Unemployment Risks and Intra-Household Insurance *
(Very preliminary draft)

Javier Fernández-Blanco †

July 6, 2017

Abstract

We consider an economy with incomplete markets and intra-household risk sharing, where households are formed by a job-seeker and an employed spouse and differ by the productivity of the spouse. We study the constrained efficient private provision of insurance within the household through the labor supply of the spouse, and what unemployment risks should be publicly insured away. Unlike the spouse's total income, neither productivity nor labor supply is observed. We characterize the directed search equilibrium, and show that the spouse's labor supply is negatively affected by unemployment benefits regardless of the search outcome of the worker in line with the empirical evidence. We also show that the optimal unemployment benefits are contingent on the household's total income as it affects the trade-off between consumption-smoothing and job search incentives. Put differently, private intrahousehold insurance shapes workers' search strategies. Moreover, we numerically explore the welfare gains of implementing a household-income-based unemployment insurance.

Keywords: Unemployment Risks, Intra-Household Risk-sharing, Directed Search, Efficient Private Insurance

JEL Codes: J08, J22, J64, J65

*We thank Nezih Guner, Jean-Baptiste Michau, Claudio Michelacci, Aysegul Sahin and Edward Schlee for their feedback. We are also grateful to numerous participants at the SEA (2015), ENTER Jamboree (2016), Istanbul SaM Workshop (2016), the Annual SaM Conference (2016), the ESEM (2016) and the SED (2017) as well as seminar participants at Oxford, Ryerson, Western Ontario, Waterloo, McMaster, CREST, and Universitat de Girona. We acknowledge financial support from the Spanish Ministry of Science and Technology under Grant No. ECO 2013-46395, ECO2015-67602-P from MINECO/FEDER, UE, and from the Spanish Ministry of Economy and Competitiveness through the Severo Ochoa Programme for Centres of Excellence in R&D (SEV-2015-0563).

†Universitat Autònoma de Barcelona, Barcelona GSE and MOVE

1 Introduction

A consistent finding in public economics is that welfare gains can be obtained from making policy instruments contingent on worker's observable characteristics. Alesina et al. (2011) analyze gender-based taxation, while Weinzierl (2011) and Farhi and Werning (2013) find that optimal income taxation varies over the life cycle, and Michelacci and Ruffo (2015) extend this result to unemployment insurance (UI) benefits. Likewise, Kleven et al. (2009) find that optimal tax rates on an individual's labor income differ by the earnings of the spouse.

The primary goal of this paper is to study the constrained efficient private intra-household insurance against unemployment risks, and what risks should be publicly insured away. Intuitively, UI benefits should be contingent on the spouse's earnings to the extent that these help reduce consumption risks and are sensitive to both income shocks and policy.

Significant consumption smoothing at the household level has been estimated in the literature. For example, Heathcote et al. (2014) find that only does 40% of individual permanent wage fluctuations pass through to household consumption.[1] Blundell et al. (2016) study the main sources of consumption insurance against a permanent fall in the husband's earnings, and estimate that, on average, adjusting the wife's labor supply accounts for approximately 50%, with assets and transfers accounting for 20% each.[2] The analysis of the effects of the husband's income on his wife's labor supply goes back at least to Mincer (1962), who showed informal evidence that wives work more if their husbands are unemployed. Blau and Kahn (2007) estimate the average cross-wage elasticity of wife's hours worked at -0.2.[3] Cullen and Gruber (1996) estimate a 6% increase in the wife' hours worked in the short run after the displacement of her husband.

Likewise, the crowding-out effects of publicly-provided insurance on private provision appear to be sizable. Two estimates of this distortion are provided by Gruber (1997) and Cullen and Gruber (2000).[4] The former finds that a 10 percent increase in the replacement

[1]Dynarski and Gruber (1997) estimate the average elasticity of total consumption with respect to male's earnings at 0.24, which implies that 76 cents are smoothed away for each dollar in male's earnings.

[2]In contrast, and because of the gender gap in labor supply elasticities, permanent shocks on wives' wages are primarily smoothed away through savings and government transfers.

[3]The evidence on cross-wage elasticity is quite limited. Using March CPS data, Blau and Kahn (2007) find that almost 80% of married women worked in 2000, and that cross-wage elasticity is negative and significant at both the extensive and intensive margins, increases with education and is twice as high for married women with children under 6. Furthermore, there is no significant difference when including likely cohabitation. Likewise, Hyslop (2001) estimates that a $1 increase in the husband's hourly wages reduces wife's annual earnings by $300 and her labor supply by 35 annual hours. We provide more references to the empirical evidence on this source of private insurance in the literature section. Devereux (2004) estimate the cross-wage elasticity to -0.4.

[4]As summarized by Heathcote et al. (2009), there is a number of private insurance sources. Engen and

rate is associated with a reduction in the consumption drop of 2.8 percentage points. The latter find that hours worked of wives increase by 30% during an unemployment spell of their husbands in the absence of UI, but each dollar of UI reduces the wife's earnings by 36-73 cents.[5]

Despite the evidence, the literature that addresses unemployment risks has forgotten to draw the labor-market-based intra-household insurance into the picture. We investigate the optimal allocation of unemployment risks in a static economy with directed search and intra-household risk sharing. Households are formed by a job-seeker and an employed spouse, and differ by the productivity of the spouse. Partial insurance against unemployment (or consumption) risks is privately arranged by pooling income and adjusting the spouse's labor supply. In line with Chetty and Saez (2010), we assume no moral hazard is generated as a result of such an arrangement. The optimal design of the public unemployment insurance scheme factors in the intra-household risk-sharing mechanism, and, hence, takes into account the household's total income. Therefore, there is a trade-off between the distortions on job creation generated by the taxes necessary to finance the public provision and the forgone leisure stemming from the private arrangements of insurance. Moreover, the optimal design of the public provision of insurance is limited by the heterogeneity across households and the informational frictions: unlike the spouse's income, neither her working time nor her productivity is observed.

We first characterize the *laissez-faire* equilibrium. Because of our assumptions on preferences, the spouse's leisure is a normal good. This result has two different empirical implications in this context. First, the labor supply of the spouses married to unemployed workers is larger than the one of those married to employed workers, which is in good accordance with the empirical evidence referred to above.[6] Second, we also find that an increase in unemployment

Gruber (2001) estimate that the negative percentage effect of UI on asset holdings is twice as large for singles as for married unemployed workers. However, as pointed out by Chetty and Finkelstein (2012), the magnitude of the effect of UI on private savings is modest, and the median wealth holdings are very low as reported by Engen and Gruber (2001) and Chetty (2008). See also Kolsrud et al. (2015) for Sweden. Kaplan (2012) documents that a large number of low-skilled youth males move back to their parents' place after job loss. Moreover, using Canadian survey data, Browning and Crossley (2001) estimate a much smaller effect of the replacement rate on household total expenditure, and a significant effect on married workers whose spouse was unemployed.

[5]The precise estimate is much larger for instrumented than for potential UI. Cullen and Gruber (1996) also find that the effects of UI on the hours conditional on working, but not their employment likelihood, are large and significant for wives of employed husbands with high unemployment risk, but not for the unemployed group. This suggests that households anticipate those risks. The responsiveness to UI also varies over the life cycle. For example, the crowd-out effects are much larger in household with small children and for young couples.

[6]We consider spouses in a frictionless labor market, and, hence, we make no distinction between the extensive and intensive margins. Nonetheless, standard frictions can be added with no qualitatively different

benefits reduces the labor supply of spouses of both unemployed and employed workers in equilibrium, which is consistent with the evidence reported by Cullen and Gruber (2000).

Consistent with the empirical evidence, we also show that, under some mild conditions, public provision of insurance crowds out the private one, and workers with high spouse's earnings are more likely to have long unemployment spells and smaller private insurance provisions.

We then address the normative questions: What is the constrained efficient allocation of risks in this economy? Can it be decentralized in the market economy? We first show that the insurance level in the equilibrium allocation falls short of the optimal level as welfare gains are obtained from redistributing resources from households with the two members employed to households with one unemployed worker. Because of the lack of redistributive instruments, two sources of private insurance are excessively at work: the intensive margin of the labor supply of the spouse and the job creation margin in the labor markets.

In the case of ex-ante homogeneous households, we show that the planner's allocation can be decentralized in the market economy. This implementation requires three fiscal instruments: unemployment benefits financed by lump sum and a proportional income tax on newly employed workers.

In Section 6, we quantitatively illustrate the welfare gains of moving from the current system to the planner's solution. Relative to the present-day system, two outcomes are of interest. First, the welfare gains differ significantly over the distribution of households. Second, productivity gains also take place since the labor supply increases (reduces) for more (less) productive spouses. Finally, we explore how far away it is from the planner's solution a simple policy consisting of a replacement rate and a dependency allowance contingent on the spouse being unemployed. As of 2015, nine states of the U.S. provide such an allowance, and its amount varies across states.[7]

In our analysis of the optimal unemployment insurance, we abstract from the persistent effects of unemployment on earnings by focusing on unemployment risks and the short run, thereby lessening the effects on the spouse's labor supply.[8] Consistent to the short run

results.

[7]The states are Connecticut, Washington D.C, Illinois, Iowa, Maine, Michigan, New Jersey, Ohio and Pennsylvania. See the Department of Labor documentation: http://www.unemploymentinsurance.doleta.gov/unemploy/pdf/uilawcompar/2015/monetary.pdf

[8]Stevens (1997) finds that earnings remain approximately 9% below expected levels 6 years after job loss. Using PSID data, Stephens (2002) documents that husbands' earnings remain about 20% lower 3-4 years after displacement, and wives' working hours keep increasing during this period. He estimates a 11% increase on average in annual working hours of wives, which includes both the intensive and extensive margins and offsets over 25% of their husbands' lost earnings, which is in line with the figure estimated by Morissette and Ostrovsky (2008) for Canada in the 1990s. Dynarski and Gruber (1997) also find a large response in wives'

approach, no frictions to adjustments in the labor supply of the spouse have been modeled. We also abstract from moral hazard problems and monitoring as well as administrative costs, features that would augment the relative costs of public insurance.

The paper proceeds as follows. After a brief summary of the related literature, Section 2 shows our data work. Section 3 describes the economy. In Section 3, we study the market equilibrium. Section 4 analyzes the planner's solution. In Section 5, we undertake a numerical exercise, and Section 5 concludes. All proofs are relegated to the Appendix.

1.1 Related Literature

This paper contributes to several branches of the labor literature. First, in the search literature, several attempts have been undertaken to examine the optimal level of unemployment benefits under various sources of private insurance. For example, in a random search model, Krusell et al. (2010) find that the sizable negative effects on job creation limit significantly the generosity of the optimal public provision of insurance in an economy where workers can insure themselves through savings. Although not focused on the optimal unemployment insurance, Acemoglu and Shimer (1999) show that private markets offer insurance against unemployment risks to job-seekers who can direct their search. In the search literature, Burdett and Mortensen (1978) were the first in stating that the participation decision of a household member depends on the employment state of the other members. To the best of our knowledge, no attempt has been done to introduce households in this framework to study the optimal insurance.[9]

Second, Ortigueira and Siassi (2013) and Choi and Valladares-Esteban (2016) quantitatively assess unemployment insurance with couples in the Aiyagari-Hugget framework, in which households are hit by exogenous employment shocks. Instead, in our setting, the public insurance scheme as well as the private provision of insurance affect the search decisions and job opportunities of the unemployed. Attanasio et al. (2005) estimate larger welfare costs of uncertainty in the husband's earnings in the absence of the ability of their wives to adjust the labor supply.

earnings following their husbands' job loss using CEX data for high-school and college graduates, but not significant using PSID data. Using CPS data, Mankart and Oikonmou (2014) estimate that wives are 7.7% more likely to participate in the labor market in the month in which their husband becomes unemployed, which is almost as high as the overall participation probability of wives.

[9]As Guler et al. (2012) point out, there has been no continuation of their work until very recently. They analyze the case of couples jointly searching for jobs in different locations. As in our setting, wage dispersion arises in equilibrium in those models as the spouse's income affects the reservation value of the job-seekers when there is perfect income pooling. They do not pay attention, however, to the efficient distribution of unemployment risks.

Our work is also closely related to the optimal income taxation literature, starting from Mirrlees (1971). Boone and Bovenberg (2004) and Hungerbühler et al. (2006) analyze optimal taxation in a frictional labor market. While in the latter, the demand side and wages are exogenous, workers are risk neutral and unemployment benefits are constant in the latter. None of these deals with two-member households. The optimal income taxation with couples is the focus of Kleven et al. (2009). In an economy with heterogeneity in spouse's wage rate and worker's participation costs, they find that optimal tax rates on an individual's income differ by the earnings of the spouse. In contrast to our setting, in their economy, unemployment is voluntary, wages are exogenous and constant across types, and the effects on the demand side of the market are overlooked, and the utility function is quasi-linear in consumption, which eliminates the income effects on labor supply of the spouse. Chetty and Saez (2010) also examine the optimal design of UI schemes in the presence of private insurance provided by employers in the form of severance pay, and derive sufficient statistic formulas that map elasticities into optimal policies.

The crowding-out effects of public intervention have also been analyzed in different settings. Krueger and Perri (2011) investigate the optimal degree of progressivity in the income tax as a public risk-sharing device in the presence of limited private insurance markets. They and Attanasio and Ríos-Rull (2000) show that the introduction of mandatory public insurance may indeed backfire and reduce total insurance. In particular, the former, Di Tella and MacCulloch (2002) and Thomas and Worrall (2007) show that public provision of insurance makes it more difficult to enforce limited commitment contracts as it increases the outside value. Instead, we have *altruistic* private insurance arrangements within the household. As these papers show, the specifics of the private insurance scheme matter for the net gains of the public provision. For example, the crowd-out effects of the public UI would have been larger if we had modeled potential inabilities of the households to smooth consumption in the short run due to e.g. habit formation or consumption commitments. Golosov and Tsyvinski (2007) also show that the optimal public provision of insurance is lower in the presence of endogenous private provision, whereas Cutler and Gruber (1996) estimate empirically crowding-out effects of medicaid expansions.

Finally, we model heterogeneity across households in the spouse's income, but other dimensions of heterogeneity are also potentially key for the design of the public provision of insurance. For example, differences across workers result from their unemployment duration and over the lifecycle as studied by Hopenhayn and Nicolini (1997) and Michelacci and Ruffo (2015), respectively.

[To be completed]

Table 1: Summary Statistics

	Full Sample		Subsample with only first spell	
	Mean	**Std. Dev.**	**Mean**	**Std. Dev.**
Age	38.358	8.803	38.193	8.871
Female	0.516	0.450	0.523	0.499
Married	0.567	0.496	0.571	0.495
No. of children under 18	1.057	1.261	1.043	1.252
Spouse employed	0.762	0.426	0.766	0.423
if men	0.640	0.480	0.646	0.480
if women	0.881	0.324	0.880	0.325
Spouse NILF	0.165	0.371	0.161	0.368
if men	0.276	0.447	0.270	0.444
if women	0.056	0.229	0.058	0.235
Spouse's earnings (if > 0)	3848.194	4013.308	3897.06	4048.526
if men	2725.897	2823.848	2828.839	3079.269
if women	4677.694	4527.658	4664.924	4466.380
HH net liquid wealth	52 361.66 (median 6.74)	856 131.40	57 732.82 (median 108.03)	982 945.20
Nonemployment duration	21.007	23.748	22.090	25.270
No. of observations	42,334		31,216	

Note.- Net liquid wealth is defined as total wealth minus home, vehicles and business equity and also net of unsecured debt.

2 Data

We use data from the Survey of Income and Program Participation (SIPP) for the U.S. for the 1996, 2001, 2004 and 2008 panels, covering from 1996:6 to 2013:6. Surveyed individuals are interviewed every four months and report for the previous four-month period a number of demographic and economic variables, in particular their labor market status, income and hours worked. We consider two labor market status: employment, E, and nonemployment, $\not E$.[10]

We restrict our dataset to individuals aged 25-55, who are quite attached to the labor market, and for which we have precise information of the duration of their unemployment. An observation is a $E\not E E$ spell. We eliminate observations with a $\not E$ spell shorter than 3

[10] We largely follow Cullen and Gruber (2000) and Chetty (2008).

Table 2: Cox Hazard Model Estimates

Spouse's log earnings	−0.064***	(.011)				
Married -no info on spouse			0.034	(.030)	0.028	(.030)
Spouse has no earnings			−0.015	(.034)	−0.008	(.034)
Spouse's earnings Q1			0.063*	(.035)	0.070**	(.035)
Spouse's earnings Q2			−0.042	(.036)	−0.036	(.036)
Spouse's earnings Q3			−0.075**	(.036)	−0.071**	(.036)
Spouse's earnings Q4			−0.073**	(.035)	−0.071**	(.036)
Spouse's earnings Q5			−0.092**	(.036)	−0.090**	(.037)
HH net liquid wealth Q1					−0.138***	(.030)
HH net liquid wealth Q2					−0.191***	(.029)
HH net liquid wealth Q3					−0.177***	(.035)
HH net liquid wealth Q4					−0.129***	(.029)
HH net liquid wealth Q5					−0.148***	(.030)
Female	−0.224***	(.023)	−0.137***	(.014)	−0.130***	(.015)
No. of children under 18	−0.033***	(.009)	−0.023***	(.006)	−0.021***	(.006)
Previous log earnings	0.083***	(.010)	0.089***	(.007)	0.089***	(.007)
No. of observations	12,410		30,140		30,140	

Note.- Subsample comprising only the first $E\not{E}E$ spell for all individuals in the sample. The first coefficient in first specification reports the elasticity of hazard rate with respect to spouse's average earnings of the previous three months prior to re-employment. The first set of coefficients in second and third specifications can be interpreted as the percentage change in hazard rate associated with the quintile of the spouse's average earnings of the previous three months prior to re-employment. All models also include log unemployment rate, a time line and monthly dummies, a quadratic polynomial of age, and dummies for seam effects, white and black, high-school, college degree and postcollege degree, homeownership, state, occupation and industry. The net liquid wealth dummies are indicator variables for whether the household wealth falls into the corresponding quintile. Standard errors are in parenthesis.

weeks. We end up with 42,334 spells. Table 2 shows the summary statistics for the whole sample. Since only 45.60% of the individuals in our sample have a single spell, dealing with all spells would overweight short spells. Therefore, we further restrict our dataset to the very first spell of all individuals in our sample, which accounts for 73.74% of the observations.[11]

We use total earned income as earnings in SIPP are defined as wages and salary and self-employment income, and includes earnings from all jobs in case of multi-jobs holders.

We estimate a Cox proportional hazard model, which is reported in Table 2. Both columns

[11] We conduct robustness checks by extending the dataset to the first two observations of all workers, accounting for almost 93% of the observations. Cullen and Gruber (2000) deals with this issue by equally weighting all the observations of a given individual so that her total weight is one.

show a negative relationship between the hazard rate of a worker and his or her spouse's wage. Notice that these estimates do not vary significantly when including information on net liquid wealth. To the extent that there is no moral hazard problems within the household, the higher exit rates from nonemployment for workers whose spouse's wage is below the median points to households facing liquidity constraints and/or consumption commitments.[12] This suggests that large consumption-related welfare gains might be expected from UI benefits if contingent on spouse's earnings. However, it may be the case that workers in the bottom part of the spouse's wage distribution are intrinsically different from their counterparts above the median, and, hence, the differences in exit rates be not explained by the intrahousehold insurance.

Figure 1 displays the change in spouse's earnings before and after a $\not{E}$ spell of the worker over the distribution of the spouse's earnings prior to the $\not{E}$ spell. Sizable differences appear for households with the spouse's prior earnings below and above the median level, with significant increases for those below the median. This indicates that the elasticity estimates that are usually reported at the sample mean must be taken cautiously as the cross-wage elasticity quite varies across the distribution. Blundell et al. (2016, Table 7) report sizable effects of permanent shocks to husband wages on the wife's participation decision, and no significant effects from transitory shocks.

[To be completed]

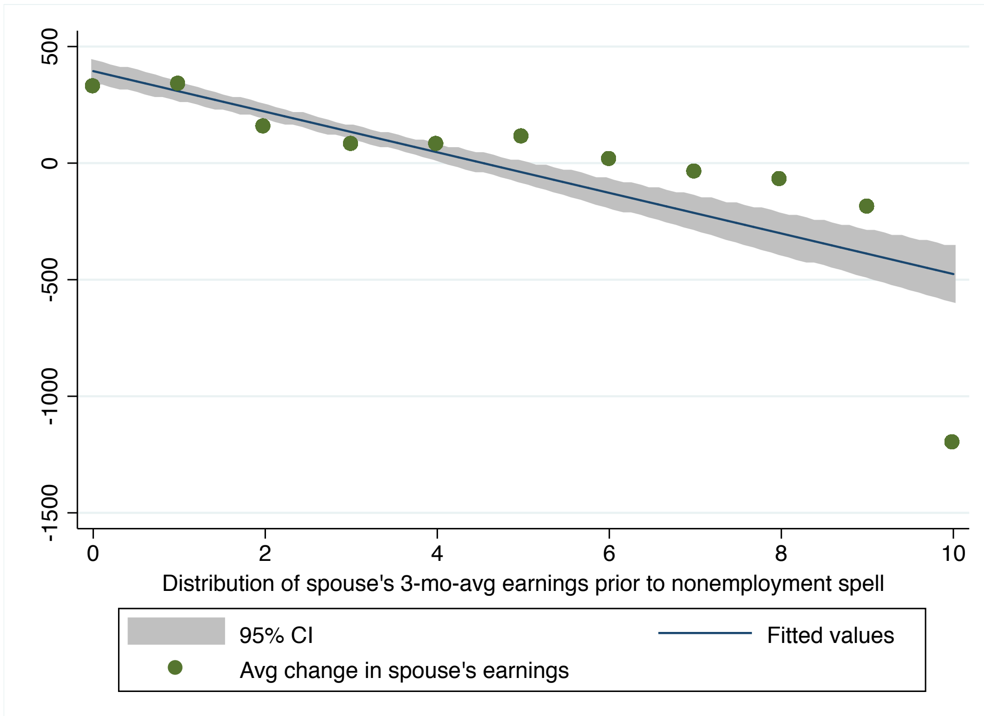
3 Benchmark Model

Consider an economy populated by a measure one of two-member households and a large continuum of risk-neutral firms. The mass of active firms is pinned down by free entry. Households are formed by an unemployed worker and her spouse endowed with market productivity $x \in [\underline{x}, \overline{x}]$, with $\underline{x} > 0$.[13] The former searches for a job, whereas the spouse chooses her labor supply $\ell \in [0,1]$. Let $F(x)$ denote the measure of households with type below level x, and it is assumed to be a differentiable cdf. Following the optimal taxation literature since Mirrlees (1971), productivity and labor supply (either hours worked or effort) are private information, whereas an individual's total earnings are observable by the government and the

[12] Although the negative cross-wage elasticity of hours worked can also be explained by substitution in home production, the estimate of having children under age 18 is not statistically significant at 10%.

[13] Alternatively, x can be interpreted as ability or hourly wage. This latter interpretation neglects general equilibrium effects. The assumption of a positive lower bound is made for expositional reasons. Single-earner households can be thought of as the case with the spouse's productivity x being arbitrarily small.

Figure 1: Change in Spouse's Earnings



planner.[14] It is convenient to think in terms of observable variables, and, hence, we model the spouse as deciding total earnings y instead of labor supply $\ell = y/x$.[15]

To begin with, we follow Guler et al. (2012) and assume that households are the decision-making units, and consumption is a public good within the household.[16] They derive utility

[14] As pointed out by Salanie (2011), if labor supply were interpreted as hours worked, the government could force employers to report them.

[15] For simplicity, we make the assumption that spouses work in a frictionless labor market and decide their labor supply at a given productivity. According to CPS data, not-in-the-labor-force wives amount to 30 percent of married women. To abstract from the underlying reasons of their non-participation decision -caring of children and elderly, etc.-, we model their market productivity at zero. Similarly, single primary earners are assumed to be married to a zero-productivity spouse. The empirical literature has not unraveled whether the increase in wives' hours results from increasing working time at the same job or from job-switching.

[16] In Section 4.4.2, we extend the analysis to a cooperative model of the household.

from consumption c and leisure of the spouse.[17] We impose the following assumptions on the utility function $\upsilon(c, y/x)$ that describes the preferences of a couple:

A1. υ is thrice continuously differentiable.

A2. υ is increasing in consumption and leisure: $\upsilon_c > 0$, $\upsilon_\ell < 0$

A3. υ is strictly concave: $\upsilon_{\ell\ell}, \upsilon_{cc} < 0$, and $\upsilon_{\ell\ell}\upsilon_{cc} - \upsilon_{c\ell}^2 > 0$.

A4. Weak complementarity: $\upsilon_{c\ell} \leq 0$.

A5. $\lim_{\ell\to 0} \upsilon_\ell < \lim_{\ell\to 0} \upsilon_c$ and $\lim_{\ell\to 1} \upsilon_c < \lim_{\ell\to 1} \upsilon_\ell$.

The first three conditions are fairly standard. We also assume that the cross-partial derivative is non-positive, meaning complementarity between consumption and leisure, which includes the case of additive separability between consumption and leisure.[18] The last condition ensures the existence of an interior solution in the household's problem. Because of our quantitative exercise in Section ???, we will pay particular attention to two widely used families of preferences that are additively separative in consumption and leisure.

$$\mathcal{F}_1 \equiv \left\{ \upsilon | \upsilon(c,\ell) = \frac{-e^{-\gamma_c c}}{\gamma_c} - \gamma \frac{e^{\gamma_\ell \ell}}{\gamma_\ell}, \gamma, \gamma_c, \gamma_\ell > 0 \right\},\ \mathcal{F}_2 \equiv \left\{ \upsilon | \upsilon(c,\ell) = \frac{c^{1-\sigma_c}}{1-\sigma_c} - \gamma \frac{\ell^{1+\sigma_\ell}}{1+\sigma_\ell}, \sigma_c, \sigma_\ell > 0 \right\}$$

Families $\mathcal{F}_1$ and $\mathcal{F}_2$ are of CARA- and CRRA-type in consumption, respectively, and convex in labor supply. It is convenient to introduce the following notation: $A_s \equiv |\frac{\upsilon_{ss}}{\upsilon_s}|$ and $P_s \equiv |\frac{\upsilon_{sss}}{\upsilon_{ss}}|$ for $s \in \{c, \ell\}$. These concepts are usually referred to as absolute risk aversion and prudence.

There are four stages. In stage one, unemployed workers direct their search. That is, they choose a submarket and place an application at cost κ.[19] A submarket or location is defined by a set of job characteristics. In stage two, firms decide on the submarket to place their vacancies, and incur cost k when posting a vacancy. Market productivity of newly employed workers is normalized to 1. As usual in the search literature, each recruiting firm holds a single vacancy. Meetings take place in stage three as described below. Some workers become

[17] We abstract from whether leisure of the household members are substitutes or complements by assuming indivisible labor supply of the unemployed worker.

[18] Weak complementarity between consumption and leisure together with strict concavity is a sufficient, but not necessary condition for the results, as shown by Chipman (1977) regarding leisure being a normal good. In particular, Attanasio and Weber (1995) and Meghir and Weber (1996) provide empirical evidence in this regard. Most macro models assume additive separability. See e.g. Kleven et al. (2009) and Heathcote et al. (2014).

[19] Labor market participation is costly to make the social planner's problem analyzed in Section 5 non-trivial.

employed, whereas some other workers remain unemployed and produce z at home. In stage four, spouses decide their total earnings y, and both production and consumption take place.

To ensure existence of equilibrium and that all jobless workers search for a job, we make the following two assumptions. First, there is a gap between net market productivity and vacancy creation costs, $1 - z > k$, to ensure that vacancy creation is a profitable activity. Second, cost κ is sufficiently small so that $\max_y \upsilon(y+1, y/\underline{x}) - \max_y \upsilon(y+z, y/\underline{x}) > \kappa$.

Matching Rates. Meetings are bilateral. Workers find a job at submarket ω with probability $\nu(q)$, where q denotes the expected queue length or ratio of job-seekers to vacancies, whereas firms fill their vacancies with probability $\eta(q)$. Since the mass of newly employed workers equals the mass of newly filled vacancies in any given submarket w, it must be the case that $\nu(q) = \frac{\eta(q)}{q}$. We assume that ν is a decreasing function to capture the intuition that it is harder to find a job in tighter labor markets, and, hence, η is assumed to be increasing. Likewise, the following limit conditions are necessary to ensure existence of equilibrium and planner's allocations: $\lim_{q \to 0} \nu(q) = \lim_{q \to \infty} \eta(q) = 1$ and $\lim_{q \to \infty} \nu(q) = \lim_{q \to 0} \eta(q) = 1$. Let $\gamma(q) \equiv \frac{q\eta'(q)}{\eta(q)}$ denote the elasticity of the job-filling rate, which is assumed to be a decreasing function.[20]

4 Market Economy

In this section, we analyze an economy in which agents make decisions in a decentralized way to maximize their utility. There are potentially infinitely many submarkets. Each submarket is defined by a single-wage offer w. Whereas firms decide whether to create a vacancy and what wage to commit to, the household's decision is twofold. First, it chooses a submarket to submit a job application. Then, after learning the search outcome, it decides the labor supply of the spouse. We start detailing this last stage and, then, proceed backwards.

Stage Four. Let w denote the income of the job-seeker at the end of the period, with $w = z$ if unemployed. We denote the household's indirect utility function by V_x, which is defined as

$$V_x(w) \equiv \max_y \upsilon\big(y + w, y/x\big) \tag{1}$$

The Weierstrass theorem together with Assumption A1 ensures that V_x is well-defined. Notice that the first order necessary condition is also sufficient because of Assumption A3. Moreover,

[20] These properties are satisfied for the usual matching functions, e.g. the Cobb-Douglas and urn-ball ones.

Assumption A5 ensures the existence of an interior solution for the first order condition. Therefore, the following equations uniquely determine the contingent earnings of the spouse, $y_x^e(w)$ and y_x^u.

$$\upsilon_c\big(y+w, y/x\big)x = -\upsilon_\ell\big(y+w, y/x\big) \tag{2}$$

$$\upsilon_c\big(y+z, y/x\big)x = -\upsilon_\ell\big(y+z, y/x\big) \tag{3}$$

As can be anticipated from the observation of the household's problem, the indirect utility function is central in the analysis of the market equilibrium. In the following lemma, we establish properties of function V_x and of the optimal total earnings of the spouse, which are inherited from the assumptions on the utility function υ and follow from using repeatedly the Implicit Function Theorem to the first order conditions. In particular, the labor supply of the spouse is larger if married to an unemployed worker than to an employed one in line with the evidence reported in the Introduction. This is because labor supply is set to equate the marginal utility of leisure and the marginal utility of consumption, which is lower for households with the two members employed, together with the complementarity between consumption and leisure. Furthermore, we conclude that the cross-wage elasticity is negative, which is in line with the evidence reported in the Introduction. In other words, the spouse's leisure is a normal good. Finally, earnings of a spouse in a single-earner household increase with productivity because of concavity of the utility function and the complementarity between consumption and leisure, whereas the spouse's earnings in two-earner households need not increase with productivity x if worker's wage raises with x because of the income effect.[21] Similarly, the overall effects of productivity on labor supply depend on whether income effects dominate the substitution effects.

Lemma 4.1 *For any productivity $x \in [\underline{x}, \overline{x}]$, function V_x is twice continuously differentiable, strictly increasing and concave in wages. Furthermore, the optimal solution y_x^e is twice continuously differentiable and strictly decreasing in wages. In particular, $y_x^e(w) < y_x^u$ for all $w > z$.*

Likewise, for any wage $w \geq z$, $V_x(w)$ increases and its derivative $V_x'(w)$ decreases with the spouse's productivity x. Furthermore, the optimal solution $y_x^u(w)$ is twice continuously differentiable and increasing in productivity x.

[21] Consider additively separable preferences for simplicity. Then,

$$\frac{\partial y_x^e(w)}{\partial x} = \frac{-\upsilon_{cc}^e \frac{\partial w}{\partial x} x^2 + \upsilon_{\ell\ell}^e \frac{y^e}{x} - \upsilon_c^e x}{\upsilon_{cc}^e x^2 + \upsilon_{\ell\ell}^e}. \tag{4}$$

The sign of the derivate is ambiguous if $\frac{\partial w}{\partial x} > 0$.

Stage Two. There is entry of firms in all submarkets as long as expected profits are positive in and out of equilibrium. That is, the following condition must hold both for all $w \in [z, 1]$:

$$\eta(q)(1 - w) \le k, \text{ and } q \le \infty, \text{ with complementary slackness.} \tag{5}$$

Intuitively, when the larger the wage, the lower the mass of vacancies posted. In the limit, no positive mass of firms commit to a wage equal to market productivity of workers.

Stage One. Job-seekers rationally anticipate the optimal behavior of firms in the second stage, and trade off a higher wage and a higher job-finding probability. The expected utility of a household with the spouse's productivity x amounts to $\big(1 - \nu(q(x))\big)V_x(z) + \nu(q(x))V_x(w)$, where $q(x)$ denotes the expected queue length in submarket w.

4.1 Equilibrium.

We now turn to the definition of equilibrium.

Definition 1 *A directed search equilibrium consists of, for all $x \in [\underline{x}, \overline{x}]$, household values U_x, earnings $y^e_x : [z, 1] \to \mathcal{R}_+$ and $y^u_x \in \mathcal{R}_+$, wages w_x, and a queue length function $Q : [z, 1] \to \mathcal{R}_+$ such that:*

i) Households optimally direct their search and choose earnings. For all $x \in [\underline{x}, \overline{x}]$,

(a) $\nu(Q(w))\big(V_x(w) - V_x(z)\big) + V_x(z) \le U_x$, $\forall w \in [z, 1]$, and
$\nu(Q(w_x))\big(V_x(w_x) - V_x(z)\big) + V_x(z) = U_x$

(b) For all $w \in [z, 1]$, $y^e_x(w)$ and y^u_x solve the respective household's problem, and, hence, satisfy conditions (2) and (3), respectively.

ii) Free entry of firms:

$\eta(Q(w))(1 - w) \le k$, $\forall w \in [z, 1]$, and $Q(w) \le \infty$, with complementary slackness. In particular, the first inequality is an equality for all w_x.

The first equilibrium condition is self-explanatory. The second condition determines the ratio of job-seekers to vacancies both on and off the equilibrium path. Workers form rational expectations about firms' decisions in stage 2. Specifically, they expect the ratio of job-seekers to firms in any submarket to be determined by the zero-profit condition. Thus, the household's problem at the beginning of the period is[22]

$$\begin{aligned} \max_{q\geq 0, w\in[z,1]} \quad & V_x(z)+\nu(q)\big(V_x(w)-V_x(z)\big) \\ \text{s. to} \quad & \text{condition (5)} \end{aligned} \tag{6}$$

4.2 Equilibrium Characterization

The following proposition states that there exists a unique equilibrium, and characterizes it. Equilibrium condition (7) is the first order condition of the household's problem. It equates the costs of creating a vacancy to the expected profits, which amount to the probability of filling a vacancy times the share $1-\gamma(q)$ of the joint value of the firm-worker pair adjusted by the marginal utility of the household. Equilibrium equation (8) is the zero-profit condition expressed in terms of wages. For notational simplicity, we denote hereafter the equilibrium queue length at wage w_x as $q_x \equiv Q(w_x)$.

Proposition 4.2 *There exists a unique equilibrium. For any given household type $x \in [\underline{x}, \overline{x}]$, the equilibrium pair (q_x, w_x) is characterized by the following system of equations*

$$k = \eta(q)(1-\gamma(q))\left(\frac{V_x(w)-V_x(z)}{V_x'(w)}+1-w\right) \tag{7}$$

$$k = \eta(q)\big(1-w\big) \tag{8}$$

The equilibrium is generically separating. In particular, there is wage dispersion in equilibrium even though workers are equally productive and firms are also homogeneous. This is because the attitudes towards unemployment risks differ across households because of the private insurance arrangement. This poses a source of heterogeneity which has been overlooked when examining wage dispersion using a Mincerian regression.

The natural question is how worker's search strategies vary with the spouse's productivity. Put differently, given that job search can be thought of as a risky asset, do attitudes towards risk differ across households? As Gollier (2004) argues, it is commonly accepted that the risk premium we are willing to pay to escape an additive risk decreases with our wealth, which is the case when preferences exhibit decreasing absolute risk aversion. In a similar fashion, we show that households of higher types apply to jobs that are harder to obtain if the absolute risk aversion of the indirect utility function, $\frac{-V_x''(w)}{V_x'(w)}$, decreases with the spouse's productivity. This is the case, in particular, for preferences of family $\mathcal{F}_2$, which are standard

[22] For expositional simplicity, we omit the participation decision because of the assumption on sufficiently small search costs.

in the macroeconomic literature. Importantly, under that condition, this result is consistent with the evidence on hazard rates reported in Table 2. Furthermore, this is also in line with the findings in Acemoglu and Shimer (1999), although notice that if preferences are CARA-type, i.e. if $\upsilon \in \mathcal{F}_1$, wages also increase with productivity x in sharp contrast to their setting with wealth.

Proposition 4.3 *If $\frac{-V_x''(w)}{V_x'(w)}$ is decreasing (increasing) in x, then w_x and q_x are increasing (decreasing) in x. In particular, if $\upsilon \in \mathcal{F}_1 \bigcup \mathcal{F}_2$, then w_x and q_x increase with x.*

4.3 Private Insurance

In this section we examine how private insurance varies across households and how it is affected by the public insurance provision. We first define the measure of private insurance as the earnings of the spouse in a single-earner household in excess of the market income earned in absence of risk, $y_x^u - y_x^e(m_x)$, where m_x is the solution of the equation

$$V_x(m_x) = \max_{q \geq 0, w \in [z,1]} \quad V_x(z) + \nu(q)\big(V_x(w) - V_x(z)\big) \quad \text{s. to} \quad \text{condition (5)} \tag{9}$$

We refer to m_x as certainty equivalent for household of type x, and can be interpreted as the income that makes it indifferent between the job search lottery that maximizes its utility and a sure amount. The continuity and monotonicity of function V_x stated in Lemma 4.1 ensure that the certainty equivalent exists and is unique and $m_x \in (z, w_x)$.

The following lemma regarding the monotonicity of the certainty equivalent with respect to the spouse's productivity can be read as a reformulation of Proposition 4.3.

Lemma 4.4 *The certainty equivalent m_x increases (declines) with x if $\frac{-V_x''(w)}{V_x'(w)}$ is decreasing (increasing) in x. In particular, if $\upsilon \in \mathcal{F}_1 \bigcup \mathcal{F}_2$, then m_x increases with x.*

Proof of Lemma 4.4.

More importantly, this result highlights that our measure of insurance captures a spurious component which is due to the extra risk that the insurance itself generates. To be precise, consider the case of $\frac{-V_x''(w)}{V_x'(w)}$ being a decreasing function of x. This is a necessary and sufficient condition for a higher productivity x be equivalent to a lower degree of risk aversion. Therefore, a higher x is associated to a higher m_x partly because of the higher risk in job search the household takes when implicitly becoming less risk-averse. We must address this concern when assessing how private insurance varies across the household distribution, which is the next question.

Do more productive spouses provide more insurance against unemployment? To address this question, we examine the variation of the relative measure of insurance, $\frac{y_x^u - y_x^e(m_x)}{y_x^e(m_x)}$, with the spouse's productivity. Notice that by doing so we would also be studying the relative change in labor supply. By differentiating the first order condition of the household's problem (1) with respect to productivity x, we obtain

$$\frac{\partial\left(\frac{y_x^u - y_x^e(m_x)}{y_x^e(m_x)}\right)}{\partial x} = \left(\frac{\frac{\partial y_x^u}{\partial x}}{y_x^u} - \frac{\frac{\partial y_x^e(m_x)}{\partial x}}{y_x^e(m_x)}\right)\frac{y_x^u}{y_x^e(m_x)} \tag{10}$$

where

$$\frac{\frac{\partial y_x^u}{\partial x}}{y_x^u} - \frac{\frac{\partial y_x^e(m_x)}{\partial x}}{y_x^e(m_x)} = \underbrace{\frac{\upsilon_{\ell\ell}^u \ell_x^u - \upsilon_c^u x + \upsilon_{c\ell}^u y_x^u}{y_x^u(\upsilon_{cc}^u x^2 + \upsilon_{\ell\ell}^u + 2\upsilon_{c\ell}^u x)} - \frac{\upsilon_{\ell\ell}^e \ell_x^e - \upsilon_c^e x + \upsilon_{c\ell}^e y_x^e}{y_x^e(m_x)(\upsilon_{cc}^e x^2 + \upsilon_{\ell\ell}^e + 2\upsilon_{c\ell}^e x)}}_{d_1(x)} + \underbrace{\frac{\upsilon_{cc}^e \frac{\partial m_x}{\partial x} x}{y_x^e(m_x)(\upsilon_{cc}^e x^2 + \upsilon_{\ell\ell}^e + 2\upsilon_{c\ell}^e x)}}_{d_2(x)}$$

The earnings difference has two components. The second term of the above expression, $d_2(x)$, captures the spurious component highlighted above. If the absolute risk aversion of the indirect utility function is decreasing in x, then $d_2(x)$ is positive. The first term, $d_1(x)$, of expression (10) is the primary object of our analysis, instead.

We restrict the analysis to preferences that are additively separable in consumption and leisure. The following lemma states sufficient conditions for the first component to be negative. The first condition says that the degree of absolute prudence is larger than the degree of absolute risk aversion in consumption, a necessary and sufficient to ensure DARA in consumption, which once again is widely accepted in the literature. The second condition is its counterpart for the labor supply of the spouse, while the monotonicity of the absolute risk aversion in labor supply is bounded by the third condition. In particular, the preferences of families $\mathcal{F}_1$ and $\mathcal{F}_2$ satisfy such conditions.

Lemma 4.5 *Consider additively separable utility functions. If A_c and A_ℓ are decreasing functions in consumption and labor, respectively, and $R(\ell) \equiv \ell A_\ell$ is a non-decreasing function in labor, then the term $d_1(x)$ is negative. Furthermore, any $\upsilon \in \mathcal{F}_1 \bigcup \mathcal{F}_2$ satisfies those conditions, and the corresponding $d_2(x)$ is positive.*

Recall that Figure 1 displayed a declining pattern of earnings change relative to prior-to-unemployment earnings. As in Table 2, the pattern appears to be non-linear, with no large differences above the median of the spouse's earnings distribution. Despite the differences between the theoretical and the empirical constructions, for the theory to be in line with the data under the conditions stated in the previous lemma, the first component of expression (10) should dominate the second one.

Does public insurance provision crowd out private insurance? We now examine the effects of an increase in the generosity of the unemployment insurance system. To capture the first-order effects abstracting from the tax-related general equilibrium effects, consider changes in parameter z. The following lemma states that a more generous public provision of insurance (i.e. a higher x) reduces the labor supply of spouses married to unemployed workers because, once again, spouse's leisure is a normal good. Furthermore, as is common in search models, wages increase and job-finding rates decrease with z. This result together with the negative relationship between the wage of the primary earner and the labor supply of the secondary earner stated in Lemma 4.1 implies that the labor supply of the spouse married to an employed worker also decreases with z. These direct and indirect negative effects of unemployment benefits on the spouse's labor supply are consistent with the empirical evidence reported by Cullen and Gruber (2000). They estimate that each \$100 in potential benefits lowers the working hours of wives of employed and unemployed workers by 5.2 and 22.7 per month, respectively. The general equilibrium effect through wages of unemployment benefits on the labor supply of the spouse in two-earner households has two components. While the positive macroeconomic effects of benefits on wages are supported by Hagedorn et al. (2015), the second component of the mechanism is in line with the negative cross-elasticities estimated by e.g. Hyslop (2001) and Blau and Kahn (2007) as reported in the Introduction.

Lemma 4.6 *Comparative Statics.*

1. *Wages, queue lengths and certainty equivalents increase with z.*
2. *The spouse's labor supply decreases with z regardless of the employment state of the worker.*

It is noteworthy, once again, that a reduction in the spouse's earnings y_x^u resulting from an increase in benefits may not imply a smaller private insurance provision as $y_x^e(w_x)$ also reduces because of the income effects of higher wages. To address the question of whether public provision crowds out private insurance, we thus consider the earnings difference $y_x^u - y_x^e(m_x)$. By differentiating the first order condition of the household's problem (1) with respect to parameter z, we obtain

$$\frac{\partial(y_x^u - y_x^e(m_x))}{\partial z} = \underbrace{\frac{-\upsilon_{cc}^u x^2 - \upsilon_{c\ell}^u x}{\upsilon_{cc}^u x^2 + \upsilon_{\ell\ell}^u + 2\upsilon_{c\ell}^u x} + \frac{\upsilon_{cc}^e x^2 + \upsilon_{c\ell}^e x}{\upsilon_{cc}^e x^2 + \upsilon_{\ell\ell}^e + 2\upsilon_{c\ell}^e x}}_{\delta_1(x)} + \underbrace{\left(\frac{\partial m_x}{\partial z} - 1\right)\frac{\upsilon_{cc}^e x^2 + \upsilon_{c\ell}^e x}{\upsilon_{cc}^e x^2 + \upsilon_{\ell\ell}^e + 2\upsilon_{c\ell}^e x}}_{\delta_2(x)} \quad (11)$$

The variation in the earnings difference as z rises comprises two terms. The second term, $\delta_2(x)$, captures the effects of an increase in z on the difference between worker's wage and unemployment benefits. To the extent that the pressure on the certainty equivalent is below one-to-one, the expression $\delta_2(x)$ is negative. By abstracting from the general equilibrium effects on wages, the term $\delta_1(x)$ amounts to the direct effects of an increase in benefits on private insurance arrangements. The following lemma states that decreasing absolute risk aversion in consumption and labor are sufficient conditions for the $\delta_1(x)$-related private insurance to be crowded out by public insurance provision when restricting to additively separable preferences. Notice that preferences that belong to family $\mathcal{F}_2$ satisfy these two conditions. The total private insurance declines with unemployment benefits if, in addition, the parameter σ_ℓ is above one. Recall that this parameter governs the Hicks elasticity, and it is the inverse of the Frisch elasticity in a dynamic model, and both elasticities have been consistently estimated below one.[23]

Lemma 4.7 *Consider additively separable preferences. If A_c and A_ℓ are decreasing functions in consumption and labor, respectively, then $\delta_1(x) < 0$. In particular, if $\upsilon \in \mathcal{F}_2$, then $\delta_1(x) < 0$; and if, additionally, $\sigma_\ell \geq 1$, then $\delta_2(x) < 0$ and $\frac{\partial(y^u - y_x^e(m_x))}{\partial z} < 0$. Furthermore, if $\upsilon \in \mathcal{F}_1$, then $\delta_1(x) = 0$.*

Unfortunately, we cannot make substantive general statements on how the crowding-out effects of the public insurance provision vary across households of different spouse's productivity. We shall address this point in the quantitative exercise.

[23]Specifically, the Hicks elasticity is equal to $\frac{1}{\sigma_\ell + \sigma_c \frac{x\ell}{w + x\ell}}$. Keane (2011, Tables 6 and 7) reports the Hicks elasticity to be widely estimated below 0.3 for both men and women, and an average Frisch elasticity at 0.85 for men and fairly low for working women (above 2 when including the extensive margin for females).

4.4 Extensions

In this section we examine two particular cases that have been considered in the literature: namely, quasi-linear preferences and a cooperative model of the household.

4.4.1 Quasi-linear Preferences.

Consider first a quasi-linear utility function in consumption, $\upsilon(c, y/x) = c + \phi(y/x)$, where function ϕ is twice continuously differentiable, decreasing and concave. Then, the indirect utility function V_x is linear in wages. As is well known, labor supply of the spouse is insensitive to the income of the job-seeker and, in particular, to unemployment benefits; hence, household's consumption increases one-to-one with benefits. The equilibrium conditions are

$$\begin{aligned} k &= \eta(q_x)(1-\gamma(q_x))(1-z), && \forall x \in [\underline{x}, \overline{x}] && (12)\\ k &= \eta(q_x)(1-w_x), && \forall x \in [\underline{x}, \overline{x}] && (13)\\ \phi'(y_x^j/x) &= -x, && \forall x \in [\underline{x}, \overline{x}], j \in \{u, e\} && (14) \end{aligned}$$

Notice that the first two equations are the counterparts of conditions (7) and (8), whereas the last one is the first order condition of the household's problem (2). It follows from the first two equilibrium conditions that all workers search in the same market regardless of their spouse's wage rate.[24] The third condition shows no income effects on the labor supply of the spouse, and an increasing income y_x^j in x.

Consider next quasi-linear preferences in leisure: $\upsilon(c, y/x) = \psi(c) - y/x$, where ψ is a twice continuously differentiable, increasing and concave function. The equilibrium conditions are the same as before, except for the last one, which is replaced by

$$\psi'(w_x + y_x^e), \psi'(z + y_x^u) \;=\; 1/x \qquad (15)$$

There is full insurance because the labor supply of the spouse adjusts to make consumption invariant to the search outcome. This intra-household way of completing markets intuitively eliminates all consumption risks. As a result, job-seekers no longer factor consumption risks in their search decisions, and just trade off job-finding rates and income, and all search in the same market regardless of productivity x.[25] Needless to say, consumption increases with

[24] Notice that the absolute risk aversion of the indirect utility function V_x is zero.

[25] Formally, productivity x determines consumption, and, hence, the objective function in problem (6) becomes $\nu(q)(w - z)$. Both the objective function and the constraint are independent of household type, and

the spouse's productivity because so does the spouse's income, and the difference $y_x^u - y_x^e$ is also constant in x.

4.4.2 Cooperative Model

The benchmark economy hosts a unitary model of the household, in which households are the decision-making units that maximize a utility function subject to a budget constraint. This modeling has been questioned on empirical and theoretical grounds. See Chiappori and Donni (2009) for a survey. Therefore, it is worth checking the robustness of our results in a cooperative model of the household. In such models, each member of the household has their own preferences, and the decision-making process is usually not made explicit. Consumption is no longer a public good. Instead, the two members of the household pool income and decide on their individual consumption and labor supply. Importantly for our analysis, cooperative models ensure Pareto efficient intra-household outcomes.

For notational simplicity, let m and f denote the index of the two members of the household. Likewise, α stands for the Pareto weight on the utility of the first member, and captures the m's relative power within the household. To abstract from the interaction between policy and intra-household power distribution, we assume that the Pareto weights do not depend on income, and in particular, they are insensitive to wage w and productivity x as well as unemployment benefits z.[26] The indirect utility function of a household of type x is

$$\begin{aligned} V_x(w) = \max_{c^f, c^m, y} \quad & \alpha \upsilon^m\big(c^m, \underline{\ell}\big) + (1-\alpha)\upsilon^f\big(c^f, y/x\big) \\ \text{s. to} \quad & c^f + c^m = y + \mathcal{I}_e w + (1-\mathcal{I}_e) z \end{aligned} \tag{16}$$

where υ^f and υ^m satisfy properties A1-A5, labor supply $\underline{\ell}$ is exogenous, and $\mathcal{I}_e$ is an indicator function that values one if the worker is employed and zero otherwise.

Notice that the first order conditions establish the following risk-sharing policy, which depends on the Pareto weights, $\alpha \upsilon_c^m = (1-\alpha)\upsilon_c^f$. That is, the marginal utility must be equal across members after adjusting for the weight distribution within the household. Likewise, the household marginal gains of an increase in wages must be equal to the marginal gains from an equivalent increase in the spouse's leisure, $V_x'(w) = \frac{-(1-\alpha)}{x}\upsilon_\ell^f$.

The following lemma states that we obtain the same results as in the benchmark case.

we are then solving the standard program in the basic directed search model with risk-neutral workers.

[26] In a *collective* model of the household instead, the Pareto weights may depend on the relative earnings, total income and other called *distribution factors* out of the model.

Lemma 4.8 *Function V_x is twice continuously differentiable, strictly increasing and concave. The optimal solution y_x^e is twice continuously differentiable, and strictly decreasing in the wage. In particular, $y_x^e(w) < y_x^u$ for all $w > z$ and x. Furthermore, the spouse's hours worked decrease with z regardless of the employment status of the worker. The equilibrium allocation is determined by equations (7) and (8).*

5 Constrained Efficiency

The main result of this section is that constrained efficiency cannot be attained in the market economy because private provision of insurance against consumption risks is inefficiently limited, and the insurance mechanisms, in the labor market through vacancy creation and within the household through the spouse's labor supply, are excessively used. We first characterize the constrained efficient allocation.

5.1 The Planner's problem

As usually assumed in the search literature, the social planner maximizes a utilitarian welfare function. It sets a mass of vacancies, assigns search strategies and transfers to workers, and faces the same coordination frictions as agents encounter in the market economy. Moreover, the planner cannot observe the type of the households and the spouse's labor supply; however, both the spouse's income y and the worker's employment state are observable.

More specifically, the planner designs a symmetric incentive compatible revelation mechanism that consists of a menu of contracts $\{(q_x, c_x^e, c_x^u, y_x^e, y_x^u)|x \in [\underline{x}, \overline{x}]\}$ indexed by the household's announcement of its type. The mechanism is symmetric in the sense that all households reporting a given type are treated identically. For any reported type x, the mechanism specifies a location where to submit an application and the associated job-finding probability, $\nu(q_x)$, consumption as well as the spouse's income contingent on the search outcome, (c_x^e, c_x^u) and (y_x^e, y_x^u).

We say that a mechanism is *feasible* if total consumption promises do not exceed total output net of vacancy creation costs, i.e. if the following resource constraint holds

$$\int_{\underline{x}}^{\overline{x}} \frac{k}{q_x} dF(x) \quad = \quad \int_{\underline{x}}^{\overline{x}} \Big(\nu(q_x)\big(1 + y_x^e - c_x^e\big) + (1-\nu(q_x))\big(z + y_x^u - c_x^u\big)\Big) dF(x) \tag{RC}$$

The ex-ante utility of a household of type x reporting type $\hat{x}$ can thus be written as

$$\mathcal{U}_x(\hat{x}) \equiv \nu(q_{\hat{x}})\upsilon\big(c_{\hat{x}}^e, y_{\hat{x}}^e/x\big) + (1-\nu(q_{\hat{x}}))\upsilon\big(c_{\hat{x}}^u, y_{\hat{x}}^u/x\big) \tag{17}$$

To simplify notation, let us denote $\mathcal{U}_x \equiv \mathcal{U}_x(x)$. The mechanism must be compatible with agents' incentives. This implies that job-seekers must truthfully reveal their types. That is, the following incentive compatibility constraints must hold.

$$\mathcal{U}_x \geq \mathcal{U}_x(x'), \qquad \forall x, x' \in [\underline{x}, \overline{x}] \qquad\qquad (\text{ICC}_x)$$

Furthermore, the value of job-search must exceed the application cost to ensure that participating in the market is desirable. That is, the following set of participation conditions must also hold.

$$\mathcal{U}_x \geq \kappa + \upsilon(c^u_x, y^u_x/x) \qquad\qquad (\text{PC}_x)$$

We will refer as the constrained efficient allocation to the feasible incentive-compatible mechanism that solves the planner's problem, which can be written as

$$\textit{Planner's problem:} \quad \max \int_{\underline{x}}^{\overline{x}} \mathcal{U}_x dF(x)$$
$$\text{s. to} \quad (\text{PC}_x),\ (\text{ICC}_x) \text{ and (RC) for all } x$$

To understand the importance of each one of these constraints, let us consider what allocation would be obtained if either one were subtracted. Obviously, resources are limited. If the participation conditions were eliminated, the planner would promise equal bundles regardless of the search outcome, and this allocation could trivially not be decentralized. If the incentive compatibility constraint were eliminated instead because types were observable and preferences were additively separable, then the planner's allocation could be decentralized in the market economy as we will comment later. However, consumption would be constant across types and higher types would produce more, yielding a declining expected utility over productivity levels. This result is well-known in the optimal taxation literature, see e.g. Mankiw et al. (2009). To see that incentive compatibility ensures that households with more productive spouses obtain higher values notice that

$$\mathcal{U}_x \;\geq\; \mathcal{U}_x(x') = \nu(q_{x'})\upsilon\big(c^e_{x'}, y^e_{x'}/x\big) + (1-\nu(q_{x'}))\upsilon\big(c^u_{x'}, y^u_{x'}/x\big) > \mathcal{U}_{x'}, \qquad \text{for } x' < x$$

where the first inequality is condition (ICC_x), and the second inequality results from function υ being strictly increasing in the household's type. The intuition underlying this result is that if the sign of the inequality were reversed, higher-type households would have incentives to misreport their type. The following lemma states existence of the planner's solution as a straightforward result of Weierstrass Theorem.

Lemma 5.1 *There exists a solution to the planner's problem. Furthermore, the expected utility in the constrained efficient allocation is monotonic over household types.*

To characterize the planner's solution, it is generally convenient to reduce the dimensionality of the problem by replacing the incentive-compatibility condition (ICC_x) by a first- and a second-order condition[27]

$$\dot{\mathcal{U}}_x = -\nu(q_x)\upsilon^e_\ell \frac{y^e_x}{x^2} - (1-\nu(q_x))\upsilon^u_\ell \frac{y^u_x}{x^2}, \qquad (FOC - ICC_x)$$

$$-\nu'(q_x)\dot{q}_x \frac{\upsilon^e_\ell y^e_x - \upsilon^u_\ell y^u_x}{x^2} - \nu(q_x)\frac{\upsilon^e_{c\ell}\dot{c}^e_x y^e_x + \dot{y}^e_x\left(\upsilon^e_{\ell\ell}y^e_x/x + \upsilon^e_\ell\right)}{x^2}$$
$$-(1-\nu(q_x))\frac{\upsilon^u_{c\ell}\dot{c}^u_x y^u_x + \dot{y}^u_x\left(\upsilon^u_{\ell\ell}y^u_x/x + \upsilon^u_\ell\right)}{x^2} \geq 0 \qquad (SOC - ICC_x)$$

where, for notational simplicity, $\upsilon^j \equiv \upsilon(c^j_x, y^j_x/x)$ for $j \in \{u, e\}$ and for all x, and $\dot{n} \equiv \frac{dn}{dx}$ denote the derivative of variable n with respect to x. These necessary conditions are local. The following lemma states that they are also sufficient.

Lemma 5.2 *The above necessary conditions are also sufficient.*

5.2 Efficiency in the market economy

The following proposition states that constrained efficiency is not achieved in the *laissez-faire* equilibrium. It is easy to see that the equilibrium allocation belongs to the feasible set of the planner's problem. That is, the resource and participation constraints hold, and the equilibrium allocation is also incentive compatible. However, efficiency gains can be obtained by redistributing resources to increase consumption of the households with an unemployed member. Put differently, the private provision of insurance against consumption risks falls short of the constrained efficient level. Moreover, inefficiency may also result from the ex-ante heterogeneity across households and the market's inability of redistributing resources among them.

[27]The FOC of the (ICC_x) indeed says that the total differential with respect to $\hat{x}$ is zero at $\hat{x} = x$,

$$\frac{d\mathcal{U}_x(\hat{x})}{d\hat{x}}_{|\hat{x}=x} = \frac{\partial\mathcal{U}_x(\hat{x})}{\partial q_{\hat{x}}}\dot{q}_{\hat{x}} + \frac{\partial\mathcal{U}_x(\hat{x})}{\partial c^e_{\hat{x}}}\dot{c}^e_{\hat{x}} + \frac{\partial\mathcal{U}_x(\hat{x})}{\partial y^e_{\hat{x}}}\dot{y}^e_{\hat{x}} + \frac{\partial\mathcal{U}_x(\hat{x})}{\partial c^u_{\hat{x}}}\dot{c}^u_{\hat{x}} + \frac{\partial\mathcal{U}_x(\hat{x})}{\partial y^u_{\hat{x}}}\dot{y}^u_{\hat{x}}{}_{|\hat{x}=x} = 0,$$

which is equivalent to the expression above. To obtain the second order condition at $\hat{x} = x$, we differentiate again with respect to $\hat{x}$. To simplify the second derivative, we benefit from this condition to hold for all x. After some tedious, but straightforward calculations, we obtain expression (SOC-ICC_x).

Proposition 5.3 *Under Assumptions A1-A5, the equilibrium allocation is not constrained efficient.*

Next, we examine the inefficiency result by looking at several particular cases.

5.3 Quasi-linear preferences in consumption

We first consider the case of a quasi-linear utility function, $v(c, y/x) = c + \phi(y/x)$, where function ϕ is decreasing and concave as assumed in Section 4.4.1. The following proposition states that the equilibrium allocation is constrained efficient. That is, the planner asks all job-seekers to search in the same location and the labor supply of the spouse is not affected by the search outcome. Importantly, worker types are separated costlessly, and the marginal rate of substitution is 1 for all households.

Proposition 5.4 *Constrained efficiency is attained in the market economy.*

Why are these preferences of interest? Certainly, these preferences do not satisfy Assumption A3. However, this case constitutes the benchmark for two different literatures. First, in the optimal income taxation literature, it has been common to assume such preferences because it greatly simplifies the problem and allows for a closed-form solution.[28] See Salanie (2011) for a summary, and Kleven et al. (2009), for the optimal taxation of couples. Furthermore, this assumption is in fair agreement with the evidence on the income elasticity of labor supply for primary earners, mostly men. In contrast, when referring to secondary earners, mostly wives, labor supply elasticity is much larger, and the empirical evidence reported in the Introduction shows significant income effects of earnings and unemployment benefits on wife's hours of work.

Second, in the standard directed search model, agents are assumed to be risk neutral. Interestingly, the equilibrium allocation is constrained efficient because wages price waiting time. See Moen (1997). Therefore, although the focus of our analysis is on secondary earners and intra-household insurance, the economy with quasi-linear preferences in consumption is also of interest as a benchmark.

The following lemma establishes that constrained efficiency is also attained in equilibrium when household members have their own (quasi-linear) preferences and cooperate when making their decisions.

[28] To make the problem nontrivial, a motive for redistribution is assumed for example by either giving a lower weight to higher-earnings agents or assuming a concave transformation of agent's utility function.

Lemma 5.5 *Consider a cooperative model of the household as the one described in Section 4.4.2 in which each member has quasi-linear preferences. Then, the equilibrium is constrained efficient.*

5.4 Economy with ex-ante homogeneous households

We now examine an economy with a degenerate distribution F. Recall that, in the market economy, the equilibrium pair (w, q) is determined by conditions (7) and (8), as stated in Proposition 4.2. For expositional purposes, we assume that neither productivity nor labor supply nor output is observable to the planner, and, hence, incentive compatibility must be taken into account.[29] The planner's problem can be written as follows

$$\begin{aligned} \max \quad & \nu(q)V(m^e) + (1-\nu(q))V(m^u) \\ \text{s. to} \quad & \frac{k}{q} = \nu(q)\big(1-m^e\big) + (1-\nu(q))\big(z-m^u\big) \\ & \kappa \leq \nu(q)\big(V(m^e) - V(m^u)\big) \end{aligned} \tag{18}$$

where function V resembles expression (1). As stated in Proposition 5.3, the equilibrium is not constrained efficient. This is because of the inability of the market economy to efficiently insure the consumption risks away. Indeed, the income of the spouse is above the constrained efficient level as private intra-household insurance tries to compensate for the lack of redistribution across households.

Nonetheless, the planner's solution can be implemented in the market economy by setting an unemployment insurance system funded by lump sum and proportional income taxes. In order not to distort the labor supply of spouses, which provides intra-household insurance, their income is not taxed.[30] Furthermore, income taxes are necessary to convey the proper search incentives to job-seekers, and this is done through wages in a directed search economy. Notice that this is the case regardless of whether the spouse's income is observed or not because the planner factors in the spouse's optimal decision by equating the marginal utility from an additional consumption unit and an additional unit of leisure. It is worth underscoring that the planner's solution makes the publicly-provided insurance be based on the intra-household insurance and, hence, on household's total potential income.

We question again whether the implementation of the planner's solution still holds when

[29] We obtain the same results if they were observable as would be the case from our set of assumptions.

[30] This not taxing on agents with a high income elasticity is along the lines of Alesina et al. (2011).

considering a cooperative, in lieu of a unitary, model of the household. The answer is yes. Notice that the planner's problem written as in (18) allows for a straightforward interpretation of function V as in expression (16) in Section 4.4.2. The proof is straightforward; hence, omitted. Importantly, the specific optimal policy is contingent on the Pareto-weights of the household model as so is the planner's allocation.

Proposition 5.6 *If households are ex-ante identical, then the spouse's income is excessively large in equilibrium. Constrained efficiency can be attained in the market economy through the implementation of a public unemployment insurance financed by a proportional income tax on newly employed workers and a lump sum tax. Furthermore, this result also holds with a cooperative model of the household.*

If workers' productivity were observable, the result of the decentralization of the planner's allocation could be extended to an economy with a non-degenerate cdf F by means of productivity-contingent tax rates. Otherwise, as assumed here, an incentive scheme, based on realized spouse's earnings, must be set to elicit information on actual intra-household insurance. We next analyze scenarios with ex-ante heterogeneous households averse to consumption risks.

5.5 Quasi-linear preferences in leisure

We start by considering quasi-linear preferences in leisure: $\upsilon(c, \ell) = \psi(c) - \ell$, where function ψ is increasing and concave. The following proposition characterizes the planner's solution.

Proposition 5.7 *The planner's allocation is characterized by the following conditions*

$$\begin{aligned} & k = \eta(q_x)(1 - \gamma(q_x))(1 - z), \;\; \text{and } c_x^e = c_x^u = c_x \\ & \dot{c}_x \geq 0, \;\; \text{and } \nu(q)\dot{y}_x^e + (1 - \nu(q))\dot{y}_x^u \geq 0 \end{aligned} \tag{19}$$

The participation condition (PC_x) is redundant. There exists a subset of positive mass in which the above inequalities are strict and $\psi'(c_x) > \frac{1}{x}$. The equilibrium mass of vacancies is constrained efficient, but labor supply is not. Furthermore, there may be bunching: $\dot{c}_x = \nu(q)\dot{y}_x^e + (1 - \nu(q))\dot{y}_x^u = 0$ within a subset of $[\underline{x}, \overline{x}]$.

Condition (19) states two outcomes: there is full intra-household insurance, and a single labor market is active. Furthermore, the constrained efficient vacancy creation is the one that maximizes output, which is inherently associated with the previous result. As full insurance is provided within the household, job-seekers behave as though they were risk-neutral agents, and, as a result, the planner aims at maximizing output to redistribute across ex-ante different households. Notice that both full private insurance and output maximization also take place in equilibrium, but there is no redistribution across households. Indeed, the equilibrium consumption level exceeds the planner's consumption, implying that labor supply of the spouse is inefficiently large.

Furthermore, there may exists a subset of individuals applying to contracts which specify the same consumption level and expected income of the spouse. Following the optimal taxation literature, we refer to this result as *bunching*.

The following lemma states that the planner's allocation can be implemented in the market economy through a system of transfers across households types. There is no redistribution within-group, instead.

Lemma 5.8 *If there is no bunching in the planner's solution, then it can be decentralized through the implementation of a tax on household's total income.*

[To be completed]

6 Quantitative Exploration

In this section, we quantitatively explore the welfare gains that would obtain from moving from the U.S. economy to another one with the optimal household-income-based insurance system. Consistently with our previous work, our benchmark hosts a unitary model of the household, and we also make the comparison with a cooperative model.

[To be completed]

7 Conclusions

There are many aspects that are left unexamined for the sake of the theoretical analysis. Important points are the extent of leisure complementarity and assortative mating by skill level (see e.g. Boskin and Sheshinski (1983)), the policy implications on family formation (see e.g. Gayle and Shephard (2016)) and inequality within the household (see e.g. the latter and Alesina et al. (2011)). We believe that for short average duration and incidence of unemployment like the ones in the U.S. economy, the policy effects on these dimensions is arguably negligent for a large proportion of the labor force. Instead, assortative mating may have a first order effect on redistribution across households in the system-financing burden.

In the trade-off examined between public and private provision of insurance, we have assumed that the only private costs amount to forgone leisure. However, there might be others such as

References

Acemoglu, D. and R. Shimer (1999): "Efficient Unemployment Insurance," *Journal of Political Economy*, 107, 893–928.

Alesina, A., A. Ichino, and L. Karabarbounis (2011): "Gender-based taxation and the division of family chores," *American Economic Journal: Economic Policy*, 3, 1–40.

Attanasio, O., H. Low, and V. Sánchez-Marcos (2005): "Female labor supply as insurance against idiosyncratic risk," *Journal of the European Economic Association*, 3, 755–764.

Attanasio, O. and J.-V. Ríos-Rull (2000): "Consumption smoothing in island economies: Can public insurance reduce welfare?" *European Economic Review*, 44, 1225–1258.

Blau, F. D. and L. M. Kahn (2007): "Changes in the Labor Supply Behavior of Married Women: 1980-2000," *Journal of Labor Economics*, 25, 393–438.

Blundell, R., L. Pistaferri, and I. Saporta-Eksten (2016): "Consumption Inequality and Family Labor Supply," *American Economic Review*, 106, 387–435.

Boone, J. and L. Bovenberg (2004): "The optimal taxation of unskilled labor with job search and social assistance," *Journal of Public Economics*, 88, 2227–2258.

Boskin, M. J. and E. Sheshinski (1983): "Optimal tax treatment of the family: Married couples," *Journal of Public Economics*, 20, 281–297.

Browning, M. and T. F. Crossley (2001): "Unemployment insurance benefit levels and consumption changes," *Journal of public Economics*, 80, 1–23.

Burdett, K. and D. Mortensen (1978): *Research in Labor Economics*, JAI Press, vol. 2, chap. Labor Supply under Uncertainty.

Chetty, R. (2008): "Moral Hazard versus Liquidity and Optimal Unemployment Insurance," *Journal of Political Economy*, 116, 173–234.

Chetty, R. and A. Finkelstein (2012): "Social insurance: Connecting theory to data," Tech. rep., National Bureau of Economic Research.

Chetty, R. and E. Saez (2010): "Optimal Taxation and Social Insurance with Endogenous Private Insurance," *American Economic Journal. Economic Policy*, 2, 85.

Chiappori, P.-A. and O. Donni (2009): "Non-unitary models of household behavior: A survey of the literature," .

Chipman, J. S. (1977): "An empirical implication of Auspitz-Lieben-Edgeworth-Pareto complementarity," *Journal of Economic Theory*, 14, 228–231.

Choi, S. and A. Valladares-Esteban (2016): "Unemployment Insurance in an Economy with Single and Married Households," *mimeo*.

Cullen, J. B. and J. Gruber (1996): "Spousal Labor Supply as Insurance: Does Unemployment Insurance Crowd Outthe Added Worker Effect?" Tech. rep., National Bureau of Economic Research.

——— (2000): "Does unemployment insurance crowd out spousal labor supply?" *Journal of Labor Economics*, 18, 546–572.

Cutler, D. M. and J. Gruber (1996): "The effect of Medicaid expansions on public insurance, private insurance, and redistribution," *The American Economic Review*, 86, 378–383.

Devereux, P. J. (2004): "Changes in relative wages and family labor supply," *Journal of Human Resources*, 39, 698–722.

Di Tella, R. and R. MacCulloch (2002): "INFORMAL FAMILY INSURANCE AND THE DESIGN OF THE WELFARE STATE*," *The Economic Journal*, 112, 481–503.

Dynarski, S. and J. Gruber (1997): "Can families smooth variable earnings?" *Brookings Papers on Economic Activity*, 229–303.

Engen, E. M. and J. Gruber (2001): "Unemployment insurance and precautionary saving," *Journal of monetary Economics*, 47, 545–579.

Farhi, E. and I. Werning (2013): "Insurance and taxation over the life cycle," *The Review of Economic Studies*, 80, 596–635.

FUDENBERG, D. AND J. TIROLE (1991): "Game theory, 1991," *Cambridge, Massachusetts*, 393.

GAYLE, G.-L. AND A. SHEPHARD (2016): "Optimal Taxation, Marriage, Home Production, and Family Labor Supply," *mimeo.*

GOLLIER, C. (2004): *The economics of risk and time*, MIT press.

GOLOSOV, M. AND A. TSYVINSKI (2007): "Optimal taxation with endogenous insurance markets," *The Quarterly Journal of Economics*, 122, 487–534.

GRUBER, J. (1997): "The Consumption Smoothing Benefits of Unemployment Insurance," *The American Economic Review*, 87, pp. 192–205.

GULER, B., F. GUVENEN, AND G. L. VIOLANTE (2012): "Joint-search theory: New opportunities and new frictions," *Journal of Monetary Economics*, 59, 352–369.

HAGEDORN, M., F. KARAHAN, I. MANOVSKII, AND K. MITMAN (2015): "Unemployment Benefits and Unemployment in the Great Recession: The Role of Macro Effects," .

HEATHCOTE, J., K. STORESLETTEN, AND G. VIOLANTE (2014): "Consumption and Labor Supply with Partial Insurance: An Analytical Framework," *THE AMERICAN ECONOMIC REVIEW*, 104, 1–52.

HEATHCOTE, J., K. STORESLETTEN, AND G. L. VIOLANTE (2009): "Quantitative macroeconomics with heterogeneous households," Tech. rep., National Bureau of Economic Research.

HOPENHAYN, H. A. AND J. P. NICOLINI (1997): "Optimal unemployment insurance," *Journal of political economy*, 105, 412–438.

HUNGERBÜHLER, M., E. LEHMANN, A. PARMENTIER, AND B. VAN DER LINDEN (2006): "Optimal redistributive taxation in a search equilibrium model," *The Review of Economic Studies*, 73, 743–767.

HYSLOP, D. R. (2001): "Rising US earnings inequality and family labor supply: The covariance structure of intrafamily earnings," *American Economic Review*, 755–777.

KAPLAN, G. (2012): "Moving back home: Insurance against labor market risk," *Journal of Political Economy*, 120, 446–512.

Keane, M. P. (2011): "Labor supply and taxes: A survey," *Journal of Economic Literature*, 49, 961–1075.

Kleven, H. J., C. T. Kreiner, and E. Saez (2009): "The optimal income taxation of couples," *Econometrica*, 77, 537–560.

Kolsrud, J., C. Landais, P. Nilsson, and J. Spinnewijn (2015): "The Optimal Timing of Unemployment Benefits: Theory and Evidence from Sweden," Tech. rep., Society for Economic Dynamics.

Krueger, D. and F. Perri (2011): "Public versus private risk sharing," *Journal of Economic Theory*, 146, 920–956.

Krusell, P., T. Mukoyama, and A. Şahin (2010): "Labour-market matching with precautionary savings and aggregate fluctuations," *The Review of Economic Studies*, 77, 1477–1507.

Mankart, J. and R. Oikonmou (2014): "Household search and the aggregate labor market," *mimeo.*

Mankiw, N. G., M. Weinzierl, and D. Yagan (2009): "Optimal Taxation in Theory and Practice," *The Journal of Economic Perspectives*, 147–174.

Michelacci, C. and H. Ruffo (2015): "Optimal life cycle unemployment insurance," *The American Economic Review*, 105, 816–859.

Mincer, J. (1962): "Labor force participation of married women: A study of labor supply," in *Aspects of labor economics*, Princeton University Press, 63–105.

Mirrlees, J. A. (1971): "An exploration in the theory of optimum income taxation," *The review of economic studies*, 175–208.

Moen, E. R. (1997): "Competitive search equilibrium," *Journal of Political Economy*, 105, 385–411.

Morissette, R. and Y. Ostrovsky (2008): "How Do Families and Unattached Individuals Respond to Layoffs," *Evidence from Canada.(PDF) Statistics Canada Catalogue no. 11F0019MIE.*

Ortigueira, S. and N. Siassi (2013): "How important is intra-household risk sharing for savings and labor supply?" *Journal of Monetary Economics*, 60, 650–666.

Salanie, B. (2011): *The economics of taxation*, MIT press.

Stephens, M. (2002): "Worker displacement and the added worker effect," Tech. rep., Journal of Labor Economics.

Stevens, A. H. (1997): "Persistent effects of job displacement: The importance of multiple job losses," *Journal of Labor Economics*, 165–188.

Thomas, J. P. and T. Worrall (2007): "Unemployment insurance under moral hazard and limited commitment: public versus private provision," *Journal of Public Economic Theory*, 9, 151–181.

Weinzierl, M. (2011): "The surprising power of age-dependent taxes," *The Review of Economic Studies*, 78, 1490–1518.

8 Appendix

8.1 Appendix. Proofs of Section 4

Proof of Lemma 4.1.

Consider the first order condition (2). Let $f(w,y) \equiv \upsilon_c\big(y+w, y/x\big)x + \upsilon_\ell\big(y+w, y/x\big)$. Notice that $\frac{\partial f(w,y)}{\partial y} < 0$ due to Assumptions A3 and A4. Therefore, the Implicit Function Theorem ensures that there exists a unique function $y_x(w)$ such that $f(w, y_x(w)) = 0$ in an open neighborhood of w. Indeed, y_x is twice continuously differentiable since so is f because of assumption A1.

To show that y_x is a strictly decreasing function, we differentiate equation (2) with respect to w, and obtain

$$\begin{aligned} \big(\upsilon_{cc}x + \upsilon_{\ell c}\big)\left(\frac{dy_x}{dw} + 1\right) + \upsilon_{c\ell}\frac{dy_x}{dw} + \upsilon_{\ell\ell}\frac{dy_x}{dw}\frac{1}{x} = 0 \\ \Leftrightarrow \frac{dy_x}{dw} = -\frac{\upsilon_{cc}x + \upsilon_{\ell c}}{\upsilon_{cc}x + 2\upsilon_{c\ell} + \upsilon_{\ell\ell}/x} < 0. \end{aligned} \tag{20}$$

Moreover, if $z < w$, then total earnings are lower if married with an employed worker, $y_x^e(w) < y_x^u$.

We now make use of these results to prove that function V_x is twice continuously differentiable. We can rewrite it as a composite function of twice continuously differentiable functions, $V_x(w) = \upsilon\big(y_x^e(w) + w, y_x^e(w)/x\big)$, and, hence, so is it.

To show that function V_x is strictly increasing and concave, we compute the first and second derivatives.

$$\begin{aligned} V_x'(w) &= \upsilon_c > 0 \\ V_x''(w) &= \upsilon_{cc}\left(\frac{dy_x^e}{dw}+1\right)+\upsilon_{c\ell}\frac{dy_x^e}{dw}\frac{1}{x} = \frac{\upsilon_{cc}\upsilon_{\ell\ell}-\upsilon_{c\ell}^2}{\upsilon_{cc}x^2+2x\upsilon_{c\ell}+\upsilon_{\ell\ell}} < 0 \end{aligned}$$

The first derivate is determined using the Envelope Theorem. To compute the second derivative, we have used expression (20). Assumptions A3 and A4 ensure that the second derivative is negative.

Finally, for a given wage w, by differentiating equation (2) with respect to productivity x and grouping terms, we obtain

$$\big(\upsilon_{cc}x+\upsilon_{\ell\ell}1/x+2\upsilon_{\ell c}\big)\frac{dy_x(w)}{dx} = \upsilon_{c\ell}\frac{y_x(w)}{x} - \upsilon_c + \upsilon_{\ell\ell}\frac{y}{x^2} \Rightarrow \frac{dy_x(w)}{dx} > 0.$$

Now, consider a constant w. Let $V_x(w) \equiv \max_y \upsilon(y+w, y/x)$. The first order condition of this maximization problem is $\upsilon_c x + \upsilon_\ell = 0$. Then,

$$\frac{\partial V_x}{\partial x} = -\upsilon_\ell \frac{y}{x^2} > 0$$

Thus, V_x is increasing in x.

Moreover, by differentiating the first order condition for a single-earner household, we obtain

$$\frac{\partial y_x^u}{\partial x} = \frac{-\upsilon_c^u + \upsilon_{c\ell}^u y_x^u + \upsilon_{\ell\ell}^u \frac{y_x^u}{x}}{x^2\upsilon_{cc}^u + 2x\upsilon_{c\ell}^u + \upsilon_{\ell\ell}^u} > 0$$

Recall that $V_x'(w) = \upsilon_c$. Therefore,

$$\begin{aligned} \frac{\partial V_x'}{\partial x} &= \upsilon_{cc}\frac{\partial y_x}{\partial x} + \upsilon_{c\ell}\left(\frac{\partial y_x}{\partial x}\frac{1}{x} - \frac{y_x}{x^2}\right) = \\ &= \frac{\big(\upsilon_{cc}+\upsilon_{c\ell}/x\big)\Big(-\upsilon_c + \upsilon_{c\ell}y_x + \upsilon_{\ell\ell}\frac{y_x}{x}\Big) - \frac{y_x}{x^2}\upsilon_{c\ell}\Big(x^2\upsilon_{cc} + 2x\upsilon_{c\ell} + \upsilon_{\ell\ell}\Big)}{x^2\upsilon_{cc} + 2x\upsilon_{c\ell} + \upsilon_{\ell\ell}} \\ &= \frac{(\upsilon_{cc}\upsilon_{\ell\ell} - \upsilon_{c\ell}^2)\frac{y_x}{x} - \upsilon_c(\upsilon_{cc} + \upsilon_{c\ell}/x)}{x^2\upsilon_{cc} + 2x\upsilon_{c\ell} + \upsilon_{\ell\ell}} < 0 \end{aligned}$$

where the last expression results after some simplifications.‖

Proof of Proposition 4.2.

Consider problem (6) of a household of type x . Notice that the constraint establishes a positive relationship between w and q. Therefore, the household's problem can be rewritten only in terms of the wage w. Since the resulting objective function is continuous in w and the domain $[z,1]$ is compact, the Weierstrass Theorem ensures the existence of a solution. The first derivative becomes

$$-\nu(q)\eta(q)\frac{1-\gamma(q)}{\gamma(q)}\frac{V_x(w)-V_x(z)}{k}+\nu(q)V_x'(w)$$

When, $w=z$, the first term of the derivative is 0, while the second term is strictly positive. Likewise, the derivative is negative at $w=1$. Furthermore, the objective function is nonnegative and values $V_x(z)$ at the two extremes of the domain. These two results together imply that the wage solution must be an interior point. The first order condition becomes

$$\text{(FOC):}\frac{V_x(w)-V_x(z)}{V_x'(w)}=\frac{k}{\eta(q)}\frac{\gamma(q)}{1-\gamma(q)}$$

The right hand side of this expression is decreasing in q and, hence, also in w. The derivative of the left hand side is

$$1-\frac{(V_x(w)-V_x(z))V_x''(w)}{V_x'(w)^2}>0$$

This expression is positive because function V_x is concave as stated in Lemma 4.1. Therefore, the solution of the first order condition must be unique, and it is also a sufficient condition.

As said in the text, equations (2) and (3) have a unique solution because of the concavity of the utility function. Therefore, there exists an equilibrium, and it is unique.‖

Proof of Proposition 4.3

We follow closely the proof of Proposition 2 in Acemoglu and Shimer (1999). Consider types x and x' such that $x'<x$. Given that $V_x(w)$ is strictly increasing and differentiable in w, there exists an inverse function V_x^{-1}, which is also differentiable. Define function $f(s)\equiv\max_y \upsilon(y+V_x^{-1}(s),y/x')$, which is twice continuously differentiable. Notice that $V_{x'}(w)=f\circ V_x(w)$. By differentiating with respect to w, we obtain

$$V_{x'}'(w)=f'(V_x(w))V_x'(w),\text{ and }V_{x'}''(w)=f''(V_x(w))V_x'(w)^2+f'(V_x(w))V_x''(w)$$

Using the first equality, we can rewrite the second expression as

$$f''(V_x(w))V_x'(w)^2=V_{x'}''(w)-V_x''(w)\frac{V_{x'}'(w)}{V_x'(w)}$$

Therefore, f is a concave (convex) function and, hence, $V_{x'}(w)$ is a concave (convex) transformation of $V_x(w)$ if and only if $\frac{-V''_{x'}(w)}{V'_{x'}(w)}$ is greater (lower) than $\frac{-V''_x(w)}{V'_x(w)}$.

To show the wage difference in the application strategies of these two household types, it suffices to follow the remaining steps in the proof of Proposition 2 in Acemoglu and Shimer (1999). Finally, the difference in queue lengths is of the same sign as the wage difference since the equilibrium zero-profit condition establishes a positive relationship between q and w.

We now show that the absolute risk aversion of the indirect utility function is decreasing in x for preferences that belong to If $\mathcal{F}_1 \bigcup \mathcal{F}_2$. Notice that, by using the first order condition of the household's problem (1), for additively separable preferences, we can write

$$\frac{-V''_x(w)}{V'_x(w)} = \frac{A_c A_\ell}{A_c x + A_\ell} = \begin{cases} \frac{\gamma_c \gamma_\ell}{\gamma_c x + \gamma_\ell} & \text{, if } \upsilon \in \mathcal{F}_1, \\ \frac{\sigma_c \sigma_\ell}{\sigma_c y + \sigma_\ell c} & \text{, if } \upsilon \in \mathcal{F}_2 \end{cases} \quad \text{as } (A_c, A_\ell) \equiv \left(\frac{-\upsilon_{cc}}{\upsilon_c}, \frac{\upsilon_{\ell\ell}}{\upsilon_\ell} \right) = \begin{cases} (\gamma_c, \gamma_\ell) \\ \left(\frac{\sigma_c}{c}, \frac{\sigma_\ell}{\ell}\right) \end{cases}$$

and Lemma 4.1 states that the spouse's income is an increasing functions of x.‖

Proof of Lemma 4.5.

When the utility function is additively separable, we can rewrite the term $d_1(x)$ in expression (10) as

$$d_1(x) = \frac{\ell^u_x A^u_\ell + 1}{y^u_x (A^u_c x + A^u_\ell)} - \frac{\ell^e_x A^e_\ell + 1}{y^e_x(m_x)(A^e_c x + A^e_\ell)} = \frac{1 + \frac{1}{\ell^u_x A^u_\ell}}{x(\frac{A^u_c}{A^u_\ell} x + 1)} - \frac{1 + \frac{1}{\ell^e_x A^e_\ell}}{x(\frac{A^e_c}{A^e_\ell} x + 1)}$$

Now, $\frac{dA_c}{dc} = A_c(A_c - P_c)$ and $\frac{dA_\ell}{d\ell} = A_\ell(P_\ell - A_\ell)$, where $P_c \equiv -\frac{\upsilon_{ccc}}{\upsilon_{cc}}$ and $P_\ell \equiv \frac{\upsilon_{\ell\ell\ell}}{\upsilon_{\ell\ell}}$. Then,

$$\frac{dA_c}{dc}, \frac{dA_\ell}{d\ell} < 0, \text{ and } c^e_x > c^u_x \text{ and } \ell^e < \ell^u \Rightarrow \frac{A^u_c}{A^u_\ell} > \frac{A^e_c}{A^e_\ell}.$$

Furthermore,

$$\frac{d(\ell A_\ell)}{d\ell} \geq 0, \text{ and } \ell^e < \ell^u \Rightarrow \frac{1}{\ell^u A^u_\ell} < \frac{1}{\ell^e A^e_\ell}.$$

Therefore, $d_1(x) < 0$.

Finally, if $\upsilon \in \mathcal{F}_1$, $A_c = \gamma_c$ and $A_\ell = \gamma_\ell$ are both constant, while $\ell A_\ell = \ell \gamma_\ell$ is a strictly increasing function. If $\upsilon \in \mathcal{F}_2$ instead, $A_c = \frac{\sigma_c}{c}$ and $A_\ell = \frac{\sigma_\ell}{\ell}$ are both strictly decreasing functions and $\ell A_\ell = \sigma_\ell$ is a constant. ‖

Proof of Lemma 4.6.

1. The proof of wages and queue lengths as increasing functions of z is analogous to its counterpart in Acemoglu and Shimer (1999). Hence, it is omitted.

Moreover, by using the Envelope Theorem and differentiating equation (9) with respect to the parameter z, we obtain

$$\frac{\partial m_x}{\partial z} = (1 - \nu(q_x)) \frac{\upsilon_c(z + y_x^u, y_x^u/x)}{\upsilon_c(m_x + y_x^e(m_x), y_x^e(m_x)/x)} > 0.$$

2. Similarly to the previous case, by differentiating equation (3) with respect to z, we obtain

$$\begin{aligned}\left(\upsilon_{cc}x + \upsilon_{c\ell}\right)\left(\frac{\partial y_x^u}{\partial z} + 1\right) + \left(\upsilon_{c\ell} + \upsilon_{\ell\ell}/x\right)\frac{\partial y_x^u}{\partial z} = 0 \\ \Leftrightarrow \frac{\partial \ell_x^u}{\partial z} = \frac{\partial y_x^u}{\partial z}/x = \frac{-x\upsilon_{cc} - \upsilon_{c\ell}}{\upsilon_{cc}x^2 + 2x\upsilon_{c\ell} + \upsilon_{\ell\ell}},\end{aligned}$$

which is negative.

Note that $\frac{\partial \ell_x^e}{\partial z} = \frac{d\ell_x^e}{dw}\frac{\partial w}{\partial z} \leq 0$ because the first factor is negative according to Lemma 4.1, while the second one was shown above to be non-negative.‖

Proof of Lemma 4.7. For additively separable preferences, we can rewrite $\delta_1(x)$ from expression (11) as

$$\delta_1(x) = \frac{-\upsilon_{cc}^u x^2}{\upsilon_{cc}^u x^2 + \upsilon_{\ell\ell}^u} + \frac{\upsilon_{cc}^e x^2}{\upsilon_{cc}^e x^2 + \upsilon_{\ell\ell}^e} = \frac{x}{-x - \frac{A_\ell^u}{A_c^u}} - \frac{x}{-x - \frac{A_\ell^e}{A_c^e}} \gtreqless 0 \Leftrightarrow \frac{A_\ell^e}{A_c^e} \lesseqgtr \frac{A_\ell^u}{A_c^u}$$

where the last expression is obtained by using the first order condition of the household's problem (1), and $A_s^i = \frac{\upsilon_{ss}^i}{\upsilon_s^i}$ for $i \in \{u, e\}$ and $s \in \{c, \ell\}$.

If $\upsilon \in \mathcal{F}_1$, then $A_c^i = \gamma_c$ and $A_\ell^i = \gamma_\ell$ for $i \in \{u, e\}$; hence, $\delta_1(x) = 0$.

Assume A_c and A_ℓ are decreasing functions. As $c_x^u < c_x^e$, $A_c^u > A_c^e$. Moreover, as $\ell_x^u > \ell_x^e$, $A_\ell^u < A_\ell^e$. Therefore, we have $\frac{A_\ell^e}{A_c^e} > \frac{A_\ell^u}{A_c^u}$. In particular, if $\upsilon \in \mathcal{F}_2$, then

$$A_c' = \frac{-\upsilon_{ccc}\upsilon_c + \upsilon_{cc}^2}{\upsilon_c^2} = A_c(A_c - P_c) \leq 0 \text{ because } P_c = \frac{\sigma_c + 1}{c} > A_c = \frac{\sigma_c}{c},$$

and

$$A_\ell' = \frac{\upsilon_{\ell\ell\ell}\upsilon_\ell - \upsilon_{\ell\ell}^2}{\upsilon_\ell^2} = A_\ell(P_\ell - A_\ell) \leq 0 \text{ because } P_\ell = \frac{\sigma_\ell - 1}{\ell} < A_\ell = \frac{\sigma_\ell}{\ell}.$$

It remains to show that if $\sigma_\ell \geq 1$, then $\delta_2(x) < 0$, i.e. $\frac{\partial m_x}{\partial z} < 1$. By using the Envelope

Theorem, we obtain

$$\frac{\partial m_x}{\partial z} = (1-\nu(q_x))\frac{V_x'(z)}{V_x'(m_x)} \leq 1 \Leftrightarrow (1-\nu(q_x))V_x'(z) \leq V_x'(m_x)$$

Let $N_x \equiv (1-\nu(q_x))z + \nu(q_x)w_x$. Notice that, because of the monotonicity and the concavity of function V_x stated in Lemma 4.1,

$$V_x(m_x) = (1-\nu(q_x))V_x(z) + \nu(q_x)V_x(w_x) < V_x(N_x) \Rightarrow m_x < N_x \text{ and } V_x'(m_x) > V_x'(N_x)$$

If $V_x''' < 0$, then

$$V_x'(N_x) > (1-\nu(q_x))V_x'(z) + \nu(q_x)V_x'(w_x) \Rightarrow V_x'(m_x) > (1-\nu(q_x))V_x'(z).$$

We are now to prove that $V_x''' < 0$ if $\upsilon \in \mathcal{F}_2$. Recall that for such preferences

$$V_x''(w) = -\upsilon_c\frac{A_c A_\ell}{A_c x + A_\ell} = \frac{-\sigma_c\sigma_\ell\gamma x^{-1-\sigma_\ell}}{y^{-\sigma_\ell}\big((\sigma_c+\sigma_\ell)y + \sigma_\ell w\big)}$$

Therefore, the third derivative of the indirect utility function is negative if and only if the denominator of the last expression is increasing in w. Its derivative with respect to w is

$$\frac{\partial y}{\partial w}y^{-\sigma_\ell}\big((1-\sigma_\ell)(\sigma_c+\sigma_\ell) - \sigma_\ell^2 w/y\big) + \sigma_\ell y^{-\sigma_\ell} \geq 0 \text{ if } \sigma_\ell \geq 1.\|$$

Proof of Lemma 4.8.

Consider the first order condition of the household problem (16), for a given x. After replacing c^m using the budget constraint, the first order conditions with respect to c^f and y are

$$f_1(w, c^f, y) = -\alpha\upsilon_c^m + (1-\alpha)\upsilon_c^f = 0 \tag{21}$$

$$f_2(w, c^f, y) = \alpha\upsilon_c^m x + (1-\alpha)\upsilon_\ell^f = 0 \tag{22}$$

Let $f(w, c^f, y) = (f_1(w, c^f, y), f_2(w, c^f, y))$. The Jacobian matrix of f is invertible since

$$\begin{aligned} |\mathbf{J}| &= \begin{vmatrix} \frac{\partial f_1}{\partial c^f} & \frac{\partial f_1}{\partial y} \\ \frac{\partial f_2}{\partial c^f} & \frac{\partial f_2}{\partial y} \end{vmatrix} = \begin{vmatrix} \alpha\upsilon_{cc}^m + (1-\alpha)\upsilon_{cc}^f & -\alpha\upsilon_{cc}^m + (1-\alpha)\upsilon_{c\ell}^f/x \\ -\alpha\upsilon_{cc}^m x + (1-\alpha)\upsilon_{c\ell}^f & \alpha\upsilon_{cc}^m x + (1-\alpha)\upsilon_{\ell\ell}^f/x \end{vmatrix} \\ &= \big((1-\alpha)\alpha\upsilon_{cc}^m\big(\upsilon_{cc}^f x^2 + \upsilon_{\ell\ell}^f + 2x\upsilon_{c\ell}^f\big) + (1-\alpha)^2\big(\upsilon_{cc}^f\upsilon_{\ell\ell}^f - (\upsilon_{c\ell}^f)^2\big)\big)/x > 0 \end{aligned}$$

Therefore, the Implicit Function Theorem ensures that, for a given x, there exists unique functions $c_x^f(w)$, $c_x^m(w)$ and $y_x(w)$ such that $f(w, c_x^f(w), y_x(w)) = 0$. Indeed, y_x is twice continuously differentiable since so is f because of assumption A1.

To show that y_x is a strictly decreasing function, we differentiate the two first order conditions (21) and (22) with respect to w, and obtain a system of two equations with unknowns $\frac{dy_x}{dw}$ and $\frac{dc_x^f}{dw}$. By manipulating them, we obtain

$$\frac{dy_x}{dw} = \frac{-\alpha \upsilon_{cc}^m \big(\upsilon_{c\ell}^f + x\upsilon_{cc}^f\big)}{\alpha \upsilon_{cc}^m \big(x\upsilon_{cc}^f + 2\upsilon_{c\ell}^f + \upsilon_{\ell\ell}^f/x\big) + (1-\alpha)\big(\upsilon_{cc}^f \upsilon_{\ell\ell}^f - (\upsilon_{c\ell}^f)^2\big)/x} < 0$$

Moreover,

$$\frac{dc_x^f}{dw} = -\frac{dy_x}{dw}\frac{\upsilon_{\ell\ell}^f/x + \upsilon_{c\ell}^f}{\upsilon_{c\ell}^f + x\upsilon_{cc}^f} > 0 \tag{23}$$

We now make use of these results to prove that function V_x is twice continuously differentiable. We can rewrite it as a composite function of twice continuously differentiable functions, $V_x(w) = \alpha \upsilon^m \big(y_x(w) + w - c_x^f(w), \underline{\ell}\big) + (1-\alpha)\upsilon^f\big(c_x^f(w), y_x(w)/x\big)$, and, hence, so is it.

To show that function V_x is strictly increasing and concave, we compute the first and second derivatives.

$$\begin{aligned}
V_x'(w) &= \alpha \upsilon_c^m \left(\frac{dy_x}{dw} + 1 - \frac{dc_x^f}{dw}\right) + (1-\alpha)\left(\upsilon_c^f \frac{dc_x^f}{dw} + \upsilon_\ell^f \frac{dy_x}{dw}\frac{1}{x}\right) \underset{(21),(22)}{=} \alpha \upsilon_c^m > 0 \\
V_x''(w) &= \alpha \upsilon_{cc}^m \left(\frac{dy_x}{dw} + 1 - \frac{dc_x^f}{dw}\right) \underset{(23)}{=} \alpha \upsilon_{cc}^m \left(\frac{dy_x}{dw}\left(\frac{x\upsilon_{cc}^f + \upsilon_{\ell\ell}^f/x + 2\upsilon_{c\ell}^f}{\upsilon_{c\ell}^f + x\upsilon_{cc}^f}\right) + 1\right) \\
&\underset{(23)}{=} \alpha \upsilon_{cc}^m \frac{(1-\alpha)\big(\upsilon_{cc}^f \upsilon_{\ell\ell}^f - (\upsilon_{c\ell}^f)^2\big)}{\alpha \upsilon_{cc}^m \big(x^2 \upsilon_{cc}^f + 2x\upsilon_{c\ell}^f + \upsilon_{\ell\ell}^f\big) + (1-\alpha)\big(\upsilon_{cc}^f \upsilon_{\ell\ell}^f - (\upsilon_{c\ell}^f)^2\big)} < 0
\end{aligned}$$

Assumptions A3 and A4 ensure that the second derivative is negative.

Let ℓ_x^e and ℓ_x^u denote hours worked by the spouse of an employed and unemployed worker, respectively. The proof of $\frac{\partial \ell_x^u}{\partial z} < 0$ is analogous to expression (23). Finally, $\frac{\partial \ell_x^e}{\partial z} = \frac{d\ell_x^e}{dw}\frac{\partial w}{\partial z} < 0$ since the second factor is positive as stated in Lemma 4.6.‖

8.2 Appendix. Proofs of Section 5

Proof of Lemma 5.2.

The proof is by contradiction.[31] Suppose that such conditions are not sufficient, and, hence, there exists $\hat{x}$ and x such that $\mathcal{U}_x(\hat{x}) > \mathcal{U}_x$. Let us suppose without loss of generality that $\hat{x} > x$. That is, $\displaystyle\int_x^{\hat{x}} \frac{\partial U_x(a)}{\partial a} da > 0$. This integral can be developed to obtain

$$\begin{aligned}
\int_x^{\hat{x}} \frac{\partial U_x(a)}{\partial a} da \;=\; & \int_x^{\hat{x}} \Bigg(\nu'(q_a)\dot{q}_a\big(v(c_a^e, y_a^e/x) - v(c_a^u, y_a^u/x)\big) \\
& + \nu(q_a)\Bigg(v_c(c_a^e, y_a^e/x)\dot{c}_a^e + v_\ell(c_a^e, y_a^e/x)\frac{\dot{y}_a^e}{x}\Bigg) \\
& + (1-\nu(q_a))\Bigg(v_c(c_a^u, y_a^u/x)\dot{c}_a^u + v_\ell(c_a^u, y_a^u/x)\frac{\dot{y}_a^u}{x}\Bigg)\Bigg) da \leq \\
& \int_x^{\hat{x}} \Bigg(\nu'(q_a)\dot{q}_a\big(v(c_a^e, y_a^e/a) - v(c_a^u, y_a^u/a)\big) \\
& + \nu(q_a)\Bigg(v_c(c_a^e, y_a^e/a)\dot{c}_a^e + v_\ell(c_a^e, y_a^e/a)\frac{\dot{y}_a^e}{a}\Bigg) \\
& + (1-\nu(q_a))\Bigg(v_c(c_a^u, y_a^u/a)\dot{c}_a^u + v_\ell(c_a^u, y_a^u/a)\frac{\dot{y}_a^u}{a}\Bigg)\Bigg) da = 0,
\end{aligned}$$

where the inequality results from the integrand being an increasing function in x because of the (local) second order condition, and the last equality is the necessary first order condition. This is a contradiction, and, hence, the local conditions are also sufficient.‖

Proof of Proposition 5.3

Let (q_x, w_x) denote the unique equilibrium pair of distributions of queue lengths and wages. We first show that it belongs to the feasible set of the planner's problem. The resource constraint (RC) holds in equilibrium because the zero-profit condition is satisfied in all submarkets. The participation condition for all households holds with strict inequality because cost κ is sufficiently small by assumption. Likewise, utility-maximizing households of type x prefer pair (q_x, w_x) than $(q_{x'}, w_{x'})$ for any x' in equilibrium; hence, the equilibrium allocation is incentive compatible.

Consider now the following alternative allocation: $c_x^e = w_x + y_x^e - \epsilon$, $c_x^u = z + y_x^u + \delta$, such that it is resource-neutral, i.e. $\displaystyle\int_{\underline{x}}^{\overline{x}} \Big(-\nu(q_x)\epsilon + (1-\nu(q_x))\delta\Big) dF(x) = 0$. Notice that this allocation yields a strictly higher value than the equilibrium one due to the concavity of the utility function v. It also belongs to the feasible set as the resource constraint holds. It is straightforward to see that the participation condition (PC_x) also holds because the participation cost κ is sufficiently small. The incentive compatibility conditions (ICC_x) also

[31] We closely follow Fudenberg and Tirole (1991, Ch. 7, p. 261).

hold for ϵ arbitrarily small because Proposition 4.2 ensures that the problem of type x households has a unique solution and the pair $(q_x, w_x - \epsilon)$ satisfies its inequality constraint.[32] Therefore, the equilibrium allocation is not solution of the planner's problem.‖

Proof of Proposition 5.4

Consider the following program

$$\max \int_{\underline{x}}^{\overline{x}} \left(\nu(q_x)\big(1 + y_x^e + \phi(y_x^e/x)\big) + (1 - \nu(q_x))\big(z + y_x^u + \phi(y_x^u/x)\big) - \frac{k}{q_x} \right) dF(x)$$

This is the planner's unconstrained problem after replacing the consumption values using the resource constraint. Then, we are to show that the equilibrium allocation is feasible and incentive compatible, and is a solution of the unconstrained problem, and, hence, must coincide with the planner's solution.

The first order conditions of the planner's unconstrained problem are, for all x,

$$\begin{aligned} \frac{k}{\eta(q)} &= \big(1 - \gamma(q)\big)(1 - z), \qquad \text{where } q \equiv q_x \\ \phi'(y_x/x) &= -x, \qquad \text{where } y_x \equiv y_x^e = y_x^u \end{aligned}$$

Notice that these equations coincide with the equilibrium conditions (12)-(14), and there exists a unique solution to this system of equations. Therefore, the equilibrium allocation is the solution of the planner's unconstrained problem. We define consumption levels as $c_x^e \equiv w + y_x/x$ and $c_x^u \equiv z + y_x/x$, where $w = 1 - \frac{k}{\eta(q)}$ is the equilibrium wage. For this allocation to be the planner's solution, it remains to show that all constraints of the planner's problem hold. First, the participation rate holds for all types as it does in equilibrium. Second, the equilibrium allocation is obviously incentive compatible. Therefore, the equilibrium allocation is constrained efficient.‖

Proof of Lemma 5.5

The proof follows very closely the one for Proposition 5.4. First, for notational simplicity, let $\tilde{\alpha} \equiv \alpha \mathcal{I}_{\alpha \geq 0.5} + (1 - \alpha)(1 - \mathcal{I}_{\alpha \geq 0.5})$.

The equilibrium allocation is determined by condition (12) and (13), whereas the equilibrium condition (14) is replaced by $\phi'(y_x/x) = -x\frac{\tilde{\alpha}}{1-\alpha}$, which leads to $y_x^e = y_x^u$.

As for the unitary model of the household, using the resource constraint, the planner's

[32]The household's problem can be equivalently formalized with an inequality instead of an equality constraint.

objective function can be rewritten as

$$\int_{\underline{x}}^{\overline{x}} \left(\nu(q_x)\big(\tilde{\alpha}\big(1+y_x^e\big)+(1-\alpha)\phi(y_x^e/x)\big)+(1-\nu(q_x))\big(\tilde{\alpha}\big(z+y_x^u\big)+(1-\alpha)\phi(y_x^u/x)\big)-\frac{k}{q_x}\right)dF(x)$$

The necessary conditions of the unconstrained problem are, for all x,

$$\begin{aligned} \frac{k}{\eta(q)} &= \big(1-\gamma(q)\big)(1-z), && \text{where } q\equiv q_x \\ \phi'(y_x/x) &= -x\frac{\tilde{\alpha}}{1-\alpha}, && \text{where } y_x \equiv y_x^e = y_x^u \end{aligned}$$

Since the equilibrium and efficiency conditions are the same, and there exists a unique solution to this system of equations, the equilibrium allocation is the solution of the planner's problem.‖

Proof of Proposition 5.6.

The first order conditions of the planner's problem (18) are

$$\begin{aligned} \nu'(q)\bigg(\big(V(m^e)-V(m^u)\big)(1+\xi_2)+\xi_1(1-m^e-z+m^u)\bigg) &= -\xi_1\frac{k}{q^2} \\ V'(m^e)(1+\xi_2)\le \xi_1 \text{ and } m^e\ge z, &\qquad \text{with comp. slackness} \\ V'(m^u)\left(1-\xi_2\frac{\nu(q)}{1-\nu(q)}\right)\le \xi_1 \text{ and } m^u\ge z, &\qquad \text{with comp. slackness} \end{aligned}$$

where ξ_1 and ξ_2 are the Lagrange multipliers of the first and second constraints, respectively. Since $m^e > m^u > z$, we can use the last two conditions to replace the multipliers in the first equation to obtain

$$\big(1-\gamma(q)\big)\left(\frac{V(m^e)-V(m^u)}{V'(m^e)}+1-m^e-z+m^u\right) = \frac{k}{\eta(q)} \tag{24}$$

Notice that $\xi_2 > 0$ because $m^e = m^u$ otherwise, and constraint (PC) would not hold. Therefore, the participation condition is binding. Let $(\hat{m}^e, \hat{m}^u, \hat{q})$ denote the planner's solution, which satisfies this necessary condition as well as the two constraints of the planner's problem with equality.

Consider the following set of fiscal instruments (b, τ, T), where the first element stands for unemployment benefits, the second one is a proportional income tax rate for newly employed workers, and the last instrument is a lump sum tax paid per household. For the tax-distorted equilibrium (w, q) to be constrained efficient, the following conditions must hold:

$$\begin{aligned} \hat{m}^u &= z + b - T & (25) \\ \hat{m}^e &= w(1-\tau) - T & (26) \\ \tau \frac{k}{\eta(\hat{q})} &= (1-\gamma(\hat{q}))(\tau + b) & (27) \\ T + \nu(\hat{q})\tau w &= (1-\nu(\hat{q}))b \end{aligned}$$

The first two conditions ensure that consumption in the decentralized economy equals its level in the planner's allocation, where the wage w is determined by equation (8) evaluated at $q = \hat{q}$. The third condition is necessary and sufficient for the equilibrium and the efficiency first order conditions (7) and (24) to be equal to one another. Finally, the last condition is the government's budget constraint. Notice that the government's equation is the same as the planner's resource constraint (18) after replacing wages using the zero-profit condition (8). Therefore, to show the implementation of the planner's solution, we need to determine a solution of the system of equations (25)-(27). We can eliminate T and write b in terms of τ by subtracting (25) from (26) to obtain

$$b = \frac{\left(1 - \frac{k}{\eta(\hat{q})}\right)(1-\tau) - z}{\hat{m}^e - \hat{m}^u}.$$

Notice that b is strictly positive as both the numerator and denominator must be strictly positive. Then, we can obtain τ from equation (27) after replacing b as

$$\tau = \frac{1 - \frac{k}{\eta(\hat{q})} - z}{(\hat{m}^e - \hat{m}^u)\left(1 - \frac{k}{\eta(\hat{q})(1-\gamma(\hat{q}))} - \frac{1 - \frac{k}{\eta(\hat{q})}}{\hat{m}^e - \hat{m}^u}\right)},$$

and then the values of b and T are uniquely determined. Finally, notice that $\tau \neq 0$ because otherwise $b = 0$ according to equation (27) and $T < 0$ due to condition (25), and, as a result, the government's budget constraint (28) would fail to hold.‖

Proof of Proposition 5.7.

We first rewrite the planner's problem

$$
\begin{aligned}
\max \quad & \int_{\underline{x}}^{\overline{x}} \mathcal{U}_x f(x) dx \\
\text{s. to} \quad & \mathcal{U}_x = \quad \nu(q_x)\upsilon(c^e_x, y^e_x/x) + (1-\nu(q_x))\upsilon(c^u_x, y^u_x/x) \\
& \dot{A}_x = \quad \left(\nu(q_x)\big(1+y^e_x - c^e_x\big) + (1-\nu(q_x))\big(z + y^u_x - c^u_x\big) - \frac{k}{q_x}\right) f(x) \qquad (28) \\
& A_{\underline{x}}, A_{\overline{x}} = \quad 0 \\
& \mathcal{U}_x \geq \quad \kappa + \upsilon(c^u_x, y^u_x/x) \qquad (29) \\
& \dot{\mathcal{U}}_x = \quad \nu(q_x)\frac{y^e_x}{x^2} + (1-\nu(q_x))\frac{y^u_x}{x^2} \qquad (30) \\
& 0 \leq \quad -\nu'(q_x)p^q_x\big(y^u_x - y^e_x\big) + \nu(q_x)p^e_x + (1-\nu(q_x))p^u_x \qquad (31) \\
& \dot{q}_x = \quad p^q_x \\
& \dot{y}^e_x = \quad p^e_x \\
& \dot{y}^u_x = \quad p^u_x
\end{aligned}
$$

The control variables are c^e_x, c^u_x, p^q_x, p^e_x, and p^u_x, whereas $\mathcal{U}$, q_x, y^e_x and y^u_x are the state variables. As usual in optimal control problems, we transform the resource constraint (RC) into differential equation (28) along with two boundary constraints. Inequality (29) is the participation condition (PC$_x$). The incentive compatibility conditions (FOC-ICC$_x$) and (SOC-ICC$_x$) become (30) and (31), respectively. The last three differential equations are state equations.

The Hamiltonian is defined as

$$
\begin{aligned}
\mathcal{H} \;=\; & \mathcal{U}_x f(x) + \lambda^1_x\left(\nu(q_x)\big(1+y^e_x - c^e_x\big) + (1-\nu(q_x))\big(z+y^u_x - c^u_x\big) - \frac{k}{q_x}\right) f(x) \\
& + \lambda^2_x\left(\nu(q_x)\frac{y^e_x}{x^2} + (1-\nu(q_x))\frac{y^u_x}{x^2}\right) + \lambda^3_x\Big(\mathcal{U}_x - \kappa - \upsilon^u_x\Big) \\
& + \lambda^4_x\Big(\mathcal{U}_x - \nu(q_x)\upsilon^e_x - (1-\nu(q_x))\upsilon^u_x\Big) \\
& + \lambda^5_x\Big(-\nu'(q_x)p^q_x\big(y^u_x - y^e_x\big) + \nu(q_x)p^e_x + (1-\nu(q_x))p^u_x\Big) \\
& + \lambda^6_x p^q_x + \lambda^7_x p^e_x + \lambda^8_x p^u_x
\end{aligned}
$$

where λ^1_x, λ^2_x, λ^6_x, λ^7_x and λ^8_x are the respective co-state variables, and the multipliers λ^3_x, $\lambda^5_x \geq 0$. To simplify notation, we denote $\upsilon^j_x \equiv \upsilon(c^j_x, y^j_x/x)$, for $j \in \{u, e\}$.

The following necessary conditions must be satisfied:

$$\begin{aligned}
\frac{\partial\mathcal{H}}{\partial q_x} &= -\dot{\lambda}_x^6 \Leftrightarrow && \lambda_x^1 f(x)\bigg(\nu'(q_x)\big(1+y_x^e-c_x^e-z-y_x^u+c_x^u\big)+\frac{k}{q_x^2}\bigg) \\
& && -\lambda_x^2\nu'(q_x)(y_x^u-y_x^e)/x^2-\lambda_x^4\nu'(q_x)\big(\upsilon_x^e-\upsilon_x^u\big) && (32)\\
& && +\lambda_x^5\big(-\nu''(q_x)p_x^q\big(y_x^u-y_x^e\big)+\nu'(q_x)(p_x^e-p_x^u)\big)=-\dot{\lambda}_x^6 \\
\frac{\partial\mathcal{H}}{\partial c_x^e} &= 0 \Leftrightarrow && \lambda_x^4=-\frac{\lambda_x^1 f(x)}{\upsilon_c^e} && (33)\\
\frac{\partial\mathcal{H}}{\partial c_x^u} &= 0 \Leftrightarrow && \lambda_x^3\upsilon_c^u=-\big(1-\nu(q_x)\big)\big(\lambda_x^4\upsilon_c^u+\lambda_x^1 f(x)\big) && (34)\\
\frac{\partial\mathcal{H}}{\partial y_x^e} &= -\dot{\lambda}_x^7 \Leftrightarrow && \lambda_x^1\nu(q_x)f(x)+\lambda_x^2\nu(q_x)/x^2+\lambda_x^4\nu(q_x)/x+\lambda_x^5\nu'(q_x)p_x^q=-\dot{\lambda}_x^7 && (35)\\
\frac{\partial\mathcal{H}}{\partial y_x^u} &= -\dot{\lambda}_x^8 \Leftrightarrow && \lambda_x^1\big(1-\nu(q_x)\big)f(x)+\lambda_x^2\big(1-\nu(q_x)\big)/x^2+\lambda_x^3/x+\lambda_x^4\big(1-\nu(q_x)\big)/x \\
& && -\lambda_x^5\nu'(q_x)p_x^q=-\dot{\lambda}_x^8 && (36)\\
\frac{\partial\mathcal{H}}{\partial A_x} &= -\dot{\lambda}_x^1 \Leftrightarrow && \lambda_x^1=\lambda^1,\forall x && (37)\\
\frac{\partial\mathcal{H}}{\partial p_x^q} &= 0 \Leftrightarrow && \lambda_x^6=\lambda_x^5\nu'(q_x)\big(y_x^u-y_x^e\big) && (38)\\
\frac{\partial\mathcal{H}}{\partial p_x^e} &= 0 \Leftrightarrow && \lambda_x^7=-\lambda_x^5\nu(q_x) && (39)\\
\frac{\partial\mathcal{H}}{\partial p_x^u} &= 0 \Leftrightarrow && \lambda_x^8=-\lambda_x^5(1-\nu(q_x)) && (40)\\
\frac{\partial\mathcal{H}}{\partial \mathcal{U}_x} &= -\dot{\lambda}_x^2 \Leftrightarrow && f(x)+\lambda_x^3+\lambda_x^4=-\dot{\lambda}_x^2 && (41)\\
\lambda_x^3\geq 0, & \quad\text{and} && 0=\lambda_x^3\Big(\mathcal{U}_x-\kappa-\upsilon_x^u\Big) && (42)\\
\lambda_x^5\geq 0, & \quad\text{and} && 0=\lambda_x^5\Big(-\nu'(q_x)p_x^q\big(y_x^u-y_x^e\big)+\nu(q_x)p_x^e+(1-\nu(q_x))p_x^u\Big) && (43)
\end{aligned}$$

and since there are neither initial nor final conditions for $\mathcal{U}$, q_x, y_x^e and y_x^u, the following transversality conditions hold

$$\lambda_{\underline{x}}^2=\lambda_{\bar{x}}^2=0,\lambda_{\underline{x}}^6=\lambda_{\bar{x}}^6=0,\lambda_{\underline{x}}^7=\lambda_{\bar{x}}^7=0,\lambda_{\underline{x}}^8=\lambda_{\bar{x}}^8=0$$

Because the inequality constraints are concave in the control variables, the inequality constraint qualification holds leading to conditions (42) and (43).

We first show that $\lambda^1>0$. Using equation (41), we can write $\lambda_x^2=-\int_{\underline{x}}^{x}\big(f(t)+\lambda_t^3+\lambda_t^4\big)dt$

because of the transversality condition $\lambda^2_{\underline{x}} = 0$. Likewise, the transversality condition implies

$$\lambda^2_{\overline{x}} = 0 = -1 - \int_{\underline{x}}^{\overline{x}} \big(\lambda^3_t + \lambda^4_t\big)dt \Leftrightarrow \lambda^1 = \frac{1}{\displaystyle\int_{\underline{x}}^{\overline{x}} \frac{1}{\upsilon^e_c}\left(1 - \frac{\upsilon^u_c - \upsilon^e_c}{\upsilon^u_c}\big(1-\nu(q_x)\big)\right) f(x)dx} > 0, \quad (44)$$

where the sum $\lambda^3_t + \lambda^4_t$ obtains from equations (33) and (34). The co-state variable is positive as it can easily be shown that $c^u_x \leq c^e_x$ and due to the concavity of function υ.

From equations (39) and (40), we obtain $\lambda^7_x\big(1-\nu(q_x)\big) = \lambda^8_x \nu(q_x)$. By differentiating this expression with respect to x, we obtain

$$\dot{\lambda}^7_x\big(1-\nu(q_x)\big) - \dot{\lambda}^8_x \nu(q_x) = -\lambda^5_x \nu'(q_x) p^q_x$$

We then subtract equation (36) multiplied by $\nu(q_x)$ from (35) times $1-\nu(q_x)$ and use this last equality to obtain $\lambda^3_x = 0$ for all x. We obtain that consumption does not vary with the employment state of the worker as it follows from equations (33) and (34) that

$$\lambda^3_x \upsilon^u_c = -\lambda^1 f(x)\big(1-\nu(q_x)\big)\left(1 - \frac{\upsilon^u_c}{\upsilon^e_c}\right) \underset{\lambda^3_x=0,\lambda^1\neq 0}{\Rightarrow} c^e_x = c^u_x = c_x, \forall x \underset{(PC_x)}{\Rightarrow} y^e_x < y^u_x, \forall x$$

Now, we differentiate equations (38) and (39) with respect to x

$$\begin{aligned} \dot{\lambda}^6_x &= \dot{\lambda}^5_x \nu'(q_x)\big(y^u_x - y^e_x\big) + \lambda^5_x \nu''(q_x) p^q_x \big(y^u_x - y^e_x\big) + \lambda^5_x \nu'(q_x)\big(p^u_x - p^e_x\big), \\ \dot{\lambda}^7_x &= -\dot{\lambda}^5_x \nu(q_x) - \lambda^5_x \nu'(q_x) p^q_x, \end{aligned}$$

and substitute out $\dot{\lambda}^6_x$ and $\dot{\lambda}^7_x$ in equations (32) and (35), respectively, to obtain

$$\begin{aligned} \big(\dot{\lambda}^5_x - \lambda^2_x/x^2\big)\nu'(q_x)\big(y^u_x - y^e_x\big) &= \lambda^1 f(x)\left(\nu'(q_x)\left(\frac{y^e_x - y^u_x}{x\upsilon^e_c} - 1 - y^e_x + z + y^u_x\right) - \frac{k}{q^2_x}\right) \\ \dot{\lambda}^5_x - \lambda^2_x/x^2 &= \lambda^1 f(x)\left(1 - \frac{1}{x\upsilon^e_c}\right) \end{aligned} \quad (45)$$

By combining these two equations, we obtain expression (19), which implies $p^q_x = \dot{q}_x = 0$.

Next, we show that $\dot{c}_x \geq 0$ and $\nu(q_x)p^e_x + (1-\nu(q_x))p^u_x \geq 0$. Notice that the first order condition of the (ICC$_x$) can be stated as follows, using $\dot{q}_x = 0$ and $c_x^e = c_x^u = c_x$:

$$\frac{d\mathcal{U}_x(\hat{x})}{d\hat{x}}_{|\hat{x}=x} = 0 \Longleftrightarrow \frac{\partial \mathcal{U}_x(\hat{x})}{\partial c_{\hat{x}}^e}\dot{c}_{\hat{x}} + \frac{\partial \mathcal{U}_x(\hat{x})}{\partial y_{\hat{x}}^e}\dot{y}_{\hat{x}}^e + \frac{\partial \mathcal{U}_x(\hat{x})}{\partial c_{\hat{x}}^u}\dot{c}_{\hat{x}} + \frac{\partial \mathcal{U}_x(\hat{x})}{\partial y_{\hat{x}}^u}\dot{y}_{\hat{x}}^u{}_{|\hat{x}=x} = 0 \Longleftrightarrow$$
$$\upsilon_c^e \dot{c}_x = \nu(q)\frac{p_x^e}{x} + (1-\nu(q))\frac{p_x^u}{x}$$

The right hand side is nonnegative because of constraint (31). Therefore, so is $\dot{c}_x$.

We now prove, by contradiction, that there exists a subset of positive mass within consumption strictly increases with spouse's productivity. Suppose that there is no subset of positive mass within $(\underline{x}, \overline{x})$ such that $\dot{c}_x > 0$ or equivalently, due to the FOC of the (ICC$-x$) together with $\dot{q}_x = 0$, $\nu(q_x)p_x^e + (1-\nu(q_x))p_x^u > 0$. It follows from expression (44) that $\lambda^1 = \upsilon_c^e$, which is constant in x, and, hence, $\lambda_x^4 = -f(x)$, because of condition (33), and $\lambda_x^2 = 0$, due to condition (41) and the transversality conditions, for all $x \in (\underline{x}, \overline{x})$ except for a zero mass set. Then, equation (45) can be rewritten as $\dot{\lambda}_x^5 = \lambda^1 f(x)\left(1 - \frac{1}{x\upsilon_c^e}\right)$. Notice that the term in parenthesis is increasing in x and $\lambda_{\underline{x}}^5 = 0$ and $\dot{\lambda}_{\underline{x}}^5 \geq 0$. It follows that $\lambda_{\overline{x}}^5 > 0$, which is a contradiction because of the transversality conditions. Therefore, there exists a subset $\mathcal{S} \subset [\underline{x}, \overline{x}]$ of positive mass in which both c_x and $\nu(q)y_x^e + (1-\nu(q))y_x^u$ strictly increase with x.

We turn to show that for all $x \in \mathcal{S}$, $\frac{1}{x\upsilon_c^e} < 1$. By definition, $\nu(q)p_x^e + (1-\nu(q))p_x^u > 0$ and $\dot{c}_x > 0$ for all $x \in \mathcal{S}$. Hence, by continuity, these inequalities hold within an open neighborhood of $x \in \mathcal{S}$. Therefore, $\lambda_x^5 = 0$ and $\dot{\lambda}_x^5 = 0$ for all $x \in \mathcal{S}$ because of condition (43). Equation (45) implies that $\frac{1}{x\upsilon_c^e} < 1$ if $\lambda_x^2 < 0$. We now show that $\lambda_x^2 < 0$ for all $x \in (\underline{x}, \overline{x})$. We use conditions (33), (41) and (44) to write

$$\dot{\lambda}_x^2 = \lambda^1 f(x) \int_{\underline{x}}^{\overline{x}} \left(\frac{1}{\upsilon_c^e(c_x, y_x^e/x)} - \frac{1}{\upsilon_c^e(c_t, y_t^e/t)} \right) f(t)dt.$$

As fraction $\frac{1}{\upsilon_c^e}$ is an nondecreasing function in x because $\dot{c}_x \geq 0$, the derivative of the integral with respect to x is positive and, hence, $\dot{\lambda}_x^2$ changes sign at most once and is almost always different from 0. Indeed, the integral is negative at $x = \underline{x}$ and positive at $x = \overline{x}$. Since $\lambda_{\underline{x}}^2 = \lambda_{\overline{x}}^2 = 0$, we have that λ_x^2 is negative and convex in $(\underline{x}, \overline{x})$.

Finally, it is immediate to see that the equilibrium allocation is not constrained efficient as equilibrium condition (15) holds neither in $\mathcal{S}$ nor out of it where $\dot{c}_x = 0$.‖

Proof of Lemma 5.8.

Let T denote a tax on household's total income. For the tax-distorted equilibrium to coincide with the planner's solution, efficiency condition (19) as well as $\psi'(c_x) > \frac{1}{x}$ must hold.

The tax-distorted counterparts of equilibrium equations (12), (13) and (15)

$$
\begin{aligned}
(1-\gamma(q))\left(\frac{V(w)-V(z)}{\psi(c_x^e)(1-T'(w+y_x^e))}+1-w\right) &= \frac{k}{\eta(q)} && (46)\\
k &= \eta(q)(1-w) && (47)\\
\psi(c_x^e)\big(1-T'(w+y_x^e)\big) &= \psi(c_x^u)\big(1-T'(z+y_x^u)\big) = \frac{1}{x} && (48)
\end{aligned}
$$

where $c_x^e \equiv w + y_x^e - T(w+y_x^e)$ and $c_x^u \equiv z + y_x^u - T(w+y_x^e)$. Furthermore, the government balances its budget, and, hence,

$$\int_{\underline{x}}^{\overline{x}} \left(\nu(q)T(w+y_x^e)+(1-\nu(q))T(z+y_x^u)\right)f(x)dx = 0$$

We now compare the planner's solution with the tax-distorted equilibrium allocation. First, for the consumption levels to be independent of the employment status, it must be the case that $c_x^e = c_x^u = c_x$. Second, given the planner's queue length, the tax-distorted equilibrium wage w is determined by the zero-profit condition (47). Third, by comparing equilibrium condition (48) with its planner's counterpart, we conclude that the tax function T must be increasing. Next, by using equation (48) to replace the marginal utility and imposing the equal-consumption condition, we can rewrite equation (46) as

$$(1-\gamma(q))\left(1-z-T(w+y_x^e)+T(z+y_x^u)\right) = \frac{k}{\eta(q)}$$

Therefore, the implementation of the planner's solution implies $T(w+y_x^e) = T(z+y_x^u)$, which leads to a pre-tax household's total income independent of the employment state, $w + y_x^e = z + y_x^u$. As w is invariant in x so is the difference $y_x^u - y_x^e$.

Finally, it is straightforward to combine the zero-profit condition and the balanced-budget constrained of the government to obtain the resource constraint of the planner. ‖