Liquidity Saving Mechanism in an Interconnected Payment Network

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Abstract

This paper studies the intraday payment behavior of banks under the real time gross settlement (RTGS) system. Our game structure is based on Bech and Garratt (2003), with additional stages for recycling reserve. This gives us a variant of the hawk–dove structure, which enables efficient reserve recycling in an equilibrium. Based on this structure, we analyze the welfare effects of introducing a liquidity saving mechanism (LSM) to the RTGS system. We focus on situations where the LSM works only locally, that is, it serves to offset only a part of the whole obligation. Such partial offsetting effectively works to separate a connected payments network into mutually disconnected subnetworks. It is apparent that disconnection of the network itself will prevent efficient recycling throughout the network. We further find that there possibly exists a strategic indirect effect which will prevent efficient recycling even within each subnetwork. We show that introducing an LSM possibly has an overall negative welfare effect. Specifically, we devise a theoretical class of core-periphery structures for underlying payment networks, and examine situations where the LSM could offset obligations only among the core banks. We show that the density of the network could have a nonlinear impact in the effects of the LSM in the sense that the negative effect is more likely for middle ranged dense networks. Our analysis suggests a policy mix wherein the utilization of the LSM should be examined in combination with other policies promoting efficient liquidity recycling, namely controlling the fee for the reserve or the ordering of banks regarding their payments.

JEL classification: C72, D85, E42, E58

Keywords:

payment system, RTGS, liquidity saving mechanism, game on network, network effect

1 Introduction

In interbank settlement systems, real-time gross settlement (RTGS) systems have largely replaced the traditional designated-time net settlement (DTNS) systems.\footnote{The World Bank (2013) documents that 116 of 139 surveyed countries had adopted the RTGS system up to 2010.} In
traditional DTNS systems, settlements are executed only at several designated times a day. Their limited frequency of settlements is viewed as a source of credit risk. RTGS systems serve to eliminate such risk by settling payments on a real-time basis. But RTGS systems have their downside. Since payments are settled on a gross basis under RTGS systems, banks’ liquidity needs are greater than those under DTNS systems. This is thought to be undesirable since liquidity is essentially a scarce resource. In this respect, some RTGS systems have developed and introduced mechanisms referred to as liquidity saving mechanisms (LSMs) that aim to equip the RTGS systems with the offsetting service. LSMs allows payments to be queued instead of being settled immediately so that payments within the queue are offset with each other. LSMs are primarily expected to reduce liquidity needs, but they could affect banks’ behaviors in the relevant strategic context.

This paper examines the welfare effects of introducing an LSM into an RTGS system in a strategic context. The key observation for our paper is that LSMs, in reality, are able to offset at best only part of the whole relevant payments. This is understandable since offsetting mechanisms essentially reintroduce credit risk, and the risk would be quite large when LSMs could offset the whole relevant payments. Departing from the existent literature that effectively examined situations where LSMs could offset all relevant payments, we analyze the consequences of introducing LSMs to serve as a partial netting mechanism.

Under our RTGS system, participant banks are engaged in a timing game, namely, the timing of making their payments in a day. Strategic consideration is relevant since incoming payments effectively serve as zero-cost financing of liquidity while external financing is costly. Banks decide the timing of making their payments, trading off the cost of financing liquidity against the cost of delaying payments.

Bech and Garratt (2003) examined banks’ intraday behaviors in a game theoretic framework for the case of two players. The paper modeled intraday settlements as two-stage games in which each player basically determines whether to make each payment early or late. The paper showed that the game structure effectively appears in dominant strategy situations, the prisoner’s dilemma, or the stag hunt, depending on relevant parameter values. Our paper examines situations including more than three players, where linkages of payments can be considerably long. It differs from the two-player setting in that there is now the possibility of recycling liquidity. Accordingly, we model intraday settlements by adding middle stages, in which banks can recycle liquidity. We find that for certain parameter values, the game structure can be characterized as a variant of the hawk–dove, the dominant strategy structure, or possible mixtures of these. The hawk–dove structure implies an equilibrium where liquidity is efficiently recycled. Introducing an LSM can change the relevant network and consequently affect the game structure. Actually, we find that introducing an LSM can have a negative welfare effect as it dismisses efficient liquidity recycling.

The key feature for the negative effect of the LSM is that banks face “heterogeneous” liquidity shock in the middle of a day. This heterogeneity brings potential conflicts between the two structures in that which game structure would appear. We observe that within a connected network, the variant of the hawk–dove structure would appear even in the presence of potential conflicts. This is depicted as a strategic spillover through the connection of a network. Now, when the network is separated into mutually disconnected
subnetworks in a way that only either type exists for some of them, the potential conflict would be released for those subnetworks, leading to the emergence of the dominant strategy structure. This means that efficient liquidity recycling is no more realized for those subnetworks.

Specifically, we introduce and examine a class of core-periphery structures for our underlying payment networks. Our core-periphery structure intends to capture the characteristics of real-world payment networks. Imakubo and Soejima (2010) executed network analysis on payment flows in Japan’s interbank money market and pointed out that some banks form a core network which serves as a hub for peripheral banks. This is termed as the core-periphery structure. A similar observation was also reported by Soramäki, Bech, Arnold, Glass, and Beyeler (2007) for the Fedwire case, and Rordam and Bech (2009) for Danish interbank money flow. In our core-periphery payment networks, participants are classified into two groups: core banks and periphery banks. The former serves as a hub to the latter. We say that periphery banks are more tightly connected to each other in a more dense network. We examine a situation where an LSM could offset only payments among core banks. Offsetting these payments leads to the emergence of a larger number of mutually disconnected subnetworks in less dense networks. We show that the density has a nonlinear impact in effects of an LSM when the LSM serves to dismiss the strategic spillover. Concretely, negative effects of an LSM are more likely for middle ranged dense networks.

Several papers have examined the effects of LSMs. Based on the game structure introduced by Bech and Garratt (2003), Martin and McAndrews (2008) examined the consequences of a queuing arrangement in situations with a larger cycle or small cycles of payments. Martin and McAndrews (2010) extended the paper to study the different designs of queuing arrangements. These papers introduced liquidity shocks that create differences in banks’ behaviors regarding delaying payments. In their settings without LSMs, there emerges an equilibrium where efficient coordination fails. They showed that even without offsetting payments, queuing availability itself can possibly have a positive effect by mitigating the coordination problem. Our paper complements their papers by focusing more on the effects of offsetting payments. In our setting, the heterogeneity of liquidity risk creates differences in banks’ incentives, but efficient liquidity recycling is still possible without LSMs. We argue that introducing an LSM could dismiss efficient liquidity recycling through offsetting payments. Willison (2005) examined the effects of offsetting in situations with a complete network, where every bank makes payments to all the other banks. Our paper differs in that it examines the consequences of the partial netting service of an LSM, while Willison (2005) effectively analyzed the full netting service of LSMs. Roberds (1999) studied the queuing arrangement with regard to the incentives of banks engaging in risk-taking behaviors. Kahn and Roberds (2001) considered the LSM in the context of a Continuous Linked Settlement (CLS).

Only a few studies on financial network analyzed the consequences of the transformation of relevant networks. Hayakawa (2016b) introduced a graph-theoretic approach, where the consequences of a shock on underlying networks were examined through a combination of basic network transformations. The paper specifically analyzed effects of the Central Clearing Counterparty (CCP) in reducing liquidity needs in a general payment network, where the consequences of introducing CCPs were analyzed as shocks on un-
derlying payment networks. The effect of the CCP was decomposed into two types, termed as the “central routing effect” and the “central netting effect”. The former refers to effects relevant to adding a node to the network, while the latter refers to the effects of netting payments. The paper showed that both effects can be negative. In our paper, we observe that introducing an LSM can have a negative effect in exactly the same manner as in the case of the CCP with respect to the “central netting effect”. The difference is that strategic incentives could also matter in the case of the LSM. Thus, departing from Hayakawa (2016b), we effectively examine the “central netting effect” in a strategic context. Our finding is that the “central netting effect” could have an additional negative effect by affecting the strategic incentive in the case of the LSMs.

Our paper is relevant to the literature on herding behavior in financial economics. Diamond and Dybvig (1983) described bank runs in the view of payoff externality among depositors, wherein the payoff externality is global in the sense that each depositor’s payoff is relevant to the number of depositors who withdraw their deposits. In contrast, our paper describes herding in the network context. In our paper, the payoff externality has a local feature wherein a bank’s payoff is relevant to the behavior of banks who have payments to the bank, but is of little relevance to the others. Still, we show that it is possible for local externality to spread globally through the linkage of payments. Thus, the network structure is relevant in our analysis.

In view of network externality, recent literature of financial contagion is also relevant to our paper. Eisenberg and Noe (2001) provided a mathematical framework arguing about the domino effect of bankruptcy through balance sheet linkage. Eboli (2013), Elliott, Golub, and Jackson (2014), and Acemoglu, Ozdaglar, and Tahbaz-Salehi (2015) have presented relevant frameworks arguing about financial contagion. They largely examined non-strategic spillover by arguing how a loss in the balance sheet of one agent affects the balance sheets of the other agents through the linkage. In a different context, this paper focuses on strategic spillover regarding the timing of payments by arguing how the change of identity and consequent change in strategy of one agent could affect the strategies of the other agents through the linkage of payment relationships.

Our paper supposes that certain monetary substance is used as a means of payment, without arguing the foundation of money. Regarding the foundation of money, Kiyotaki and Wright (1989) proposed a search model where money could serve as a medium of exchange in an equilibrium. The following researches examined relevant models to study

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2Devenow and Welch (1996) surveyed the literature on herding in financial economics. They classified rational herding models into three types in terms of the relevant effects: Payoff externalities models, Principal-agent models, and Cascade models.

3In Diamond and Dybvig (1983), the key assumption for the possibility of a bank run was that deposit contracts cannot be contingent on the number of depositors. The paper specifically assumed a sequential service constraint in withdrawal of bank deposits. Herding behavior in bank runs was also examined in view of the informational aspect, as in Gorton (1985), and Chari and Jagannathan (1988).

conditions of such monetary equilibrium. Our model is based on those researches in the sense that we effectively suppose certain monetary equilibrium.

The rest of this paper is organized as follows. Section 2 presents the environment, and defines our game, equilibrium, and welfare. Section 3 examine the RTGS system without the LSM. Section 4 analyzes the effects of our LSM. Section 5 argues policy implications of the analysis. Section 6 concludes. The Appendix shows some of the proofs.

2 Set up

2.1 The Base Setting

We consider an interbank settlement system with \(N\) banks and one settlement institution. The banks are risk-neutral strategic agents, while the settlement institution is as a non-strategic agent. We examine settlements within a day, which consists of four periods: 1, 2, 3, and 4. At the beginning of a day, the banks are given certain amount of payments to make and receive among them, as described later in detail. Also, all the banks initially have zero reserves. Banks can borrow reserves from the Central Bank (CB), and there need be returned at the end of the day. They can borrow reserves to make the payments in periods 1 and 4, but not in periods 2 and 3. In periods 2 and 3, banks can make payments only either with their holdings at the beginning of each period or with what they receive within each period. We call periods 2 and 3 the “recycling” periods. In periods 1 and 4, we suppose passive lending by the CB such that only the necessary amount is lent each time. When banks decide to make their payments within each period, the payments are sent unilaterally to the payment system but they arrive in a random order excluding the same arrival. Reserves are borrowed only if they are necessary under the realized order. Note that even when a bank has zero reserves when it decides to make a payment, it does not necessarily borrow reserves. For example, suppose two banks belong to a cycle of length two, which means that the two banks are to make payments to each other, and both decide to make payments in the same period with zero reserves. Here, either bank is randomly chosen to make its payment first and needs to borrow, while the other makes its payment afterward with the reserves it receives. We assume that banks cannot lend their holding reserve to others so that we preclude excess borrowing. Borrowing one unit of reserves from the CB costs banks \(x\) amount of the fee. When banks do not make their payments in period 1 and delay them, they must pay the delay cost. Delay costs are denoted as \(\gamma_1, \gamma_2, \gamma_3,\) and \(\gamma_4\), where a bank incurs \(\gamma_t\) when its payments are made in period \(t + 1\) for \(t = 1, 2, 3,\) and \(\gamma_4\) is the cost when a bank has not made its payments within the day. Banks decide the timing of their payments trading off the cost of making payments and the cost of delaying them.

We further suppose a liquidity shock in the middle of the day, which is a pair of payments to and from the settlement institution. At the end of period 2, banks that face the liquidity shock must make one unit of payment to the settlement institution. In turn, they receive one unit of payment from the settlement institution at the beginning of

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\(^5\)Kiyotaki, Lagos, and Wright (2016) provides a recent survey for the literature of foundation of money. The relevant researches argued that the key conditions of a monetary equilibrium are limited commitment/information as well as the lack of double–coincidence.
period 3. Payments to and from the settlement institution cannot be delayed. For banks who face the liquidity shock, necessary amounts of reserves are lent by the CB. Although we assume that these outgoing and incoming shocks are paired to affect each bank, our results largely hold for cases in which the two shocks occur independently. Importantly, for our analysis, there exists heterogeneity in liquidity shock: each bank faces liquidity shock either with probability $\sigma^l$ or $\sigma^h$, with $0 < \sigma^l < \sigma^h < 1$. Banks with probability type $\sigma^h$ are more likely to face liquidity shock than those with type $\sigma^l$. The distribution of heterogeneity is described later in detail, together with our description of the payment network structure. Table 1 shows the definitions of the parameters.

The banks have series of decisions in a day. At the beginning of period 1, they decide whether to make the payment or not. If they decide to make the payment, no further decisions are needed in the later periods. If they do not make the payment in period 1, then at the beginning of period 2, they decide whether they are willing to make payments. Note that they cannot make payments when they have no reserves held or received in period 2. At the beginning of period 3, if they have not made their payments, they decide whether they are willing to make payments in the same way as they did in period 2. Decision in period 4 is the same as that in period 1, but all the banks will decide to make the payment with our supposition of the sufficiently large $\gamma_4$.

The timings of the events are summarized in Figure 1. First, nature chooses the banks that receive a liquidity shock. Then, banks make their decisions. After that, payments are executed in period 1 with no delay cost. Then, in period 2, payments are executed with delay cost $\gamma_1$. At the end of period 2 and the beginning of period 3, payments are made between banks and a settlement institution. Then, in period 3, payments are made with delay cost $\gamma_2$. Payments in period 4 are made with delay cost $\gamma_3$, and if payments are not executed in a day, the cost becomes $\gamma_4$. At the end of the day, banks pay their borrowing fees to the CB in line with their maximum negative balance on the day.

Table 1: Parameters

<table>
<thead>
<tr>
<th>$x$</th>
<th>Cost of borrowing</th>
</tr>
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<tbody>
<tr>
<td>$0 &lt; \gamma_1 &lt; \gamma_2 \leq \gamma_3 &lt; \gamma_4$</td>
<td>Delay costs</td>
</tr>
<tr>
<td>$0 &lt; \sigma^l &lt; \sigma^h &lt; 1$</td>
<td>Probability of a liquidity shock</td>
</tr>
</tbody>
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2.2 Core-periphery network

We denote the set of banks as $I$, with $|I| = N$. At the beginning of a day, we suppose the banks are given payments that form a core-periphery network defined as follows.

**Definition 1.** core-periphery network

We say payments among banks $I$ form a core-periphery network with $(K, W)$ when

1. banks are divided into $K$ core banks $I_{core} = \{i_{core,k}\}_{k=1,2,...,K}$ and $K$ groups of periphery banks $I_{per} = \{I_{per,k}\}_{k=1,2,...,K}$ where $|I_{per,k}| = W$ for every $k = 1, 2, ..., K$,
2. each core bank has two units of payments to make such that one is to a core bank and the other is to a periphery bank. Similarly, it has two units of payments to receive,
where, again, one is from a core bank, and the other is from a periphery bank,
(3) payments among \( K \) core banks forms a cycle with their \( K \) payments,
(4) each periphery bank has one unit of payment to make and also one unit to receive,
and
(5) for each \( k = 1, 2, \ldots K \), payments among \( I_{\text{per}, k} \) constitute a path, and each of the ends
is connected to a core bank with a payment.

We denote a set of core-periphery networks with \((K, W)\) as \( \Psi_{(K,W)} \), and a set of whole
core periphery networks as \( \Psi \). \( K \) simply indicates the number of core banks, while \( W \) is
interpreted as the “width” of the network. Using graph-theoretic notations, we consider
a directed network \((V, A)\) with a set of vertices \( V \) and a set of arrows \( A \), where for \( a \in A \),
\( a = (v, v') \) with \( v, v' \in V \). Figure 2 shows examples of core-periphery networks \( \Psi_{(3,4)} \) along with the graph-theoretic representation.

In order to simplify our later analysis, which calculates the amount of borrowed reserve,
we focus on core-periphery networks that have no twisted nature in the sense of the vertex-twisted relation defined as below. A network \((V, A)\) is closed when the number of incoming
arrows and outgoing arrows are the same for every vertex. Note that \( K \)–core-periphery networks are all closed in this sense. For a closed network \((V, A)\), we consider a numbering function on \( V \) as \( s : V \to \{1, 2, \ldots, |V|\} \). Take a cycle \((V', A') \subset (V, A)\) wherein there is no
vertex \( v \in V' \) included in more than two arrows. We call such a cycle as non-punctured.
For a given numbering \( s \) on \((V, A)\), for the cycle \( c = (V', A')\), we derive the vertex-reverse number \( r^{\text{v-re}}(c, s) = \sum_{a'=(v,v') \in A'} 1_{\{s(v)<s(v')\}} \) with the indicator function \( 1_{\{\}} \). The minimum possible value for \( r^{\text{v-re}}(c, s) \) is 1 when the numbers increase as we follow the direction of the arrows, while the maximum possible value is \(|V'| - 1\) as the numbers increase in exactly the opposite direction.
Definition 2. No vertex-twisted cycles

For a closed network \((V,A)\), take every non-punctured cycle that constitutes a set \(C\). We say \((V,A)\) has no vertex-twisted cycles when we can take a numbering \(s : V \rightarrow \{1,2,\ldots,|V|\}\) such that \(vtw(c,s) = 1\) for every \(c \in C\).

Each core-periphery network illustrated in Figure 2 has no vertex-twisted cycles. We move to define density on each \(\Psi(K,W)\).

Definition 3. Density

For a core-periphery network \(\psi \in \Psi(K,W)\), remove a cycle of payments among core banks. The rest of the payments now form several numbers of disconnected cycles of payments, and we call each cycle a core-separated cycle. Let \(d(\psi)\) denote the density for \(\psi\), which indicates the number of core-separated cycles. Further, for \(\psi,\psi' \in \Psi(K,W)\), we say \(\psi\) is more dense than \(\psi'\) if \(d(\psi) < d(\psi')\).

For \(\psi \in \Psi(K,W)\), we have \(d(\psi) \in \{1,2,\ldots,K\}\). Note that for each \(K, W,\) and \(n = 1,2,\ldots,K\), we always have at least one network \(\psi \in \Psi(K,W)\) such that \(d(\psi) = n\) and it has no vertex-twisted cycles.

Figure 2: Examples of core-periphery networks \(\Psi(3,4)\)

In each network, there are three core banks \(I_{\text{core}} = \{a,b,c\}\), and three groups of periphery banks. The density for each network is 1, 2, and 3 from the left to the right. The network on the far left is the most dense.

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6The notion of the vertex-twisted relation was introduced by Hayakawa (2014) and discussed further by Hayakawa (2016a). The relation was studied in the context of a liquidity problem which is essentially the same problem examined in this paper. The author formalized the problem in a general class of networks and showed a characterization in terms of network topology. The vertex-twisted relation is shown as one of key topologies in the characterization.

7For the network shown on the left-hand side of Figure 2, suppose we reverse the direction of the arrows among the core banks. Then, the generated network has vertex-twisted cycles with regard to the inner cycle formed with payments of the core banks and the outer cycle formed with payments of the periphery banks and the core banks.
2.3 Network structure and Liquidity shock

For each core-periphery network $\psi \in \Psi_{(K,W)}$, we suppose that there exists $L \in \{1,2,\ldots,K\}$ where every bank in $I_{per,L}$ faces a liquidity shock with probability $\sigma^l$, and the other periphery banks and all the core banks face the shock with probability $\sigma^h$. This assumption on the distribution is made in order to highlight our results regarding the negative effect of the LSM.

In reality, the higher liquidity shock would be more probable for banks that have more payments to make and receive. Our assumption is consistent with this view in that we suppose the core banks always face higher liquidity shock. Our negative effect comes from the supposition that there is a groups of periphery bank $I_{per,k}$ with $k \notin L$, wherein all the banks face higher liquidity shock. Denote the average liquidity shock probability as $\bar{\sigma} = \frac{1}{N}(N^h\sigma^h + N^l\sigma^l)$, where $N^l = |I_{per,L}|$, $N^h = N - N^l$. We suppose $\sigma^l, \sigma^h$ are the actual ratios of banks that face the liquidity shock.

2.4 Game and Equilibrium

First, we formulate a game without an LSM, which we refer to as the “intraday liquidity management game with recycling (ILMGR)”, which essentially adds a stage of recycling reserve to the games introduced by Bech and Garratt (2003), which were referred to as the “intraday liquidity management games”.

We refer to the identity of each bank in terms of two aspects: whether it is a core bank or a periphery bank, and whether it faces a high or a low liquidity shock. We simply suppose that the network structure regarding $\psi$ and the identity of each bank is common knowledge. Let $A_i$ denote the set of available actions for bank $i \in I$. For the statement of available actions, we focus on meaningful actions in the sense that they are not eliminated by simple backward induction. At the beginning of period 4, when a bank has not made some of its payments, it is always preferable to make the payment than not make it since we suppose sufficiently high delay cost $\gamma_4$. So, all the banks choose to make their payments in period 4. At the beginning of period 3, for banks that still have payments to make choose whether or not to recycle the reserves that they hold or that they might receive in the period. When they choose not to recycle, their delay costs increase from $\gamma_2$ to $\gamma_3$. Thus, all the banks choose to recycle. At the beginning of period 2, banks similarly choose whether or not to recycle the reserves, but both choices are not simply eliminated. At the beginning of period 1, banks choose whether to make payments or not. Both choices are not eliminated.

The set of available actions for each periphery bank $j \in I_{per}$ is now defined as $A_j = \{P,R,H\}$, where $P$ indicates making a payment in period 1, and $R$ denotes making a payment in period 1 but recycling in period 2. $H$ denotes not making a payment in period 1 and not recycling in period 2. We focus on pure strategies, $s_i \in A_i$ for each bank $i \in I$. Throughout the paper, we suppose banks choose $P$ when they are indifferent to $R$ or $H$, and choose $R$ if they are indifferent to $H$.

For core banks, the set of their actions is larger since each has two payments to make and receive. We let the set of available actions for the core banks$^8$ $i \in I_{core}$ be

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$^8$Note that actions of core banks cannot be dependent on the order they receive their payments in our setting since we do not explicitly examine the timing of arrivals of payments within period 2 and 3.
\[ A_i = \{ P, P_c R, P_p R, R_{cc}, R_{cp}, P_c H, P_p H, R_c H_{cc}, R_p H_{cc}, R_c H_{cp}, R_p H_{cp}, H_{cc}, H_{cp} \} \]. Here, \( P \) indicates that both payments are made in period 1. \( P_c R(P_p R) \) indicates that the payment is made to a core(periphery) bank in period 1, and a reserve is recycled in period 2 to a periphery(core) bank. \( R_{cc} \) indicates that neither payment is made in period 1, and reserves are recycled in period 2 and 3 in a way that a payment received from a core bank goes to a core bank while that from a periphery bank goes to a periphery bank. \( R_{cp} \) indicates recycling such that a payment received from a core bank goes to a periphery bank.

\[ P_c H(P_p H) \] indicates that the payment is made to a core(periphery) bank in period 1, no reserve is recycled in period 2, and a reserve is recycled in period 3 to a periphery(core) bank. Each of \( R_{cc} \), \( R_{cp} \), \( R_{cc} \), and \( R_{cp} \) specify the action where a bank recycles only one reserve in period 2 (as specified in the former part, \( R_c \) or \( R_p \)), and recycles in period 3 (as specified in the latter part, \( H_{cc} \) or \( H_{cp} \)). \( H_{cc} \) and \( H_{cp} \) indicate that recycling occurs only in period 3 for each specified route.

Let \( S = \times_{i \in I} s_i \in S \) denote a strategy profile. Denote \( S_{-i} = \times_{i' \in I \setminus i} s_{i'} \). Given a structure of payment network \( \psi \in \Psi \), the payoff function for bank \( i \) is denoted as \( \pi_i : A_i \times \psi \to R \), which is equal to the negative value of the expected settlement cost function, \( c_i : A_i \times \psi \to R \), that is, \( \pi_i(s_i, S_{-i}, \psi) = -c_i(s_i, S_{-i}, \psi) \). We consider Nash equilibrium for this game.

**Definition 4. Nash Equilibrium**

A strategy profile \( S^* = (s_i^*, S_{-i}^*) \in S \) is a Nash equilibrium for the “intraday liquidity management game with recycle” (ILMGR) under a given structure of payment network \( \psi \) if and only if

\[ E[\pi_i(s_i^*, S_{-i}^*, \psi)] \geq E[\pi_i(s_i', S_{-i}^*, \psi)] \]  

for every \( s_i' \in A_i \) and \( i \in I \).

### 2.5 Welfare

In our subsequent analysis, we suppose that the CB can control the intraday lending fee \( x \) before the game starts. The CB is supposed to set the fee level after considering into social welfare, which we define as the negative of socially valued aggregate costs. Regarding the cost of liquidity, we suppose the true cost of intraday liquidity which is referred to as the implicit market interest rate \( r \).\(^{10}\) We define social welfare \( W(S, \psi) \) under a strategy profile \( S \) and a network structure \( \psi \in \Psi \) as below.

\[ W(S, \psi) = -\max(M(S, \psi)) \times r - \sum_{i \in I} \sum_{t=1}^{4} \gamma_i t(S) \]  

\(^9\)Regarding the actions specified with \( R_{cc} \) and \( R_{cp} \), the routes of recycling payments need be the same between periods 2 and 3. We ignore situations where the routes are different, since they are not essential to our analysis and to avoid cumbersome notations.

\(^{10}\)The true cost of intraday liquidity of the CB is associated with the potential loss on its balance sheet; a large loss could shake the credibility of the CB.
There, $M(S, \psi)$ denotes the probability distribution regarding aggregate borrowing amounts,\textsuperscript{11} and the operator $\max(.)$ derives the maximum possible borrowing amounts under strategy profile $S$ and network structure $\psi$. We suppose the worst case scenario for our social welfare. This is partly an analytical supposition that serves to magnify the impact of the LSM regarding liquidity cost in analyzing both the positive and the negative effect. For delay costs under a strategy profile $S$, for $t = 1, 2, 3, 4$, for $i \in I$, we have $\gamma_{i,t}(S) = \gamma_t$ if bank $i$ pays delay cost $\gamma_t$, and $\gamma_{i,t} = 0$ otherwise. Since transfers among banks and the CB are canceled out, there is no lending fee $x$ explicitly shown in our social welfare.

We define the first-best strategy profiles.

**Definition 5.** First-best strategy profile

A strategy profile $S$ under a network structure $\psi \in \Psi$ attains first-best if and only if

$$W(S, \psi) \geq W(S', \psi)$$

for every $S' \in S$.

For our welfare analysis, we make two assumptions throughout the paper.

**Assumption 1.**

$$\frac{\hat{\mu}}{r} < 1.$$  

This assumption ensures that successful recycling in period 2 is worth examining in terms of social welfare.

**Assumption 2.**

$$\frac{\hat{\mu}}{r} + \sigma^h < \frac{\bar{\sigma}}{r}.$$  

The above assumption states that social welfare is higher when banks choose to recycle reserves instead of holding them in period 2. Our focus is situations where recycling is desirable.

**Assumption 3.**

$$\frac{2K}{N} < \tilde{\sigma} < \frac{N-K}{N}.$$  

This assumption is a technical requirement that allows us to compare social welfare among core-periphery network structures. The former part, $\frac{2K}{N} < \tilde{\sigma}$, ensures that liquidity shock is sufficiently large such that in any network structure, the required amount of reserve when all the payments are made in period 1 is always equal to the number of existent payments in terms of social welfare. The latter part, $\tilde{\sigma} < \frac{N-K}{N}$, ensures that the liquidity shock is not as high as the level where there is no welfare gain by recycling reserves.

\textsuperscript{11}We consider probability distributions for aggregate borrowing amounts since we suppose that payments made within periods 1 and 4 arrive at the system in a random order.
3 RTGS without LSM

3.1 First best strategy profiles

We focus on two types of strategy profiles.
1. (All-early) $s_i = P$ for every $i \in I$.
2. (1-recycle) $s_j = P$ for one periphery bank $j \in I_{\text{per}}$, while $s_{j'} = R$ for every $j' \neq j \in I_{\text{per}}$, and $s_i \in \{R_{cc}, R_{cp}\}$ for every $i \in I_{\text{core}}$.

For the second type, referred to as the 1-recycle strategy profile, we confine ourselves to strategy profiles where the route choices $\{s_i\}_{i \in I_{\text{core}}}$ are appropriate in the sense that a reserve is recycled to settle all the relevant payments. We call them as successful 1-recycle strategy profiles. The next lemma ensures that such a strategy profile actually exists for any network structure $\psi \in \Psi$.

Lemma 1. Successful 1-recycle strategy profile

Within 1-recycle strategy profiles, for any network structure $\psi \in \Psi$, there always exists a strategy profile where a reserve is always recycled to settle all the payments.

Proof. See Appendix A.1.

Denote $S^1$ and $S^2$ as the representative strategy profile of all-early, successful 1-recycle.

Lemma 2.

For a given set of banks $I = \{I_{\text{core}} = \{i_{\text{core}}\}_{k=1,2,...,K}, \{I_{\text{per},k}\}_{k=1,2,...,K}\}$ with their identities regarding liquidity shock, we have

$W(S^1, \psi) = -(N + K)r$, and

$W(S^2, \psi) = -(1 + N\bar{\sigma})r - (N + K - 1)\gamma_1$.

Proof. See Appendix A.2.

Since the lemma shows that the welfare for $S^1$ and $S^2$ does not depend on $\psi$, we refer the respective welfare as $W(S^1)$ and $W(S^2)$.

Proposition 1. First-best strategy profiles

Under Assumptions 1 and 2, $S^1$ attains first-best if $\frac{N + K - 1}{N}(1 - \frac{\gamma_1}{r}) \leq \bar{\sigma} < \frac{N + K}{N}$, while $S^2$ attains first-best if $\frac{2K}{N} < \bar{\sigma} \leq \frac{N + K - 1}{N}(1 - \frac{\gamma_1}{r})$.

Proof. See Appendix A.3.

3.2 Equilibrium, Best Response, and Spillover Effect

Next, we examine conditions for each of $S^1$ and $S^2$ to be an equilibrium. We assess relevant payoffs. For a core bank $i \in I_{\text{core}}$, we call banks that have payments to make to bank $i$ as the payers for bank $i$. We denote a pair of strategies of payers for a core bank $i$ as $s^p_i = (s_j, s_{j'})$, where $j$ and $j'$ refer to a core bank and a periphery bank respectively.
There, the strategy of the core bank \( s_j \) is denoted in the same way as that of the periphery bank \( s_{j'} \) such that \( P \) indicates that bank \( j \) makes the payment to bank \( i \) in period 1, and \( R \) indicates that bank \( j \) does not make the payment in period 1 but recycles to bank \( i \) in period 2. In these notations, we have \( s_i^P \in \{(P, P), (P, R), (R, P), (R, R)\} \). We examine payoff of core bank \( i \) under a strategy profile \( S_{-i} \), where there is a periphery bank \( j \) with \( s_j = P \), and it surely receives payment until period 2 with successful recycling, as is actually the case in both \( S_1 \) and \( S_2 \). We simplify the notation of the set of actions for the core bank \( i \) to denote as \( A_i = \{P; PR; PH; R; RH; H\} \), where we omit their route choices by supposing that they are appropriately chosen under each strategy profile.

If \( s_i^P = (P, P) \), then,

\[
\pi_i(P, S_{-i}, \psi) = -\left(\frac{1}{6}2x + \frac{1}{2}x + \frac{1}{3}\sigma^h x\right) = -\frac{5}{6}x - \frac{1}{3}\sigma^h x, \tag{4}
\]
\[
\pi_i(PR, S_{-i}, \psi) = -\frac{1}{3}x - \frac{2}{3}\sigma^h x - \gamma_1, \tag{5}
\]
\[
\pi_i(PH, S_{-i}, \psi) = -\frac{1}{3}x - \gamma_2, \tag{6}
\]
\[
\pi_i(R, S_{-i}, \psi) = -\sigma^h x - 2\gamma_1, \tag{7}
\]
\[
\pi_i(RH, S_{-i}, \psi) = -\gamma_1 - \gamma_2, and \tag{8}
\]
\[
\pi_i(H, S_{-i}, \psi) = -2\gamma_2. \tag{9}
\]

In the above, for the case of \( s_i = P \), in period 1 bank \( i \) borrows two units of reserve with probability \( \frac{1}{6} \), one unit with \( \frac{3}{6} \), and none with \( \frac{2}{6} \). When the liquidity shock hits the bank, it needs to borrow only when it has no reserve holdings. The bank would hold no reserve at the time of liquidity shock only if it has borrowed nothing in period 1. For the case of \( s_i = PR \), in period 1 bank \( i \) borrows one unit with probability \( \frac{1}{3} \) and none with \( \frac{2}{3} \). For the case of \( s_i = PH \), bank \( i \) always holds a reserve at the beginning of period 2, and since it does not recycle within the period, the reserve protects the bank from the liquidity shock. Similar logic can be used to derive payoffs for the other cases.

If \( s_i^R \in \{(P, R), (R, P)\} \), then

\[
\pi_i(P, S_{-i}, \psi) = -\frac{1}{3}2x - \frac{2}{3}x = -\frac{4}{3}x, \tag{11}
\]
\[
\pi_i(PR, S_{-i}, \psi) = -\frac{1}{2}x - \frac{1}{2}\sigma^h x - \gamma_1 \tag{12}
\]
\[
\pi_i(PH, S_{-i}, \psi) = -\frac{1}{2}x - \gamma_2 \tag{13}
\]
\[
\pi_i(R, S_{-i}, \psi) = -\sigma^h x - 2\gamma_1, \tag{14}
\]
\[
\pi_i(RH, S_{-i}, \psi) = -\gamma_1 - \gamma_2, and \tag{15}
\]
\[
\pi_i(H, S_{-i}, \psi) = -2\gamma_2. \tag{16}
\]

In the above, for the case of \( s_i = P \), in period 1, bank \( i \) borrows either two units or one unit of reserves. Thus, bank \( i \) has at least one unit of reserves at the time of the liquidity shock.
If \( s^p_i = (R, R) \), then

\[
\begin{align*}
\pi_i(P, S_{-i}, \psi) &= -2x, \\
\pi_i(PR, S_{-i}, \psi) &= -x - \gamma_1 \\
\pi_i(PH, S_{-i}, \psi) &= -x - \gamma_2 \\
\pi_i(R, S_{-i}, \psi) &= -2\gamma_1 - \sigma^h x, \\
\pi_i(RH, S_{-i}, \psi) &= -\gamma_1 - \gamma_2, \text{ and} \\
\pi_i(H, S_{-i}, \psi) &= -2\gamma_2.
\end{align*}
\]

(18) \quad (19) \quad (20) \quad (21) \quad (22) \quad (23)

In the above, note that we examine payoffs for strategy profiles with successful recycling. For periphery bank \( i \in I_{\text{per}} \), we similarly define \( s^p_i \in \{P, R, H\} \).

If \( s^p_i = P \), then

\[
\begin{align*}
\pi_i(P, S_{-i}, \psi) &= \frac{1}{2}x - \frac{1}{2}\sigma_i x, \\
\pi_i(R, S_{-i}, \psi) &= -\gamma_1 - \sigma_i x, \text{ and} \\
\pi_i(H, S_{-i}, \psi) &= -\gamma_2.
\end{align*}
\]

(24) \quad (25) \quad (26) \quad (27)

The type regarding liquidity shock for the periphery bank is denoted with \( \sigma_i \in \{\sigma^l, \sigma^h\} \).

If \( s^p_i \neq P \), then

\[
\begin{align*}
\pi_i(P, S_{-i}, \psi) &= -x, \\
\pi_i(R, S_{-i}, \psi) &= -\sigma_i x - \gamma_1 \cdot 1_{\{S_{-i} \in S^a_{-i}\}} - \gamma_2 \cdot 1_{\{S_{-i} \in S^b_{-i}\}}, \text{ and} \\
\pi_i(H, S_{-i}, \psi) &= -\gamma_2 - \sigma_i x \cdot 1_{\{S_{-i} \in S^h_{-i}\}}.
\end{align*}
\]

(28) \quad (29) \quad (30)

\( S^a_{-i} \) denotes a set of strategy profiles except for bank \( i \), which makes at least one payment in period 1. \( S^b_{-i} \) denotes a strategy profile where \( s^p_{i'} = R \) for every \( i' \in I \setminus i \). For our purpose, we focus on \( S_{-i} \in \{S^a_{-i}, S^b_{-i}\} \). For the case with \( s_i = R \) and \( S_{-i} \in S^a_{-i} \), bank \( i \) successfully recycles a reserve in period 2. But when \( S_{-i} \in S^b_{-i} \), no bank makes the payment in period 1, and thus, bank \( i \) recycles only in period 3 after a reserve is brought by the later part of the liquidity shock.

In the next lemma, we show conditions of equilibrium for \( S^1 \) and \( S^2 \).

**Lemma 3.** Equilibrium \( S^1 \) and \( S^2 \)

1. (All-early) Strategy profile \( S^1 \) is an equilibrium if the following condition (A) is satisfied.

\[
\text{Condition (A)} \quad i) \quad \frac{\gamma_1}{x} + \sigma^h \leq \frac{2\gamma_2}{x}.
\]

\( ^{12} \)In Appendix A.5, we present several other types of equilibria.
\[ ii) \frac{1}{2}(1 - \frac{2}{3}\sigma^a) \leq \frac{21}{x}. \]
\[ iii) \frac{1}{2}(1 - \sigma^l) \leq \frac{21}{x}. \]

2. \textbf{(1-recycle)} There exists a strategy profile \( S^2 \) which constitutes an equilibrium if the following \textbf{Condition (B)} is satisfied.

\textbf{Condition (B)}

\[ i) \frac{21}{x} + \sigma^h \leq \frac{21}{x}. \]
\[ ii) 1 - \sigma^l \leq \frac{21}{x}. \]
\[ iii) \frac{21}{x} \leq \frac{1}{2}(1 - \sigma^l). \]
\[ iv) \frac{21}{x} \leq 1 - \sigma^h. \]

\textit{Proof.} See Appendix A.4.

We proceed to observe relevant best responses, which clarify the equilibrium conditions in the above lemma, and serve to elaborate our relevant findings. We observe the best response for each bank \( i \) within \( S_{-i} \subseteq \{S_{-i}^c, S_{-i}^b\} \), where all banks choose to recycle in period 2. We do not specify the route choices for \( S_{-i} \) by supposing that the route choices are made so that recycling is successful whenever possible. We can classify the best response of periphery banks into three types. \((B1)\) refers to the type of \textit{unconditional payment}, where the best response for a periphery bank \( i \) is \( s_i = P \) regardless of the other banks’ choices. \((B2)\) refers to \textit{opportunistic payment}, where the best response for a periphery bank \( i \) is \( s_i = P \) only if \( s_i^p \in \{P, Pr\} \) or \( S_{-i} = S_{-i}^b \). \((B3)\) denotes the \textit{sacrificial payment} type, where the best response for a periphery bank \( i \) is \( s_i = P \) only if \( S_{-i} = S_{-i}^b \).

For core bank \( i \in I_{core} \), we confine ourselves to \( s_i^p \in \{(P, P), (R, R)\} \) with successful recycling since we suppose a periphery bank makes a payment under \( S^2 \). We have five types of best responses for core banks. \((B1,0)\) indicates the best response for a core bank \( i \) such that \( s_i = P \) for \( s_i^p \in \{(P, P), (R, R)\} \). \((B2,1)\) corresponds to \( s_i = P \) if \( S_{-i}^c = (P, P) \), while \( s_i = Pr \) if \( s_i^p = (R, R) \). \((B2,2)\) is the best response such that \( s_i = P \) if \( s_i^p = (P, P) \), while \( s_i = Pr \) if \( s_i^p = (R, R) \). \((B2,3)\) is the best response such that \( s_i = Pr \) if \( s_i^p = (P, P) \), while \( s_i = R \) if \( s_i^p = (R, R) \). \((B2,4)\) corresponds to \( s_i = R \) if \( s_i^p = (P, P) \), while \( s_i = R \) if \( s_i^p = (R, R) \).

\textbf{Observation 1} (Best Response).

\[ i) \text{With sufficiently large } \gamma_2, \text{ the best response for periphery bank } i \text{ with } \sigma_i \in \{\sigma^l, \sigma^h\} \text{ is } \]
\begin{align*}
(B1) & \text{ if } 1 - \sigma_i \leq \frac{21}{x}, \\
(B2) & \text{ if } \frac{1}{2}(1 - \sigma_i) \leq \frac{21}{x} \leq 1 - \sigma_i, \\
(B3) & \text{ if } \frac{21}{x} \leq \frac{1}{2}(1 - \sigma_i).
\end{align*}

\[ ii) \text{We confine ourselves within strategy profiles } S^1 \text{ and } S^2. \text{ Further, for } S^2, \text{ we suppose } s_j = P \text{ for } j \in I_{per}, \text{ which is not a payer to any core bank. Then, with sufficiently large } \gamma_2, \text{ the best response for core bank } i \text{ with } \sigma_i = \sigma^h \text{ is } \]
\begin{align*}
(B_c,0) & \text{ if } 1 \leq \frac{21}{x}, \\
(B_c,1) & \text{ if } 1 - \sigma^h \leq \frac{21}{x} \text{ and } \frac{1}{2}(1 - \frac{1}{2}\sigma^h) \leq \frac{21}{x}, \\
(B_c,2a) & \text{ if } \frac{21}{x} \leq 1 - \sigma^h \text{ and } \frac{1}{2}(1 - \frac{1}{2}\sigma^h) \leq \frac{21}{x}, \\
(B_c,2b) & \text{ if } 1 - \sigma^h \leq \frac{21}{x} \text{ and } \frac{21}{x} \leq \frac{1}{2}(1 - \frac{1}{2}\sigma^h), \\
(B_c,3) & \text{ if } \frac{21}{x} \leq \frac{1}{2}(1 - \frac{1}{2}\sigma^h), \frac{21}{x} \leq 1 - \sigma^h, \text{ and } \frac{1}{2}(1 - \sigma^h) \leq \frac{21}{x}, \\
(B_c,4) & \text{ if } \frac{21}{x} \leq \frac{1}{3}(1 - \sigma^h).\]

15
Figure 3 shows the best responses for periphery banks in a two-dimensional presentation regarding parameters $\gamma_1$ and $\sigma^l, \sigma^h$. We refer to $\gamma_1$ as the relative cost of delay and $\gamma_2$ is referred as the relative cost of large delay. From the first two figures, we observe that higher liquidity shock $\sigma^l, \sigma^h$ is accompanied by banks making earlier payments. It is because within our successful recycling strategy profiles, making a payment with borrowing in period 1 means that a bank will surely receive a reserve before the liquidity shock. Borrowing a reserve in period 1 serves as an insurance for the liquidity shock. Such insurance gets less attractive for smaller relative cost of delay.

![Figure 3: Relevant best responses for periphery banks](image)

Note: Relevant best responses of periphery banks are presented with sufficiently large $\gamma_2$.

Figure 4 similarly shows relevant best responses for core banks in the same two-dimensional representation. For core banks, which have two payments, the insurance for liquidity shock matters only for making one payment. Thus, whether making both payments ($B_c, 0$) constitutes a best response or not does not depend on $\sigma^l, \sigma^h$. It is the best response when the relative cost of delay is sufficiently large $\gamma_1 > 1$. In the case of $S^2$, where $s^P_i = (R, R)$ for core bank $i \in I_{core}$, the logic of the insurance is evident: a payment is made when a larger shock is experienced ($B_{c1}$ and $B_{c2b}$), and not otherwise ($B_{c2a}$, $B_{c3}$, and $B_{c4}$). In the case of $S^1$, where $s^P_i = (P, P)$, the insurance is less attractive for making additional payment since it has less increase in probability of borrowing a reserve. Thus, the marginal increase of the liquidity shock leads to lower marginal increase in the attractiveness of the insurance, as confirmed in the figure wherein the dividing lines $\gamma_1 = \frac{1}{3}(1 - \sigma^h)$ and $\gamma_2 = \frac{1}{3}(1 - \frac{1}{3}\sigma^h x)$ gets steeper. We obtain the latter line by a parallel shift of the former line to the right since making the second additional payment is possible to be accompanied with ex-post “excessive” insurance when the bank borrows two reserves. Note that for fixed $\sigma^h$, the occurrence of ($B_{c2b}$) or ($B_{c2a}$) depends on the level of $\sigma^h$.

Figure 5 summarizes the relationship between our equilibria and relevant best re-
Figure 4: Relevant best responses for core banks
Note: Relevant best responses of core banks are presented with sufficiently large $\gamma_2$.

Figure 5: Equilibrium, Best Response, and Spillover Effect
Note: The figure shows equilibria, best responses (BRs) of periphery banks (peri) and core banks (core), and existence of “spillover effect” (Spillover) for $0 < \frac{\gamma_2}{x} \leq 1$, with $(a) = \frac{1}{2}(1 - \sigma^h)$, $(b) = \frac{1}{3}(1 - \sigma^h)$, $(c) = \frac{1}{2}(1 - \frac{2}{3}\sigma^h)$, $(d) = \frac{1}{2}(1 - \sigma^l)$, $(e) = 1 - \sigma^h$, and $(f) = 1 - \sigma^l$. Specifically, consistent parameter values are: $\sigma^h = 0.45$, $\sigma^l = 0.2$, and $\frac{\gamma_2}{x} = 1.5$. We have $(a) = 0.183$, $(b) = 0.275$, $(c) = 0.35$, $(d) = 0.4$, $(e) = 0.55$, and $(f) = 0.8$. 
responses in a one-dimensional figure regarding parameter $\frac{\sigma}{x}$ under specific values for $\sigma^h(<\frac{3}{4})$, $\sigma^l$, and $\gamma_2$. The figure also shows an additional row, “Spillover”, which we explain later. Observe that the dividing line between whether $S^1$ or $S^2$ constitutes an equilibrium is determined by the line denoting best responses for periphery banks with low shocks in terms of (B2) and (B3). The corresponding dividing line for periphery banks with high shocks is placed on the left of the previous line. This implies that if there were no periphery banks with low shock, the dividing line for equilibria would shift to the left. The row “Spillover” shows how far the dividing line would shift. The dark area shows an equilibrium for $S^2$, where a periphery bank is the sole payer in period 1. Here, when a periphery bank receives a payment in period 1, the receiver’s identity regarding liquidity shock needs to be of the low shock type $\sigma^l$. This is because the best response for periphery banks with high shock in the area is $(B2)$, thus letting the bank make a payment in period 1 if its payer makes a payment. If, departing from our core-periphery structure, a periphery bank makes a payment in period 1 and all the consecutive receivers were all periphery banks with high shocks, all the consecutive banks would also make a payment in period 1. However, in our setting, the existence of a periphery bank with a low shock among the chain of banks with high shocks prevents such a potential “domino” effect of making payments. Conversely, we observe a “spillover” effect that enhances recycling reserves. Further, the “spillover” effect is welfare improving when $S^2$ is an efficient strategy profile, as we will confirm soon. The following proposition clarifies our “spillover” effect.

**Proposition 2. Spillover effect**

Under Condition (B) and when $\frac{1}{2}(1 - \sigma^h) \leq \gamma_2$, there exists a strategy profile within $S^2$ that constitutes an equilibrium with $s_j = P$ where $j \in I_{per}$ and $j + 1 \in I_{per}$ if and only if $\sigma_{j+1} = \sigma^l$.

**Proof.**

When $\sigma_{j+1} = \sigma^l$, Lemma 3 ensures that there exists $S^2$, which constitutes an equilibrium. Concretely, there is an equilibrium within $S^2$, where $\sigma_j = \sigma_{j-1} = \sigma^l$ with $j, j - 1 \in I_{per}$. Suppose $\sigma_{j+1} = \sigma^h$. Then, the best response of $j + 1$ is $s_{j+1} = P$, which contradicts that there is an equilibrium within $S^2$. 

### 3.3 Regimes

We focus on two types of parameter values for our analyses, where each is termed as a *Regime*. Each *Regime* is characterized with each strategy profile $S^1$ and $S^2$ such that each strategy profile attains the first-best and constitutes an equilibrium at the same time. The notion of *Regime* supposes that fee level $x$ is to be set by the CB so as to maximize welfare, given all the other parameter values are constant.

**Definition 6. Regimes**

Under Assumptions 1 and 2,

i) *Liquidity Non-Precious Regime* refers to parameter values that satisfy Conditions (A) and $\frac{N + K - 1}{N}(1 - \gamma_2) < \tilde{\sigma} < \frac{N - K}{N}$, where strategy profile $S^1$ is an equilibrium and also attains first-best.
ii) **Liquidity Precious Regime** refers to parameter values that satisfy Conditions (B) and 
\[ \frac{2K}{N} < \bar{\sigma} \leq \frac{N+K-1}{N}(1 - \frac{\gamma_1}{r}), \]
where there exists a strategy profile \( S^2 \) that is an equilibrium and also attains first-best.

The former part is referred to as the **Liquidity Non-Precious Regime** since the relevant conditions are more likely to be satisfied with larger value of \( \frac{2r}{\gamma_1} \), which mentions relatively smaller value of the liquidity cost \( r \) compared to the value of \( \gamma_1 \). The latter part is oppositely referred to as the **Liquidity Precious Regime** since the relevant conditions are more likely to be satisfied with relatively larger value of the liquidity cost.

The next lemma ensures that the **Liquidity Non-Precious Regime** is always attainable with a sufficiently low fee level, and the **Liquidity Precious Regime** is attainable when the marginal delay cost \( \frac{2r}{\gamma_1} \) is sufficiently high.

**Lemma 4. Attainability of regimes under fee control**

i) When \( S^1 \) attains first-best, \( S^1 \) constitutes an equilibrium with sufficiently low fee level \( x \).

ii) When \( S^2 \) attains first-best, there exists a strategy profile \( S^2 \) that constitutes an equilibrium with an appropriate fee level for a sufficiently large \( \gamma_2 \) such that 
\[ \max(2, \frac{1-\gamma_1}{1-\sigma^h}, 1 + \frac{2\sigma^h}{1-\sigma^h}, \frac{1}{1-\sigma^h}) < \frac{\gamma_1}{\gamma_2}. \]

**Proof.**

i) is clear from Condition (A). For ii), condition (B) is rewritten as: (B1) \( x < \frac{\gamma_1}{\sigma^h} \), (B2) \( x < \frac{\gamma_1}{1-\sigma^h} \), (B3) \( \frac{\gamma_1}{1-\sigma^h} < x \), and (B4) \( \frac{\gamma_1}{1-\sigma^h} < x \). The condition reduces to 
\[ \max(\frac{2\gamma_1}{1-\sigma^h}, \frac{\gamma_1}{1-\sigma^h}) < x < \min(\frac{2\gamma_1}{\sigma^h}, \frac{2\gamma_1}{1-\sigma^h}) \].
This reduces to 
\[ \max(2, \frac{1-\gamma_1}{1-\sigma^h}, 1 + \frac{2\sigma^h}{1-\sigma^h}, \frac{1}{1-\sigma^h}) < \frac{\gamma_1}{\gamma_2}. \]
The condition is consistent with Assumption 2: \( \gamma_1 + \sigma^hr < \gamma_2 \), and it is also consistent with the condition for \( S^2 \) being the first-best: 
\[ \frac{N+K-1}{N}(1 - \frac{\gamma_1}{r}) < \bar{\sigma} < \frac{N-K}{N}. \]
\[ \square \]

We analyze the effect of LSM under either of the two regimes, which amounts to examining a rather short-term effect of the introduction of the LSM supposing either of the two regimes had been realized with appropriate fee control. We do not explicitly examine how fee \( x \) is optimally set after the LSM is introduced. The purpose of this short-term analysis is to highlight the possible negative effect of the LSM.

## 4 Introduction of the LSM

Under a core-periphery network \( \psi \in \Psi_{(K,W)} \), we examine the effect of introducing the LSM that provides a partial netting service such that cycles with less than or equal to \( K \) number of payments are possibly offset. Observe that in this setting it is possible to offset only payments among core banks. We suppose that the LSM works only in period 1 since it would have no role in the other periods under \( S^1 \) and \( S^2 \).

We add a new **queuing** period just before period 1, where each bank has an option to put each payment into the central **queue**. Among those queued payments, if there is a cycle with less than or equal to \( K \) number of payments, they are all offset with no borrowing fee. If a payment is queued but not offset, it is released to the original bank at the end of the queuing period. From period 1 onwards, we maintain all the previous
settings. Even with the option of queuing, Assumption 2 ensures that holding a reserve is worse than recycling in period 2. Accordingly, we focus on strategy profiles where no reserve is held in period 2. Since it is always weakly better to utilize the option of queuing, we maintain our notations for their actions. Thus, in our regimes with the LSM, the set of actions for both core banks and periphery banks is essentially the same. We maintain the same notation for actions of periphery banks. For core banks, we omit irrelevant notations since payments among core banks are always offset. The set of actions for both core banks and periphery banks is denoted as \(\{P, PR, PH\}\), where the payment to the core bank is fixed as \(P\). Note that for periphery banks, the queuing option has no effect since each of their payments is included in cycles larger than \(K\) payments.

We focus on three types of strategy profiles, each of which is shown to constitute an equilibrium for certain parameter values.

1. (All-early) \(s_i = P\) for every \(i \in I\).
2. \((d(\psi))-\text{recycle}\) \(s_i = P, R\) for every \(i \in I_{\text{core}}\), and there exists just one periphery bank \(j \in I_{\text{per}}\) with \(s_j = P\) for each core-separated cycle, and for any other periphery bank \(j' \in I_{\text{per}}\), \(s_{j'} = R\).
3. (Limited recycle) Separate \(I\) into \(I = \cup_{n=1,2,\ldots}d(\psi)I^n\) so that each constitutes a separated cycle of payments if payments among core banks are to be offset. Denote \(I^L\) when it satisfies \(I_{\text{per,L}} \subset I^L\). With these notations, \(s_i = P, R\) for every \(i \in I_{\text{core}} \cap I^L\), and \(s_i = P\) for every \(i' \in I_{\text{core}} / I^L\). There exists one periphery bank \(j \in I_{\text{per}}\) with \(s_j = P\) for \(j \in I_{\text{per}} \cap I^L\), while for the other periphery banks \(j' \in I_{\text{per}} \cap I^L\), \(s_{j'} = R\). For every periphery bank \(j'' \in I_{\text{per}}\) that does not belong to \(I^L\), \(s_{j''} = P\).

Denote \(S_{\text{net}}^1, S_{\text{net}}^2, S_{\text{net}}^3\) as each representative strategy profile. Note that in \(S_{\text{net}}^2\), the number of banks that make a payment in period 1 is \(d(\psi)\). In order to perceive \(S_{\text{net}}^3\), consider core-periphery networks within \(\Psi(3,4)\) shown in Figure 6, where \(I_{\text{per,L}}\) shows the set of periphery banks with low liquidity shock. Figure 7 shows each associated payment network supposing payments among core banks are offset. On the left-hand side of the figure, \(I^L = I\) since all banks belong to one connected payment network. In that case, \(S_{\text{net}}^3\) states that any one periphery bank takes \(P\), while all the other periphery banks take \(R\). For the network shown in middle of the figure, \(I^L\) refers to banks that belong to a cycle of payments including core banks \(b\) and \(c\). It is stated that one periphery bank \(j \in I^L\) takes \(P\) while the other \(j' \in I^L\) takes \(R\), and the core banks \(b\) and \(c\) take \(P, R\). For the other banks that belong to another cycle of payments including \(a\), all the periphery banks take \(P\), and the core bank \(a\) takes \(P\). For the network shown on the right-hand side of the figure, \(S_{\text{net}}^3\) mandates recycling with one early payment within \(I^L = I_{\text{per,L}} \cup b\) while for \(I \setminus I^L\), all the banks choose early payments.

For the relevant social welfare, under Assumption 3 we have:
\[
W(S_{\text{net}}^1) = -Nr, \\
W(S_{\text{net}}^2) = -(d(\psi) + \sigma N)r - (N - d(\psi))\gamma_1, \text{ and} \\
W(S_{\text{net}}^3) = -(1 + \sigma N^l + \min(N^h, n^h(\psi) + \sigma^h N^h))r - (N - 1 - n^h(\psi))\gamma_1,
\]
where \(N^l = |I_{\text{per,L}}|\), \(N^h = N - N^l\), and \(n^h(\psi) = N^h - |I^L \setminus I_{\text{per,L}}|\).

Note that \(n^h(\psi)\) is later shown to be interpreted as the number of banks with high shock that had enjoyed the spillover effect under the RTGS without the LSM but no more after the LSM is introduced.
Figure 6: Examples for core-periphery networks $\Psi_{(3,4)}$: before offset

Figure 7: Examples for core-periphery networks $\Psi_{(3,4)}$: after offset

Note: For the original networks shown in Figure 6, this figure shows only the payments which cannot be offset by the LSM.
We first examine payoffs of core bank \( i \) under \( S_{-i} \) such that it surely receives payment until period 2. If \( s^p_i = (P, P) \), then

\[
\pi_i(P, S_{-i}, \psi) = -\frac{1}{2}x - \frac{1}{2}\sigma^h x, \tag{31}
\]

\[
\pi_i(PR, S_{-i}, \psi) = -\sigma^h x - \gamma_1, \text{ and} \tag{32}
\]

\[
\pi_i(PH, S_{-i}, \psi) = -\gamma_2. \tag{33}
\]

If \( s^p_i = (P, R) \), and \( j \in I_{per,L} \) with \( s_j = P \), then

\[
\pi_i(P, S_{-i}, \psi) = -x, \tag{35}
\]

\[
\pi_i(PR, S_{-i}, \psi) = -\sigma^h x - \gamma_1, \text{ and} \tag{36}
\]

\[
\pi_i(PH, S_{-i}, \psi) = -\gamma_2. \tag{37}
\]

The payoffs for the periphery banks are all the same as before. We observe that payoffs for core banks are effectively the same as those for periphery banks with high shock. The next lemma confirms that each type of strategy profile constitutes an equilibrium.

**Lemma 5. Netting equilibrium**

1. (All-early) \( S^1_{net} \) is an equilibrium if \( \frac{\gamma_1}{x} + \sigma^h < \frac{\gamma_2}{x}, \frac{1}{2}(1 - \sigma^l) < \frac{\gamma_1}{x} \).
2. (d(\psi)-recycle) There exists an equilibrium within \( S^2_{net} \) if \( \frac{\gamma_1}{x} + \sigma^h < \frac{\gamma_2}{x}, \frac{\gamma_1}{x} < \frac{1}{2}(1 - \sigma^h) \), and \( 1 - \sigma^l < \frac{\gamma_2}{x} \).
3. (Limited recycle) There is an equilibrium within \( S^3_{net} \) if \( \frac{\gamma_1}{x} + \sigma^h < \frac{\gamma_2}{x}, \frac{1}{2}(1 - \sigma^h) < \frac{\gamma_1}{x} < \frac{1}{2}(1 - \sigma^l), \frac{\gamma_1}{x} < 1 - \sigma^h \), and \( 1 - \sigma^l < \frac{\gamma_2}{x} \).

**Proof.**

As the proof is almost the same as that for Lemma 3, it is omitted.

Note that the difference in equilibrium conditions between \( S^2_{net} \) and \( S^3_{net} \) comes from the incentive of periphery banks with high shock. Observe that when the spillover effect existed, introduction of the LSM let \( S^3_{net} \) type of equilibrium emerge. We proceed to formally state effects of the LSM regarding the three types of strategy profiles.

**Proposition 3. Effects of LSM 1**

Under the Liquidity Non-Precious Regime,

a) \( S^1_{net} \) is an equilibrium under the RTGS with the LSM, but \( S^2_{net} \) and \( S^3_{net} \) are not.

b) \( W(S^1) < W(S^1_{net}) \).

**Proof.**

The proof for part a) is evident by comparing the two sets of conditions. For part b),

\[
W(S^1_{net}) - W(S^1) = -Nr - (-((N+K)r)) = Kr > 0. \tag{38}
\]

The above proposition states that under the Liquidity Non-Precious Regime, the introduction and use of the LSM largely improve welfare.
Proposition 4. Effects of LSM 2.1

Under the Liquidity Precious Regime with \( \frac{n}{x} < \frac{1}{2}(1-\sigma^h) \),
a) \( S_{net}^2 \) is an equilibrium for the RTGS with the LSM.
b) \( W(S^2) \leq W(S_{net}^2) \) if and only if \( \frac{d(\psi)-1}{K+d(\psi)-1} \leq \frac{n}{r} \).

Proof.
The proof of part a) is evident by comparing the sets of conditions. For part b),
\[
W(S_{net}^2) \geq W(S^2) \iff -(d(\psi) + \hat{\sigma}N)r - (N - d(\psi))\gamma_1 \geq -(1 + \sigma^lN^l + \sigma^hN^h)r - (N + K - 1)\gamma_1 \iff (d(\psi) - 1)r \leq (K + d(\psi) - 1)\gamma_1.\]

This proposition states that under the Liquidity Precious Regime, introducing the LSM does not always improve welfare. Further, welfare is more likely to decrease when the payment network is less dense. Density matters since activation of netting only allows recycling among the remaining payments, and a less dense network means that the remaining payments form a larger number of core-separated cycles.

Proposition 5. Effects of LSM 2.2

Under the Liquidity Precious Regime with \( \frac{n}{x} \geq \frac{1}{2}(1-\sigma^h) \),
a) \( S_{net}^3 \) is an equilibrium for the RTGS with the LSM.
b) \( W(S^2) \leq W(S_{net}^3) \) if and only if \( \min((1 - \sigma^h)N^h, n^h(\psi)) \leq \frac{n}{r} \).

Proof.
The proof of part a) is straight. For part b),
\[
W(S_{net}^3) \geq W(S^2) \iff -(1 + \sigma^lN^l + \min(N^h, n^h(\psi) + \sigma^hN^h))r - (N - 1 - n^h(\psi))\gamma_1 \geq -(1 + \sigma^lN^l + \sigma^hN^h)r - (N + K - 1)\gamma_1 \iff \min((1 - \sigma^h)N^h, n^h(\psi))r \leq (K + n^h(\psi))\gamma_1.\]

In the part b) of the proposition, we observe that the condition is less likely to be satisfied for larger \( n^h(\psi) \) as long as the value is below \((1 - \sigma^h)N^h\) since it indicates larger liquidity cost for \( S_{net}^3 \). However, when \( n^h(\psi) \) is larger than \((1 - \sigma^h)N^h\), even larger \( n^h(\psi) \) no more indicates larger liquidity cost for \( S_{net}^3 \) and thus the conditions is more likely to be satisfied.

4.1 Remarks

Information structure
We supposed that banks surely knows which other banks will make payments to them. In order to be more realistic, we could extend our analysis by introducing uncertainty at such points. When their locations on the underlying network structure are sufficiently stable, banks could learn the likely identities of their neighbors through their daily experiences. In such cases, the essence of our analysis will still hold.

Intraday lending: fee and collateral
In real-world payment systems, it is typical to require collateral beforehand for intraday lending. Thus, whether collateral should be imposed and in what form is another policy tool in addition to controlling the level of the lending fee. The difference is that collateral is provided beforehand the lending, and so, banks will provide the maximum possible reserve shortage. In our model, suppose that the sole policy tool is to take collateral and the level of collateral is also flexibly controlled (from low cost to high cost). Then, there is
almost no incentive for banks to delay and recycle reserves because, in any case, they need to provide the collateral beforehand under positive probability of liquidity shock. Thus, even when recycling reserves is desirable in terms of social welfare, it cannot be attained by the solely taking collateral. The situation could be interpreted such that social welfare is impaired by fully insuring the credit cost of the CB with possibly excessive collateral cost for the part of the banks. In this case, in order to improve social welfare, fee control or some other policies that promote recycling need be utilized. In contrast, when no delay of payments is desirable, the CB can attain its goal by taking collateral at a sufficiently small cost to the banks. This can be socially desirable and also help insure the credit cost of the CB.

5 Policy implications

The policy implications of this paper are twofold regarding the extent of the “preciousness” of liquidity, and how the payment network tends to be formed. When liquidity is sufficiently not precious in the sense that the unit cost of liquidity is relatively lower than the unit cost of payment delay, reserves should be lent at a sufficiently small fee so as to encourage earlier payment. In this situation, introducing the LSM tends to be socially better through reducing liquidity cost. In contrast, when liquidity is precious enough, reserves should be lent at a sufficiently high fee so as to promote efficient recycling of reserves. But once reserves are efficiently recycled, the introduction of the LSM possibly has a negative effect as it dismisses efficient recycling. The negative effect tends to be large when the underlying network is connected less tightly with respect to banks identified as periphery banks within the core-periphery network structure.

Controlling the fee level would be easy when liquidity is not precious, but it would not be necessarily easy for the opposite case. In order to promote efficient recycling, the fee level should not be too low or too high. Too high a level would cause serious payment delays. In normal times, when the underlying payment network structure is stable, it would be possible to find by trial. However, in unstable situations controlling the fee level could entail a risk. In such a case, namely sufficient uncertainty, lowering the fee to a low enough level and introducing the LSM may be a prudent policy.

Our point is that the utilization of the LSM should be examined together with the policies that promote recycling of reserves. In this respect, controlling the fee level would not be the sole method to promote such recycling. With sufficient information about the underlying payment network, we could classify banks into specific groups, such as their locations in the network or relevant identities. Then, we could divide the morning time into several time zones, and let each group make payments in a specific time zone. This exogenous ordering of banks would also help promote efficient recycling.\footnote{In real world payment systems, it is typical to separate a day into several time zones so that each type of payment is made in each time zone. Sufficient correlation of payment types with the identities of the banks also partly helps promote efficient recycling.} \footnote{Regarding methods to promote efficient recycling, slicing payments into smaller units could also be effective in combination with appropriate ordering of the payments. Hayakawa (2014) and Hayakawa (2016a) provided relevant analyses in the graph-theoretic framework.}

\[\text{24}\]
6 Concluding Remarks

This paper analyzed the effect of introducing the LSM in a strategic context. We showed that the LSM could have a negative effect when it serves partial netting. The negative effect can arise when the LSM separates the underlying payment network into disconnected subnetworks. The disconnection dismisses efficient recycling of reserves, and further, it also dismisses the spillover effect that sustains efficient recycling in each subnetwork.

The fact that the LSM could have a negative effect in dismissing potentially efficient recycling of reserves suggests the need for an effective policy mix, one which combines tools to reduce liquidity (e.g., the LSM) and tools to promote recycling reserves. The former tools would perform better when prompt settlements are more important, while the latter desirable when liquidity is more precious.

There are several limitations of this study. We analyzed a stylized model of an LSM in order to shed a light on its potential negative effect. In order to argue the effect of the LSM quantitatively, the model will need to be elaborated in more detailed procedures, namely with respect to the timing of releasing queued payments, or the frequency of utilizing the LSM. We examined a stylized class of core-periphery structures, and argued the effect of the LSM regarding the topology of density. It will be useful to examine density for real payment networks in order to argue the extent of the potential negative effect of LSMs. These tasks remain for future researches.

Beyond the issue of the payment system, our analysis also provides an insight into network externality. In our formulated game structure, each agent’s payoff is primary relevant to the payoffs of its direct counterparty. However, the chain of those local relationships could mean that each payoff is relevant to the others within the connected network. In such situations, a replacement of an agent or some shock in the middle of the chain could spillover throughout the chain. This observation could be useful in examining other topics relevant to networks formed with bilateral strategic linkages.
A Appendix

A.1 Proof of Lemma 1

We prove the lemma in a constructive manner. For a given $\psi \in \Psi$, within 1-recycle strategy profiles, we determine strategies of core banks in the following manner. Suppose $j_1 \in I_{\text{per},1}$ is the sole periphery bank that makes a payment in period 1. The reserve is recycled in period 2. Follow the recycled reserve until it reaches a core bank that we denote as $i_1 \in I_{\text{core}}$. Let the bank’s strategy be $s_{i_1} = R_{cc}$. Then, it recycles the reserve to a periphery bank. Follow the recycled reserve until it reaches the next core bank $i'$. Let this bank’s strategy be $s_{i'} = R_{cc}$ if its payment is not to a periphery bank within the group of $I_{\text{per},1}$. When this is actually the case, the core bank recycles the reserve to a periphery bank. Continue to follow the recycled reserve, and let the strategies of core banks on the route be $R_{cc}$ until we reach a core bank that has to make a payment to some $j' \in I_{\text{per},1}$. In such a case, let the core bank’s strategy be $R_{cp}$. Then, the core bank makes a payment to the core bank $i_1$. Since $s_{i_1} = R_{cc}$, it recycles the reserve to a core bank. Follow the recycled reserve until it reaches a core bank whose strategy is not determined yet. When it does not reach such a core bank, terminate the procedure. When it reaches such a core bank, denote the core bank as $i_2$. Now let $s_{i_2} = R_{cp}$. Then, the core bank $i_2$ recycles to a periphery bank. We return to the beginning of the procedure until it terminates.

For a given $\psi$, denote a cycle of payments as a periphery cycle when it is a cycle without a payment between core banks. In the above procedure, each periphery cycle is settled one by one in such a way that it proceeds along with the cycle with payments among the core banks.

A.2 Proof Lemma 2

Since there are $|I| = N$ banks with $K$ core banks, the total number of payments is $N + K$. For any network structure within $\Psi_{(K; W)}$, we have at most $K + 1$ number of non-punctured cycles. When all the banks make payments in period 1, consider the worst scenario in which the borrowing amounts are maximized. Observe that in those situations, at most one bank within each cycle does not borrow a reserve.\(^\text{15}\)

Under the all-early strategy profiles with a representative $S^1$, every bank makes payments in period 1. From the above observation, within the banks that face a higher liquidity shock, the maximum possible number of banks that do not borrow in period 1 is $K$, while it is 1 within banks facing a lower liquidity shock. At the end of period 2, the number of banks facing a higher and a lower liquidity shock is $N^h\sigma^h$ and $N^l\sigma^l$ respectively. In the worst case scenario, we suppose that banks who do not hold reserves are the first to have a liquidity shock. The first part of Assumption 3 states that $2K < N^h\sigma^h + N^l\sigma^l$, which derives $K < N^h\sigma^h$. Thus, we have $\max(M(S, \psi)) = N + K$. Since there is no delay cost, we have $W(S^1, \psi) = -(N + K)r$.

Under the successful 1-recycle strategy profiles with a representative $S^2$, only one periphery bank makes a payment in period 1. All payments are settled successfully before

\(^{15}\)The detailed analysis is shown in Hayakawa (2016a). It explicitly defined the relevant liquidity problem in a general form, namely as a graph problem, and demonstrated in various ways how cycles affect the problem.
the liquidity shock at the end of period 2. In the worst scenario, the liquidity shock increases the borrowing amounts by $N^h \sigma^h + N^l \sigma^l$. Thus, the total borrowing amount is $1 + N^h \sigma^h + N^l \sigma^l$. Since total delay costs are $(N + K - 1) \gamma_1$, we have $W(S^2 \psi) = -((1 + \sigma^l N^l + \sigma^h N^h)r + (N + K - 1) \gamma_1)$.

A.3 Proof of Proposition 1

From Lemma 6, it is enough to compare $W(S^1)$ and $W(S^2)$. $W(S^1) \leq W(S^2)$ if $(N + K)r - (1 + N\bar{\sigma})r - (N + K - 1) \gamma_1 \geq 0$, or, $(N + K - 1 - N\bar{\sigma})r \geq (N + K - 1) \gamma_1$. We have, $\bar{\sigma} \leq \frac{N + K - 1}{N - 1}(1 - \frac{n}{r})$. Combining this with Assumption 3, we complete our proof.

Lemma 6.

Under Assumption 1, 2, and 3, either $S^1$ or $S^2$ attains first-best.

Proof.

Suppose some number of payments $0 \leq n < N + K$ is made in period 1, and all the other payments are not recycled in period 2 but held instead. Now, we change the strategy profile so that one payment is recycled instead of being held in period 2. This improves welfare by at least $\min(\gamma_2, \gamma_2 + \sigma^h r) - \max(\gamma_1, \gamma_1 + \sigma^h r) = \gamma_2 - \gamma_1 - \sigma^h$, which is positive by Assumption 2. When we continue to replace payments to be recycled instead of holding them in period 2, each replacement improves the welfare in a similar manner. Thus, we examine strategy profiles where there is no holding of a reserve in period 2.

Then, compare two strategy profiles where only choices of routes are different while the number of payments made in period 1 remains the same. Suppose the recycling is successful for one strategy profile and not for the other. The latter always attains less welfare since the total delay cost increases by at least $\gamma_2 - \gamma_1$, which is supposed to be positive. Thus, it suffices to focus on strategy profiles where recycling is always successful when at least one payment is made in period 1.

In terms of social welfare, the remaining candidate strategy profiles essentially differ in the number of payments made in period 1. Consider a strategy profile where every bank does not make any payment in period 1. The total cost is $N\bar{\sigma}r + (N + K) \gamma_2$ since the liquidity resulting from the liquidity shock is successfully recycled in period 3. Then, we compare it with $S^2$ whose total cost is $(1 + N\bar{\sigma})r + (N + K - 1) \gamma_1$. Assumption 1 ensures that $S^2$ attains larger welfare.

Now, replace $S^2$ such that $k > 1$ payments are made in period 1 followed by successful recycling in period 2. This decreases welfare by $r - \gamma_1$ as long as it generates additional liquidity cost in the worst case scenario. Assumption 1 ensures that $r - \gamma_1 > 0$. Note that an additional liquidity cost emerges as long as the liquidity shock is sufficiently small. Suppose this additional liquidity cost emerges until $k$ but not in $k + 1$. Then, social welfare increases by $\gamma_1$. When we increase the number one by one (as $k + 2, k + 3, \ldots$), the welfare always increases by the same amount. Since this procedure reaches $S^1$, either $S^1$ or $S^2$ attains the first best. \hfill \Box

A.4 Proof of Lemma 3

Case for $S^1$

a). Periphery bank $i$ has no incentive to deviate from $s_i = P$ if
\[
\pi_i(P, S_{-i}, \psi) \geq \pi_i(R, S_{-i}, \psi) \iff \frac{1}{2} x + \frac{1}{2} \sigma_i x \leq \gamma_1 + \sigma_i x, \text{ and }
\]
\[
\pi_i(P, S_{-i}, \psi) \geq \pi_i(H, S_{-i}, \psi) \iff \frac{1}{2} x + \frac{1}{2} \sigma_i x \leq \gamma_2.
\]
Thus, we have conditions \(\frac{1}{2}(1 - \sigma') \leq \frac{2}{x} \) and \(\frac{1}{2}(1 + \sigma^h) \leq \frac{2}{x} \).

b. Core bank \(i\) has no incentive to deviate from \(s_i = P\) if

1. \(\pi_i(P, S_{-i}, \psi) \geq \pi_i(PR, S_{-i}, \psi) \iff \frac{3}{6} x + \frac{1}{3} \sigma^h x \leq \frac{1}{3} x + \frac{2}{3} \sigma^h x + \gamma_1, \)
2. \(\pi_i(P, S_{-i}, \psi) \geq \pi_i(PH, S_{-i}, \psi) \iff \frac{5}{6} x + \frac{1}{3} \sigma^h x \leq \frac{1}{3} x + \gamma_2, \)
3. \(\pi_i(P, S_{-i}, \psi) \geq \pi_i(R, S_{-i}, \psi) \iff \frac{5}{6} x + \frac{1}{3} \sigma^h x \leq \sigma^h x + \gamma_2, \)
4. \(\pi_i(P, S_{-i}, \psi) \geq \pi_i(RH, S_{-i}, \psi) \iff \frac{5}{6} x + \frac{1}{3} \sigma^h x \leq \gamma_1 + \gamma_2, \)
and
5. \(\pi_i(P, S_{-i}, \psi) \geq \pi_i(H, S_{-i}, \psi) \iff \frac{5}{6} x + \frac{1}{3} \sigma^h x \leq 2 \gamma_2. \)

Since we suppose \(\gamma_1 < \gamma_2\), Condition (5) is eliminated by comparing with Condition (4).

When \(\gamma_1 + \sigma^h x \leq \gamma_2\), we can eliminate Condition (4) by comparing with Condition (3).

Similarly, Condition (2) is eliminated by comparing with Condition (1) since \(2 \frac{2}{3} \sigma^h x + \gamma_1 < \sigma^h x + \gamma_1 \leq \gamma_2\).

Now, Condition (3) is satisfied when \(\gamma_1 + \sigma^h x \leq \gamma_2\) by comparing with Condition (1). This is satisfied when \(\frac{1}{2}(1 - \sigma') \leq \frac{2}{x} \) since \(\frac{1}{3}(1 - \sigma') < \frac{1}{2}(1 - \sigma') \).

The remaining condition, Condition (1), is satisfied with the additional condition \(\frac{1}{2} - \frac{1}{3} \sigma^h x \leq \frac{2}{x} \).

From a) and b), we have four conditions: \(\frac{1}{2}(1 + \sigma^h) \leq \frac{2}{x} \), \(\frac{1}{2} - \frac{1}{3} \sigma^h \leq \frac{2}{x} \), and \(\frac{1}{2} - \frac{1}{3} \sigma^h \leq \frac{2}{x} \).

By combining the second and the last condition, the first condition is satisfied as \(\frac{1}{2}(1 + \sigma^h) < 1 - \frac{1}{2} \sigma^h + \sigma^h \leq \frac{2}{x} \).

Thus, \(S^1\) is an equilibrium if \(\frac{2}{x} + \sigma^h \leq \frac{2}{x} \), \(\frac{1}{2} - \sigma^h \leq \frac{2}{x} \), and \(\frac{1}{2} - \frac{1}{3} \sigma^h \leq \frac{2}{x} \).

Case for \(S^2\)

Within \(S^2\), we focus on a strategy profile where \(s_i = P\) for periphery bank \(i \in I_{per}\) with \(\sigma_i\). Also, the bank that receives a payment from bank \(i\) is a periphery bank \(j \in I_{per}\) with \(\sigma_j\).

a). For periphery bank \(i\) with \(s_i = R\), there is no incentive to deviate if

1. \(\pi_i(R, S_{-i}, \psi) \geq \pi_i(P, S_{-i}, \psi) \) with \(s_i = P \iff \sigma_i x + \gamma_1 \leq \frac{1}{2} x + \frac{1}{2} \sigma_i x, \)
2. \(\pi_i(R, S_{-i}, \psi) \geq \pi_i(P, S_{-i}, \psi) \) with \(s_i = P \iff \sigma_i x + \gamma_1 \leq x, \) and
3. \(\pi_i(R, S_{-i}, \psi) \geq \pi_i(H, S_{-i}, \psi) \) \(\iff \sigma_i x + \gamma_1 \leq \gamma_2. \)

Note that in the case of (3), the bank cannot recycle a reserve in period 2 when it deviates to take \(H\) since it is the sole bank that can make a payment in period 1. Thus, it faces a liquidity shock when it deviates. Condition (1) is satisfied with \(\frac{2}{x} \leq \frac{1}{2}(1 - \sigma') \). Condition (2) is satisfied with \(\frac{2}{x} \leq \frac{2}{x} \) and \(\frac{1}{2} - \frac{1}{3} \sigma^h \leq \frac{2}{x} \).

b). For periphery bank \(i\) with \(s_i = P\), there is no incentive to deviate if

1. \(\pi_i(P, S_{-i}, \psi) \geq \pi_i(R, S_{-i}, \psi) \iff x \leq \sigma_i x + \gamma_2, \) and
2. \(\pi_i(P, S_{-i}, \psi) \geq \pi_i(P, S_{-i}, \psi) \) \(\iff x \leq \sigma_i x + \gamma_2. \)

Both conditions are satisfied with \(\frac{1}{2} \leq \frac{2}{x} \) for bank \(i\) with \(\sigma_i = \sigma^l\).

c). Core bank \(i\) has no incentive to deviate from \(s_i = R\) when \(s_i^R = (R, R)\) if

1. \(\pi_i(R, S_{-i}, \psi) \geq \pi_i(P, S_{-i}, \psi) \iff \sigma^h x + 2 \gamma_1 \leq 2 x, \)
2. \(\pi_i(R, S_{-i}, \psi) \geq \pi_i(PR, S_{-i}, \psi) \iff \sigma^h x + 2 \gamma_1 \leq x + \gamma_1, \)
3. \(\pi_i(R, S_{-i}, \psi) \geq \pi_i(PH, S_{-i}, \psi) \iff \sigma^h x + 2 \gamma_1 \leq x + \gamma_2, \)
4. \(\pi_i(R, S_{-i}, \psi) \geq \pi_i(RH, S_{-i}, \psi) \iff \sigma^h x + 2 \gamma_1 \leq \gamma_1 + \gamma_2, \) and
5. \(\pi_i(R, S_{-i}, \psi) \geq \pi_i(H, S_{-i}, \psi) \iff \sigma^h x + 2 \gamma_1 \leq 2 \gamma_2. \)
Conditions (2) and (3) are satisfied with \( \frac{\gamma_1}{x} \leq 1 - \sigma^h \). Conditions (4) and (5) are satisfied with \( \frac{\gamma_1 + \sigma^h}{x} \leq \frac{\gamma_2}{x} \). Condition (1) is satisfied with \( 2 \frac{\gamma_1 + \sigma^h}{x} \leq 2 \frac{\gamma_2}{x} \). Note that \( \frac{\gamma_1}{x} + \sigma^h \leq \frac{\gamma_2}{x} \) is sufficient for Condition (1) since \( \sigma^h \leq \frac{\gamma_2}{x} - \frac{\gamma_1}{x} < 2 \left( \frac{\gamma_2}{x} - \frac{\gamma_1}{x} \right) \).

Thus, there is an equilibrium within \( S_2 \) if \( \frac{\gamma_1}{x} \leq \frac{1}{2} \left( 1 - \sigma^l \right) \), \( \frac{\gamma_1}{x} \leq 1 - \sigma^h \), \( \frac{\gamma_1}{x} + \sigma^h \leq \frac{\gamma_2}{x} \), and \( 1 - \sigma^l \leq \frac{\gamma_2}{x} \).

**A.5 The other type of equilibrium under RTGS without LSM**

Under an RTGS without the LSM, the 1-recycle equilibrium coexists with other types of equilibria, namely inefficient ones. We introduce the relevant terminologies. For a core-periphery network, the separation procedure is defined as follows. For payments which regard to core banks, match one payment to be received and one payment to be made. We have a pair of matches. Separate each core bank into two banks, each having matched payments. When we separate all the core banks into two, we have independent cycles of payments. Every separation and matching derives a set of independent cycles.

For a separation, we denote a set of separated cycles as \( \mathcal{C} \), and a set of core (periphery) banks included in a cycle \( c \in \mathcal{C} \) as \( \mathcal{I}_{\text{core}}(c) \), \( \mathcal{I}_{\text{per}}(c) \).

**Lemma 7. Multi-recycles**

(Multi-recycles) For any separation \( \mathcal{C} \) excluding core-separation, there exists an equilibrium wherein for every \( c \in \mathcal{C} \), \( s_j = P \) for some \( j \in \mathcal{I}_{\text{per}}(c) \), \( s_{j'} = R \) for every \( j' \neq j \in \mathcal{I}_{\text{per}}(c) \), and \( s_i = R \) for every \( i \in \mathcal{I}_{\text{core}}(c) \), if the following condition is satisfied. \( \sigma^h + \frac{\gamma_1}{x} \leq \frac{\gamma_2}{x} \), \( 1 - \sigma^l \leq \frac{\gamma_2}{x} \), and \( 1 - \sigma^l \leq \frac{1}{2} \left( 1 - \sigma^h \right) \).

**Proof.**

Under the strategy profile stated in the lemma, all banks successfully make payments until period 2, after which each core bank appropriately takes either \( R_{cc} \) or \( R_{cp} \). Then, there is no incentive for each core banks to switch between \( R_{cc} \) and \( R_{cp} \). Further, when any of the banks making a payment in period 1 deviates, the banks within the relevant cycle cannot successfully recycle reserves in period 2. In this sense, the incentives to deviate are almost the same as for those in the case of \( S_2 \). The difference is that there exist some periphery banks with high shock that make payments in period 1. Considering this, we present the conditions shown in the lemma.

We also observe other equilibria, as shown below. The former part states that there exists an equilibrium where strategies of periphery banks are differentiated by their types of liquidity shock. Again, there is no spillover effect. The latter part states that there is an equilibrium within \( S_2 \), where a core bank is the sole payer. Although not shown below, it is clear that there is an equilibrium within \( S_2 \) where a periphery bank with high shock is the sole payer.

**Lemma 8.**

1. (High-early, low-recycle)
There exists an equilibrium where $s_j = R$ for every $j \in I_{\text{per}, L}$, $s_{j'} = P$ for every $j' \in I_{\text{per}} \setminus I_{\text{per}, L}$, and $s_i \in \{P, P_cR, P_pR, R_{cc}, R_{cp}\}$ for every $i \in I_{\text{core}}$, if $1 \leq \frac{2h_i}{x} = 1 - \sigma_x \leq \frac{2x}{s} \leq \frac{1}{2}(1 - \sigma^i)$, and $\sigma_x \leq \frac{2a_i}{s} - \frac{2x}{s}$.

2. (1-recycle with a core bank)

There exists an equilibrium where $s_i = P_pR$ for $i \in I_{\text{core}}$, $s_{i'} \in \{R_{cc}, R_{cp}\}$ for every $i' \neq i \in I_{\text{core}}$, and $s_j = R$ for every $j \in I_{\text{per}}$ if $\gamma_2$ is sufficiently large and $\frac{2a_i}{s} \leq \frac{1}{2}(1 - \sigma^i)$.

**Proof.**

The proof is almost the same as that for Lemma 3.

1. (High-early, low-recycle)

a). For periphery bank $j$ with $s_j = R$, there is no incentive to deviate if $\frac{2a_i}{s} \leq \frac{1}{2}(1 - \sigma^i)$ and $\frac{2a_i}{s} + \sigma^i \leq \frac{2a_i}{s}$.

b). For periphery bank $j'$ with $s_{j'} = P$, there is no incentive to deviate if $1 < \frac{2a_i}{s}$ and $1 - \sigma^h \leq \frac{2a_i}{s}$.

c). For core bank $i$ with $s_i = P$, there is no incentive to deviate (hoisting) if $\frac{1}{2}(1 - \sigma^h) \leq \frac{2a_i}{s}$.

Thus, we have, $1 \leq \frac{2a_i}{s}, 1 - \sigma^h \leq \frac{2a_i}{s} \leq \frac{1}{2}(1 - \sigma^i), \sigma^i \leq \frac{2a_i}{s} - \frac{2x}{s}$.

2. (1-recycle)

a). For periphery bank $j$ with $s_j = R$, there is no incentive to deviate if $\frac{2a_i}{s} \leq \frac{1}{2}(1 - \sigma^h)$ and $\frac{2a_i}{s} + \sigma^h \leq \frac{2a_i}{s}$.

b). For core bank $i$ with $s_i = R$ under $s_i^p = (R, R)$, there is no incentive to deviate if $\frac{2a_i}{s} \leq 1 - \sigma^h, \frac{2a_i}{s} + \sigma^h \leq \frac{2a_i}{s}$, and $2\frac{2a_i}{s} + \sigma^h \leq 2\frac{2a_i}{s}$.

c). For core bank $i$ with $s_i = R$ under $s_i^p = (P, R)$, there is no incentive to deviate if $\pi_i(R, S_{i-1}, \psi) \geq \pi_i(P, S_{i-1}, \psi) \Leftrightarrow \sigma^h x + 2\gamma_1 \leq \frac{1}{2}x, \gamma_1 \leq \frac{2a_i}{s}$.

(2) $\pi_i(R, S_{i-1}, \psi) \geq \pi_i(PR, S_{i-1}, \psi) \Leftrightarrow \sigma^h x + 2\gamma_1 \leq \frac{1}{2}x + \frac{1}{2}\sigma^h x + \gamma_1, \gamma_1 \leq \frac{2a_i}{s}$.

(3) $\pi_i(R, S_{i-1}, \psi) \geq \pi_i(PH, S_{i-1}, \psi) \Leftrightarrow \sigma^h x + 2\gamma_1 \leq \frac{1}{2}x + \gamma_2, \gamma_1 \leq \frac{2a_i}{s}$.

(4) $\pi_i(R, S_{i-1}, \psi) \geq \pi_i(RH, S_{i-1}, \psi) \Leftrightarrow \sigma^h x + 2\gamma_1 \leq \gamma_1 + \gamma_2$, and

(5) $\pi_i(R, S_{i-1}, \psi) \geq \pi_i(S, S_{i-1}, \psi) \Leftrightarrow \sigma^h x + 2\gamma_1 \leq \gamma_2$.

The conditions reduce to: $\frac{2a_i}{s} \leq \frac{1}{2}(1 - \sigma^h), \frac{2a_i}{s} \leq \frac{1}{4} - \frac{1}{2}\sigma^h + \frac{1}{2}\gamma_1, \gamma_1 \leq \frac{2a_i}{s}$.

d). For core bank $i$ with $s_i = PR$, there is no incentive to deviate if $\pi_i(PR, S_{i-1}, \psi) \geq \pi_i(P, S_{i-1}, \psi) \Leftrightarrow x + \gamma_1 \leq 2x$.

(2) $\pi_i(PR, S_{i-1}, \psi) \geq \pi_i(PH, S_{i-1}, \psi) \Leftrightarrow x + \gamma_1 \leq x + \gamma_2$.

(3) $\pi_i(PR, S_{i-1}, \psi) \geq \pi_i(R, S_{i-1}, \psi) \Leftrightarrow x + \gamma_1 \leq 2\gamma_2 + \sigma^h x$.

(4) $\pi_i(PR, S_{i-1}, \psi) \geq \pi_i(RH, S_{i-1}, \psi) \Leftrightarrow x + \gamma_1 \leq 2\gamma_2 + \sigma^h x$.

The conditions reduce to $\frac{2a_i}{s} \leq \frac{1}{2}(1 - \sigma^h)$.

When $\gamma_2$ is sufficiently large, the relevant conditions of a), b), c), and d) reduce to $\frac{2a_i}{s} \leq \frac{1}{2}(1 - \sigma^h)$.
References


