with others in the work place and to learn from them. Regarding employment, hiring subsidies (e.g. in the form of subsidising vacancy posting costs and facilitating the job search process) are often considered both in the academic literature on policies for unemployment (e.g. Shi and Wen (1999), Heijdra and Ligthart (2002), Campolmi *et al.* (2011) and Jung and Kuester (2015)) but also considered in policy setting (e.g. Temporary Assistance for Needy Families (TANF) and Hiring Incentives to Restore Employment (HIRE) Act). Our framework allows us to evaluate the effects of such policies on employment and wage/earnings inequality and our results show that these are important.

In particular, we first find that increases in government spending to enhance unskilled productivity via OJL is beneficial to employed unskilled workers and decreases earnings inequality between employed skilled and unskilled labour. However, it increases unskilled unemployment and thus increases labour income inequality within the group of unskilled labour. This happens because following the rise in labour productivity, firms find it optimal to employ fewer, more productive workers, and make savings from posting vacancies. Hence, this policy has ambiguous effects on the welfare of unskilled population as a whole.

Second, we find that vacancy subsidies work to increase unskilled employment and returns to unskilled workers, providing a more comprehensive support mechanism to low-income, low-skill labour. However, they do lead to an increase in unemployment for skilled workers and to a short-run reduction in skilled wages and labour income. This happens because the displacement of capital and the increase in the productivity of skilled workers via OJL, following the rise in the unskilled labour input, lead to reductions in the marginal product of skilled labour, which puts downward pressure on the employment and returns to skilled labour. Hence this policy is not *Pareto* improving either.

Thus, while these policies have many benefits, and vacancy subsidies are preferable for the unskilled, they are not without their short-comings. An obvious suggestion is to complement these with further interventions. In particular, a natural combined policy would be to increase vacancy subsidies for both unskilled and skilled workers. Indeed, we find that it is possible to increase skilled vacancy subsidies to nullify the negative effects on skilled employment following an increase in unskilled vacancy subsidies, and these have to increase by 1% of the increase of unskilled vacancy subsidies. Hence, a small intervention in the skilled labour market can complement the intervention in the unskilled labour market to improve employment for all workers.

We finally evaluate the importance of OJL for the above results by considering a model without OJL and find that omitting the OJL channel from the analysis of vacancy subsides biases downwards the benefits to unskilled workers in terms of higher labour earnings and biases upwards benefits to skilled labour. Furthermore, omitting *positive* spill-over learning effects across workers in OJL leads to a failure to capture the *negative* effects of the unskilled vacancy subsidy on skilled labour. In both cases, subsidies to unskilled workers appear now as a *Pareto* improving intervention.

The rest of the paper is organised as follows. Section 2 summarises the stylised facts and Section 3 sets out the model structure. Section 4 presents the calibration and cyclical properties of the model. Section 5 analyses the results and Section 6 concludes.

2 Skilled and unskilled labour markets

The evolution of wage inequality has been well documented in the U.S. data. The literature on the skill premium has demonstrated that there are significant differences in the wages between skilled and unskilled labour, and that the skill premium has increased in recent decades to its highest levels in a century. For instance, Castro and Coen-Pirani (2008) and Heathcote *et al.* (2010) analyse more recent decades, while Goldin and Katz (2008) and Acemoglu and Autor (2011) also analyse longer time series and historical data. Castro and Coen-Pirani (2008) have also extensively analysed the cyclical behaviour of returns to labour for skilled and unskilled workers and they find that both have low (but positive) correlations with output. Moreover, the hourly wages have significant volatility, which is higher for skilled than for unskilled labour. Castro and Coen-Pirani (2008) also report that the skill premium is effectively uncorrelated with output and has lower volatility than output (see also Lindquist (2004) and Pourpourides (2011) for similar results).

The evidence in the data and existing literature also points to a higher unemployment rate for unskilled workers. For instance, Fallick and Fleischman (2004), Pilossoph (2012) and Hagedorn *et al.* (2016) document higher job separation rates and higher unemployment for unskilled versus skilled workers.[5] We also demonstrate differences between employment and unemploy-

[5] The literature has documented further differences between the skilled and unskilled labour markets. Cahuc *et al.* (2006) find that skilled workers have higher bargaining power, while Pissarides (1994), Acemoglu (2001) and Krause and Lubik (2006 and 2010)

ment for skilled and unskilled workers using data from the Current Population Survey, Table A-4. In particular, we examine data on the employment status of the civilian population 25 years and over by educational attainment (see www.bls.gov/webapps/legacy/cpsatab4) to measure the number of employed and unemployed workers in each skill category for the period 1992-2011. Workers are "skilled" if they have obtained at least a college degree. These data are monthly, and we average over the relevant quarters to obtain quarterly data following the methods in Shimer (2010) and Prescott *et al.* (2009). Employment and unemployment rates are then obtained by defining the relevant ratios of employed and unemployed workers over the sum of workers in each skill group.

In Figure 1, subplot (1,1), we plot the unemployment rates for skilled, s_t^s, and unskilled workers, s_t^u. Over the whole period, s_t^u is higher than s_t^s and the series follow fairly similar patterns, characterised by an overall decline starting in 1992, interrupted by increases associated with the recessions in 2001 and 2008.

[Figure 1 here]

Existing data and empirical work also provide empirical evidence on the differences between the labour markets for skilled and unskilled workers in business cycle frequencies. In Figure 1 we summarise differences between cyclical employment (skilled, $e_t^{s,c}$ and unskilled, $e_t^{u,c}$) and cyclical unemployment (skilled, $s_t^{s,c}$ and unskilled, $s_t^{u,c}$) over the period of 1992:1-2011:4. We also plot cyclical output, y_t^c, obtained using per capita quarterly output data from the U.S. NIPA accounts. The data reported in subplots (1,2) and (1,3) are first logged and then de-trended using the Hodrick-Prescott (HP) filter with a smoothing parameter of 1600.

Starting with unemployment, Figure 1 shows that the fluctuations of skilled unemployment are more pronounced than those of unskilled employment, and, as expected, both series tend to be counter-cyclical. These findings are consistent with those reported in Hagedorn *et al.* (2016), for an earlier time period (1979-2003). In contrast, the cyclical fluctuations in unskilled employment are more pronounced, at least after about 2001, relative to skilled employment. This finding is interesting since for the period 1984-2003, Castro and Coen-Pirani (2008) find that skilled employment is more volatile.[6] Subplot (1,3) suggests that developments after 2001 have lead to a reversal of this relationship compared to the period between the early 1980s and early 2000s. Moreover, subplot (1,3) suggests that these series are pro-

suggest that the flow cost of posting a vacancy is higher in 'good' jobs.

[6] Castro and Coen-Pirani (2008) also find that for 1979-1983 unskilled employment is more volatile than skilled employment.

cyclical.

Table 1 reports output correlations and standard deviations relative to output for the labour market statistics. As can be seen, employment (unemployment) are both strongly pro-cyclical (counter-cyclical) and the correlations are similar for skilled and unskilled labour. On the other hand, while employment is less volatile than output, unemployment is significantly more volatile than output. Moreover, while skilled unemployment is more volatile than unskilled unemployment, unskilled employment has a standard deviation that is twice as large as that of skilled employment.

Table 1: Data moments

corr.	value	rel. s.d.	value
$\rho(e^{s,c}, y^c)$	0.864	$\frac{\sigma(e^{s,c})}{\sigma(y^c)}$	0.386
$\rho(e^{u,c}, y^c)$	0.854	$\frac{\sigma(e^{u,c})}{\sigma(y^c)}$	0.703
$\rho(s^{s,c}, y^c)$	-0.872	$\frac{\sigma(s^{s,c})}{\sigma(y^c)}$	13.195
$\rho(s^{u,c}, y^c)$	-0.900	$\frac{\sigma(s^{u,c})}{\sigma(y^c)}$	10.524
$\rho(e^{c}, y^c)$	0.865	$\frac{\sigma(e^{c})}{\sigma(y^c)}$	0.553
$\rho(s^{c}, y^c)$	-0.901	$\frac{\sigma(s^{c})}{\sigma(y^c)}$	11.161

3 The model

The parts of the model relating to capital-skill complementarity in production and to search and matching frictions and wage bargaining in the labour market are standard in macroeconomic research employing models with these features. We nevertheless summarise in this section the relevant key modeling equations and briefly discuss the model solution and the implied conditions that determine the equilibrium outcomes. This helps to facilitate a coherent exposition of the framework that brings these features together with endogenous labour productivity in the form of OJL in the presence of different labour markets for skilled and unskilled workers. Moreover, when analysing the equilibrium conditions and key relationships in the model, we also discuss properties of these that further the understanding of the effects of policy interventions in following sections.

3.1 Capital-skill complementarity

There are N firms which operate in competitive product markets. To produce a single output, firms use capital, which they lease from the household, and skilled and unskilled workers. The production technology is characterised by capital-skill complementarity (see e.g. Goldin and Katz (2008) for historical evidence on the empirical relevance of this technology in the 20th century). In particular, a representative firm produces output, y_t^f, using a constant elasticity of substitution (CES) specification following e.g. Krusell *et al.* (2000):

$$y_t^f = A_t \left\{ \theta \left(l_t^{f,u} \right)^{\alpha} + (1-\theta) \left[\rho \left(k_t^f \right)^{\nu} + (1-\rho) \left(l_t^{f,s} \right)^{\nu} \right]^{\frac{\alpha}{\nu}} \right\}^{\frac{1}{\alpha}} \tag{1}$$

where $A_t > 0$ is the level of total factor productivity (TFP); $\alpha, \nu < 1$ are the parameters determining the factor elasticities, i.e. $1/(1-\alpha)$ is the elasticity of substitution between capital and unskilled labour and between skilled and unskilled labour, whereas $1/(1-\nu)$ is the elasticity of substitution between capital and skilled labour; $0 < \theta, \rho < 1$ are the factor share parameters; k_t^f is the quantity of capital used by the firm; and $l_t^{f,s}$ and $l_t^{f,u}$ denote the quantities of skilled and unskilled labour respectively.

3.2 Skilled and unskilled workers

There is a representative household whose members include skilled and unskilled workers who offer distinct services in the respective labour markets. They can find a job within the skill sector in which they belong or remain unemployed. In the latter case, they search for a job for the next time period within their skill sector. The inability to change skill status is motivated by empirical evidence suggesting that over the business cycle, the share of college educated or skilled population has low volatility and is effectively uncorrelated with output. In particular, using the data in Acemoglu and Autor (2011), we find that the standard deviation of the cyclical component of the skilled population share, relative to that of output, is 0.29, while its correlation with output is -0.23.[7] Following the literature on search frictions and unemployment in macroeconomic DGE models since Mertz (1995) (see e.g. Rogerson and Shimer (2011) for a review of macroeconomic models with unemployment), we also assume that the head of the household makes all decisions on behalf of its members and provides complete consumption insurance.

The numbers of skilled and unskilled members for the representative household are denoted as N^s and N^u, respectively. The total size of the

[7] This is obtained using annual data for the share of college educated population measured in efficiency units, 1963-2008, from Acemoglu and Autor (2011) and GDP per capita data from the U.S. National Income and Product Accounts (NIPA). The cyclical component of the series is obtained using the HP-filter with a smoothing parameter of 100.

household is normalised to be N and is thus given as: $N = N^s + N^u$. The respective population shares of skilled and unskilled members within a household are defined as: $n^s = N^s/N$ and $n^u = N^u/N$. We assume that population and its composition remain constant.

For each skill type of household members, $i = s, u$, the number of members/workers can be further decomposed into employed and unemployed members, such that:

$$N^i = N_t^{i,e} + N_t^{i,s} \tag{2}$$

where $i = s, u$ for skilled and unskilled labour; and $N_t^{i,e}$ is the number of employed members and $N_t^{i,s}$ is the number of unemployed members, who are searching for a job. By normalising by N^i, we have:

$$1 = e_t^i + s_t^i \tag{3}$$

where $e_t^i \equiv \frac{N_t^{i,e}}{N^i}$ is the employment rate and $s_t^i \equiv \frac{N_t^{i,s}}{N^i}$ is the unemployment rate or the share of workers searching for a job.

3.3 Search and matching

Each unemployed worker needs to search for a job in the skilled or unskilled sector, given her skill level, and can be matched with a firm that posts vacancies in that sector. As in the standard search-and-matching literature (see e.g. Pissarides (1986) and Blanchard and Diamond (1989)), the matching technology is represented by a Cobb-Douglas (CD) function for both skilled and unskilled labour:

$$M_t^i = \chi^i \left(S_t^i\right)^{\eta^i} \left(V_t^i\right)^{1-\eta^i} \tag{4}$$

where, M_t^i is the aggregate new matches at t; $S_t^i = N^i s_t^i$ denotes the aggregate number of unemployed searching in labour market i; $V_t^i = N v_t^i$ denotes the aggregate number of job vacancies created by firms in labour market i; $\chi^i > 0$ represents the constant efficiency of matching for labour type i; $0 < \eta^i < 1$ denotes the elasticity of searches for labour type i. In addition, we define the vacancy-to-unemployed ratio, $z_t^i = V_t^i/S_t^i = v_t^i/\left(n^i s_t^i\right)$, as the tightness of type i labour market. The smaller the ratio of z_t^i, the tighter the labour market and therefore the harder for unemployed workers to match with job vacancies.

The probability at which aggregate job searches lead to a new job match in type i labour market is given by:

$$p_t^i = \frac{M_t^i}{S_t^i} = \chi^i \left(S_t^i\right)^{\eta^i - 1} \left(V_t^i\right)^{1-\eta^i} = \chi^i \left(z_t^i\right)^{1-\eta^i} \tag{5}$$

and its inverse, $1/p_t^i$, measures the duration of type i search.

The probability at which a job vacancy can be matched with an unemployed household member is calculated by:

$$q_t^i = \frac{M_t^i}{V_t^i} = \chi^i \left(S_t^i\right)^{\eta^i} \left(V_t^i\right)^{-\eta^i} = \chi^i \left(z_t^i\right)^{-\eta^i} \tag{6}$$

and its inverse, $1/q_t^i$, measures the duration of type i job vacancy.

3.4 Household

The head of the representative household makes all decisions on behalf of its members by guaranteeing equal consumption to each of them, with the objective of maximising household welfare.

3.4.1 Problem

The household maximises discounted lifetime utility, U:

$$U = E \sum_{t=0}^{\infty} \beta^t u_t \tag{7}$$

where E denotes expectations; and $0 < \beta < 1$ denotes the constant rate of time preference. The instantaneous utility function of the household (see e.g. Shimer (2010)) is given by:

$$u_t = \ln(c_t) - n^s \xi e_t^s - n^u \xi e_t^u \tag{8}$$

where $\xi > 0$ is the preference parameter that measures the disutility cost of employment and c_t is the household's average (or per capita) private consumption. As is common in the literature, the disutility cost captures the reduction in the time available for home production when a member finds employment. Hence, the specification in equation (8) assumes that all members consume c_t and that if a member is unemployed, her utility is given by $\ln(c_t)$, whereas if a member is employed, her utility is given by $\ln(c_t) - \xi^i$, so that u_t measures average utility for the household.

The budget constraint of the household is:

$$\begin{aligned} c_t + i_t + b_{t+1}^h = \left[r_t - \tau^k \left(r_t - \delta\right)\right] k_t + \pi_t + T_t + \\ + R_{t-1}^b b_t^h + \left(1 - \tau^s\right) n^s w_t^s e_t^s Z_t^s + \left(1 - \tau^u\right) n^u w_t^u e_t^u Z_t^u \end{aligned} \tag{9}$$

where i_t is household's average private investment; b_{t+1}^h is the value of government bonds bought at period t; r_t is the gross return to physical capital;

τ^k is the tax rate on capital income; $0 < \delta < 1$ is the constant depreciation rate of physical capital; k_t is the average physical capital held by the household at the beginning of t; π_t is average dividends received from the firms; T_t is lump-sum transfers paid to the household; $R^b_{t-1} = \left(1 + r^b_{t-1}\right)$ is the gross return to bonds; τ^i is the labour income tax; w^i_t is the bargained wage; and Z^i_t represents labour productivity associated with OJL. Hence, Z^i_t is the effective labour supply per worker, so that $n^s e^s_t Z^s_t$ $(n^u e^u_t Z^u_t)$ is effective skilled (unskilled) labour supply and, since an employed worker works all of her time, $w^s_t Z^s_t$ $(w^u_t Z^u_t)$ is the effective skilled (unskilled) wage and also earnings for employed skilled (unskilled) labour.

We assume that Z^i_t increases with the level of employment or alternatively, as a decreasing function of unemployment, Z^i_t can be related to skill erosion due to not working.[8] We allow for both skill types to learn on-the-job from their own and the other skill type. We let the workers of each type internalise the effect of their own employment on their labour productivity but maintain the assumption that the employment of the other worker type is taken as given (denoted with a bar over the variable):[9]

$$Z^s_t = \Omega^s(1 + g^{z,s})\left(e^s_t\right)^{\zeta^s}\left(\overline{e}^u_t\right)^{1-\zeta^s} \tag{10}$$

$$Z^u_t = \Omega^u(1 + g^{z,u})\left(\overline{e}^s_t\right)^{\zeta^u}\left(e^u_t\right)^{1-\zeta^u} \tag{11}$$

where $g^{z,i}$ is public expenditure to support OJL.[10]

The capital stock evolves according to:

$$k_{t+1} = (1 - \delta)\,k_t + \tilde{A}^k_t i_t \tag{12}$$

The capital evolution equation allows for an exogenous process, $\tilde{A}^k_t$, capturing an investment-specific technological (IT) change, which has been shown to contribute to output fluctuations (see e.g. Greenwood *et al.* (2000), as well as the changes in the skill premium (see e.g. Krusell *et al.* (2000), Lindquist (2004), and Pourpourides (2011)). The stochastic process for investment-specific technology, $\tilde{A}^k_t$ is:

$$\widetilde{A}^k_{t+1} = \left(\widetilde{A}^k\right)^{1-\rho_{A^k}}\left(\widetilde{A}^k_t\right)^{\rho_{A^k}} e^{\varepsilon^{A^k}_{t+1}} \tag{13}$$

[8] See, for example, Davis and von Wachter (2011) and Pollack (2013) for the effects of unemployment on labour productivity and Laureys (2014) and Doppelt (2016) for search and matching models with skill depreciation due to unemployment.

[9] Note that in this formulation, labour productivity is increasing and concave with respect to employment and bounded between zero and $\Omega^i(1 + g^{z,i})$, where $i = s, u$.

[10] We will later allow for $g^{z,u}$ to vary exogenously over time, following a deterministic AR(1) process, to capture the effects of policies that have persistence, as opposed to being one-off. Since the processes for $g^{z,s}$ and $g^{z,u}$ are not used when calibrating the model and evaluating its predictions relative to the data, we drop time subscripts here to simplify the presentation.

where $\widetilde{A}^k > 0$; $0 < \rho_{A^k} < 1$; and $\varepsilon_{t+1}^{A^k} \sim iidN\left[0, \left(\sigma_{A^k}\right)^2\right]$.

By using equation (12) and defining as $A_t^k \equiv \frac{1}{\widetilde{A}_t^k}$, we can rewrite the budget constraint of household:

$$\begin{array}{l} c_t + A_t^k k_{t+1} + b_{t+1}^h = \widetilde{r}_t k_t + \pi_t + T_t + \\ + R_{t-1}^b b_t^h + (1-\tau^s)\, n^s w_t^s e_t^s Z_t^s + (1-\tau^u)\, n^u w_t^u e_t^u Z_t^u \end{array} \tag{14}$$

where $\widetilde{r}_t = r_t - \tau^k (r_t - \delta) + A_t^k (1-\delta)$, is the net return to physical capital after depreciation and tax. Note that A_t^k measures the effective price of investment, since A_t^k units of investment are needed to create one unit of capital in the next period.

Employment for type $i = s, u$ worker evolves according to:

$$e_{t+1}^i = p_t^i s_t^i + \left(1 - \gamma_t^i\right) e_t^i \tag{15}$$

where $0 < \gamma_t^i < 1$ is the rate of job separation for type i labour. The stochastic process for the job separation rate, γ_t^i, is:

$$\gamma_{t+1}^i = \left(\gamma^i\right)^{1-\rho_{\gamma^i}} \left(\gamma_t^i\right)^{\rho_{\gamma^i}} e^{\varepsilon_{t+1}^{\gamma^i}} \tag{16}$$

where $\gamma^i > 0$; $0 < \rho_{\gamma^i} < 1$; and $\varepsilon_{t+1}^{\gamma^i} \sim iidN\left[0, \left(\sigma_\gamma^i\right)^2\right]$.

The household takes as given initial conditions for k_0 and b_0^h, observes the realisation of exogenous shocks at period 0 (i.e. it observes the values of $A_0^k, \gamma_0^s, \gamma_0^u$) and chooses decision rules $\left\{c_t, k_{t+1}, b_{t+1}^h\right\}_{t=0}^\infty$ to maximise (7) subject to the constraints (3) and (14), by taking as given the processes for exogenous variables $\left\{A_t^k, \gamma_t^s, \gamma_t^u\right\}_{t=1}^\infty$; labour productivity $\{Z_t^s, Z_t^u\}_{t=0}^\infty$; policy variables $\{T_t\}_{t=1}^\infty$; factor prices $\{w_t^s, w_t^u, r_t, r_{t-1}^b\}_{t=0}^\infty$; profits $\{\pi_t\}_{t=0}^\infty$; and employment $\{e_t^s, e_t^u\}_{t=0}^\infty$.

3.4.2 First-order conditions (FOCs)

The recursive form of the household's problem is:

$$\begin{array}{l} V\left(k_t, b_t^h, e_t^s, e_t^u\right) = \max\limits_{c_t, k_{t+1}, b_{t+1}^h} \{(\ln c_t - n^s \xi e_t^s - n^u \xi e_t^u) + \\ + \beta E_t V\left(k_{t+1}, b_{t+1}^h, e_{t+1}^s, e_{t+1}^u\right)\} \end{array} \tag{17}$$

where $V(.)$ is the value function.[11] Replacing c_t making use of the budget constraint (14) gives:

$$\begin{array}{l} V\left(k_t, b_t^h, e_t^s, e_t^u\right) = \max\limits_{k_{t+1}, b_{t+1}^h} [\ln[\widetilde{r}_t k_t - A_t^k k_{t+1} - b_{t+1}^h + \pi_t + R_{t-1}^b b_t^h + \\ + T_t + (1-\tau^s)\, n^s w_t^s e_t^s Z_t^s + (1-\tau^u)\, n^u w_t^u e_t^u Z_t^u] - n^s \xi e_t^s - n^u \xi e_t^u + \\ + \beta E_t V(k_{t+1}, b_{t+1}^h, e_{t+1}^s, e_{t+1}^u)]. \end{array} \tag{18}$$

[11] To simplify notation we suppress the state variables associated with aggregate and stochastic processes in the value function throughout the paper.

The envelope condition for capital stock, k_t is:

$$V_k\left(k_t, b_t^h, e_t^s, e_t^u\right) = \frac{\widetilde{r}_t}{c_t} \tag{19}$$

and the first order condition for k_{t+1} is:

$$\beta E_t V_k\left(k_{t+1}, b_{t+1}^h, e_{t+1}^s, e_{t+1}^u\right) = \frac{A_t^k}{c_t} \tag{20}$$

which equates the discounted expected marginal benefit to the marginal cost of investment.

Finally, substituting the one-period lead of the envelope condition (19) into the first-order condition for capital (20) gives the consumption Euler:

$$E_t\left(\beta \frac{c_t \widetilde{r}_{t+1}}{c_{t+1}}\right) = A_t^k \tag{21}$$

which shows that the expected discounted return on investing in capital must equal its price. Note that the return is discounted using the stochastic discount factor $\beta \frac{c_t}{c_{t+1}}$. The envelope condition for government bonds, b_t is:

$$V_b\left(k_t, b_t^h, e_t^s, e_t^u\right) = \frac{R_{t-1}^b}{c_t} \tag{22}$$

and the first order condition for b_{t+1} is:

$$\beta E_t V_b\left(k_{t+1}, b_{t+1}^h, e_{t+1}^s, e_{t+1}^u\right) = \frac{1}{c_t}. \tag{23}$$

Substituting the one-period lead of the envelope condition (22) into the first-order condition for government bonds (23) gives the bonds Euler, which has a similar interpretation as the Euler for capital:

$$E_t\left(\beta \frac{c_t\left(1 + r_t^b\right)}{c_{t+1}}\right) = 1 \tag{24}$$

The FOCs for the household's problem are given by equations (14), (21) and (24). These determine $\left\{c_t, k_{t+1}, b_{t+1}^h\right\}_{t=0}^{\infty}$ given initial conditions, processes for exogenous variables quantities that are determined at the aggregate level and by wage bargaining.

3.5 Firms

There is a representative firm which leases capital from the household and employs skilled and unskilled workers to produce a single good, with the objective of maximising profits.

3.5.1 Problem

To hire workers, the firm needs to post vacancies one period before the jobs are required. In particular, the evolution of the number of workers per skilled type employed by the firm is given by the job transition function which links the future number of filled jobs, $l_{t+1}^{f,i}$, to the net hiring, $q_t^i v_t^i$, plus the current stock of filled jobs, $(1-\gamma_t^i)\, l_t^{f,i}$:

$$l_{t+1}^{f,i} = q_t^i v_t^i + \left(1-\gamma_t^i\right) l_t^{f,i}. \tag{25}$$

Given that posting vacancies is costly, the profit function of the firm is:

$$\pi_t^f = y_t^f - r_t k_t^f - w_t^s l_t^{f,s} - \left(1-\tau^{v,s}\right) \varphi^s v_t^s - w_t^u l_t^{f,u} - \left(1-\tau^{v,u}\right) \varphi^u v_t^u \tag{26}$$

where $\varphi^s, \varphi^u > 0$ stand for the constant resource costs of opening a new skilled and unskilled vacancy respectively; and $\tau^{v,i}$, $i = s, u$ refer to the vacancy subsidies.[12]

The employment evolution equations in (25) imply that profit maximisation is intertemporal, since expenditure on posting vacancies today will increase profits tomorrow. Therefore, the objective of the firm at time period $t = 0$ is to maximise the present value of its lifetime profits, which is given by:

$$\begin{aligned} & y_0^f - r_0 k_0^f - w_0^s l_0^{f,s} - \left(1-\tau^{v,s}\right) \varphi^s v_0^s - w_0^u l_0^{f,u} - \left(1-\tau^{v,u}\right) \varphi^u v_0^u + \\ & + E \sum_{t=1}^{\infty} \prod_{i=1}^{t} \widetilde{r}_i^{-1} \{ y_t^f - r_t k_t^f - w_t^s l_t^{f,s} - \left(1-\tau^{v,s}\right) \varphi^s v_t^s - w_t^u l_t^{f,u} - \\ & - \left(1-\tau^{v,u}\right) \varphi^u v_t^u \} \end{aligned} \tag{27}$$

where y_0^f and y_t^f are given by the CES production function in (1) at time 0 and t respectively.

Since profits are returned to the household, $t+1$ returns are converted to present value terms by the stochastic discount factor from the household's optimisation problem, (21). For $i = s, u$, the firm chooses $\left\{ k_t^f, v_t^i, l_{t+1}^{f,i} \right\}_{t=0}^{\infty}$ to maximise (27) subject to (25) , taking factor prices $\{w_t^i, r_t\}_{t=0}^{\infty}$; matching probabilities $\{q_t^i\}_{t=0}^{\infty}$; exogenous job separation rates $\{\gamma_t^i\}_{t=0}^{\infty}$; and initial conditions for $\{l_0^{f,i}\}$ as given. The variable, A_t is determined by the following stochastic process:

$$A_{t+1} = (A)^{1-\rho_A} \left(A_t\right)^{\rho_A} e^{\varepsilon_{t+1}^A} \tag{28}$$

where $A > 0$; $0 < \rho_A < 1$; and $\varepsilon_{t+1}^A \sim iidN\left[0, (\sigma_A)^2\right]$.

[12] As with $g^{z,u}$, we will later allow for $\tau^{v,s}$ and $\tau^{v,u}$ to vary exogenously over time, following a deterministic AR(1) process, to capture the effects of policies that have persistence as opposed to being one-off. As above, since these processes are not used when calibrating the model and evaluating its predictions relative to the data, we again drop time subscripts.

3.5.2 First-order conditions

The firm's problem is written in recursive form as:

$$\begin{aligned} J\left(l_t^{f,s}, l_t^{f,u}\right) &= \max_{k_t^f, v_t^s, v_t^u} [y_t^f - r_t k_t^f - w_t^s l_t^{f,s} - (1-\tau^{v,s})\,\varphi^s v_t^s - \\ & -w_t^u l_t^{f,u} - (1-\tau^{v,u})\,\varphi^u v_t^u] + E_t \widetilde{r}_{t+1}^{-1} J(q_t^s v_t^s + \\ & + (1-\gamma_t^s)\, l_t^{f,s}, q_t^u v_t^u + (1-\gamma_t^u)\, l_t^{f,u}) \end{aligned} \tag{29}$$

where $J(.)$ is the value function. The FOCs for k_t^f, v_t^s and v_t^u are:

$$\begin{aligned} r_t = \frac{1}{\alpha} A_t \left\{ \theta \left(l_t^{f,u}\right)^\alpha + (1-\theta) \left[\rho \left(k_t^f\right)^\nu + (1-\rho) \left(l_t^{f,s}\right)^\nu\right]^{\frac{\alpha}{\nu}} \right\}^{\frac{1}{\alpha}-1} \times \\ \times (1-\theta) \frac{\alpha}{\nu} \left[\rho \left(k_t^f\right)^\nu + (1-\rho) \left(l_t^{f,s}\right)^\nu\right]^{\frac{\alpha}{\nu}-1} \rho\nu \left(k_t^f\right)^{\nu-1} \equiv mpk_t \end{aligned} \tag{30}$$

$$(1-\tau^{v,s})\,\varphi^s = E_t \widetilde{r}_{t+1}^{-1} q_t^s J_{l^{f,s}} \left(l_{t+1}^{f,s}, l_{t+1}^{f,u}\right) \tag{31}$$

$$(1-\tau^{v,u})\,\varphi^u = E_t \widetilde{r}_{t+1}^{-1} q_t^u J_{l^{f,u}} \left(l_{t+1}^{f,s}, l_{t+1}^{f,u}\right) \tag{32}$$

stating respectively that the marginal cost of capital is equal to its marginal benefit and that the marginal costs of creating skilled and unskilled vacancies are equal to the expected return of hiring one additional skilled and unskilled worker next period.[13]

The envelope condition for skilled employment, $l_t^{f,s}$ is:

$$J_{l^{f,s}} \left(l_t^{f,s}, l_t^{f,u}\right) = mpl_t^s - w_t^s + (1-\gamma_t^s)\, E_t \widetilde{r}_{t+1}^{-1} J_{l^{f,s}} \left(l_{t+1}^{f,s}, l_{t+1}^{f,u}\right) \tag{33}$$

where $mpl_t^s = A_t \left\{ \theta \left(l_t^{f,u}\right)^\alpha + (1-\theta) \left[\rho \left(k_t^f\right)^\nu + (1-\rho) \left(l_t^{f,s}\right)^\nu\right]^{\frac{\alpha}{\nu}} \right\}^{\frac{1}{\alpha}-1} \times$ $\times (1-\theta) \left[\rho \left(k_t^f\right)^\nu + (1-\rho) \left(l_t^{f,s}\right)^\nu\right]^{\frac{\alpha}{\nu}-1} (1-\rho) \left(l_t^{f,s}\right)^{\nu-1}$. After substituting for the continuation value, $\widetilde{r}_{t+1}^{-1} J_{l^{f,s}} \left(l_{t+1}^{f,s}, l_{t+1}^{f,u}\right)$, using the first-order condition for v_t^s in (31) this condition becomes:

$$J_{l^{f,s}} \left(l_t^{f,s}, l_t^{f,u}\right) = mpl_t^s - w_t^s + (1-\gamma_t^s) \frac{\varphi^s}{q_t^s} (1-\tau^{v,s}). \tag{34}$$

[13] Note that the latter with be explained below in more detail.

Finally, to obtain the FOC for the firm, we first lead equation (34) by one period and substitute it into equation (31) to obtain:

$$\left(1-\tau^{v,s}\right)\varphi^{s}=E_{t}\widetilde{r}_{t+1}^{-1}q_{t}^{s}\left[mpl_{t+1}^{s}-w_{t+1}^{s}+\left(1-\gamma_{t+1}^{s}\right)\frac{\varphi^{s}}{q_{t+1}^{s}}\left(1-\tau^{v,s}\right)\right]. \quad (35)$$

Working, similarly for unskilled employment, we have:

$$\left(1-\tau^{v,u}\right)\varphi^{u}=E_{t}\widetilde{r}_{t+1}^{-1}q_{t}^{u}\left[mpl_{t+1}^{u}-w_{t+1}^{u}+\left(1-\gamma_{t+1}^{u}\right)\frac{\varphi^{u}}{q_{t+1}^{u}}\left(1-\tau^{v,u}\right)\right] \quad (36)$$

where $mpl_{t}^{u}=A_{t}\left\{\theta\left(l_{t}^{f,u}\right)^{\alpha}+(1-\theta)\left[\rho\left(k_{t}^{f}\right)^{\nu}+(1-\rho)\left(l_{t}^{f,s}\right)^{\nu}\right]^{\frac{\alpha}{\nu}}\right\}^{\frac{1}{\alpha}-1}\times$ $\times\theta\left(l_{t}^{f,u}\right)^{\alpha-1}$. Conditions (35) and (36) equate the marginal cost of posting a job vacancy to the expected discounted marginal benefit for skilled and unskilled jobs respectively. The benefit is comprised of two elements. First, the increase in profits associated with hiring an extra worker, $mpl_{t+1}^{i}-w_{t+1}^{i}$, $i=s,u$, and second the saving associated with not having to post a job vacancy in the next period, $\left(1-\gamma_{t+1}^{i}\right)\frac{\varphi^{i}}{q_{t+1}^{i}}$. Note also that the expected benefit of posting a job vacancy increases with the probability at which the job vacancy can be matched with an unemployed household member, q_{t}^{i}.

For $i=s,u$, the FOCs for the firm's problem are given by equations (25), (26), (30), (35) and (36), which determine the paths for $\left\{l_{t+1}^{f,i},\pi_{t}^{f},k_{t}^{f},v_{t}^{s},v_{t}^{u}\right\}_{t=0}^{\infty}$, given exogenous processes, $\{A_{t},\gamma_{t}^{i}\}_{t=0}^{\infty}$; variables that are determined at the aggregate level, $\{r_{t},q_{t}^{i}\}_{t=0}^{\infty}$, wage bargaining variables $\{w_{t}^{i}\}_{t=0}^{\infty}$; and initial conditions for $\{l_{0}^{f,i}\}$.

3.6 Wage Bargaining

We assume that once a worker/household member is matched with a firm, the household and the firm bargain over the wage rate. The equilibrium wage is determined by a Nash bargain. In particular, the equilibrium wage rate maximises the Nash product:

$$\left[\widetilde{V}_{e^{i}}\left(\widetilde{w}_{t}^{i}\right)\right]^{\phi^{i}}\left[\widetilde{J}_{l^{f,i}}\left(\widetilde{w}_{t}^{i}\right)\right]^{1-\phi^{i}} \quad (37)$$

where ϕ^{i} measures the power of the household/worker relative to the firm in the Nash bargain; $\widetilde{V}_{e^{i}}\left(\widetilde{w}_{t}^{i}\right)$ is the value of a successful bargain at wage $\widetilde{w}_{t}^{i}$ for the household and $\widetilde{J}_{l^{f,i}}\left(\widetilde{w}_{t}^{i}\right)$ is the value of a successful bargain at wage $\widetilde{w}_{t}^{i}$ for the firm.

3.6.1 Household's valuation of employment

The valuation of the household for an additional member being employed at wage w_t^i is given by the envelope conditions of (18) for e_t^s and e_t^u respectively:

$$\begin{aligned} V_{e^s}\left(k_t, b_t^h, e_t^s, e_t^u\right) &= \tfrac{(1-\tau^s)n^s w_t^s Z_t^s}{c_t} + \frac{(1-\tau^s)n^s w_t^s e_t^s \frac{\partial Z_t^s}{\partial e_t^s}}{c_t} - \\ -n^s\xi + (1-\gamma_t^s - p_t^s)\,\beta E_t V_{e^s}\left(k_{t+1}, b_{t+1}^h, e_{t+1}^s, e_{t+1}^u\right) \end{aligned} \tag{38}$$

$$\begin{aligned} V_{e^u}\left(k_t, b_t^h, e_t^s, e_t^u\right) &= \tfrac{(1-\tau^u)n^u w_t^u Z_t^u}{c_t} + \frac{(1-\tau^u)n^u w_t^u e_t^u \frac{\partial Z_t^u}{\partial e_t^u}}{c_t} - \\ -n^u\xi + (1-\gamma_t^u - p_t^u)\,\beta E_t V_{e^u}\left(k_{t+1}, b_{t+1}^h, e_{t+1}^s, e_{t+1}^u\right). \end{aligned} \tag{39}$$

We next consider the marginal value to a household of allowing a small number of its members, $\epsilon_t^s > 0$, to be paid an arbitrary wage, $\widetilde{w}_t^s$, in period t, assuming that the wage reverted to the equilibrium wage w_{t+1}^s from next period. In these circumstances the value function of household in equation (18) becomes:

$$\begin{aligned} \widehat{V}\left(\widetilde{w}_t^s, \epsilon_t^s\right) = \max_{k_{t+1}, b_{t+1}^h} \{ &\ln \widetilde{r}_t k_t - A_t^k k_{t+1} - b_{t+1}^h + \pi_t + R_{t-1}^b b_t^h + T_t + \\ &+ (1-\tau^s)\, n^s w_t^s e_t^s Z_t^s + (1-\tau^s)\, n^s \widetilde{w}_t^s \epsilon_t^s Z_t^s + (1-\tau^u)\, n^u w_t^u e_t^u Z_t^u) - \\ &- n^s\xi\left(e_t^s + \epsilon_t^s\right) - n^u \xi e_t^u\} + \beta E_t V\{k_{t+1}, b_{t+1}^h, [p_t^s\left(1 - e_t^s - \epsilon_t^s\right) + \\ &+ (1-\gamma_t^s)\left(e_t^s + \epsilon_t^s\right)], [p_t^u\left(1 - e_t^u\right) + \left(1-\gamma_t^u\right) e_t^u]\}. \end{aligned} \tag{40}$$

Differentiating $\widehat{V}\left(\widetilde{w}_t^s, \epsilon_t^s\right)$ with respect to ϵ_t^s and evaluating the derivative at $\epsilon_t^s = 0$ to derive the marginal value of a skilled worker employed at an arbitrary wage, $\widetilde{w}_t^s$ gives:

$$\begin{aligned} \widehat{V}_{\epsilon^s}\left(\widetilde{w}_t^s, 0\right) = \tfrac{(1-\tau^s)n^s \widetilde{w}_t^s Z_t^s}{c_t} + \frac{(1-\tau^s)n^s w_t^s e_t^s \frac{\partial Z_t^s}{\partial \epsilon_t^s}}{c_t} - n^s\xi + (1-\gamma_t^s - p_t^s) \times \\ \times \beta E_t V_{e^s}\left(k_{t+1}, b_{t+1}, e_{t+1}^s, e_{t+1}^u\right), \end{aligned} \tag{41}$$

where note that the second term appears because an increase in employment by ϵ_t^s also increases productivity for existing workers via OJL. If we combine the expression for $\widetilde{V}_{e^s}\left(\widetilde{w}_t^s\right) \equiv \widehat{V}_{\epsilon^s}\left(\widetilde{w}_t^s, 0\right)$ with the envelope condition for e_t^s in equation (38) we obtain:

$$\widetilde{V}_{e^s}\left(\widetilde{w}_t^s\right) = \frac{(1-\tau^s)\, n^s}{c_t}\left(\widetilde{w}_t^s - w_t^s\right) Z_t^s + V_{e^s}\left(k_t, b_t^h, e_t^s, e_t^u\right). \tag{42}$$

Equivalently, we can derive the marginal value of an unskilled worker employed at an arbitrary wage, $\widetilde{w}_t^u$:

$$\widetilde{V}_{e^u}\left(\widetilde{w}_t^u\right) = \frac{(1-\tau^u)\, n^u}{c_t}\left(\widetilde{w}_t^u - w_t^u\right) Z_t^u + V_{e^u}\left(k_t, b_t^h, e_t^s, e_t^u\right). \tag{43}$$

3.6.2 Firm's valuation of employment

We work similarly to obtain the firm's valuation of agreeing to employment at a wage $\widetilde{w}_t^i$. In particular, assume that the firm pays a small fraction, $\psi_t^s > 0$, of employed workers an arbitrary wage $\widetilde{w}_t^s$ at time period t, and that the wage rate will return to the equilibrium wage w_{t+1}^s from the next period. The value function of firm given by equation (29) can thus be modified to:

$$\begin{aligned} \widehat{J}\left(\widetilde{w}_t^s, \psi_t^s\right) &= \max_{v_t^s, v_t^u}\{y_t^f - r_t k_t^f - \left(w_t^s l_t^{f,s} + \widetilde{w}_t^s \psi_t^s\right) - \left(1-\tau^{v,s}\right) \times \\ &\times \varphi^s v_t^s - w_t^u l_t^{f,u} - \left(1-\tau^{v,u}\right)\varphi^u v_t^u + E_t \widetilde{r}_{t+1}^{-1} J([q_t^s v_t^s + (1-\gamma_t^s) \times \\ &\times (l_t^{s,f} + \psi_t^s)], [q_t^u v_t^u + \left(1-\gamma_t^u\right) l_t^{f,u}])\}. \end{aligned} \tag{44}$$

We next differentiate $\widehat{J}\left(\widetilde{w}_t^s, \psi_t^s\right)$ with respect to ψ_t^s and evaluate it at $\psi_t^s = 0$ to obtain the marginal profit of employing a skilled worker at $\widetilde{w}_t^s$:

$$\begin{aligned} \widehat{J}_{\psi^s}\left(\widetilde{w}_t^s, 0\right) &= A_t\{\theta\left(l_t^{f,u}\right)^\alpha + (1-\theta)\left[\rho\left(k_t^f\right)^\nu + (1-\rho)\times\right. \\ &\times \left(l_t^{f,s}\right)^\nu]^{\frac{\alpha}{\nu}}\}^{\frac{1}{\alpha}-1}(1-\theta)\left[\rho\left(k_t^f\right)^\nu + (1-\rho)\left(l_t^{f,s}\right)^\nu\right]^{\frac{\alpha}{\nu}-1} \times \\ &\times (1-\rho)\left(l_t^{f,s}\right)^{\nu-1} - \widetilde{w}_t^s + \left(1-\gamma_t^s\right) E_t \widetilde{r}_{t+1}^{-1} J_{l^{f,s}}\left(l_{t+1}^{f,s}, l_{t+1}^{f,u}\right). \end{aligned} \tag{45}$$

We then combine this with the envelope condition for $l_t^{f,s}$ in equation (34) to get the marginal profit of employing a skilled worker at an arbitrary wage, $\widetilde{w}_t^s$, at time t, and the equilibrium wage thereafter:

$$\widetilde{J}_{l^{f,s}}\left(\widetilde{w}_t^s\right) = w_t^s - \widetilde{w}_t^s + J_{l^{f,s}}\left(l_t^{f,s}, l_t^{f,u}\right) \tag{46}$$

where $\widetilde{J}_{l^{f,s}}\left(\widetilde{w}_t^s\right) \equiv \widehat{J}_{\psi^s}\left(\widetilde{w}_t^s, 0\right)$.

Similarly, we can derive the respective condition for unskilled workers:

$$\widetilde{J}_{l^{f,u}}\left(\widetilde{w}_t^u\right) = w_t^u - \widetilde{w}_t^u + J_{l^{f,u}}\left(l_t^{f,s}, l_t^{f,u}\right). \tag{47}$$

3.6.3 (Nash) equilibrium wage

The first-order condition of the Nash bargain (37) with respect to $\widetilde{w}_t^s$ is:

$$\begin{aligned} 0 &= \phi^s\left[\widetilde{V}_{e^s}\left(\widetilde{w}_t^s\right)\right]^{\phi^s-1}\left[\widetilde{J}_{l^{f,s}}\left(\widetilde{w}_t^s\right)\right]^{1-\phi^s}\frac{\partial \widetilde{V}_{e^s}(\widetilde{w}_t^s)}{\partial \widetilde{w}_t^s} + \\ &+ \left(1-\phi^s\right)\left[\widetilde{V}_{e^s}\left(\widetilde{w}_t^s\right)\right]^{\phi^s}\left[\widetilde{J}_{l_t^{f,s}}\left(\widetilde{w}_t^s\right)\right]^{-\phi^s}\frac{\partial \widetilde{J}_{l_t^{f,s}}(\widetilde{w}_t^s)}{\partial \widetilde{w}_t^s}. \end{aligned} \tag{48}$$

Substituting the derivatives of equations (42) and (46) with respect to $\widetilde{w}_t^s$ as well as the expressions for $\widetilde{V}_{e^s}(\widetilde{w}_t^s)$ and $\widetilde{J}_{l^{f,s}}(\widetilde{w}_t^s)$ from (42) and (46) respectively into (48) and evaluating at $w_t^s = \widetilde{w}_t^s$ gives:

$$\phi^s \frac{(1-\tau^s)\, n^s}{c_t} J_{l^{f,s}}\left(l_t^{f,s}, l_t^{f,u}\right) Z_t^s = (1-\phi^s)\, V_{e^s}\left(k_t, b_t, e_t^s, e_t^u\right). \tag{49}$$

Working as described in Appendix A, we can derive the wage equations (A3) - (A4), which can alternatively be written as:

$$\begin{aligned}&(1-\tau^s)\, Z_t^s w_t^s = [\phi^s\{(1-\tau^s)\, Z_t^s \left[mpl_t^s + (1-\gamma_t^s)\tfrac{\varphi^s}{q_t^s}(1-\tau^{v,s})\right] - \\ &-(1-\gamma_t^s - p_t^s)\, E_t (1-\tau^s)\, Z_{t+1}^s A_t^k \tfrac{\varphi^s}{q_t^s}(1-\tau^{v,s})\} + (1-\phi^s)\,\xi c_t]\times \\ &\times \left[\phi^s + \frac{(1-\phi^s)\left(Z_t^s + e_t^s \frac{\partial Z_t^s}{\partial e_t^s}\right)}{Z_t^s}\right]^{-1}\end{aligned} \tag{50}$$

$$\begin{aligned}&(1-\tau^u)\, Z_t^u w_t^u = [\phi^u\{(1-\tau^u)\, Z_t^u \left[mpl_t^u + (1-\gamma_t^u)\tfrac{\varphi^u}{q_t^u}(1-\tau^{v,u})\right] - \\ &-(1-\gamma_t^u - p_t^u)\, E_t (1-\tau^u)\, Z_{t+1}^u A_t^k \tfrac{\varphi^u}{q_t^u}(1-\tau^{v,u})\} + (1-\phi^u)\,\xi c_t]\times \\ &\times \left[\phi^u + \frac{(1-\phi^u)\left(Z_t^u + e_t^u \frac{\partial Z_t^u}{\partial e_t^u}\right)}{Z_t^u}\right]^{-1}\end{aligned} \tag{51}$$

where $\frac{\partial Z_t^s}{\partial e_t^s} = \zeta^s \Omega^s (1+g^{z,s})\left(e_t^s\right)^{\zeta^s - 1}\left(\overline{e}_t^u\right)^{1-\zeta^s}$; and $\frac{\partial Z_t^u}{\partial e_t^u} = (1-\zeta^u)\,\Omega^u(1+g^{z,u})\left(\overline{e}_t^s\right)^{\zeta^u}\left(e_t^u\right)$. Note that the final terms in (50)-(51) are less than one.[14] These imply that OJL creates a channel which tends to reduce the Nash bargained wage, relative to the case of no OJL. When the workers internalise the effect of employment on their productivity and thus on their returns, they are willing to work for a lower wage rate.

These equations are generalisations of wage equations under Nash bargaining obtained in the literature (see e.g. Shimer (2010)). For $i = s, u,$ the return of an additional worker to the household is given by $(1-\tau^i)\, Z_t^i w_t^i$, i.e. the after-tax effective (or productivity-adjusted) wage. In equilibrium, this is equal to a weighted average of the effective marginal product of labour under search and matching, i.e. $(1-\tau_t^i)\, Z_t^i \left[mpl_t^i + (1-\gamma_t^i)\frac{\varphi^i}{q_t^i}(1-\tau^{v,i})\right] - (1-\gamma_t^i - p_t^i)\, E_t\left(1-\tau_{t+1}^i\right) Z_{t+1}^i A_t^k \frac{\varphi^i}{q_t^i}(1-\tau^{v,i})$, and the marginal rate of substitution between consumption and leisure, MRS^i, i.e. ξc_t, with the weights given by the bargaining power of the worker.

[14]To see this, first note that $\widetilde{Z} \equiv \frac{\left(Z_t^i + e_t^i \frac{\partial Z_t^i}{\partial e_t^i}\right)}{Z_t^i} > 1$, since $e_t^i \frac{\partial Z_t^i}{\partial e_t^i} > 0$. Then, note that $\phi^i + \left(1-\phi^i\right)\widetilde{Z} > 1 \Rightarrow \phi^i + \widetilde{Z} - \phi^i \widetilde{Z} - 1 > 0 \Rightarrow$
$\left(\widetilde{Z} - 1\right) - \phi^i\left(\widetilde{Z} - 1\right) > 0 \Rightarrow 1 > \phi^i$, which is true. Hence, $\left[\phi^i + \left(1-\phi^i\right)\widetilde{Z}\right]^{-1} < 1$.

The MRS^i follows the common definition of the ratio of the marginal utility of leisure, ξ, over the marginal utility of consumption, $1/c_t$. The effective marginal product of labour measures the additional after-tax productivity-adjusted output generated by moving a worker from unemployment to employment. It is comprised of (i) the direct after-tax increase in output provided by an additional skilled worker, mpl_t^i; (ii) the additional savings in terms of resources that would be required to post a vacancy if the matched job survives, $(1-\gamma_t^i)\frac{\varphi^i}{q_t^i}(1-\tau^{v,i})$, where $(1-\gamma_t^i)$ is the probability that a worker will remain in place in the next period and $\frac{\varphi^i}{q_t^i}(1-\tau^{v,i})$ is the cost per job posting multiplied by the duration that the job needs to be posted, $\frac{1}{q_t^i}$;[15] and (iii) the increase in job-posting costs for the firm implied by the decrease in future successful matches due to the increase in employment, $(1-\gamma_t^i-p_t^i)\,E_t\left(1-\tau_{t+1}^i\right)Z_{t+1}^i A_t^k \frac{\varphi^i}{q_t^i}(1-\tau^{v,i})$.[16]

3.7 Government budget and market clearing

Our policy experiments below focus on supply-side fiscal interventions in the form of increased vacancy subsidies and spending to support labour productivity via OJL. However, we allow for a government with a rich and realistic policy menu. This aids calibration and helps to capture the effect of the distortions implied by the different taxes in the relevant markets, both for the steady-state quantities and for the transmission channels of exogenous productivity and policy on endogenous outcomes. In particular, the government budget constraint is:

$$\begin{aligned} & g_t + g^{z,s} + g^{z,u} + \tau^{v,s}\varphi^s v_t^s + \tau^{v,u}\varphi^u v_t^u + R_{t-1}^b b_t + T_t = \\ & = b_{t+1} + \tau^k (r_t - \delta)\, k_t + \tau^s n^s w_t^s e_t^s Z_t^s + \tau^u n^u w_t^u e_t^u Z_t^u \end{aligned} \tag{52}$$

where g_t is the per-capita government consumption. Transfers are set to zero in the steady-state. Their presence in equation (52) allows us to study off-steady-state dynamics in response to changes to shocks by partialling out government financing issues and ensuring stability of debt.

[15]Note that from (35) - (36), $\frac{\varphi^i}{q_t^i}\left(1-\tau_t^{v,i}\right)$ is also equal to the expected benefit to the firm from posting a job.

[16]Note that an increase in current employment increases future unemployment (and thus the requirement for the firm to post a vacancy to fill the lost job) by $\frac{\partial s_{t+1}}{\partial e_t} = \left(1-\gamma_t^i - p_t^i\right)$. This is because there is reduction in the number of workers who search for jobs. Further note that these costs need to be discounted by the price of transferring resources between periods, A_t^k, which equals, from (21), expected future returns to investment discounted by the stochastic discount factor.

The capital markets clear when the supply is equal to the demand for capital per capita:

$$k_t = k_t^f. \tag{53}$$

In the skilled and unskilled labour markets, the equality of per capita labour supply and demand is given by:

$$n^s e_t^s Z_t^s = l_t^{f,s} \tag{54}$$

and

$$n^u e_t^u Z_t^u = l_t^{f,u}. \tag{55}$$

Moreover, dividends paid to the household must equal profits:

$$\pi_t = \pi_t^f, \tag{56}$$

and debt issued by the government must equal household demand:

$$b_t^h = b_t. \tag{57}$$

Finally, in the goods markets, the economy's per capita resource constraint is satisfied:

$$y_t^f = c_t + A_t^k k_{t+1} - A_t^k (1-\delta) k_t + g_t + g^{z,s} + g^{z,u} + \varphi^s v_t^s + \varphi^u v_t^u. \tag{58}$$

3.8 Decentralized equilibrium (DE)

Given initial conditions for $\left\{k_0, b_0, e_0^s, e_0^u, A_0, A_0^k, \gamma_0^s, \gamma_0^u\right\}$ and processes for $\left\{A_t, A_t^k, \gamma_t^s, \gamma_t^u\right\}_{t=1}^{\infty}$, and a process for either $\{b_{t+1}\}_{t=0}^{\infty}$ or $\{T_t\}_{t=0}^{\infty}$, a decentralized equilibrium is defined as stochastic processes for prices, $\{w_t^s, w_t^u, r_t, r_{t-1}^b\}_{t=0}^{\infty}$, matching probabilities, $\{p_t^s, p_t^u, q_t^s, q_t^u\}_{t=0}^{\infty}$, private allocations, $\{c_t, \pi_t, k_{t+1}, b_{t+1}^h, e_{t+1}^s, e_{t+1}^u, \pi_t^f, k_t^f, v_t^s, v_t^u, l_{t+1}^{f,s}, l_{t+1}^{f,u}\}_{t=0}^{\infty}$, and for one policy instrument, i.e. either $\{b_{t+1}\}_{t=0}^{\infty}$ or $\{T_t\}_{t=0}^{\infty}$ that is not exogenously determined, such that (i) households and firms undertake their respective optimization problems, taking aggregate outcomes and economic policy as given, under search and matching in the labour market as outlined above; (ii) wage rates for both types of labour are determined by a Nash bargain for matched household members and firms; (iii) all budget constraints are satisfied; and (iv) all markets clear. Finally note that in equilibrium, we have $\overline{e}_t^s = e_t^s$ and $\overline{e}_t^u = e_t^u$.

Using Walras' law we drop the household's budget constraint, so that the DE consists of the following equations: (i) the search and vacancy matching probabilities in equations (5) and (6); (ii) the consumption and bonds Euler equations (21) and (24); (iii) the firm's optimality conditions given by equations (25) for $(i = s, u)$, (26), (30), (35) and (36); (iv) the wage equations

(50) and (51); and (v) the market clearing conditions in (53), (54), (55), (56), (57) and (58).[17]

4 Quantitative implementation

In the following section we first discuss the model calibration followed by the quantitative predictions of the model regarding the steady-state and near steady-state dynamics. We consider two model variants, depending on the assumptions we make regarding the labour productivity technology, as captured by Z_t^i, for $i = s, u$. In particular, we first consider a base case without OJL, so that $Z_t^i = \Omega^i = 1$. We then choose Ω^i in the case of OJL so that the level of labour productivity in the steady-state, Z^i, is the same across the two models. To calibrate the model and solve for the steady-state we set $T_t = 0$ for all t and let government debt be the residual variable in the government budget constraint.

4.1 Model Calibration

Table 2 reports the values for the structural parameters of the base model without OJL based on a quarterly calibration. The table indicates how each parameter is obtained by referring to various sources. This includes calculations using: (i) the data; (ii) estimates and assumptions from other studies in the literature; and (iii) calibration to target steady-state values for the relevant endogenous variables of the model. We summarise at the end of this sub-section the changes in parameters required for the OJL model.

4.1.1 Population shares, policy, discount and depreciation rates

We use data from Acemoglu and Autor (2011) for the period (1963-2008) to calculate the population share of skilled workers, $n^s = 0.45$. Consistent with the range used in the literature, the time discount factor, $\beta = 0.99$, is set to give an annual return to capital, net of depreciation, of about 4%. Similarly, the depreciation rate, $\delta = 0.022$, is calibrated to target a quarterly steady-state capital to output ratio of about 8 which on an annual basis is consistent with a ratio of around 2. Following Uhlig (2010) we set the tax rate on capital income to 36%. Moreover, we choose the two labour income

[17]Note that when the market clearing conditions (54) and (55) and the matching probabilities in equations (5) and (6) are imposed on the employment evolution equations (15) and (25) the latter become identical. Hence, we drop the employment evolution equations (15) from the household's problem from the DE.

tax rates to be $\tau^s = 35\%$ and $\tau^u = 25\%$, which imply a weighted average close to the 28% labour income tax rate used in Uhlig (2010). The level of government spending is set so that the debt to output ratio is 0.63 or in quarterly terms 2.52 (as in Uhlig (2010)).

Table 2: Model Parameters

Parameter	Value	Definition	Source
$0 < n^s < 1$	0.450	population share of skilled workers	data
$0 \leq \tau^k < 1$	0.360	tax rate on capital income	estimate
$0 \leq \tau^s < 1$	0.350	tax rate on skilled labour income	estimate
$0 \leq \tau^u < 1$	0.250	tax rate on unskilled labour income	estimate
$g > 0$	0.425	per-capita government consumption	calibration
$0 < \beta < 1$	0.990	time discount factor	calibration
$0 \leq \delta \leq 1$	0.022	depreciation rate of capital stock	calibration
$\frac{1}{1-\nu}$	0.669	capital to skilled labour elasticity	estimate
$\frac{1}{1-\alpha}$	1.669	capital to unskilled labour elasticity	estimate
$0 < \theta < 1$	0.493	share of composite input to output	calibration
$0 < \rho < 1$	0.820	share of capital to composite input	calibration
$\xi > 0$	0.100	disutility cost of employment	calibration
$0 < \gamma^s < 1$	0.028	skilled job separation rate	calibration
$0 < \gamma^u < 1$	0.045	unskilled job separation rate	calibration
$0 < \eta^s < 1$	0.600	elasticity of skilled search	assumption
$0 < \eta^u < 1$	0.500	elasticity of unskilled search	assumption
$0 < \phi^s < 1$	0.600	bargaining power of skilled workers	assumption
$0 < \phi^u < 1$	0.500	bargaining power of unskilled workers	assumption
$\varphi^s > 0$	0.900	unit cost of posting skilled job	calibration
$\varphi^u > 0$	0.820	unit cost of posting unskilled job	calibration
$0 < \tau^{v,s}, \tau^{v,u} < 1$	0.010	job vacancy subsidy	assumption
$0 < g^{z,s}, g^{z,u} < 1$	0.000	public expenditure to support OJL	assumption
$\chi^s > 0$	0.800	skilled matching efficiency	calibration
$\chi^u > 0$	0.600	unskilled matching efficiency	calibration
$0 < \zeta^s, \zeta^u < 1$	0.500	elasticity of learning	assumption

4.1.2 Production

The elasticities of substitution between skilled labour and capital and between unskilled labour and capital have been estimated by Krusell *et al.* (2000). We use their estimates, so that $\nu = -0.495$ and $\alpha = 0.401$. To ensure the skill premium and labour share in income are consistent with the data, θ and ρ respectively are calibrated to 0.493 and 0.82 (see, e.g. Lindquist (2004), He and Liu (2008), Pourpourides (2011) and He (2012) who use a similar approach to calibrating the production function). The target value for the skill premium of approximately 1.68 is obtained from Acemoglu and Autor (2011) for the period (1963-2008). We measure the labour income share using data from National Income and Product Accounts Table 1.10, 1959-2013, which gives a value of approximately 0.66. Finally, the parameters capturing steady-state TFP and investment-specific technical change, i.e. A and A^k are normalised to unity.

4.1.3 Utility function and job separation rates

Following Shimer (2010) we set the disutility of employment parameter, $\xi = 0.1$, to imply an aggregate unemployment rate of about 5%. Also note that Shimer (2005) reports an average employment exit probability of 0.034. Given this and the assumption that skilled labour has a lower job separation rate (see, e.g. Fallick and Fleischman (2004), Hagedorn *et al.* (2016), and Pilossoph (2012)) we set the job separation rates, $\gamma^s = 0.028$ and $\gamma^u = 0.045$, to approximately match the sectoral unemployment rates of 3% and 7% respectively using the CPS data analysed in Section 2.

4.1.4 New matches and bargaining power

The values used for the elasticities of new matches with respect to search time, $\eta^s = 0.6$ and $\eta^u = 0.5$, are within the range of econometric evidence reported in Petrongolo and Pissarides (2001). To ensure that the Hosios (1990) condition is satisfied we set the relative bargaining power of workers in the skilled and unskilled sectors respectively to $\phi^s = 0.6$ and $\phi^u = 0.5$ (see, Cahuc *et al.* (2006) who find that skilled workers have higher bargaining power).

4.1.5 Job posting costs and subsidy

Pissarides (1994), Acemoglu (2001) and Krause and Lubik (2006 and 2010) suggest that the flow cost of posting a vacancy is higher in 'good' jobs. Following these studies, we assume that the job posting for skilled is greater than that for unskilled labour, i.e. $\varphi^s > \varphi^u$. These parameters are calibrated to ensure aggregate job costs as a share of GDP of about 2.5%, which coheres with Arseneau and Chugh (2012), and aggregate labour market tightness of about unity, which is the value used in Pissarides (1998) and Campolmi and Gnocchi (2016). Also following Campolmi *et al.* (2011) we set the vacancy subsidy rate to 1%.

4.1.6 Matching efficiency and OJL

Consistent with an aggregate unemployment rate of 5% and an average employment exit probability of 0.034, Shimer's calibration (2010, see p. 67) implies a job finding probability about 0.65. Following this approach for each labour market gives us target probabilities of $p^s = 0.828$ and $p_t^u = 0.591$ which we obtain by calibrating $\chi^s = 0.8$ and $\chi^u = 0.6$. The job finding probabilities in turn imply unemployment durations of about 1.21 and 1.69 quarters for skilled and unskilled respectively. The calibration also suggests that the job filling rate is higher for the skilled versus the unskilled consistent with Krause and Lubik (2006 and 2010). As explained above, we present the models results below both without and with learning. In the former, $Z_t^s = Z_t^u = 1$ in equations (10-11). In the latter, we set the exponents $\zeta^s = \zeta^u = 0.5$ and calibrate Ω^s and Ω^u so that in the steady-state the Z functions are equal to unity as under no learning. This requires that $\Omega^s = \Omega^u = 1.055$. We also report results when there are no spill-over effects in OJL, so that $\zeta^s = 1$ and $\zeta^u = 0$. In this case, we find that $\Omega^s = 1.031$, $\Omega^u = 1.068$. Finally, given that public expenditures to support OJL, $g^{z,s}$ and $g^{z,u}$, have been introduced with the aim of undertaking counterfactual policy analysis, we set their steady-state values to zero.

4.2 Steady-state

The steady-state implied by the above calibration is reported in Table 3 for the base model without learning. These results show that great ratios are well in line with the U.S. data. Moreover, the remaining values cohere with the targets discussed in the calibration above.

Table 3: Steady-state base model

$\frac{c}{y}$	$\frac{k}{y}$	$\frac{g}{y}$	$\frac{b}{y}$	$\frac{we}{y}$	$\frac{v}{y}$	s^s	s^u	s
0.571	8.198	0.225	2.520	0.660	0.024	0.033	0.071	0.054
$\frac{w^s}{w^u}$	$\widetilde{r}$	r^b	z^s	z^u	p^s	p^u	q^s	q^u
1.680	1.010	0.010	1.089	0.971	0.828	0.591	0.760	0.609

For the model under OJL, following the same calibration strategy outlined above, we re-calibrate $\theta = 0.500$, $\varphi^s = 1.600$, $\varphi^u = 1.520$, and $g = 0.401$, so that this model implies effectively the same steady-state with the base model.[18] Note that as discussed in Section 3.6.3, under OJL bargained wages

[18] In later analysis we will examine the quantitative implications of not allowing for spillovers in the OLG model. For the two versions of the OJL model implied by this analysis, note that the steady-state ratios differ from those in Table 3 in most cases in the second or third decimal point (see Appendix B, Table 1).

tend to be lower and thus unemployment is lower. Therefore, to maintain the same level of unemployment and labour market tightness in the steady-state, job-posting costs need to increase.

4.3 Stochastic processes

When undertaking the model simulations we draw the four processes discussed above from a multivariate normal distribution, denoted $\mathbf{x} = N(\overline{\mathbf{x}}, \mathbf{\Sigma})$ where $\mathbf{x} = [\varepsilon_t^A, \varepsilon_t^{A^k}, \varepsilon_t^{\gamma^s}, \varepsilon_t^{\gamma^u}]$, $\overline{\mathbf{x}}$ is the vector of means and $\mathbf{\Sigma}$ is the variance-covariance matrix of shocks. The parameters of stochastic processes driving the model are reported in Table 4.

Table 4: Stochastic processes

Parameter	Value	Definition	Source
σ_A	0.008	SD of TFP	calibration
ρ_A	0.950	AR(1) coefficient of TFP	assumption
σ_{A^k}	0.0047	SD of IT	estimate
ρ_{A^k}	0.6015	AR(1) coefficient of IT	estimate
σ_{γ^s}	0.073	SD of skilled separation rate	data
ρ_{γ^s}	0.740	AR(1) coef. of skilled separation rate	data
σ_{γ^u}	0.073	SD of unskilled separation rate	data
ρ_{γ^u}	0.740	AR(1) coef. of unskilled separation rate	data
$\rho(\varepsilon_t^{\gamma^s}, \varepsilon_t^{\gamma^u})$	0.980	Job separation rate shock correlation	calibration

The autocorrelation parameter of TFP is set equal to 0.95, following Gertler and Trigari (2009), and Arseneau and Chugh (2012). As in the literature, the volatility parameter, σ_A, is calibrated to match the standard deviation of HP-filtered output, 0.011. Regarding investment-specific technical change, we use the estimates from Pourpourides (2011), which implies setting ρ_{A^k}, to 0.6015 and σ_{A^k}, to 0.0047. Given the lack of sectoral data for the job separation rates, we apply the same quarterly autocorrelation, ρ_{γ^s} and $\rho_{\gamma u}$, and standard deviation, σ_{γ^s} and σ_{γ^u}, parameters for skilled and unskilled using data from the Job Openings and Labor Turnover Survey (JOLTS) for the period 2001:1-2014:2. Finally, the correlation between job separation shocks, $\rho(\varepsilon_t^{\gamma^s}, \varepsilon_t^{\gamma^u})$, is calibrated to match the correlation between HP-filtered skilled and unskilled employment/unemployment rates in the data.[19]

[19] Note that not allowing for this correlation only affects this target.

4.4 Model solution

To ensure the stability of public debt when solving the dynamic model, we set debt in each time period equal to its steady-state value and let transfers act as the residual policy instrument.[20] Importantly, this allows us to focus in the next section on the dynamic effects of spending to support OJL and changes in vacancy subsidies by partialling out alternative forms of government financing. This is because transfers offer a neutral financing instrument which acts as a common base for all policy interventions.

Following Shimer (2010), we present results under shocks to TFP and the job separation rates but we also consider investment-specific technological change, given the importance attached to skill-biased technical change in explaining the behaviour of the skill premium in the literature. The results for the sectoral variables discussed in Section 2 are presented in Table 5. To obtain these results we first solve a first-order approximation of the dynamic system of equations characterising the DE around the steady-state, by implementing the perturbation methods in Schmitt-Grohé and Uribe (2004). We then simulate time paths under shocks to total factor productivity, the job separation rates and investment-specific technological change, as indicated. We conduct 10,000 simulations of 80 periods, to match the time periods for the sectoral employment, unemployment and wage data (see Section 2), initialised from the steady-state.

4.5 Second moments

For each simulation, we HP-filter the logged series and then compute the required moments and report the means of these moments across the simulations in Table 5. Note that we report moments for effective wages below, since these are closer to the data definitions. However, the results are similar if bargained wages are used instead for the model moments.

4.5.1 Model predictions and data

As can be seen in Table 5, both model variants capture several stylised facts regarding the differences between the skilled and unskilled labour market with respect to the cyclical properties of employment, unemployment and returns to work. In particular, consistent with the data, the calibrated models predict that: (i) in both skilled and unskilled labour markets, unemployment is significantly more volatile than output, whereas employment is less volatile

[20] See Arseneau and Chugh (2012) who employ a similar approach to avoid issues relating to alternative methods of government financing when considering exogenous policy.

than output; (ii) unemployment for skilled labour is more volatile than unemployment for unskilled labour, whereas employment for unskilled is more volatile than employment for skilled; (iii) in both skilled and unskilled labour markets, employment is pro-cyclical and unemployment counter-cyclical; (iv) there are no important differences in the correlations with output between skilled and unskilled labour, neither for employment nor for unemployment; (v) wages have high volatility and wages for skilled and unskilled have relatively similar correlation with output.

Table 5: Data vs. Model moments

		Shocks to A, γ^i		Shocks to A, A^k, γ^i	
	Data	Base	OJL	Base	OJL
$\rho(e^s, e^u)$	0.961	0.960	0.961	0.960	0.961
$\rho(e^s, y)$	0.864	0.252	0.447	0.249	0.444
$\rho(e^u, y)$	0.854	0.264	0.462	0.261	0.459
$\rho(s^s, s^u)$	0.952	0.960	0.961	0.960	0.961
$\rho(s^s, y)$	-0.872	-0.252	-0.447	-0.249	-0.444
$\rho(s^u, y)$	-0.900	-0.264	-0.462	-0.261	-0.459
$\rho(w^s Z^s, y)$	0.350	0.961	0.812	0.944	0.771
$\rho(w^u Z^u, y)$	0.270	0.953	0.911	0.901	0.770
$\rho(e, y)$	0.865	0.263	0.461	0.260	0.458
$\rho(s, y)$	-0.901	-0.263	-0.461	-0.260	-0.458
$\frac{\sigma(e^s)}{\sigma(y)}$	0.386	0.248	0.228	0.248	0.228
$\frac{\sigma(e^u)}{\sigma(y)}$	0.703	0.485	0.446	0.484	0.446
$\frac{\sigma(s^s)}{\sigma(y)}$	13.195	7.334	6.572	7.318	6.565
$\frac{\sigma(s^u)}{\sigma(y)}$	10.524	6.374	5.789	6.359	5.784
$\frac{\sigma(w^s Z^s)}{\sigma(y)}$	0.960	0.963	0.892	0.982	0.944
$\frac{\sigma(w^u Z^u)}{\sigma(y)}$	0.660	0.954	0.878	1.008	1.040
$\frac{\sigma(e)}{\sigma(y)}$	0.553	0.373	0.343	0.372	0.343
$\frac{\sigma(s)}{\sigma(y)}$	11.161	6.583	5.957	6.567	5.952

Note: The correlations and relative volatilities for wages in the data are from Table 2 in Castro and Coen-Pirani (2008), for the period 1984:1-2003:4. Since for both model and actual data we present statistics for the cyclical component, the c superscript used in Figure 1 and Table 1 has been dropped.

Both models under-predict, quantitatively, the volatility of employment in the two sectors, which is expected given the results in Shimer (2005 and 2010). However, the predicted volatility of employment in the sectoral model is significantly improved compared to the canonical one-sector model. In particular, the two models predict an employment volatility which ranges from 62% to 67% of the volatility of employment in the data, whereas in the canonical models in e.g. Shimer (2010), this ratio is typically about 25%.[21] Hence, the models predict quantitatively meaningful and relevant employment volatilities, although they still do not match those observed in the data. Moreover, both models over-predict the relative volatility of unskilled wages, but the latter is high in the data as well, so the difference is quantitative.

The above results are generally similar for both the base model and the model with OJL, although the base model predicts marginally higher volatilities for employment and unemployment. Where the base model has a bigger challenge is with respect to the magnitudes of the predicted correlations of employment (and unemployment) and wages, relative to the data. In particular, in the data the employment/unemployment-output correlations are very strong, whereas the wage correlations are clearly lower. While the model predicts the correct direction for the correlations, it suggests employment/unemployment-output correlations which are too low and wage-output correlations which are too high. In this respect, the model with OJL presents an important improvement, as it moves the model significantly in the right direction. We explain why this happens in the next sub-section.

4.5.2 The mechanism of on-the-job learning

Both model versions incorporate a base mechanism that works to generate positive (negative) employment-output (unemployment-output) correlations and also positive wage-output correlations, in response to exogenous shocks. Exogenous productivity shocks, A_t and A_t^k, raise the marginal products of labour[22] and thus, via equations (50)-(51) increase the bargained wage (see the discussion in Section 3.6.3) and via equations (35)-(36) increase employment. To see the latter, note that for equations (35)-(36) to hold after an increase in the marginal product of labour, vacancies must also increase, so that the probabilities of matches, q_t^s and q_t^u, decrease. In other words, the increased labour productivity creates incentives for firms to pay a higher wage and to hire more workers. As a result, employment and wages are positively correlated with output, which increases (decreases) following positive (negative) exogenous productivity shocks. Thus, the models predict that following productivity shocks, output, employment and wages move in the same

[21] Shimer (2010, p. 95) describes standard deviations of employment relative to output of a magnitude of about 45%, to "finally generate interesting fluctuations in employment".

[22] We refer to positive exogenous productivity shocks as an example here. Obviously, effects are reversed for negative shocks.

direction. Shocks to the job separation rates γ_t^s and γ_t^u have the opposite effects by reducing employment and putting downward pressure on wages. The latter occurs due to the fall in the expected future benefit of hiring an additional worker (see equations (50)-(51)). Moreover, shocks to γ_t^s and γ_t^u also lead to a fall in output, due to the reduction in inputs, so that again output, employment and wages move in the same direction.

4.5.3 Base model

Regarding the relationship between wages and output, in the base model without OJL the correlations between output and wages are near-perfect. This is because the dynamics of both the marginal product of labour (the key determinant of wages) and output directly follow the dynamic paths of the exogenous productivity shocks. The link is not as strong following job separation shocks, which affect output and wages indirectly, via changes in employment. Quantitatively, the importance of exogenous productivity shocks for the patterns of returns to labour and output dominates and leads to the very high correlations between them in Table 5.

Regarding the relationship between employment/unemployment and output, as noted already, employment/unemployment are not strongly correlated with output. On one hand, exogenous productivity shocks work directly on output, whereas their effect on employment takes place through the workings of the labour market as analysed above. On the other hand, the effect of job separation shocks on employment is direct but its effect on production is second-order, via the reduction in labour inputs and the production function. For both reasons, the correlations between employment/unemployment and output are low.

4.5.4 OJL model

In our model, OJL creates a feedback effect from employment to labour productivity, Z_t^s and Z_t^u, as the rise in employment also raises Z_t^s and Z_t^u. Regarding the relationship between employment/unemployment and output, this additional propagation mechanism works to further stimulate the increase in output following A_t and A_t^k shocks, and increases the persistence of the responses to employment. This mechanism thus works in the same direction on both employment and output and increases the strength of the relationship between output and employment/unemployment. The feedback effect from employment to Z_t^s and Z_t^u also increases the strength of the co-movement between output and employment/unemployment following shocks to the job separation rates γ_t^s and γ_{u_t}. In this case, it acts to amplify the effect of shocks to employment on output, by affecting the latter via labour productivity in addition to changes in the number of workers.

Regarding the relationship between wages and output, because Z_t^s and Z_t^u affect the marginal products of labour,[23] the dynamics of the latter (and hence also the dynamics of wages) also reflect the dynamics of Z_t^s and Z_t^u, and thus their dependence on exogenous productivity is reduced. These additional dynamics affecting wages are small quantitatively, but do reduce the nearly one-to-one relationship between wages and output in the model without OJL that was explained earlier.

To summarise, the model without OJL predicts marginally higher volatilities for employment and unemployment, and in this respect is slightly closer to the data. On the other hand, the model with OJL predicts significantly higher correlations with output, for both skilled and unskilled employment (unemployment), with correlation coefficients that are nearly twice as high. Moreover, it predicts lower correlations with output for both skilled and unskilled wages. Given in particular the difficulty of the base model, without OJL, to predict a reasonably high cyclicality for skilled and unskilled employment, this improvement is important in providing support in favour of the mechanism of endogenous labour productivity associated with OJL.

5 Supply-side fiscal interventions in the unskilled labour market

We next consider the effect of targeted labour market fiscal policies. In particular, temporary spending policies to increase the productivity of workers and temporary increases in job-posting subsidies to reduce the costs of hiring.[24] These policies directly aim at improving outcomes for unskilled relative to skilled workers in terms of increasing employment and returns to work. Since we focus on post-schooling age workers in business cycle frequencies, we examine policies which intend to increase productivity of unskilled workers while in the labour market. Thus, we do not evaluate education policies at the school level.[25]

[23]The relationship between the marginal product of labour and Z_t^s and Z_t^u will be analysed in more detail in the next section.

[24]Note that in all exercises reported below, the AR(1) parameters in the first-order Markov processes for the vacancy subsidies and $g_t^{z,u}$ are all set to 0.90. For all policy experiments below we plot the percent deviations of the key economic variables from their respective steady-states.

[25]The effects of policies related to education are expected to affect school-age population, as opposed to more mature workers (recall that we used data on workers over 25 years of

5.1 Policies to improve productivity of unskilled

In Figure 2 we plot the effects of a temporary increase in government spending to enhance lifelong learning and continuous professional development of unskilled workers while on their jobs, $g_t^{z,u}$. In particular, $g_t^{z,u}$ is increased from 0 to 0.01, implying for our calibration a fiscal policy that would cost about half a percentage point of GDP. As expected, this intervention increases the productivity of the unskilled workers, Z_t^u, (subplot (1,4)). However, this also leads to a decrease in the unskilled bargained wage, w_t^u, (subplot (2,2)), which will be further explained below. However, the effect of the increase in Z_t^u on the earnings for employed unskilled workers, $w_t^u Z_t^u$, outweighs the effect of the reduction in w_t^u, so that $w_t^u Z_t^u$ increases (subplot (2,4)). As expected, aggregate output, y_t, also increases (subplot (1,1)), with the increase being about 30% of the increase in Z_t^u. In fact, y_t increases by less than the increase in $g_t^{z,u}$, implying a form of crowding-out and thus declines in consumption, c_t, (subplot (1,2)) and capital (subplot (1,3)).

[Figure 2 here]

After the rise in $g_t^{z,u}$, w_t^u falls for three reasons. First, the rise in Z_t^u contributes to a reduction in the marginal product of unskilled labour, mpl_t^u, (subplot (4,3)), which, as discussed under equation (51) in Section 3.6.3, tends to reduce w_t^u. In particular, note that while mpl_t^u, defined after equation (36), is a positive function of effective skilled labour, $l_t^{f,s} = n^s e_t^s Z_t^s$, and capital, k_t, it is decreasing in the effective quantity of unskilled labour, $l_t^{f,u} = n^u e_t^u Z_t^u$, for the model calibration.[26] Hence, the increase in Z_t^u tends

age). Moreover, education policies are expected to work on a longer horison than policies to improve on-the-job skill-acquisition (see e.g. Goldin and Katz (2008) for a discussion of education reforms and Angelopoulos *et al.* (2017) for a recent study evaluating the effect of education policies on wage inequality over longer horizons).

[26] The effect of skilled labour input and capital on mpl_t^u can be analytically signed for the parameter restrictions in this model. Moreover, the model calibration and implied quantities for the endogenous variables also imply that mpl_t^u is decreasing in effective unskilled labour input. To see this, note that

$$\frac{\partial mpl_t^u}{\partial l_t^{f,u}} = \theta \left(l_t^{f,u}\right)^{\alpha-2} A_t \left\{ \theta \left(l_t^{f,u}\right)^{\alpha} + (1-\theta)\left[\rho\left(k_t^f\right)^{\nu} + (1-\rho)\left(l_t^{f,s}\right)^{\nu}\right]^{\frac{\alpha}{\nu}} \right\}^{\frac{1}{\alpha}-1} (\alpha-1) \times$$
$$\times \left[1 - \left\{ \theta \left(l_t^{f,u}\right)^{\alpha} + (1-\theta)\left[\rho\left(k_t^f\right)^{\nu} + (1-\rho)\left(l_t^{f,s}\right)^{\nu}\right]^{\frac{\alpha}{\nu}} \right\}^{-1} \theta\left(l_t^{f,u}\right)^{\alpha}\right],$$

where $\left\{ \theta \left(l_t^{f,u}\right)^{\alpha} + (1-\theta)\left[\rho\left(k_t^f\right)^{\nu} + (1-\rho)\left(l_t^{f,s}\right)^{\nu}\right]^{\frac{\alpha}{\nu}} \right\} = \left(y_t^f\right)^{a} > 1$ (see Appendix B, Table 1). This implies that the term in square brackets in the above expression is positive and thus the derivative is negative.

to increase $l_t^{f,u}$ and thus the mpl_t^u, in turn putting downward pressure on w_t^u. Second, the fall in capital, k_t, (subplot (1,3)) also contributes to the decline in mpl_t^u and thus works as above. The third effect also works via equation (51), this time via c_t which falls, given the crowding-out effects pointed out above. As discussed under equation (51) in Section 3.6.3, the decrease in c_t tends to lower w_t^u because the marginal utility of consumption is higher and thus the household's valuation of unskilled labour income increases. This implies that unskilled labour is more willing to work for a lower wage.

The results so far indicate an increase in $g_t^{z,u}$ and $w_t^u Z_t^u$ reduce earnings inequality between employed labour, $\frac{w_t^s Z_t^s}{w^u Z_t^u}$, (subplot (3,2)). However, the increase in Z_t^u also leads to a rise in unemployment for the unskilled, s_t^u (subplot (4,1)). The decline in the mpl_t^u discussed above requires, via equation (36) that q_t^u is increased, or, instead, via equation (6) that labour market tightness, z_t^u, is reduced.[27] In turn, this implies that unskilled vacancies, v_t^u, fall (subplot (3,3)), so that s_t^u increases. Intuitively, given the increase in $l_t^{f,u}$ following the rise in Z_t^u, and since the cost of posting vacancies to hire additional workers has not changed, the firm finds it optimal to reduce vacancies postings, which leads to increased unemployment. The firms employ fewer, more productive workers, and make savings from posting vacancies.

Therefore, while increasing $g_t^{z,u}$ is beneficial to employed unskilled workers and decreases earnings inequality between employed skilled and unskilled labour, it increases unskilled unemployment and thus increases labour income inequality within the group of unskilled labour. Hence, this policy has ambiguous effects on the welfare of unskilled population as a whole.

The increase in $g_t^{z,u}$ also affects skilled labour. There are different channels for this effect. On one hand, the decrease in the employment of unskilled, e_t^u, lowers OJL for the skilled workers, Z_t^s, (subplot (2,1)) and this decline in Z_t^s sets in motion effects for skilled labour opposite to those analysed above for the unskilled following the rise in Z_t^u. In particular, the decline in Z_t^s tends to increase mpl_t^s[28] and thus increase the bargained skilled wage, w_t^s, while increasing skilled vacancies, v_t^s, and skilled employment, e_t^s.[29] Second, and

[27] Since w_t^u also falls, that would create, via equation (36), incentives to decrease q_t^u. However, note that in equation (36) an equal proportional increase or decrease in mpl_t^u and w_t^u (as seen in Figure 2), implies a bigger absolute difference between the two, since $mpl_t^u > w_t^u$ (see Appendix B, Table 1). Hence, the effects of the changes in mpl_t^u dominate.

[28] The effect of unskilled labour input and capital on mpl_t^s can be analytically signed to be positive for the parameter restrictions in this model. Moreover, the model calibration and implied quantities for the endogenous variables also imply that mpl_t^s is decreasing in effective skilled labour input, by evaluating the quantity $\frac{\partial mpl_t^s}{\partial l_t^{f,s}}$ for the parameters and steady state solution that they imply (see Appendix B, Table 1).

[29] As for the unskilled, since mpl_t^s and w_t^s move proportionately in Figure 2, and $mpl_t^s >$

working in the same direction, the increase in Z_t^u tends to increase $l_t^{f,u}$ and mpl_t^s, which again works to increase w_t^s and υ_t^s and to decrease unemployment for skilled, s_t^s.[30]

On the other hand, the reduction in k_t (subplot (1,3)) especially hurts skilled workers, who are complementing capital in production. In particular, the fall in k_t tends to reduce mpl_t^s, w_t^s, υ_t^s, and e_t^s via the same channels as those analysed above for unskilled labour.

In the short-run, the positive effects dominate so that for skilled workers we observe increases in mpl_t^s (subplot (4,4)), w_t^s (subplot (2,3)), earnings for employed skilled workers, $w_t^s Z_t^s$, (subplot (3,1)), υ_t^s (subplot (3,4) and a fall in s_t^s subplot (4,2) respectively). After 10 quarters, when the accumulated reduction in k_t has become big enough and the increase in unskilled labour productivity, Z_t^u, has fallen below 50% of its original increase, the negative effects dominate and mpl_t^s falls below its steady-state value and the above positive effects for skilled earnings, $w_t^s Z_t^s$, and employment, e_t^s are reversed.

5.2 Policies to increase employment of unskilled

We next analyse the effects of increasing subsidies for posting unskilled job vacancies, $\tau_t^{v,u}$, from 1% to 2%. The dynamic effects are summarised in Figure 3. As expected, this policy creates incentives for increasing υ_t^u (subplot (3,3)), because the cost of posting vacancies in equation (36) is reduced.[31] In turn this leads to reductions in s_t^u (subplot (4,1)).

[Figure 3 here]

The increase in e_t^u also increases Z_t^u (subplot (1,4)) and thus, given the rise in $l_t^{f,u}$, since both e_t^u and Z_t^u increase, the mpl_t^u decreases (subplot (4,3)). The rise in $\tau_t^{v,u}$ and fall in mpl_t^u create, via the bargaining process in equation (51), pressure for a decrease in the w_t^u. On the other hand, the rise in υ_t^u and the fall in s_t^u decreases q_t^u in equation (6), which tends to increase w_t^u. This is further supported by the rise in c_t, at least after the first few quarters (subplot (1,2)). The latter effects dominate so that w_t^u is increased and thus

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w_t^s in levels, the effects of dominate in equation (35) and determine the change in q_t^s and υ_t^s.

[30] The reduction in c_t also tends to decrease the w_t^s and thus, via equation (35), lower, q_t^s, and raise υ_t^s and e_t^s.

[31] Note that a reduction in the left-hand side of equation (36) requires a reduction in q_t^u, hence an increase in vacancies and a decrease in unemployment, since q_t^u is inversely related with labour market tightness. Intuitively, it is cheaper for the firm to post vacancies, hence the quantity of vacancies posted must increase to reduce its expected marginal benefit to the level of the lower new marginal cost.

$w_t^u Z_t^u$ (subplot (2,4)) rises, leading to reductions in earnings inequality for employed unskilled workers, $\frac{w_t^s Z_t^s}{w_t^u Z_t^u}$, (subplot (3,2)).

Thus, with respect to unskilled labour, support for hiring both reduces unemployment and increases the earnings of those employed, making it a good policy for an overall reduction in inequality. However, this is not a *Pareto* improving policy since $w_t^s Z_t^s$ (subplot (3,1)) is initially reduced and s_t^s (subplot (4,2)) is increased for about 15 quarters following the policy change.

The increase in $l^{f,u}t$ (given the rise in e_t^u and Z_t^u) tends to increase the mpl_t^s, whereas the reduction in k_t (subplot (1,3)), which is substituted in production by the relatively cheaper unskilled labour, tends to lower mpl_t^s. OJL creates an additional effect on mpl_t^s, via the spill-over productivity effects for skilled workers, arising from the increased employment of the unskilled. In particular, Z_t^s is increased (subplot (2,1)) following the rise in e_t^u and this tends to increase $l_t^{f,s}$ and thus contributes to lowering mpl_t^s (subplot (4,4)) following the rise in $\tau_t^{v,u}$. As analysed earlier, the overall reduction in mpl_t^s leads to a reduction in the w_t^s (subplot (2,3)), which is initially greater than the increase in Z_t^s, so that $w_t^s Z^s$ is reduced (subplot (3,1)). Moreover, the fall in mpl_t^s also leads to a fall in v_t^s (subplot (3,4)) via equation (35) and a rise in s_t^s (subplot (4,2)). The situation is gradually improved for skilled workers as k_t starts to increase as it recovers towards the steady-state. Given capital-skill complementarity, this has strong effects on improving mpl_t^s and thus on increasing returns and employment of skilled workers.

What is particularly interesting in this analysis is that in the environment with search and matching frictions and wage bargaining, positive spill-over effects to the productivity of skilled workers from the unskilled, actually work to decrease employment and labour income of skilled in the short-run, when the employment of the unskilled is increased.

5.3 Policies to increase employment of skilled and unskilled

Although increases in $\tau_t^{v,u}$ increase both the probability of being employed and earnings when employed for unskilled workers, they are not *Pareto* improvements as analysed above. Therefore, an intervention is also needed in the skilled labour market to counteract the negative employment and earnings implications for skilled labour by the rise in $\tau_t^{v,u}$. Naturally, this can take the form of concurrent increases in $\tau_t^{v,s}$, alongside the rise in $\tau_t^{v,u}$. In Figure 4 we plot the responses of a policy experiment where $\tau_t^{v,s}$ is increased by as much is required, following an increase in $\tau_t^{v,u}$ from 1% to 2%, so that the negative effects of the latter on unskilled unemployment are neutralised. More specifically, this requires that $\tau_t^{v,s}$ is increased from 1% to 1.01%.

[Figure 4 here]

As can be seen in Figure 4, the effects on unskilled employment and labour earnings are effectively the same as those in Figure 3. However, the increase in $\tau_t^{v,s}$, working via the same channels as those for the effects of the increase in $\tau_t^{v,u}$ for unskilled workers, tends to increase employment for skilled, thus leading to very small overall deviations from the steady-state for skilled unemployment.

5.4 The importance of spillovers and OJL

To gain an appreciation for the roles played by spillovers and OJL in the above analysis, we first examine the contribution of spill-over effects when $g_t^{z,u}$ and $\tau_t^{v,u}$ are each increased. We next contrast the effects of increases in $\tau_t^{v,u}$ in the model with OJL versus without OJL.

We first find that spill-over effects are not important qualitatively in the $g_t^{z,u}$ exercise. For example, when we set $\zeta^s = 1$ and $\zeta^u = 0$ in equations (10) and (11) the effects of the increase in $g_t^{z,u}$ are similar to those reported in Figure 2.[32] This further points to the importance of capital for the skilled labour market, which was noted in Section 5.1. Although the spill-over effects associated with OJL contribute to the movements of the marginal products of labour for skilled workers, the dynamic path of the capital stock, as discussed earlier, is on balance the key factor determining the developments in the skilled labour market in this case and dominate.

However, there are quantitative differences regarding the effects of $\tau_t^{v,u}$ on the economy in the presence of spillovers. To highlight the importance of the endogenous productivity channel associated with OJL we next examine the two cases. First, no spill-overs which is defined when $\zeta^s = 1$ and $\zeta^u = 0$ in equations (10) and (11). Second, no OJL which is defined when $Z_t^s = Z_t^u = 1$. The results of a temporary one percentage point increase in $\tau_t^{v,u}$ are shown in Figure 5.

[Figure 5 here]

As can be seen in Figure 5, while the effects for unskilled labour are qualitatively very similar with those presented in Figure 3, the effects on skilled labour are reversed. In particular, in both cases, returns to skilled labour (both w_t^s and $w_t^s Z_t^s$) and e_t^s increase. Therefore, in terms of the earlier analysis, under OJL with spill-over effects, the increase in e_t^u is critical for the fall

[32] We do not present these results here to save on space but they are available on request.

in Z_t^s, and in turn for the reduction in mpl_t^s, resulting in the fall in returns and employment for skilled workers. Moreover, although the qualitative effects of the increase in $\tau_t^{v,u}$ on returns and employment for unskilled labour are similar across the three versions of the model in Figures 3 and 5, there are quantitative differences regarding wages and earnings for the workers between the models with and without OJL. In particular, under OJL, the positive effects of increased employment on output and capital are amplified, via the propagation mechanism of endogenous labour productivity, which was analysed earlier. As a result, under OJL, the response of y_t and c_t are more positive, thus working to increase wages (via the bargaining process) and earnings by more than without OJL.

To summarise, omitting the OJL channel from the analysis of labour market effects of subsidies to job posting costs biases downwards the benefits to unskilled workers in terms of increased labour earnings and biases upwards benefits to skilled labour, so that the policy appears as a *Pareto* improving intervention. Furthermore, omitting *positive* spill-over learning effects across workers in OJL leads to a failure to capture the *negative* effects effects of $\tau_t^{v,u}$ for skilled labour, thus making increases in $\tau_t^{v,u}$ appear as a *Pareto* improving intervention.

6 Conclusions

This paper evaluated the effects of supply side fiscal interventions on sectoral productivity and earnings inequality in a business cycle model with search and matching frictions. We extended the canonical model by including capital-skill complementarity in production, labour markets with skilled and unskilled workers as well as OJL within and across skill types. These extensions capture key characteristics of skilled and unskilled labour markets in the data.

Our first policy result is that increases in government spending to enhance unskilled productivity via OJL is beneficial to employed unskilled workers and decreases earnings inequality between employed skilled and unskilled labour. However, it increases unskilled unemployment and thus increases labour income inequality within the group of unskilled labour. This happens because following the rise in labour productivity, firms find it optimal to employ fewer, more productive workers, and make savings from posting vacancies. Hence, this policy has ambiguous effects on the welfare of unskilled population as a whole.

Second, we find that vacancy subsidies work to increase unskilled employment and returns to unskilled workers, providing a more comprehensive support mechanism to low-income, low-skill labour. However, they do lead to an increase in unemployment for skilled workers and to a short-run reduction in skilled wages and labour income. This happens because the displacement of capital and the increase in the productivity of skilled workers via OJL, following the rise in the unskilled labour input, lead to reductions in the marginal product of skilled labour, which puts downward pressure on the employment and returns to skilled labour. Hence this policy is not *Pareto* improving either.

Thus, while these policies have many benefits, and vacancy subsidies are preferable for the unskilled, they are not without their short-comings. An obvious suggestion is to complement these with further interventions. In particular, a natural combined policy would be to increase vacancy subsidies for both unskilled and skilled workers. Indeed, we find that it is possible to increase skilled vacancy subsidies to nullify the negative effects on skilled employment following an increase in unskilled vacancy subsidies, and these have to increase by 1% of the increase of unskilled vacancy subsidies. Hence, a small intervention in the skilled labour market can complement the intervention in the unskilled labour market to improve employment for all workers.

References

[1] Acemoglu, D. (2001). 'Good jobs versus bad jobs', *Journal of Labor Economics*, 19, 1–21.

[2] Acemoglu, D. and D. Autor (2011). 'Skills, tasks and technologies: implications for employment and earnings', in O. Ashenfelter and D. Card (eds.), *Handbook of Labor Economics*, 4b, 1043-1171, Amsterdam: Elsevier.

[3] Angelopoulos, K., Malley, J., and A. Philippopoulos (2017). 'Human capital accumulation and transition to skilled employment', *Journal of Human Capital,* (forthcoming, vol. 11, no. 1).

[4] Arseneau, D. and S. Chugh (2012). 'Tax smoothing in frictional labor markets', *Journal of Political Economy*, 120, 926-985.

[5] Blanchard, O. and P. Diamond (1989). 'The beveridge curve', *Brookings Papers on Economic Activity*, 1, 1-60.

[6] Cahuc, P., Postel-Vinay, F. and J. Robin (2006). 'Wage bargaining with on-the-job search: theory and evidence', *Econometrica*, 74, 323-364.

[7] Campolmi, A. and S. Gnocchi (2016). 'Labor market participation, unemployment and monetary policy', *Journal of Monetary Economics*, 79, 17–29.

[8] Campolmi A., Faia, E. and R. Winkler (2011). 'Fiscal Calculus and the Labor Market', *The B.E. Journal of Macroeconomics*, 11, 1-27.

[9] Castro, R. and D. Coen-Pirani (2008). 'Why have aggregate skilled hours become so cyclical since the mid-1980s?', *International Economic Review*, 49, 135-184.

[10] Davis, S. and T. von Wachter (2011). 'Recessions and the costs of job loss', *Brookings Papers on Economic Activity*, 43, 1-72.

[11] Dolado, J., Jansen, M. and J. Jimeno (2009). 'On-the-job search in a matching model with heterogeneous jobs and workers', *Economic Journal*, 119, 200-228.

[12] Doppelt, R. (2016). 'Skill flows: a theory of human capital and unemployment', working paper.

[13] Fallick, B. and C. Fleischman (2004). 'Employer-to-employer flows in the U.S. labor market: the complete picture of gross worker flows', Working Paper 2004-34, Federal Reserve Board of Governors.

[14] Gertler, M. and A. Trigari (2009). 'Unemployment fluctuations with staggered Nash wage bargaining', *Journal of Political Economy*, 117, 38-86.

[15] Goldin, C. and L. Katz (2008). *The race between education and technology*, Harvard University Press.

[16] Greenwood, J., Hercowitz, Z. and P. Krusell (2000). 'The role of investment-specific technological change in the business cycle', *European Economic Review*, 44, 91-115.

[17] Hagedorn, M., Manovskii, I. and S. Stetsenko (2016). "Taxation and unemployment in models with heterogeneous workers," *Review of Economic Dynamics*, 19, 161-189.

[18] He, H. and Z. Liu (2008). 'Investment-specific technological change, skill accumulation, and wage inequality', *Review of Economic Dynamics*, 11, 314-334.

[19] He, H. (2012). 'What Drives the Skill Premium: Technological change or demographic variation?', *European Economic Review*, 56, 1546-1572.

[20] Heathcote, J., F. Perri and G. Violante (2010). 'Unequal we stand: An empirical analysis of economic inequality in the United States, 1967–2006', *Review of Economic Dynamics*, 13, 15-51.

[21] Heijdraa, B. and J. Ligthart (2002). 'The hiring subsidy cum firing tax in a search model of unemployment', *Economics Letters*, 75, 97-108.

[22] Hosios, A. (1990). 'On the efficiency of matching and related models of search and unemployment', *Review of Economic Studies*, 57, 279-298.

[23] Jung, P. and K. Kuester (2015). 'Optimal Labor-Market Policy in Recessions', *American Economic Journal*: *Macroeconomics*, 7, 124-156.

[24] Katz, L. and K. Murphy (1992). 'Changes in relative wages, 1963-1987: supply and demand factors', *The Quarterly Journal of Economics*, 107, 35-78.

[25] Krause, M. and T. Lubik (2006). 'The cyclical upgrading of labour and on-the-job search', *Labour Economics*, 13, 459-77.

[26] Krause, M. and T. Lubik (2010). 'On-the-job search and the cyclical dynamics of the labor market', Federal Reserve Bank of Richmond Working Paper Series WP, 10–12.

[27] Krusell, P., Ohanian, L., RÌos-Rull J. and G. Violante (2000). 'Capital-skill complementarity and inequality: a macroeconomic analysis', *Econometrica*, 68, 1029-1053.

[28] Laureys, L. (2014). 'The cost of human capital depreciation during unemployment', Bank of England Working Paper, No. 505.

[29] Lindquist, M. (2004). 'Capital–skill complementarity and inequality over the business cycle', *Review of Economic Dynamics*, 7, 519-540.

[30] Merz, M. (1995). 'Search in the labor market and the real business cycle', *Journal of Monetary Economics*, 36, 269-300.

[31] Petrongolo, B. and C. Pissarides (2001). 'Looking into the black box: a survey of the matching function', *Journal of Economic Literature,* 39, 390-431.

[32] Pilossoph, L. (2012). A multisector equilibrium search model of labor reallocation, Ph.D. thesis, The University of Chicago.

[33] Pissarides, C. (1986). 'Vacancies and unemployment in Britain', *Economic Policy*, 3, 499-559.

[34] Pissarides, C. (1994). 'Search unemployment with on the job search', *Review of Economic Studies* 61, 457–476.

[35] Pissarides, C. (1998). 'The impact of employment tax cuts on unemployment and wages: the role of unemployment benefits and tax structure', *European Economic Review,* 42, 155-183.

[36] Pollak, R. (2013). 'Allocating household time: when does efficiency imply specialization?' NBER Working Papers 19178.

[37] Pourpourides, P. (2011). 'Implicit contracts and the cyclicality of the skill-premium', *Journal of Economic Dynamics and Control*, 35, 963-979.

[38] Prescott, C.,Ueberfeldt, A., and S. Cociuba (2009). 'U.S. hours and productivity behavior: using CPS hours worked data: 1947-III to 2009-III,' Mimeo, November 2009.

[39] Quadrini, V. and V. Rios-Rull (2015). 'Inequality in Macroeconomics", in A.B. Atkinson and F.J. Bourguignon (Eds.), Handbook of Income Distribution, Volume 2B, 1229-1302.

[40] Rogerson, R. and R. Shimer (2011). 'Search in macroeconomic models of the labor market', Handbook of Labor Economics, Elsevier.

[41] Shi, S. and Q. Wen (1999). 'Labor market search and the dynamic effects of taxes and subsidies', *Journal of Monetary Economics*, 43, 457-495.

[42] Shimer, R. (2005). 'The cyclical behavior of equilibrium unemployment and vacancies', *American Economic Review*, 95, 25-49.

[43] Shimer, R. (2010). *Labor Markets and Business Cycles*, CREI Lectures in Macroeconomics, Princeton University Press.

[44] Uhlig, H. (2010). 'Some fiscal calculus', *American Economic Review*, 100, 30-34.

Appendix A: Derivation of Wage Equations

Using (49) and its one-period lead to eliminate $V_{e^s}(k_t, e_t^s, e_t^u)$ and $V_{e^s}(k_{t+1}, e_{t+1}^s, e_{t+1}^u)$ in the envelope condition for skilled employment (38) yields an expression whose both sides are multiplied by $c_t(1-\phi^s)$ and divided by n^s to give:

$$\begin{aligned} &\phi^s(1-\tau^s) J_{l^{f,s}}\left(l_t^{f,s}, l_t^{f,u}\right) Z_t^s = (1-\tau^s) w_t^s Z_t^s (1-\phi^s) - \\ &-\xi(1-\phi^s) c_t + (1-\phi^s)(1-\tau^s) w_t^s e_t^s \tfrac{\partial Z_t^s}{\partial e_t^s} + \\ &+ (1-\gamma_t^s - p_t^s) \beta E_t \phi^s \tfrac{(1-\tau^s)c_t}{c_{t+1}} J_{l^{f,s}}\left(l_{t+1}^{f,s}, l_{t+1}^{f,u}\right) Z_{t+1}^s. \end{aligned} \tag{A1}$$

Multiplying $E_t \frac{c_t}{c_{t+1}}$ by $\frac{\widetilde{r}_{t+1}}{\widetilde{r}_{t+1}}$ in $(A1)$, substituting for $\widetilde{r}_{t+1}^{-1} J_{l^{f,s}}\left(l_{t+1}^{f,s}, l_{t+1}^{f,u}\right)$ using (33) and $J_{l^{f,s}}\left(l_t^{f,s}, l_t^{f,u}\right)$ using (34), and substituting $\frac{A_t^k}{c_t}$ for $\beta E_t\left(\frac{\widetilde{r}_{t+1}}{c_{t+1}}\right)$ in (A1) using the Euler (21) yields:

$$\begin{aligned} &\phi^s\left[mpl_t^s - w_t^s + (1-\gamma_t^s)\tfrac{\varphi^s}{q_t^s}(1-\tau^{v,s})\right] = w_t^s(1-\phi^s) - \tfrac{\xi(1-\phi^s)c_t}{(1-\tau^s)Z_t^s} + \\ &+ (1-\phi^s) w_t^s e_t^s \tfrac{\partial Z_t^s}{\partial e_t^s} + (1-\gamma_t^s - p_t^s) E_t \phi^s \left[\tfrac{\varphi^s}{q_t^s}(1-\tau^{v,s})\right] \tfrac{Z_{t+1}^s}{Z_t^s} A_t^k. \end{aligned} \tag{A2}$$

Simplifying the resulting expression gives the Nash wage rate for skilled workers:

$$\begin{aligned} w_t^s \;=\; & [\phi^s\{mpl_t^s + [1-\gamma_t^s - (1-\gamma_t^s - p_t^s) \times \\ & \times E_t A_t^k \left(\frac{Z_{t+1}^s(1-\tau^s)}{Z_t^s(1-\tau^s)}\right)] \frac{\varphi^s}{q_t^s}(1-\tau^{v,s})\} + \frac{\xi(1-\phi^s)c_t}{(1-\tau^s)Z_t^s}\Bigg] \times \\ & \times \left[\phi^s + \frac{(1-\phi^s)\left(Z_t^s + e_t^s \frac{\partial Z_t^s}{\partial e_t^s}\right)}{Z_t^s}\right]^{-1} \end{aligned} \tag{A3}$$

Working similarly, we can derive the Nash wage rate for unskilled workers:

$$\begin{aligned} w_t^u \;=\; & [\phi^u\{mpl_t^u + [1-\gamma_t^u - (1-\gamma_t^u - p_t^u) \times \\ & \times E_t A_t^k \left(\frac{Z_{t+1}^u(1-\tau^u)}{Z_t^u(1-\tau^u)}\right)] \frac{\varphi^u}{q_t^u}(1-\tau^{v,u})\} + \frac{\xi(1-\phi^u)c_t}{(1-\tau^u)Z_t^u}\Bigg] \times \\ & \times \times \left[\phi^u + \frac{(1-\phi^u)\left(Z_t^u + e_t^u \frac{\partial Z_t^u}{\partial e_t^u}\right)}{Z_t^u}\right]^{-1} \end{aligned} \tag{A4}$$

Alternatively, the wage equations that result from Nash bargaining can be written as in (50)-(51) of the main text.

Appendix B: Steady-States All Models

Appendix Table 1

	base	OJL spillover	OJL no spillover
$\frac{c}{y}$	0.571	0.559	0.554
$\frac{k}{y}$	8.198	8.174	8.167
$\frac{g}{y}$	0.225	0.217	0.216
$\frac{b}{y}$	2.520	2.520	2.392
$\frac{we}{y}$	0.660	0.636	0.628
$\frac{v}{y}$	0.024	0.044	0.051
s^s	0.033	0.034	0.030
s^u	0.071	0.072	0.064
s	0.054	0.0545	0.0486
$\frac{w^s}{w^u}$	1.680	1.681	1.706
$\widetilde{r}$	0.010	0.010	0.010
r^b	0.010	0.010	0.010
z^s	1.089	1.023	1.340
z^u	0.971	0.946	1.220
p^s	0.828	0.807	0.892
p^u	0.591	0.584	0.663
q^s	0.760	0.789	0.671
q^u	0.609	0.617	0.543
y	1.893	1.847	1.857
c	1.080	1.033	1.029
k	15.515	15.101	15.168
w^s	1.689	1.591	1.580
w^u	1.006	0.946	0.926
mpl^s	1.734	1.667	1.670
mpl^u	1.079	1.081	1.079
$\frac{\partial mpl^s}{\partial k^f}$	0.065	0.064	0.064
$\frac{\partial mpl^s}{\partial l^{f,s}}$	-2.995	-2.907	-2.902
$\frac{\partial mpl^s}{\partial l^{f,u}}$	0.592	0.584	0.581
$\frac{\partial mpl^u}{\partial k^f}$	0.013	0.013	0.013
$\frac{\partial mpl^u}{\partial l^{f,s}}$	0.592	0.584	0.581
$\frac{\partial mpl^u}{\partial l^{f,u}}$	-0.896	-0.889	-0.879

Figure 1: Stylised facts (1992:1−2011:4)

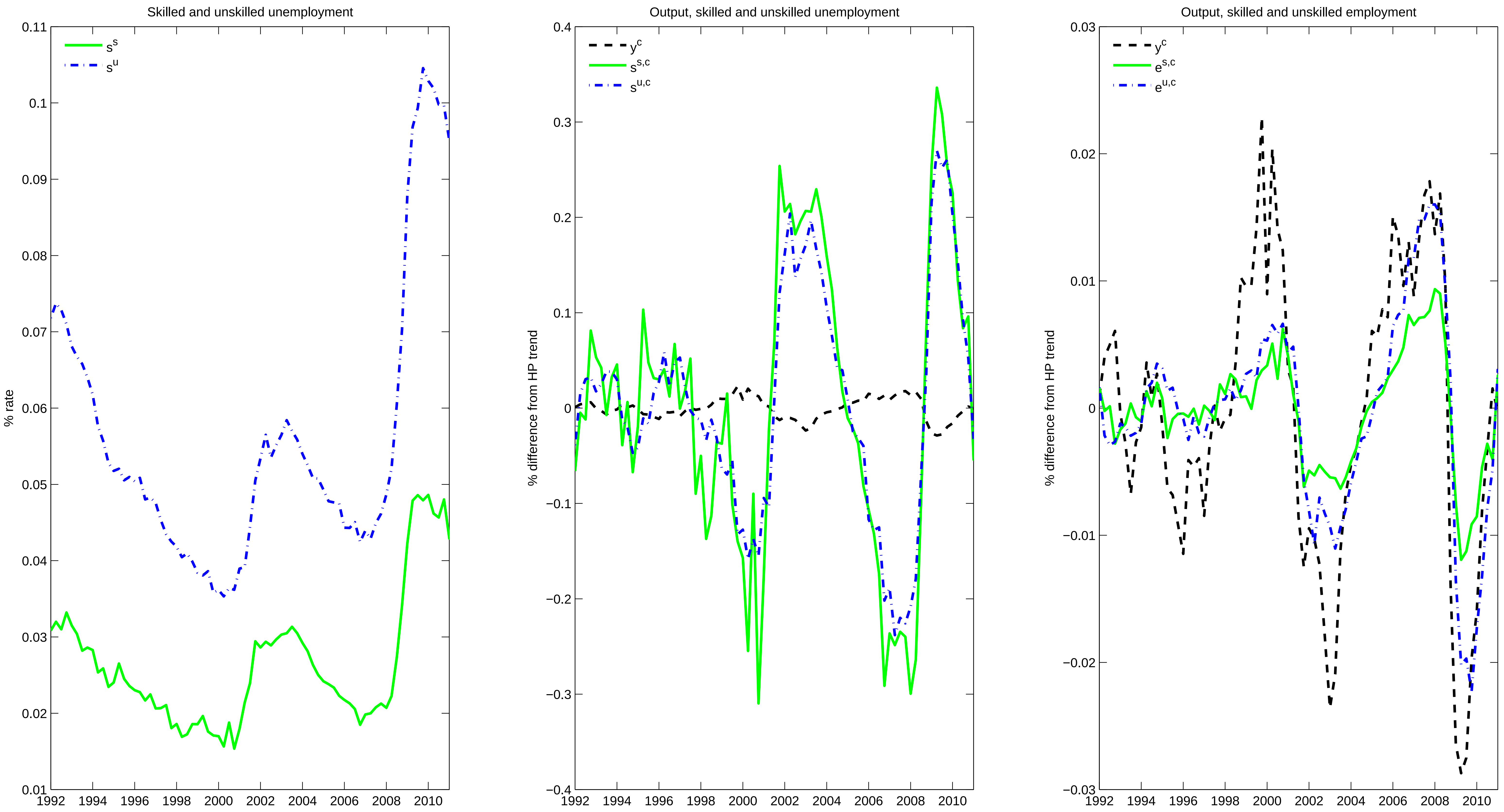

Figure 2: Temporary 1 percentage point rise in $g^{z,u}$

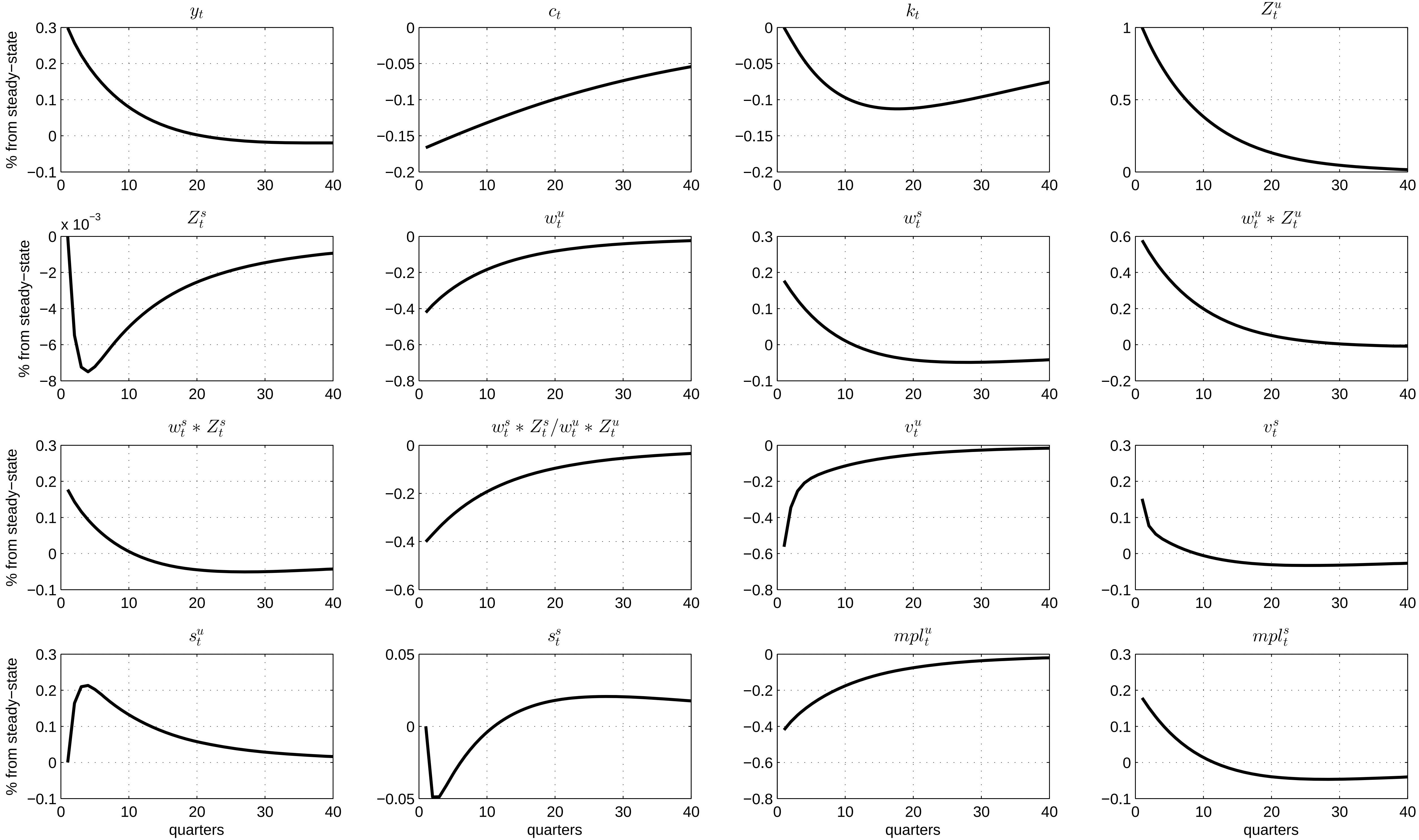

Figure 3: Temporary 1 percentage point rise in $\tau^{v,u}$

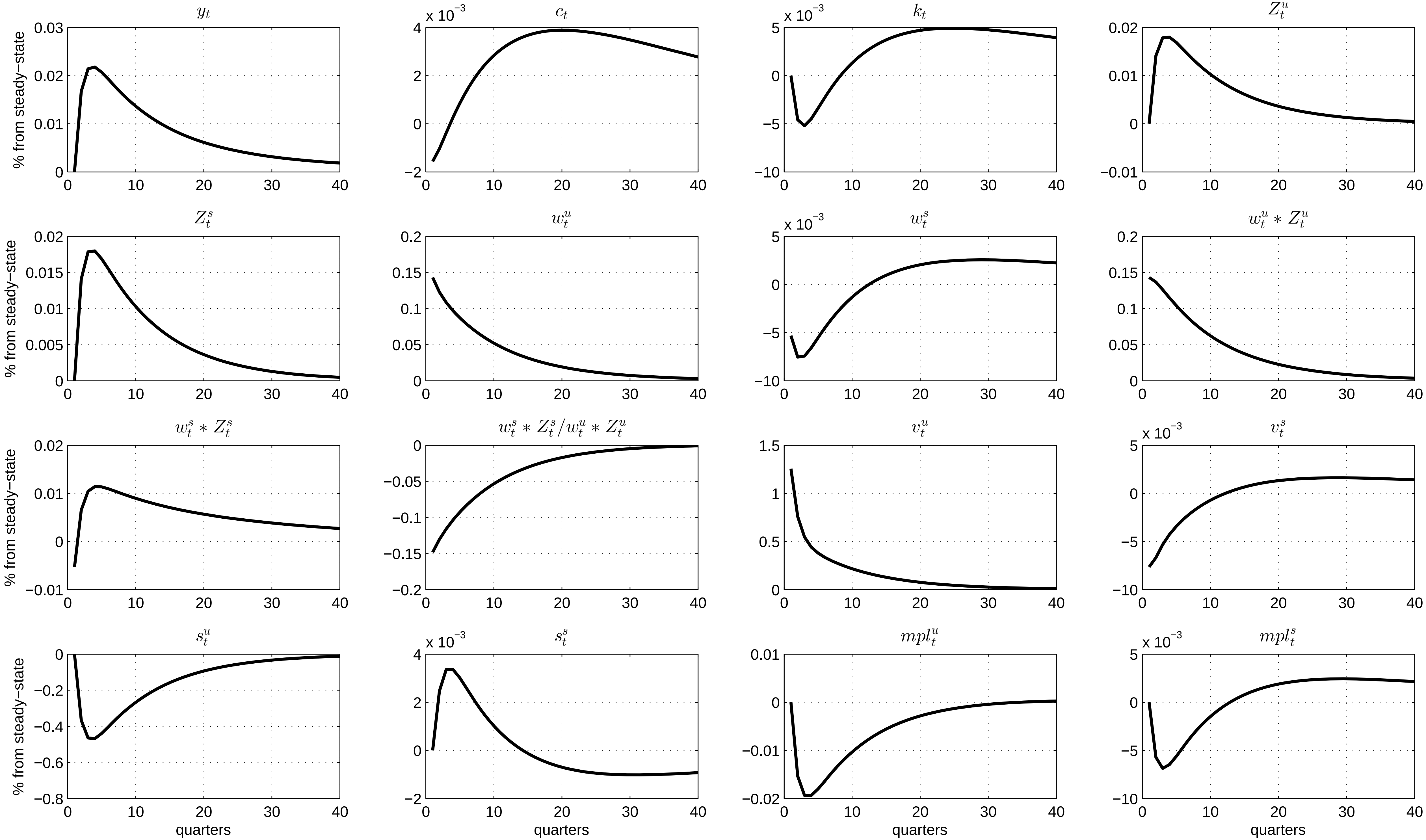

Figure 4: Temporary 1 percentage point rise in $\tau^{v,u}$ and 0.01 percentage point rise in $\tau^{v,s}$

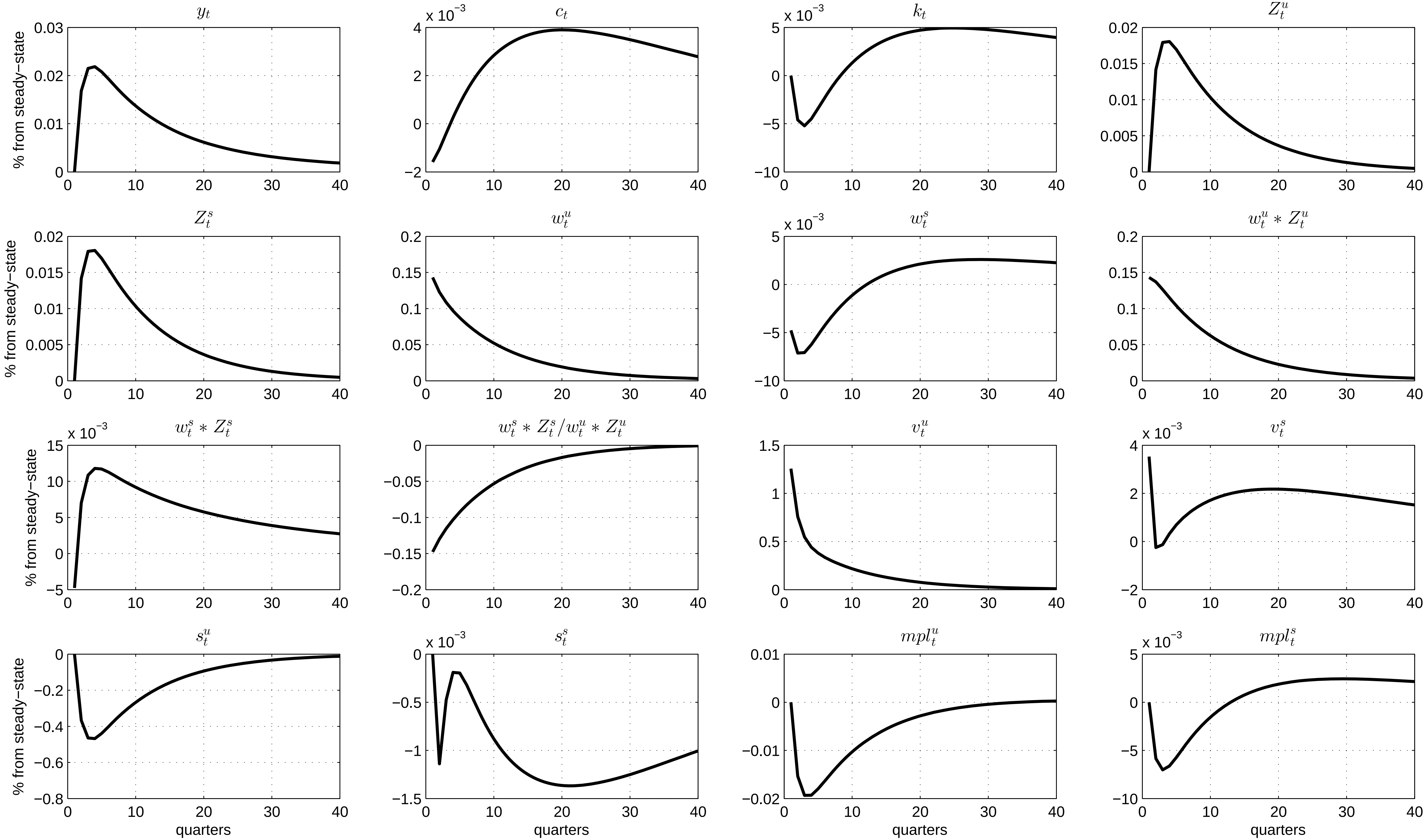

Figure 5: Temporary 1 percentage point rise in $\tau^{v,u}$