Jobs for the boys! Meritocracy, public sector pay and human capital accumulation*

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Abstract

We set a model with search and matching frictions to understand the effects of non-meritocratic hiring and wages in the public sector on unemployment and education decisions. If the public sector hiring is non-meritocratic, a smaller education premium in the public sector reduces incentives for education. More meritocratic hiring increases unemployment, particularly when the public sector wages are high relative to the private sector.

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1 Introduction

Public sector employment is a major element of the labour market. On average, in European economies, public sector employment represents 18% of total employment. It seems undisputable that public sector employment and wage policies affect aggregate labour market outcomes.

Governments hire workers to produce public goods, but they do not face the same competitive forces as private firms. As a result, governments use their employment and wage policies to accomplish a multitude of, sometimes conflicting, goals: attain budgetary targets [Poterba and Rueben (1998); Gyourko and Tracy (1989)], implement macroeconomic stabilization policy [Keynes (1936); Holm-Hadulla, Kamath, Lamo, Pérez, and Schuknecht (2010); Lamo, Pérez, and Schuknecht (2013)], redistribute resources [Alesina, Baqir, and Easterly (2000); Alesina, Danninger, and Rostagno (1999); Wilson (1982)], and satisfy interest groups for electoral gains [Borjas (1984); Matschke (2003); Gelb, Knight, and Sabot (1991)]. In particular, this paper builds upon the observation that, in several countries, the government hiring and pay do not follow meritocratic practices.

Scoppa (2009) finds evidence of non-meritocratic public sector hiring in Italy, where the probability of working there in the public sector is 44% higher for individuals whose parent also works in the public sector. Martins (2010) finds that, in Portugal, between 1980 and 2008, over the months preceding an election, appointments in state-owned firms increased significantly compared to private sector firms. Hiring also increased after elections, but only if a new government took office. Fafchamps and Labonne (2014), following the 2007 and 2010 municipal elections in Philippines, find that individuals who share one or more family names with a local elected official are more likely to be employed in better-paying occupations, compared to individuals with family name of the loosing candidates. Furthermore, the magnitude of the effect is consistent with a preferential treatment of relatives as managers in the public sector.

Another dimension of non-meritocracy relates to public sector pay. Several papers that use micro level data to estimate public sector wage differentials relative to the private sector, find that, on average, the public sector pays higher wages, specially to low-skilled workers. This wage compression across education groups in the public sector imply that the returns of education are much lower in the public sector. Examples include: Katz and Krueger (1991) for the United States, Postel-Vinay and Turon (2007) or Disney and Gosling (1998) for the United Kingdom, and by Christofides and Michael (2013), Castro, Salto, and Steiner (2013) and Giordano *et al.* (2011) for several European countries.

We aim to study the implication of the lack of meritocratic hiring and wage compression in the public sector on labour market outcomes and the incentives to accumulate human capital. Non-meritocratic hiring and a pay that is not linked to the productivity of workers in the public sector might reduce the returns to education. Furthermore, the pursue of rent-seeking activities might entail negative effects on overall productivity.

We set up a search model, where workers can search for jobs in either the private or the public sector. Employment and wages in the private sector are determined through the usual channels of profit maximization and Nash bargaining. This ensures that hiring and wage setting in the private sector are meritocratic: more productive (educated/skilled) workers have better chances of finding a job, better bargaining position in wage setting, and receive higher wages. In the public sector, by contrast, wages are chosen exogenously. We will take into account evidence of cronyism in hiring workers in the public sector, by assuming that job seekers can use their personal relationships and connections to find a public sector job. We will assume that, prior to enter the labour market, workers have to pay a cost to get "connections" that is draw from an exogenous distribution of across workers. In our setting, non-meritocratic means that the government will reserve some of its jobs only to workers that have "connections". If such practices are in place, in equilibrium, workers with stronger connections can more easily find public sector jobs.

In this setup, we will incorporate a human capital accumulation decision. Prior to entering the labour market, individuals also decide whether to invest in education. Investing in education is costly but yields returns, as those highly educated are more productive and, thus, benefit from higher job finding rate and wages in both sectors. We will assume that workers are heterogeneous with respect to their education costs, reflecting either different learning abilities or financial constraints. Thus, only a fraction of individuals - those whose benefits exceeds the costs - will invest in education.

The model provides two main findings. The first main finding is that the interaction between non-meritocratic hiring and government policies is determinant for the education decision. If all public sector hiring is meritocratic, the government employment, wages and separation rates, do not affect the educational composition of the labour force. This happens because any improvement in the value of working in the public sector is fully neutralized by an increase of the unemployed queueing for public sector jobs and the consequent reduction in job finding rate. However, if a fraction of jobs are devoted to connected workers, this limited mobility will reduce the flow from the private to the public sector and, hence, government policies will affect the incentives for education. In particular, any government policy that raises the value of working in the public sector for unskilled workers, relative to educated workers, will reduce the proportion of educated workers in the labour force.

The second main finding is perhaps surprising. Although clearly inefficient, the recruitment through "connections" reduces unemployment, specially when the public sector offers high wages relative to the private sector. When the value of working in the public sector is high compared to the private sector (either because of high wages or low separation rate), more unemployed are going to queue for these jobs, moving away from the private sector. If most of these jobs are only available through "connections", then the unconnected unemployed are not going to try to search, and only the few "connected" will.

This paper contributes to the recent labour market search literature that analyses the role and effects of public sector employment and wages. Burdett (2012) includes a public sector in a job-ladder framework where firms post wages. Bradley, Postel-Vinay, and Turon (2017) further introduce on-the-job search and transitions between the two sectors to study the effects of public sector policies on the distribution of private sector wages. Albrecht, Robayo-Abril, and Vroman (2016) consider heterogeneous human capital and match specific productivity in a Diamond-Mortensen-Pissarides model. Michaillat (2014) shows, that the crowding out effect of public sector employment is lower during recessions, giving rise to higher government spending multipliers. These papers assume that unemployed randomly search across sectors and, hence, the public sector policies only affect the equilibrium by affecting the outside option of the unemployed and their reservation wage.

Hörner, Ngai, and Olivetti (2007) study the effect of turbulence on unemployment when wages in the public sector are insulated from this volatility. Quadrini and Trigari (2007) analyse the effects of exogenous business cycle rules on volatility. Gomes (2015) has emphasise the role of the public sector wage policy in achieving the efficient allocation, while Afonso and Gomes (2014) highlighted the interactions between private and public wages. As in the our model, these papers assume that the unemployed can chose whether to search in the private or public sectors, and that this fraction will vary depending on the government hiring, separation and wage policies. This mechanism is discussed in length in Gomes (2015).

With the exception of Albrecht, Robayo-Abril, and Vroman (2016), all these papers do not consider explicitly the heterogeneity in terms of education. This is quite an oversight, given that the government hires predominantly workers with college degree and that the public sector wage premium varies substantially with education. Other papers that consider heterogeneity include Gomes (2016) that examines the effects of a public sector wage reform that eliminates the wage premium-gap for all types of public sector workers. Domeij and Ljungqvist (2016) study, in a frictionless labour market how the public employment hiring of skilled and unskilled workers in Sweden and the US can explain the different evolutions of the skill premium in the two countries. In a model of occupational choice, Gomes and Kuehn (2014) analyse the effects of skill biased hiring in the public sector on occupational choice of entrepreneurs and firm size.

All these papers take the education endowment as exogenous. In our model, the choice of education is endogenous and hence it can be affected by the government policies. This feature is only present in one other paper. Wilson (1982) argues that the government might use its employment policy to change equilibrium prices and redistribute income among different groups. In his model, the government policies affect the education choice. We show that this is only the case when the government hiring is non-meritocratic.

2 General set up

We consider a search and matching model with private sector firms and a public sector. Workers can be either employed and producing or unemployed and searching for a job. Each private sector firm is endowed with a single vacancy which can be vacant or filled (job). At each instant, τ individuals are born and die so that the population is constant and normalized to unity. All agents are risk neutral and discount the future at a common rate $r > 0$, time is continuous.

An agent can be either low or high educated. All individuals are born low-educated, but prior to entering the labor market, they can become high educated by paying a schooling cost ϵ. The schooling cost is distributed across individuals according to the cumulative distribution function $\Xi^{\epsilon}(\cdot)$ on $[0, \bar{\epsilon}]$. Heterogeneity with respect to schooling cost reflects either different learning abilities or the existence of financial constraints.

In parallel, all workers can become "connected", by paying a cost c. The cost is distributed across individuals according to the cumulative distribution function $\Xi^{c}(\cdot)$ on $[0, \bar{c}]$. "Connected" workers are as productive as workers with no connections, but might have priority - higher job finding rate - for public sector jobs.

An endogenous proportion of the population (consisting of those whose schooling cost is sufficiently low) will become high educated; another fraction (consisting of those whose connection cost is sufficiently low) will become 'connected. If both costs are low, workers will become "connected" and high educated, while the rest will remain either low educated or "unconnected" or both. Variables are therefore indexed by the subscript $x = [g, p]$, where g refers to the public (government) sector and p to the private sector, and two superscript $i = [l, h]$ and $j = [c, u]$, where c refers to "connected", u refers to "unconnected", h refers to high- and l to low-educated.

In each of the two sectors there are two labor markets segmented by education. In the "high-education" market firms open vacancies suited for the high-educated workers, whereas in the "low-education" market vacancies are suited for low-educated workers; high-educated individuals direct their search towards type-h jobs whereas, low-educated workers direct their search towards type-l jobs. A searching (unemployed) worker of type i receives a flow of income b_i, which can be considered as the opportunity cost of employment. The output y_i^x of a match between a worker and a firm depends on the worker's education and high-educated individuals (jobs) are more productive than low-educated individuals (jobs) ($y_h^x > y_l^x$), which is independent of the "connections" status.

2.1 The Private sector

The two sectors (private and public) differ substantially in two aspects: hiring practices and wage-setting. The rate at which high- and low-educated workers are hired into private-sector jobs is endogenous and depends on firm profits and job entry. In particular, firms in each of the two labor markets of the private sector open vacancies and search for suitable workers until all rents are exhausted. The rate at which type-i workers find private-sector jobs of type i depends positively on the tightness, $\theta_i = \frac{v_i^p}{u_i^p}$, where v_i^p is the measures of private-sector vacancies of type i and u_i^p the fraction of type i workers that are unemployed and searching in the private sector. Workers of type i are hired into private-sector jobs (of type i) at Poisson rate $m(\theta_i)$ and private-sector firms fill type i vacancies at rate $q(\theta_i) = \frac{m(\theta_i)}{\theta_i}$.

Wages in the private sector, denoted as w_i^p, are also endogenous, depend on match surplus and differ by education level (i). They are determined by Nash bargaining and are such that the worker get a share β of match surplus while the rest goes to the firm. With higher match surplus, firms expect to generate larger profits from creating jobs, firm entry is higher, workers can more easily find jobs and can also earn higher wages. Hence, the private-sector hiring and wage-setting procedures are in a sense meritocratic: if due to their higher productivity high-educated workers generate higher rents to firms than low-educated workers, then high-educated workers will also enjoy higher job offer and wage rates than low-educated workers. Any differences in wage and job offer rates between the two types of workers reflect nothing but differences in their productivity (match surplus).

A vacant firm bears a recruitment cost κ_i specific to education, related to the expenses of keeping a vacancy open and looking for a worker. When a vacancy and a worker are matched, they bargain over the division of the produced surplus. The surplus that results from a match is known to both parties. After an agreement has been reached, production commences immediately. Matches in the private sector with workers of type i dissolve at the rate s_i^p. Following a job destruction, the worker and the vacancy enter the corresponding sector/market and search for new match.

2.2 Government

In the public sector, by contrast, policies are taken to be exogenous. To produce some government services, the government hires an exogenous number of workers, with and without education (e_h^g, e_l^g). In each period the government has to hire enough workers to compensate the workers that exogenously separate or retire. That means hiring $(s_h^g + \tau)e_h^g$ skilled and $(s_l^g + \tau)e_l^g$ unskilled workers, where s_i^g is the separation rate. A fraction μ_i of these workers will be hired through "connections". The matching function in the public sector is $M_{i,j}^g = \min\{v_{i,j}^g, u_{i,j}^g\}$. We assume that the number of searchers in each segment of the public sector, $u_{i,j}^g$, is at least equal to the number of job openings, $v_{i,j}^g$, meaning that $M_{i,j}^g = v_{i,j}^g$ and no vacancies in the public sector remain unfilled. [1] Since the government's objec-

[1]Alternatively, we could assume that the hiring cost of the government is independent of the number of hires or market tightness, and that the government can target exactly its number of workers. In the Appendix we illustrate the case of a Cobb Douglas matching function under these assumptions.

tive is maintain employment levels (e_h^g, e_l^g) by hiring enough workers to replace those that separate or retire, it follows that $v_{i,u}^g = (1-\mu_i)(s_i^g+\tau)e_i^g$ and $v_{i,c}^g = \mu_i(s_i^g+\tau)e_i^g$. Connected and unconnected workers of type-i find public-sector jobs at rate $m_{i,c}^g = \frac{\mu_i(s_i^g+\tau)e_i^g}{u_{i,c}^g}$ and $m_{i,u}^g = \frac{(1-\mu_i)(s_i^g+\tau)e_i^g}{u_{i,u}^g}$, respectively. For the moment we do not assume any bias in terms of education and set $\mu_h = \mu_l = \mu$. Moreover, we assume that μ is given and independent of conditions in either the private or the public sector. We relax these assumptions later.

Finally, the public sector wages, $(\bar{w}_h^g, \bar{w}_l^g)$, are other exogenous policy variables reflecting non-meritocracy in the public sector.

2.3 Value functions, Free entry, Wages

Let U_i^p and E_i^p be the values (expected discounted lifetime incomes) associated with unemployment (searching for a job) and employment, respectively, in the private sector of a worker of education level $i = [h, l]$. These are defined by

$$(r+\tau)U_i^p = b_i + m(\theta_i)\left[E_i^p - U_i^p\right] \tag{1}$$

$$(r+\tau)E_i^p = w_i^p - s_i^p\left[E_i^p - U_i^p\right] \tag{2}$$

The values associated with unemployment in the public sector of a worker of education level $i = [h, l]$ with and without connections are given respectively by:

$$(r+\tau)U_{i,u}^g = b_i + m_{i,u}^g\left[E_{i,u}^g - U_{i,u}^g\right] \tag{3}$$

$$(r+\tau)U_{i,c}^g = b_i + m_{i,c}^g\left[E_{i,c}^g - U_{i,c}^g\right] \tag{4}$$

while the wage in the public sector does not depend on connections, the values of being employed are different for workers with and without connection:

$$
\begin{aligned}
(r+\tau)E_{i,u}^g &= w_i^g - s_i^g\left[E_{i,u}^g - U_{h,u}^g\right] && (5)\\
(r+\tau)E_{i,c}^g &= w_i^g - s_i^g\left[E_{i,c}^g - U_{h,c}^g\right] && (6)
\end{aligned}
$$

On the private-sector firm side, let J_i^p be the value associated with a job by a worker of type i and V_i^p be the value associated with posting a private-sector vacancy and searching for a type i worker to fill it. These values are given by

$$
\begin{aligned}
rJ_i^p &= y_i^p - w_i^p - (s_i^p + \tau)\left[J_i^p - V_i^p\right] && (7)\\
rV_i^p &= \kappa_i + q(\theta_i)\left[J_i^p - V_i^p\right] && (8)
\end{aligned}
$$

In equilibrium, free entry drives the value of a private vacancy to zero. This implies the following two conditions:

$$V_i^p = 0, \quad i = [h,l] \tag{9}$$

Wages are then determined by Nash bargain between the matched firm and the worker. The outside options of the firm and the worker are the value of a vacancy and the value of being unemployed, respectively. Let $S_i^p \equiv J_i^p - V_i^p + E_i^p - U_i^p$ denote the surplus of a match with a type i worker. With Nash-bargaining the wage w_i^p is set to a level such that the worker gets a share β of the surplus, and the share $(1-\beta)$ goes to the firm. This implies two equilibrium conditions of the following form:

$$\beta S_i^p = E_i^p - U_i^p \qquad (1-\beta)S_i^p = J_i^p - V_i^p \text{ for } i = [h,l] \tag{10}$$

Setting $V_i^p = 0$ in (8) and imposing the Nash bargaining condition in (10) gives:

$$\frac{\kappa_i}{q(\theta_i)} = (1-\beta)S_i^p \text{ for } i = [h,l] \tag{11}$$

Using (1)-(7) together with (10) and the free-entry condition $V_i^p = 0$ we can write:

$$S_i^p = \frac{y_i^p - b_i}{r + \tau + s_i^p + \beta m(\theta_i)} \tag{12}$$

and the free-entry condition as

$$\frac{\kappa_i}{q(\theta_i)} = \frac{(y_i^p - b_i)(1-\beta)}{r + \tau + s_i^p + \beta m(\theta_i)} \text{ for } i = [h, l] \tag{13}$$

The equation in (13) gives the two job creation conditions that determine the numbers of type-h and type-l vacancies in the private sector. The job creation condition sets the expected costs of having a vacancy (left-hand-side) equal to the expected gain from a job (right-hand-side) and it can be used to determine the equilibrium market tightness θ_i and in turn the rates at which workers find jobs in the private sector, $m(\theta_i)$.

Imposing the free-entry condition (11) for private-sector vacancy creation, the Nash bargaining solution implies

$$w_i^p = b_i + \beta(y_i^p - b_i + \kappa_i \theta_i), \quad i = [h, l] \tag{14}$$

Lemma 1 *Tightness and wages in the private sector, in both low- and high-educated submarkets, are independent of any government policy*

This lemma is a useful intermediate result and follows straightforwardly from equations (13) and (14). It implies that government policies will only affect the equilibrium either by affecting the education decision of the new born, or by affecting the scale of the private sector through the number of unemployed searching for a private sector job. Given a constant tightness, policies that make the public sector more attractive, will drain unemployed from the private sector and reduce, one-to-one the number of vacancies, leaving private wages unchanged.

2.4 Decisions and Allocations

We can summarize the six options of the newborn as

$$
\begin{aligned}
(r+\tau)U_i^p &= b_i + \frac{m(\theta_i)}{r+\tau+s_i^p+m(\theta_i)}[w_i^p - b_i] && (15)\\
(r+\tau)U_{i,u}^g &= b_i + \frac{m_{i,u}^g}{r+\tau+s_i^g+m_{i,u}^g}[w_i^g - b_i] && (16)\\
(r+\tau)U_{i,c}^g &= b_i + \frac{m_{i,c}^g}{r+\tau+s_i^g+m_{i,c}^g}[w_i^g - b_i] && (17)
\end{aligned}
$$

These six options are depicted in Figure 1. Workers, either with high or low education, can search without connections in either the public or the private sector. In equilibrium the values of these two options have to equate:

$$U_i = U_{i,u}^g = U_i^p \tag{18}$$

This equation determines the number of searchers in the public sector unconnected sector, $u_{i,u}^g$, which is the variable that compensates any asymmetry in the value of the job in the two sectors.

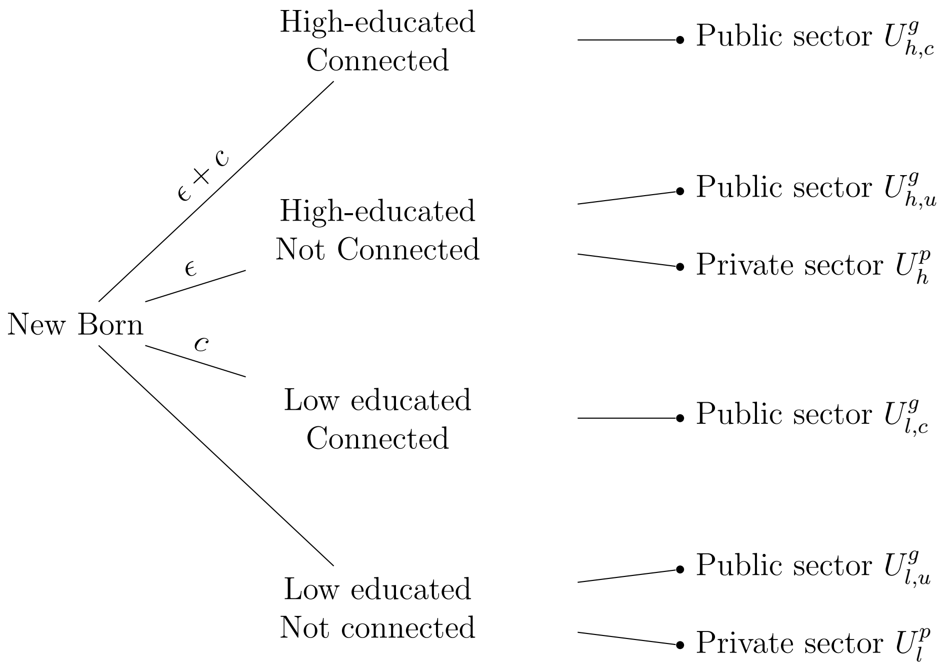
Alternatively, workers can use connections to find jobs in only the public sector. In what follows we therefore drop the superscript g in $U_{i,c}^g$ and set $U_{i,c} \equiv U_{i,c}^g$. The newborn will choose the option that given his ϵ and c gives the highest value between:

$$Max\{U_l, U_h - \epsilon, U_{l,c} - c, U_{h,c} - c - \epsilon\} \tag{19}$$

A worker of type $i = [h, l]$ and connections cost c will choose to obtain connections only if the benefit, $U_{i,c} - U_i$, exceeds the cost. That is, only if $c \leq U_{i,c} - U_i$. The threshold level of c at which a worker of type i is indifferent between using connections to find a public-sector job or not, is therefore given by

$$\tilde{c}_i = U_{i,c} - U_i \tag{20}$$

Figure 1: Decision of newborn



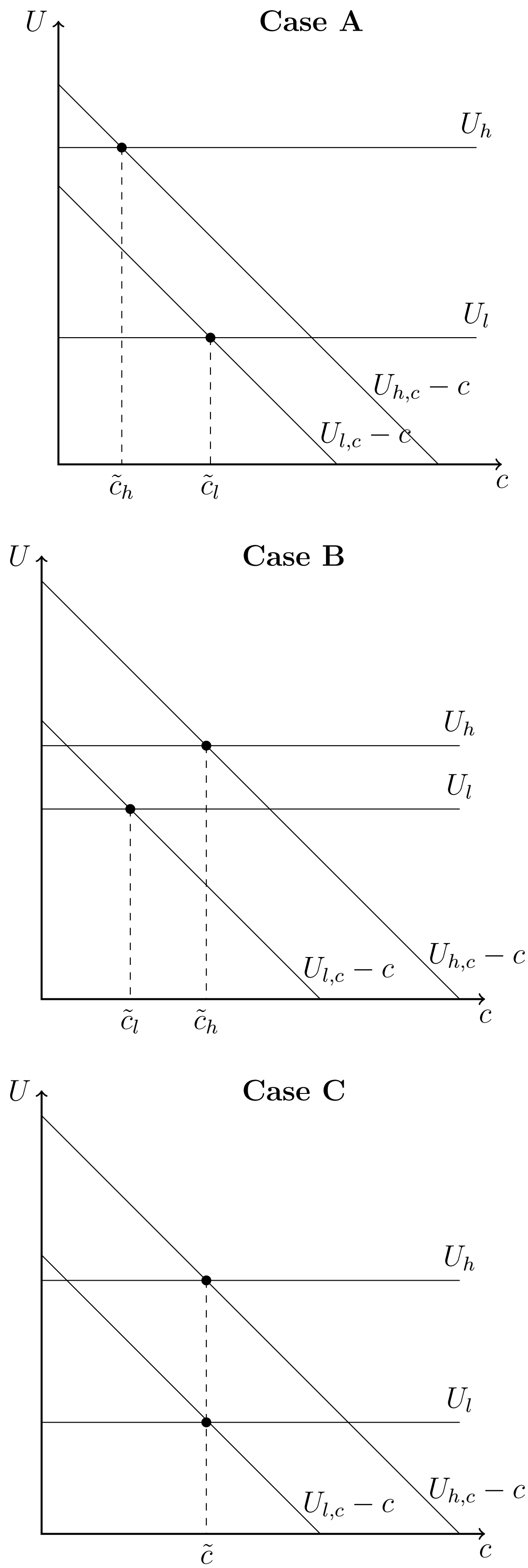
Lemma 2 *There exist a public sector "unconnected" market for workers of type i where no vacancy is left unfilled, provided the public sector pays a sufficient high wage* $\bar{w}^g_i \geq \underline{w}^g_{i,u}$. *There exists a public sector "connected" market for workers of type i where no vacancy is left unfilled, provided the public sector pays a sufficient high wage* $\bar{w}^g_i \geq \underline{w}^g_{i,c} > \underline{w}^g_{i,u}$

The exact expression for $\underline{w}^g_{i,u}$ and $\underline{w}^g_{i,c}$ are in Appendix. This lemma states that the public sector needs to pay a sufficient high wage for people to accept to search there. However, with a connection sector, this wage has to be higher, to compensate the costs of acquiring connections. In what follows, we assume that these conditions are always satisfied.

As shown in Figure 2 we can have three different cases, each having different implications for how the existence of public-sector hiring through connections alters worker's incentives to invest in education.

In the first case - case A in Figure 2 - the benefit from investing in education is smaller if the worker is planning to use connections to find a public-sector job than if not. That is, $U_h - U_l \geq U_{h,c} - U_{l,c}$ and low-educated workers have more incentive to use connections than high-educated workers ($\tilde{c}_h < \tilde{c}_l$). Case A could reflect a situation where public-sector wages

Figure 2: Decision Thresholds



are relatively flat across worker qualifications (i.e., public-sector wages are compressed), whereas in the private sector, wages increase steeply with workers' qualifications. In such cases, those seeking to use their connections in order to find jobs in the public sector, where wages are compressed, will have smaller incentive to invest in education, while those lacking connections (i.e. those whose connection cost is high) will have more incentive to opt for education. In other words, in such cases education "substitutes" for the lack of connections and vice versa. More specifically, the two thresholds $\tilde{c}_h$ and $\tilde{c}_l$ can be used to divide workers into three groups that differ in terms of incentives to obtain higher education. In the first group are workers whose connection cost c is low: $c < \tilde{c}_h (< \tilde{c}_l)$. As can be seen in the Figure for these workers, $U_{i,c} - c > U_i$ for $i = [h, l]$, and irrespective of their education level, using their connections in order to find a job in the public sector always yields a higher payoff to them than not using them. For these workers, the net benefit from investing in education is given by $\tilde{\epsilon}_c = U_{h,c} - U_{l,c}$. Next, is the group of workers whose connection cost c lies between $\tilde{c}_h$ and $\tilde{c}_l$. For these workers, attempting to find a job in the public sector through connections is worthwhile only if they remain low-educated. That is, $U_{l,c} - c > U_l$ but $U_{h,c} - c < U_h$. If they invest in eduction they are better off not using connections, thereby saving the connection cost. Their benefit from education is therefore $\tilde{\epsilon}_m(c) \equiv U_h - (U_{l,c} - c)$, which is evidently increasing in c. In the last group are the workers who will never use connections to find jobs in the public sector because their cost of doing so is very high. These are the workers with $c > \tilde{c}_l (> \tilde{c}_h)$. As can be seen, for these workers $U_i > U_{i,c} - c$ for $i = [h, l]$. If they choose to become high-educated they will obtain a payoff of $\tilde{\epsilon}_u \equiv U_h - U_l$.

In the opposite case - case B in Figure 2 - the education premium is larger in the public sector, reflecting relatively small wage compression in the public sector. In this case education "complements" the use of connections: workers targeting public-sector jobs through the use of their connections have more incentive to become high-educated ($U_{h,c} - U_{l,c} \geq U_h - U_l$), which also implies that high-educated workers have more incentive to use connections in order to find a job in the public sector compared to low-educated workers ($\tilde{c}_h > \tilde{c}_l$). As above, the low-connection-cost workers, those with $c < \tilde{c}_l (< \tilde{c}_h)$ will always choose to target public-sector jobs through connections and will yield a payoff $\tilde{\epsilon}_c$ from becoming high-educated. At the opposite end are those with $c > \tilde{c}_h (> \tilde{c}_l)$; the workers that will never use connections and will yield payoff $\tilde{\epsilon}_u$ from investing in education. In between are the workers with $\tilde{c}_l < c < \tilde{c}_h$. As can be seen in the Figure for these workers $U_{l,c} - c < U_l$ and $U_{h,c} - c > U_h$; thus, they will use connections if they become high-educated, but will not if they remain low-educated. Investing in education yields to them a benefit of $\tilde{\epsilon}_m(c) \equiv U_{h,c} - c - U_l$.

Finally, case C is the knife-edge case where the payoff from being high-educated is the same in both sectors. Having connections does not alter a worker's payoff from investing in education ($U_{h,c} - U_{l,c} = U_h - U_l$) and high- and low-educated workers both have equal incentives to use connections ($\tilde{c}_h = \tilde{c}_l$). In this case all (connected or unconnected) workers will obtain a payoff of $\tilde{\epsilon} = U_{h,c} - U_{l,c} = U_h - U_l$ from investing in education.

It can be easily verified that we can write:

$$\tilde{\epsilon}_c = \tilde{\epsilon}_u + \tilde{c}_h - \tilde{c}_l \tag{21}$$

$$\tilde{\epsilon}_m(c) = \tilde{\epsilon}_u + c - \tilde{c}_l \quad c \in [\tilde{c}_h, \tilde{c}_l], \text{ if } \tilde{c}_h < \tilde{c}_l \text{ (case A)} \tag{22}$$

$$\tilde{\epsilon}_m(c) = \tilde{\epsilon}_u + \tilde{c}_h - c \quad c \in [\tilde{c}_l, \tilde{c}_h], \text{ if } \tilde{c}_h > \tilde{c}_l \text{(case B)} \tag{23}$$

and $\tilde{\epsilon}_c \leq \tilde{\epsilon}_m \leq \tilde{\epsilon}_u$, if $\tilde{c}_h < \tilde{c}_l$ (case A), $\tilde{\epsilon}_c \geq \tilde{\epsilon}_m \geq \tilde{\epsilon}_u$, if $\tilde{c}_h > \tilde{c}_l$ (case B) and $\tilde{\epsilon}_c = \tilde{\epsilon}_m = \tilde{\epsilon}_u$, if $\tilde{c}_h = \tilde{c}_l = \tilde{c}$ (case C). A worker will invest in education only if the benefit exceeds the cost (ϵ). The eduction benefit can be either $\tilde{\epsilon}_u, \tilde{\epsilon}_m$ or $\tilde{\epsilon}_c$, depending on the worker's connection cost (c). Evidently, in case B, where the education premium is relatively large in the public sector, the benefit from education is higher for those belonging in the low-connection-cost group; these are the workers who among the three groups are more likely to become high-educated. Workers in the intermediate group follow and those with the least incentives to become high-educated are those in the high-connection-cost group. The opposite holds in case A, where wages in the public sector are compressed relative to the private sector. In that case education substitutes for the lack of connections and those most likely to invest in education are those whose connection cost is high. In case C incentives to invest in education are independent of workers' connection cost and equal fractions of connected and unconnected workers will invest in education.

Worker's decisions about whether to use connections and invest in education will determine their selection into four groups: the high- and low-educated that use connections to find public-sector jobs ($L^g_{h,c}$ and $L^g_{l,c}$) and the high- and low-educated that do not use connections ($L_{h,u}$ and $L_{l,u}$) as depicted in Figure 3. For each of the cases (A,B and C), discussed above, we can measure each of these four groups' share in the labor force as:

$$L^g_{h,c} = \begin{cases} \Xi^\epsilon(\tilde{\epsilon}_c)\Xi^c(\tilde{c}_h), & \text{if } \tilde{c}_h < \tilde{c}_l \text{ (Case A)} \\ \Xi^\epsilon(\tilde{\epsilon}_c)\Xi^c(\tilde{c}_l) + \int_{\tilde{c}_l}^{\tilde{c}_h} \Xi^\epsilon(\tilde{\epsilon}_m(c))d\Xi^c(c), & \text{if } \tilde{c}_l < \tilde{c}_h \text{ (Case B)} \\ \Xi^\epsilon(\tilde{\epsilon}_u)\Xi^c(\tilde{c}), & \text{if } \tilde{c}_h = \tilde{c}_l \text{ (Case C)} \end{cases} \tag{24}$$

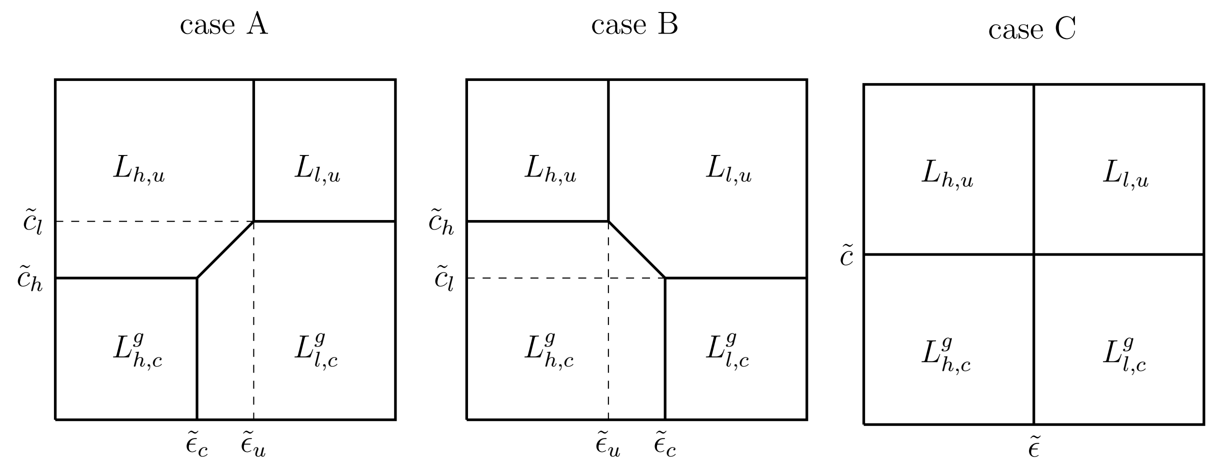
$$L^g_{l,c} = \begin{cases} (1-\Xi^\epsilon(\tilde{\epsilon}_c))\Xi^c(\tilde{c}_h) + \int_{\tilde{c}_l}^{\tilde{c}_h}(1-\Xi^\epsilon(\tilde{\epsilon}_m(c)))d\Xi^c(c), & \text{if } \tilde{c}_h < \tilde{c}_l \text{ (Case A)} \\ (1-\Xi^\epsilon(\tilde{\epsilon}_c))\Xi^c(\tilde{c}_l), & \text{if } \tilde{c}_l < \tilde{c}_h \text{ (Case B)} \\ (1-\Xi^\epsilon(\tilde{\epsilon}_u))\Xi^c(\tilde{c}), & \text{if } \tilde{c}_h = \tilde{c}_l \text{ (Case C)} \end{cases} \tag{25}$$

$$L_{h,u} = \begin{cases} \int_{\tilde{c}_h}^{\tilde{c}_l} \Xi^\epsilon(\tilde{\epsilon}_m(c))d\Xi^c(c) + (1-\Xi^c(\tilde{c}_l))\,\Xi^\epsilon(\tilde{\epsilon}_u) & \text{if } \tilde{c}_h < \tilde{c}_l \text{ (Case A)} \\ (1-\Xi^c(\tilde{c}_h))\,\Xi^\epsilon(\tilde{\epsilon}_u), & \text{if } \tilde{c}_l < \tilde{c}_h \text{ (Case B)} \\ (1-\Xi^c(\tilde{c}))\,\Xi^\epsilon(\tilde{\epsilon}), & \text{if } \tilde{c}_l = \tilde{c}_h \text{ (Case C)} \end{cases} \tag{26}$$

$$L_{l,u} = \begin{cases} (1-\Xi^\epsilon(\tilde{\epsilon}_u))(1-\Xi^c(\tilde{c}_l)), & \text{if } \tilde{c}_h < \tilde{c}_l \text{ (Case A)} \\ (1-\Xi^\epsilon(\tilde{\epsilon}_u))(1-\Xi^c(\tilde{c}_h)) + \int_{\tilde{c}_l}^{\tilde{c}_h}(1-\Xi^\epsilon(\tilde{\epsilon}_m(c)))d\Xi^c(c), & \text{if } \tilde{c}_l < \tilde{c}_h \text{ (Case B)} \\ (1-\Xi^c(\tilde{c}))(1-\Xi^\epsilon(\tilde{\epsilon}_u)), & \text{if } \tilde{c}_h = \tilde{c}_l \text{ (Case C)} \end{cases} \tag{27}$$

$L_h = L^g_{h,c} + L_{h,u}$ gives the share of high-educated in the labor force and $L_l = 1 - L_h = L^g_{l,c} + L_{l,u}$ is the share of low-educated. Among the workers (low- or high-educated) that choose not to use connections some will search in the private sector ($L^p_{i,u}$) and some in the public sector ($L^g_{i,u}$). Hence, $L_{i,u} = L^p_{i,u} + L^g_{i,u}$.

Figure 3: Allocations



Using (10)-(13)and (15)-(17) we can write:

$$\tilde{c}_i = \frac{1}{r+\tau}\left[\frac{\frac{\mu(s^g_i+\tau)e^g_i}{u^g_{i,c}}}{r+\tau+s^g_i+\frac{\mu(s^g_i+\tau)e^g_i}{u^g_{i,c}}}[w^g_i - b_i] - \frac{\beta\kappa_i\theta_i}{(1-\beta)}\right] \tag{28}$$

$$\tilde{\epsilon}_u = \frac{1}{r+\tau}\left[b_h - b_l + \frac{\beta\kappa_h\theta_h}{(1-\beta)} - \frac{\beta\kappa_l\theta_l}{(1-\beta)}\right] \tag{29}$$

Definition 3 *A steady state equilibrium consists of a set of cut-off costs $\{\tilde{c}_h, \tilde{c}_l, \tilde{\epsilon}_h, \tilde{\epsilon}_l\}$, private sector tightness $\{\theta_h, \theta_l\}$, and unemployed searching in each market $\{u^p_h, u^p_l, u^g_{h,c}, u^g_{l,c}, u^g_{h,u}, u^g_{l,u}\}$, such that, given some exogenous government policies $\{w^g_h, w^g_l, e^g_h, e^g_l, \mu\}$, the following apply.*

1. *Private sector firms satisfy the free-entry condition (13) $i = [h, l]$.*

2. *Private sector wages are the outcome of Nash Bargaining (14) $i = [h, l]$.*

3. *Newborns decide optimally their investments in education and connections (equation 19), and the population shares are determined by equations (24)-(27).*

4. *The search between the public and private sector of the unconnected unemployed satisfies equation (18).*

5. *Flows between private employment and unemployment are constant*

$$(s_h^p + \tau)e_h^p = m(\theta_h)u_h^p \tag{30}$$

$$(s_l^p + \tau)e_l^p = m(\theta_l)u_l^p \tag{31}$$

6. *All markets clear:*

$$L_{h,u} = e_h^p + (1-\mu)e_h^g + u_h^p + u_{h,u}^g \tag{32}$$

$$L_{l,u} = e_l^p + (1-\mu)e_l^g + u_l^p + u_{l,u}^g \tag{33}$$

$$L_{h,c} = \mu e_h^g + u_{h,c}^g \tag{34}$$

$$L_{l,c} = \mu e_l^g + u_{l,c}^g \tag{35}$$

$$L_{h,u} + L_{l,u} + L_{h,c} + L_{l,c} = 1 \tag{36}$$

3 Main results

3.1 Effects of non-meritocratic hiring on unemployment

Proposition 1 *An increase in μ will lead to an increase in the number of (low- and high-educated) workers that are either searching or are employed in the private sector (i.e. will increase, $L^p = 1 - L^g$ where $L^g = L_{h,u}^g + L_{l,u}^g + L_{h,c}^g + L_{l,c}^g$ is the total number of workers that are either employed or are searching in the public sector). At the same time, it will decrease the total number of workers of each type that are searching in the public sector ($u_i^g = u_{i,u}^g + u_{i,c}^g$), but will leave the number of workers employed in the public sector (e_i^g) intact. These effects are larger the larger the public sector premium and the larger the size of the public sector.*

Intuition: With a higher fraction of public sector jobs being reserved for workers with connections the value of trying to find (searching for) a public-sector job without connections decreases. Workers have more incentive to direct their search towards the private sector or obtain connections. Since it is costly to obtain connections some of them, those whose connection cost is high will abandon search in the public sector and will search for private sector jobs instead.

3.2 Effects on the educational composition of the labor force

Proposition 2 *If $\mu = 0$ then public-sector wages, separation rates etc., have no impact on the the composition of the labor force in terms of education – the educational composition of the labor force is independent of what is happening in the public sector. But, if $\mu > 0$ then an increase in w_h^g, s_h^g, e_h^g (decrease in w_l^g, s_l^g, e_l^g) – or in general anything that improves public sector premium of the high- relative to the low educated (i.e. anything that increases $\tilde{\epsilon}_c$ and $\tilde{c}_h$ relative to $\tilde{c}_l$) will increase L_h and decrease L_l – will increase the proportion of high-educated in the labor force.*

Intuition: If $\mu = 0$ then free mobility between the public and the private sector keeps the education premium fixed. Consider for instance an increase in w_h^g. Such an increase will increase the value of searching for a high-education job in the public sector. Consequently, some high educated workers will quit searching for private-sector jobs and direct their search towards the public sector. The increased congestion due to the arrival of additional job seekers in the public sector will lower the job finding rate, and will push the value of searching for a high-education job in the public sector back to its initial level. Incentives to invest in education will remain intact.

On the other hand, If $\mu > 0$, mobility between the two sectors (public, private) becomes more difficult (costly). A fraction μ of public-sector jobs are reserved for connected workers. But in order to obtain connections a worker needs to pay a cost. Limited mobility will now reduce the inflow of searchers from the private towards the public sector. As a result, the same increase in w_h^g will now increase the value of searching for a high-education job in the public sector, it will increase the benefit from investing in education and will induce a higher fraction of the labor force to become high-educated.

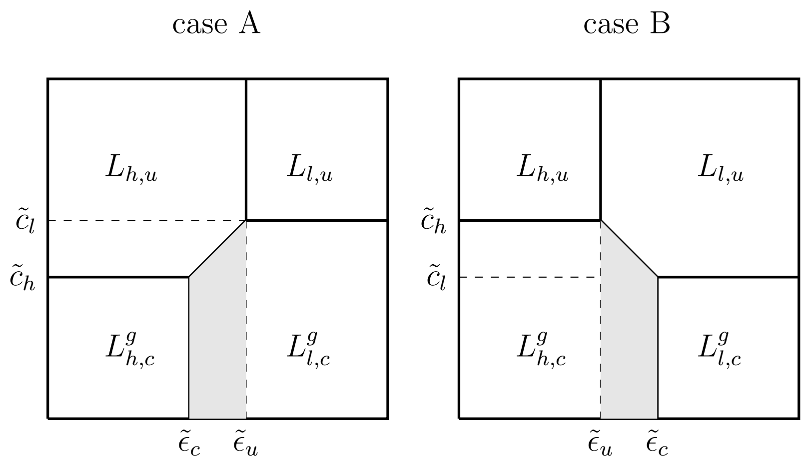
Proposition 3 *If $\tilde{c}_h < \tilde{c}_l$ ($\tilde{c}_l < \tilde{c}_h$) – Case A (Case B)– the existence of public-sector hiring through connections ($\mu > 0$) worsens (improves) the educational composition of the labor force: decreases (increases) L_h. If $\tilde{c}_h = \tilde{c}_l$ – Case C– the existence of public-sector hiring through connections ($\mu > 0$) has no impact on the educational composition of the labor force*

Intuition: If it pays relatively more to be high-educated in the private than in the public sector (Case A, where wages in the public sector are compressed), then those that can benefit the most from the use of connections in order to find public sector jobs are the low educated. When a fraction of public sector jobs is reserved for those that have connections, then those that benefit the most are the low educated. For some workers, those whose connection cost is low, it will be worthwhile to substitute education with connections and try to target low-education job in the public sector using their connections. As a result, the fraction of high-educated falls. In the opposite case (case B), it pays more to be high-educated in the public than in the private sector. When the option of using connections to find jobs in the public sector is allowed then workers with connections will have more incentive to become educated. The shaded areas in Figure 4 show the increase and decrease, respectively, in fraction of high-educated workers in the labor force once public-sector hiring through connections is introduced.

3.3 When the lack of meritocracy is bounded: a limit to μ

Here we relax the assumption that μ is completely isolated from labor marker conditions. We show that in situations where the public sector wage premium is not large enough to generate queues, changes in the supply of connected job searchers could influence the size of

Figure 4: Lack of Meritocracy and the Educational Composition



the connected sector.

Let $\bar{\mu}_i$ be the maximum fraction of type-i public-sector vacancies that can be filled through connections.[2] The government will able to use connections at the maximum, i.e. will be able to fill a fraction $\bar{\mu}_i$ of type-i jobs through connections provided that it pays a sufficiently high wage to attract enough connected job searchers. According to Lemma 2 there exists a wage, $\underline{\underline{w}}^g_{i,c}$, at which the government will be able to attract exactly $\bar{\mu}_i(s^g_i + \tau)e^g_i$ connected job searchers, i.e., $u^g_{i,c} = \bar{\mu}_i(s^g_i + \tau)e^g_i$. Hence, for any wage $w^g_i \geq \underline{\underline{w}}^g_{i,c}$ the government will be able to fill a fraction $\bar{\mu}_i$ of jobs through connections and public sector hiring will be the least meritocratic possible. The exact expression for $\underline{\underline{w}}^g_i$ is in the Appendix.

If the government wage is lower, but still high enough to attract some connected job searchers, that is, if $\underline{\underline{w}}^g_{i,c} > w^g_i > \underline{w}^g_{i,u}$, the number of connected job searchers will be lower, but still positive: $0 < u^g_{i,c} < \bar{\mu}_i(s^g_i + \tau)e^g_i$. In this case also, a connected sector exists ($\mu_i > 0$) but the government is restricted to fill only a fraction $\mu_i < \bar{\mu}_i$ of type-i vacancies through connections, where μ_i is such that $u^g_{i,c} = \mu_i(s^g_i + \tau)e^g_i$. The remaining vacancies $(1 - \mu_i)$ will be filled by unconnected workers. Using (34) and (35) we can solve for μ_i and write:

$$\mu_i = \frac{L^g_{i,c}}{e^g_i(s^g_i + \tau + 1)} \tag{37}$$

[2] In principle one could set $\bar{\mu}_i = 1$, but we prefer to generalize and assume that $\bar{\mu}_i$ can take any value below 1, reflecting different degrees of tolerance against non-meritocratic practices.

It states that there are no connected workers queuing for jobs: the total number of connected workers $L^g_{i,c}$ equals the new hires $\mu_i(s^g_i+\tau)e^g_i$ plus those already employed in the public sector $\mu_i e^g_i$.

In the limiting case where $w^g_i = \underline{w}^g_{i,u}$, the wage is not high enough to compensate for the connection cost and no worker has incentive to use connections in order to find a public sector job; hence $u^g_{i,c} = 0$, which means $\mu_i = 0$.

To sum up, we state that

$$\mu_i = \bar{\mu}_i \qquad \text{if } w^g_i \geq \underline{\underline{w}}^g_{i,c} \tag{38}$$

$$\mu_i = \frac{L^g_{i,c}}{e^g_i(s^g_i+\tau+1)} \qquad \text{if } \underline{\underline{w}}^g_{i,c} > w^g_i > \underline{w}^g_{i,u} \tag{39}$$

$$\mu_i = 0 \qquad \text{if } w^g_i = \underline{w}^g_{i,u} \tag{40}$$

Proposition 4 *Provided that the public sector wage is large enough to attract some connected job searchers of type-i, but not high enough to generate queues, the fraction of vacancies of type-i that the government fills through connections, μ_i, is larger the larger the public sector wage w^g_i and smaller the larger the size of public sector employment, e^g_i.*

Intuition: The government fill higher fraction of jobs through connections when the public-sector wage is higher because then the supply of connected job searchers is higher. A larger size of public sector employment means that the number of workers that the government needs to hire each period in order to replace those that separate due to retirements or other reasons is also larger, while the number of connected workers that search for them is smaller. Hence, the proportion of government jobs filled by connected job searchers is smaller.

3.4 Efficiency - INCOMPLETE

The social planner's problem and the first order conditions are shown in Appendix. There are four types of inefficiencies in this model: i) the existence of a "connections" sector that propels newborns to take on rent seeking activities; ii) the existence of queues for public sector jobs (given the assumption that the vacancy cost is sunk, independent of the number of hires or the tightness); iii) the usual thick-market and congestion externalities in both high- and low-education markets and iv) the fact that the new born might not internalize the returns of education (hold-up problem).

Eliminating the connection sector requires that $\mu = 0$ and $L_{h,c} = L_{l,c} = 0$. To avoid the queues in the public sector the government should set a public sector wage for high and low ability such that $u^g_{i,u} = (s^g_i + \tau)e^g_i$, in other words, that at any instance the job-finding rate for government jobs should be 1, which implies setting $\underline{w}^g_i$. The Hosios condition in the private sector bargaining guarantee that both the thick market and the congestion externalities are internalized. For the unemployed to internalize the returns of education, the government might need to give a subsidy.

4 Conclusion

This paper provides a benchmark model to understand how the public sector hiring and wage policies affect education decision. In particular, this paper builds upon the observation that, in several countries, the government hiring and pay do not follow meritocratic practices.

Our results provide insights that can explain several European cross-country facts. First, the existence of a "connection" market for public jobs requires that public sector wages are very high compared to the private sector. This result is consistent with evidence that South European countries that are know for having non-meritocratic hiring have a higher public sector wage premium, while Nordic countries where the government follows more meritocratic hiring tend to have lower or negative public sector wage premium. The second result that non-meritocratic hiring actually lowers unemployment, specially in the presence of a high public sector wage premium, might also explain why these practices are so accepted in South European countries. Given the high public sector wage premium, the presence of non-meritocracy hiring might be constraint efficient.

Finally, we have shown that the government policies only affect the education decision if there is a “connection” sector. In this case, a smaller education premium in the public sector reduce the incentives of the newborn to become high-educated. Again, this fact can help explaining why South European countries have lower education attainment than Nordic Countries. The next step would be to calibrate the model to a southern European country, and evaluate quantitatively these effects.

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5 Appendix

Lemma 2

We consider that the public sector unconnected labour market for workers of type i breaks down if the government is not able to hire enough workers to replace the workers that have lost their job. At the limit, it means the government needs to post a wage, defined as $\underline{w}^g_{i,u}$, such that it attracts at least $(1-\mu_i)(s^i_g+\tau)e^g_i$ job searches. This means $u^g_{i,u} = (1-\mu_i)(s^i_g+\tau)e^g_i$ and the job-finding rate is 1.

$$b_i + \frac{1}{r+\tau+s^g_i+1}[\underline{w}^g_{i,u} - b_i] = (r+\tau)U^p_i$$

Substituting the $(r+\tau)U^p_i$ by equation 15 we get

$$\underline{w}^g_{i,u} = \frac{(r+\tau+s^g_i+1)m(\theta^*_i)}{r+\tau+s^p_i+m(\theta^*_i)}[w^{p,*}_i - b_i] + b_i$$

where θ^*_i and $w^{p,*}_i$ are the equilibrium tightness and wages in the private sector.

If $\mu_i = 0$ then no connected sector exists and all workers hired into the public sector are unconnected. If, on the other hand, a connected sector exists then a share μ_i of workers hired into the public sector will be hired through connections. For the existence of a connected sector, through which the government is able to hire a fraction μ_i of its employees the government needs to attract at least $\mu_i(s^i_g+\tau)e^g_i$ connected job searchers. This means that it has to pay a higher wage, $\underline{w}^g_{i,c}$, which compensates connected workers for the cost of getting connections.

$$\underline{w}^g_{i,c} = \underline{w}^g_{i,u} + \Xi^{c,-1}(\mu_i(s^g_i+\tau)e^g_i)(r+\tau+s^g_i+1)$$

where $\Xi^{c,-1}$ is the inverse of the distribution of "connection" cost. What it means is that, at the margin, the government has to pay high enough wages such that a sufficient high mass of unemployed decide to pay the cost.

Notice that $\underline{w}^g_{i,c}$ is increasing in μ_i, while $\underline{w}^g_{i,u}$ is independent of μ_i. If $\mu_i = 0$ then we get $\underline{w}^g_{i,c} = \underline{w}^g_{i,u}$, whereas if $\mu_i = \bar{\mu}_i$ then $\underline{w}^g_{i,c} = \underline{\underline{w}}^g_{i,c}$ where

$$\underline{\underline{w}}^g_{i,c} = \underline{w}^g_{i,u} + \Xi^{c,-1}(\bar{\mu}_i(s^g_i+\tau)e^g_i)(r+\tau+s^g_i+1)$$

Proof of Proposition 1

Proof.

Fist, we show that $\frac{dL^p}{d\mu} > 0$:

Let $L^g_i = L^g_{i,u}+L^g_{i,c}$ denote the total number of workers of skill type i that are either employed or are searching in the public sector. Using the conditions (18) and (20) to solve, respectively, for $L^g_{i,u}$ and $L^g_{i,c}$, and then adding them up gives:

$$L^g_i = e^g_i\left[\lambda + (1-\lambda)\left(\frac{(w^g_i - b_i)(1-\beta)}{\beta\kappa_i\theta_i}\right)\left(\frac{\beta\kappa_i\theta_i + (1-\mu)(1-\beta)\tilde{c}_i}{\beta\kappa_i\theta_i + (1-\beta)\tilde{c}_i}\right)\right] \tag{41}$$

where $\lambda = \frac{r+\tau}{r+s_i^g+\tau}$. Note that equation (13) can be solved for the equilibrium value of θ_i which is independent of μ; thus $\frac{d\theta_i}{d\mu} = 0$. Given this, we can write:

$$\frac{dL_i^g}{d\mu} = \frac{\partial L_i^g}{\partial \mu} + \frac{\partial L_i^g}{\partial \tilde{c}_i}\frac{d\tilde{c}_i}{d\mu} \tag{42}$$

and using (24) and (28) we can derive that for $i = [h, l]$, $j = [h, l]$ and $j \neq i$:

$$\frac{d\tilde{c}_i}{d\mu} = \frac{\Delta_i + B_i \Delta_j}{1 - B_i B_j} \tag{43}$$

where

$$\begin{aligned} B_i &= \frac{\frac{\partial A_{i,c}}{\partial L_{i,c}^g}\frac{\partial L_{i,c}^g}{\partial \tilde{c}_j}}{r + \tau - \frac{\partial A_{i,c}}{\partial L_{i,c}^g}\frac{\partial L_{i,c}^g}{\partial \tilde{c}_i}} \\ \Delta_i &= \frac{\frac{\partial A_{i,c}}{\partial \mu}}{r + \tau - \frac{\partial A_{i,c}}{\partial L_{i,c}^g}\frac{\partial L_{i,c}^g}{\partial \tilde{c}_i}} \\ A_{i,c} &\equiv \frac{\frac{\mu(s_i^g+\tau)e_i^g}{L_{i,c}^g - \mu e_{i,c}}}{r + \tau + s_i^g + \frac{\mu(s_i^g+\tau)e_i^g}{L_{i,c}^g - \mu e_{i,c}}}\left(w_i^g - b_i\right) \end{aligned} \tag{44}$$

From the expression for $A_{i,c}$ in (44) we get that $\frac{\partial A_{i,c}}{\partial L_{i,c}^g} < 0$, $\frac{\partial A_{i,c}}{\partial \mu} > 0$ and from (24)-(25) that $\frac{\partial L_{i,c}^g}{\partial \tilde{c}_i} > 0$ and $\frac{\partial L_{i,c}^g}{\partial \tilde{c}_j} < 0$ so that $\Delta_i > 0$ and $1 > B_i > 0$. Therefore, it must be the case that $\frac{d\tilde{c}_i}{d\mu} > 0$. Moreover, it can be easily verified from (41) that $\frac{\partial L_i^g}{\partial \mu} < 0$ and $\frac{\partial L_i^g}{\partial \tilde{c}_i} < 0$, implying from (42) that $\frac{dL_i^g}{d\mu} < 0$. Given that $L^p = 1 - L_h^g - L_u^g$, it follows that $\frac{dL^p}{d\mu} > 0$.

After some substitutions we can write:

$$\frac{dL_i^g}{d\mu} = -e_i^g \left[\frac{\tilde{c}_i(r+\tau)}{\tilde{c}_i(r+\tau) + \frac{\beta\kappa_i\theta_i}{1-\beta}}\right]\left[\frac{s_i^g}{r+\tau+s_i^g}\right]\left[\frac{w_i^g - b_i}{\frac{\beta\kappa_i\theta_i}{1-\beta}}\right] \tag{45}$$

It is increasing in w_i^g, e_i^g and in $\tilde{c}_i$, which is given in (28), and which, as shown below, is also increasing in w_i^g and e_i^g. This means that a given increase in μ will have a larger negative effect on L_i^g and thus a larger positive effect on L^p when the public sector wage (w_i^g) and employment (e_i^g) are large.

Next we show that $\frac{du_i^g}{d\mu} < 0$ while $\frac{de_i^g}{d\mu} = 0$. The number of workers searching in the public sector with and without connections are given, respectively, by $u_{i,c}^g = L_{i,c}^g - \mu e_i^g$ and $u_{i,u}^g = L_{i,u}^g - (1-\mu)e_i^g$. By adding them up we get $u_i^g = u_{i,u}^g + u_{i,c}^g = L_i^g - e_i^g$. The number of type i workers employed in the public sector, e_i^g, is given by (**??**) and (**??**) and it is independent of μ. Since $\frac{de_i^g}{d\mu} = 0$ and $\frac{dL_i^g}{d\mu} < 0$ then $\frac{du_i^g}{d\mu} < 0$. ■

Proof of Proposition 2

Proof. Consider first case $\mu = 0$. If $\mu = 0$ then $\tilde{c}_h = \tilde{c}_l = 0$. Adding up $L_{h,u}$ and $L^g_{h,c}$ using equations (24) and (26) (case C) we get $L_h = \Xi^\epsilon(\tilde{\epsilon}_u)$ and $L_l = 1 - \Xi^\epsilon(\tilde{\epsilon}_u)$. Both depend only on $\tilde{\epsilon}_u$ which as can be verified from (29) it is independent of public sector wages or separation rates and depends only on private sector parameters.

Consider now $\mu > 0$. Using (24) and (28) we can derive for $x_i = [w^g_i, s^g_i, e^g_i]$, $i = [h, l]$, $j = [h, l]$ and $j \neq i$ that

$$\frac{d\tilde{c}_i}{dx_i} = \frac{\frac{\partial A_{i,c}}{\partial x_i}}{r + \tau - \frac{\partial A_{i,c}}{\partial L^g_{i,c}}\left[\frac{\partial L^g_{i,c}}{\partial \tilde{c}_i} + B_j \frac{\partial L^g_{i,c}}{\partial \tilde{c}_j}\right]} \tag{46}$$

$$\frac{d\tilde{c}_j}{dx_i} = B_j \frac{d\tilde{c}_i}{dx_i} \tag{47}$$

where B_j and $A_{i,c}$ are as defined above. From the expression for $A_{i,c}$ in (44) we know that $\frac{\partial A_{i,c}}{\partial x_i} > 0$. From (24)-(25) we can derive $\frac{\partial L^g_{i,c}}{\partial \tilde{c}_i}$ and $\frac{\partial L^g_{i,c}}{\partial \tilde{c}_j}$ and obtain:

$$\left[\frac{\partial L^g_{h,c}}{\partial \tilde{c}_h} + B_l \frac{\partial L^g_{h,c}}{\partial \tilde{c}_l}\right] = \begin{cases} \xi^\epsilon(\tilde{\epsilon}_c)\Xi^c(\tilde{c}_h)\left[1 - B_l\right] + \xi^c(\tilde{c}_h)\Xi^\epsilon(\tilde{\epsilon}_c), \text{ if } \tilde{c}_h < \tilde{c}_l \\ \xi^\epsilon(\tilde{\epsilon}_c)\Xi^c(\tilde{c}_l)\left[1 - B_l\right] + \xi^c(\tilde{c}_h)\Xi^\epsilon(\tilde{\epsilon}_u) + \int_{\tilde{c}_l}^{\tilde{c}_h} \Xi^\epsilon(\tilde{\epsilon}_m(c))d\Xi^c(c), \text{ if } \tilde{c}_l < \tilde{c}_h \\ 0, \text{ if } \tilde{c}_h = \tilde{c}_l \end{cases} \tag{48}$$

$$\left[\frac{\partial L^g_{l,c}}{\partial \tilde{c}_l} + B_h \frac{\partial L^g_{l,c}}{\partial \tilde{c}_h}\right] = \begin{cases} \xi^\epsilon(\tilde{\epsilon}_c)\Xi^c(\tilde{c}_h)\left[1 - B_h\right] + \xi^c(\tilde{c}_l)\left(1 - \Xi^\epsilon(\tilde{\epsilon}_u)\right) + \int_{\tilde{c}_h}^{\tilde{c}_l} \Xi^\epsilon(\tilde{\epsilon}_m(c))d\Xi^c(c), \text{ if } \tilde{c}_h < \tilde{c}_l \\ \xi^\epsilon(\tilde{\epsilon}_c)\Xi^c(\tilde{c}_l)\left[1 - B_h\right] + \xi^c(\tilde{c}_l)\left(1 - \Xi^\epsilon(\tilde{\epsilon}_c)\right), \text{ if } \tilde{c}_l < \tilde{c}_h \\ 0, \text{ if } \tilde{c}_l = \tilde{c}_h \end{cases} \tag{49}$$

Since, as shown above, $0 < B_i < 1$, the expressions in (48) and (49) are in all cases positive. Moreover, $\frac{\partial A_{i,c}}{\partial L^g_{i,c}} < 0$ thus the denominator of the expressions in (46) is also always positive. We can therefore conclude that:

$$\frac{d\tilde{c}_i}{dx_i} > 0 \text{ and } \frac{d\tilde{c}_j}{dx_i} > 0 \text{ for } x_i = [w^g_i, s^g_i, e^g_i],\ i = [h, l],\ j = [h, l] \text{ and } j \neq i \tag{50}$$

With (21)-(23) substituted in (24) and (26) we can derive an expression for $L_h (= L_{h,u} + L^g_{h,c})$ in terms of only $\tilde{c}_h$, $\tilde{c}_l$ and model parameters so that:

$$\frac{dL_h}{dx_i} = \frac{\partial L_h}{\partial \tilde{c}_h}\frac{d\tilde{c}_h}{dx_i} + \frac{\partial L_h}{\partial \tilde{c}_l}\frac{d\tilde{c}_l}{dx_i} \tag{51}$$

Using (47) we can write:

$$\frac{dL_h}{dx_h} = \frac{d\tilde{c}_h}{dx_h}\left[\frac{\partial L_h}{\partial \tilde{c}_h} + B_l \frac{\partial L_h}{\partial \tilde{c}_l}\right] \tag{52}$$

$$\frac{dL_h}{dx_l} = \frac{d\tilde{c}_l}{dx_l}\left[\frac{\partial L_h}{\partial \tilde{c}_h} B_h + \frac{\partial L_h}{\partial \tilde{c}_l}\right] \tag{53}$$

Next we derive expressions for the terms in the brackets:

$$\left[\frac{\partial L_h}{\partial \tilde{c}_h} + B_l \frac{\partial L_h}{\partial \tilde{c}_l}\right] = \begin{cases} \xi^\epsilon(\tilde{\epsilon}_c)\Xi^c(\tilde{c}_h)\left[\frac{r+\tau-\frac{\partial A_{l,c}}{\partial L_{l,c}}\xi^c(\tilde{c}_l)(1-\Xi^\epsilon(\tilde{\epsilon}_u))}{r+\tau-\frac{\partial A_{l,c}}{\partial L^g_{l,c}}\frac{\partial L^g_{l,c}}{\partial \tilde{c}_l}}\right], & \text{if } \tilde{c}_h < \tilde{c}_l \\ \xi^\epsilon(\tilde{\epsilon}_c)\Xi^c(\tilde{c}_l)\left[1-B_l\right] + \int_{\tilde{c}_l}^{\tilde{c}_h} \xi^\epsilon(\tilde{\epsilon}_m(c))d\Xi^c(c), & \text{if } \tilde{c}_l < \tilde{c}_h, \end{cases} \tag{54}$$

$$\left[\frac{\partial L^g_h}{\partial \tilde{c}_l} + B_h \frac{\partial L_h}{\partial \tilde{c}_h}\right] = \begin{cases} -\xi^\epsilon(\tilde{\epsilon}_c)\Xi^c(\tilde{c}_h)\left[1-B_h\right] - \int_{\tilde{c}_h}^{\tilde{c}_l} \xi^\epsilon(\tilde{\epsilon}_m(c))d\Xi^c(c), & \text{if } \tilde{c}_h < \tilde{c}_l \\ -\xi^\epsilon(\tilde{\epsilon}_c)\Xi^c(\tilde{c}_l)\left[\frac{r+\tau-\frac{\partial A_{h,c}}{\partial L^g_{h,c}}\xi^c(\tilde{c}_h)\Xi^\epsilon(\tilde{\epsilon}_u)}{r+\tau-\frac{\partial A_{h,c}}{\partial L^g_{h,c}}\frac{\partial L^g_{h,c}}{\partial \tilde{c}_h}}\right], & \text{if } \tilde{c}_l < \tilde{c}_h \end{cases} \tag{55}$$

The terms in (54) are positive while the terms in (55) negative. Using (50) we can therefore conclude that:

$$\frac{dL_i}{dx_i} > 0, \frac{dL_i}{dx_j} < 0, \; x_i = [w^g_i, s^g_i, e^g_i], \; i = [h,l], \; j = [h,l] \text{ and } j \neq i \tag{56}$$

■

Proof of Proposition 3

Proof. If $\mu = 0$ then $\tilde{c_h} = \tilde{c_h} = 0$ and from (24) and (26) we obtain

$$L_h = \Xi^\epsilon(\tilde{\epsilon}_u) \tag{57}$$

while if $\mu > 0$ we get

$$L_h = \Xi^c(\tilde{c}_h)\Xi^\epsilon(\tilde{\epsilon}_c) + \int_{\tilde{c}_h}^{\tilde{c}_l} \Xi^\epsilon(\tilde{\epsilon}_m(c))d\Xi^c(c) + (1-\Xi^c(\tilde{c}_l))\,\Xi^\epsilon(\tilde{\epsilon}_u), \text{ if } \tilde{c}_h < \tilde{c}_l \tag{58}$$

$$L_h = \Xi^c(\tilde{c}_l)\Xi^\epsilon(\tilde{\epsilon}_u) + \int_{\tilde{c}_l}^{\tilde{c}_h} \Xi^\epsilon(\tilde{\epsilon}_m(c))d\Xi^c(c) + (1-\Xi^c(\tilde{c}_h))\,\Xi^\epsilon(\tilde{\epsilon}_u), \text{ if } \tilde{c}_l < \tilde{c}_h \tag{59}$$

Subtracting (58) from (57) we obtain

$$L^{\mu=0}_h - L^{\mu>0}_h = -\int_{\tilde{c}_h}^{\tilde{c}_l} \left[\Xi^\epsilon(\tilde{\epsilon}_u) - \Xi^\epsilon(\tilde{\epsilon}_m(c))\right] d\Xi^c(c) - \Xi^c(\tilde{c}_h)\left[\Xi^\epsilon(\tilde{\epsilon}_u) - \Xi^\epsilon(\tilde{\epsilon}_c)\right] < 0 \tag{60}$$

As shown above, $\tilde{\epsilon}_c \leq \tilde{\epsilon}_m \leq \tilde{\epsilon}_u$, if $\tilde{c}_h < \tilde{c}_l$, implying that the terms in the brackets are positive. Subtracting (59) from (57) we obtain

$$L^{\mu=0}_h - L^{\mu>0}_h = \int_{\tilde{c}_l}^{\tilde{c}_h} \left[\Xi^\epsilon(\tilde{\epsilon}_m(c)) - \Xi^\epsilon(\tilde{\epsilon}_u)\right] d\Xi^c(c) + \Xi^c(\tilde{c}_l)\left[\Xi^\epsilon(\tilde{\epsilon}_c) - \Xi^\epsilon(\tilde{\epsilon}_u)\right] > 0 \tag{61}$$

As shown above, $\tilde{\epsilon}_c \geq \tilde{\epsilon}_m \geq \tilde{\epsilon}_u$, if $\tilde{c}_h > \tilde{c}_l$, implying that the terms in the brackets are positive.

■

Cobb Douglas Matching Function

Suppose that the matching function in the public sector is Cobb Douglas: $M^g_{i,j} = (v^g_{i,j})^\alpha (u^g_{i,j})^{1-\alpha}$. For the government to be able to maintain constant employment matches in each period should be equal to the number of workers that retire or separate. That is, $M^g_{i,j} = e^g_i(s^g_i + \tau)$ and the worker's matching rate becomes $m^g_{i,j} = \frac{M^g_{i,j}}{u^g_{i,j}} = \frac{e^g_i(s^g_i+\tau)}{u^g_{i,j}}$, as in the text. The only difference in this case is that the government has to post a high enough number of vacancies to guarantee number of matches equal to $M^g_{i,j} = e^g_i(s^g_i + \tau)$. Solving for $v^g_{i,j}$ we obtain $v^g_{i,j} = \frac{\left(e^g_i(s^g_i+\tau)\right)^{\frac{1}{\alpha}}}{(u^g_{i,j})^{\frac{1-\alpha}{\alpha}}}$.

Competitive Search in the Private Sector

Suppose that as in the text markets in the private sector are segmented by skill; there are therefore two private-sector markets, the skilled and the unskilled. However, we depart from the assumptions of Nash bargaining and random search in the private sector. Instead, as in Moen (1997) we introduce a competitive search equilibrium in the private sector. To this end, we assume that each of the two private-sector markets consists of submarkets. In each submarket there is a subset of unemployed workers and firms with vacant jobs that are searching for each other. The number of matches in submarket n of skill type i is $m(v_{i,n}, u_{i,n}) = (v_{i,n})^\alpha (u^p_{i,n})^{(1-\alpha)}$, $m(\theta_{i,n})$ is the job finding rate and $q(\theta_{i,n})$ the job filling rate. For a worker of skill type i in submarket n

$$(r+\tau)U^p_{i,n} = b_i + m(\theta_{i,n})\left[E^p_{i,n} - U^p_{i,n}\right] \tag{62}$$

$$(r+\tau)E^p_{i,n} = w^p_{i,n} + s^p_i m(\theta_{i,n})\left[E^p_{i,n} - U^p_{i,n}\right] \tag{63}$$

Unemployed workers of skill type i are free to move between the submarkets of market i. They will choose to search for a job in the submarket that yields the highest expected income. Since workers of the same skill type are ex-ante identical and movement across submarkets is free, this means that $U^p_{i,n} = U^p_i$. Using (62) and (62) we can write:

$$m(\theta_{i,n}) = \left(\frac{(r+\tau)U^p_i - b_i}{w^p_{i,n} - (r+\tau)U^p_i}\right)(r + \tau + s^p_i) \tag{64}$$

The values of vacancies and filled jobs in submarket n of market i satisfy

$$rV^p_{i,n} = -\kappa_i + q(\theta_{i,n})\left[J^p(w^p_{i,n}) - V^p_{i,n}\right] \tag{65}$$

$$rJ^p(w^p_{i,n}) = y^p_i - w^p_{i,n} + (s^p_i + \tau)\left[V^p_{i,n} - J^p(w^p_{i,n})\right] \tag{66}$$

Solving for $V^p_{i,n}$ gives

$$rV^p_{i,n} = \frac{-\kappa_i(r + s^p_i + \tau) + q(\theta_{i,n})(y^p_i - w^p_i)}{r + q(\theta_{i,n}) + s^p_i + \tau} \tag{67}$$

In a competitive search equilibrium a market maker determines the number of submarkets in each market and the wage in each submarket. The wage is chosen to maximize the value of a vacancy. All vacancies in the same submarket offer the same wage. Taking the derivative of (67) with respect to $w^p_{i,n}$ we get

$$-(1-\alpha)(r+s^p_i+\tau)\frac{d\theta_{i,n}}{dw^p_{i,n}}\left[p-w^p_{i,n}+\kappa_i\right]=\theta_{i,n}(r+s^p_i+\tau)+m(\theta_{i,n}) \tag{68}$$

Using (64) we obtain

$$\frac{d\theta_{i,n}}{dw^p_{i,n}}=-\left(\frac{\theta_{i,n}}{w^p_{i,n}-(r+\tau)U^p_i}\right)\frac{1}{\alpha} \tag{69}$$

Combining (68) with (69) we get

$$w^p_{i,n}=(1-\alpha)y^p_{i,n}+\alpha(r+\tau)U^p_i \tag{70}$$

There is free entry of vacancies in each submarket. Setting $V^p_{i,n}=0$ in (67) gives:

$$\frac{\kappa_i}{q(\theta_{i,n})}=\frac{y^p_i-w^p_i}{r+s^p_i+\tau} \tag{71}$$

Using (64) and (71) we can substitute for $(r+\tau)U^p_i$ in (70) and obtain

$$w^p_{i,n}=b_i+(1-\alpha)\left(y^p_{i,n}-b_i+\theta_{i,n}\kappa_i\right) \tag{72}$$

Combining (71) and (72) we get the job creation condition in each submarket

$$\frac{\kappa_i}{q(\theta_{i,n})}=\frac{\alpha(y^p_{i,n}-b_i)}{r+s^p_i+\tau+(1-\alpha)m(\theta_{i,n})} \tag{73}$$

Notice that if $y^p_{i,n}=y^p_i$, meaning that productivity is the same across all submarkets of market i then $\theta_{i,n}=\theta_i$ and $w^p_{i,n}=w^p_i$. All submarkets in market i offer the same wage and job finding rate. If in addition the Hosios condition holds, i.e. $1-\alpha=\beta$, then job creation, market tightness and the Nash bargaining wage in the Benchmark model described in the text (see equations (13) and (14) are identical to those derived under competitive search.

Efficiency

It is straightforward that the existence of a "connections" sector in the public sector is inefficient. Eliminating the connection sector requires that $\mu=0$ and $L_{h,c}=L_{l,c}=0$. Given that any queues in the public sector is inefficient (given the assumption that the vacancy cost is sunk, independent of the number of hires or the tightness), the government should set a public sector wage for high and low ability such that $u^g_{i,u}=(s^g_i+\tau)e^g_i$, in other words, the job-finding rate for government jobs should be 1, which implies setting $\underline{w}^g_i$.

Given an exogenous number of public sector workers and the minimum number of unemployed searching in the public sector, the social planner's problem becomes

$$\max\int_0^\infty e^{-(r+\tau)t}\{H[(1-e^g_h-u^g_h-u^p_h)y_h-u^p_hb_h-\theta_h\kappa_hu^p_h]+(1-H)[(1-e^g_l-u^g_l-u^p_l)y_l-u^p_lb_l-\theta_l\kappa_lu^p_l]-$$

$$\tau \int_0^{\epsilon^*} \epsilon \Xi^\epsilon(\epsilon) d\epsilon \} dt$$

s.t

$$\dot{u}_h^p = s_h^p(1 - e_h^g - u_h^g - u_h^p) - q(\theta_h)\theta_h u_h^p$$
$$\dot{u}_l^p = s_l^p(1 - e_l^g - u_l^g - u_l^p) - q(\theta_l)\theta_l u_l^p$$
$$\dot{H} = \tau \Xi^\epsilon(\epsilon^*) - \tau H$$

If we set the Hamiltonian

$$\mathcal{H} = \int_0^\infty e^{-(r+\tau)t} \{ H[(1-e_h^g-u_h^g-u_h^p)y_h - u_h^p b_h - \theta_h \kappa_h u_h^p] + (1-H)[(1-e_l^g-u_l^g-u_l^p)y_l - u_l^p b_l - \theta_l \kappa_l u_l^p] -$$

$$\tau \int_0^{\epsilon^*} \epsilon \Xi^\epsilon(\epsilon) d\epsilon \} dt + \phi_1[s_h^p(1 - e_h^g - u_h^g - u_h^p) - q(\theta_h)\theta_h u_h^p] + \phi_2[s_l^p(1 - e_l^g - u_l^g - u_l^p) - q(\theta_l)\theta_l u_l^p] +$$

$$\phi_3[\tau \Xi^\epsilon(\epsilon^*) - \tau H]$$

The six optimality conditions are

$$\frac{\partial \mathcal{H}}{\partial \theta_h} = 0 \implies -e^{-(r+\tau)t} u_h^p \kappa_h H + \phi_1[-q(\theta_h)u_h^p - q'(\theta_h)\theta_h u_h^p]$$
$$\frac{\partial \mathcal{H}}{\partial \theta_l} = 0 \implies -e^{-(r+\tau)t} u_l^p \kappa_l (1-H) + \phi_2[-q(\theta_l)u_l^p - q'(\theta_l)\theta_l u_l^p]$$
$$\frac{\partial \mathcal{H}}{\partial \epsilon^*} = 0 \implies -e^{-(r+\tau)t} \tau[\epsilon^* \Xi^\epsilon(\epsilon^*)] + \phi_3 \tau \Xi'^\epsilon(\epsilon^*)$$
$$\frac{\partial \mathcal{H}}{\partial u_h^p} = -\dot{\phi}_1 \implies e^{-(r+\tau)t}[-y_h + b_h - \theta_h \kappa_h]H + \phi_1[-s_h^p - q(\theta^h)\theta^h] = -\dot{\phi}_1$$
$$\frac{\partial \mathcal{H}}{\partial u_l^p} = -\dot{\phi}_2 \implies e^{-(r+\tau)t}[-y_l + b_l - \theta_l \kappa_l](1-H) + \phi_2[-s_l^p - q(\theta^l)\theta^l] = -\dot{\phi}_2$$
$$\frac{\partial \mathcal{H}}{\partial H} = -\dot{\phi}_3 \implies e^{-(r+\tau)t}[((1-e_h^g-u_h^g-u_h^p)y_h - u_h^p b_h - \theta_h \kappa_h u_h^p) - ((1-e_l^g-u_l^g-u_l^p)y_l - u_l^p b_l - \theta_l \kappa_l u_l^p)] = -\dot{\phi}_3$$