WHAT DO WE KNOW ABOUT INVESTMENT UNDER UNCERTAINTY?

Alan Carruth[†], Andy Dickerson[†] and Andrew Henley[‡]

February 1998

Abstract

Recent theoretical developments relating to investment under uncertainty have highlighted the importance of irreversibility for the timing of investment expenditures and their expected returns. This has subsequently stimulated a growing empirical literature which examines uncertainty and threshold effects on investment behaviour. This paper presents a review of this literature. A variety of methods have been used to investigate the empirical implication of irreversibility in investment, the majority focusing on the relationship between investment flows and proxy measures of uncertainty. A general conclusion is that increased uncertainty, at both aggregate and disaggregate levels, leads to lower investment rates. This suggests that there is an irreversibility effect, under which greater uncertainty raises the value of the "option" to delay a commitment to investment. This effect appears to dominate any positive impact on investment arising from the fact that greater uncertainty, under certain circumstances, increases the marginal profitability of capital. The methods used raise a number of issues which call into question the reliability of the findings, and these are addressed in the paper. However, if such irreversibility effects are present, then their omission from traditional investment models casts doubt on the efficacy of such specifications.

JEL Classification: D81, D92, E22

Keywords: Investment, Uncertainty, Irreversibility

Acknowledgements: Financial support for this work is gratefully acknowledged from ESRC Research Grant R000221709. We should like to thank Heather Gibson for comments on an earlier draft.

Correspondence Address: Professor Alan Carruth, Department of Economics, Keynes College, University of Kent, Canterbury, Kent CT2 7NP, UK. Tel: +44 (0) 1227 827449, e-mail: A.A.Carruth@ukc.ac.uk

[†] Department of Economics, University of Kent at Canterbury, Canterbury, CT2 7NP, UK.
[‡] Department of Economics, University of Wales Aberystwyth, Penglais, Aberystwyth, SY23 3DB, UK.

TABLE OF CONTENTS

1. Introduction

Given the enormous importance of investment fluctuations in explaining economic performance, it comes as no surprise that analysis of investment decisions remains a subject of key methodological and empirical concern to economists. Earlier reviews of this empirical research have been provided by the classic survey of Dale Jorgensen (1971), and by Robert Chirinko (1993) who surveys the literature since Jorgensen.[1] However, within the broad corpus of work on investment, the analysis of investor behaviour when investments are irreversible has attracted considerable recent attention. In part, this is testimony to the importance of Avinash Dixit and Robert Pindyck's influential work *Investment under Uncertainty* (1994), and the subsequent detailed re-examination of the empirical evidence on investment that it has motivated.[2] Their particular contribution is to examine how "options" pricing theory can inform our understanding of investment behaviour when decision makers face uncertainty about future prices and returns and when their decisions are irreversible. Their key insight is that there will exist an "option" value to delay an investment decision in order to wait the arrival of new information about market conditions.

The existence of this "option" value drives a wedge between the conventional net present value (NPV) calculation of the current worth of an investment project and the current worth of the project to the decision maker. At a given point in time, for an investment to be made, its NPV must be sufficiently larger than zero to cover the value to the decision maker of delaying the decision and keeping the investment option alive. This leads to a focus on the importance of the timing of investment decisions, the role of uncertainty in influencing that timing, and hence on the level of investment activity at a given point in time for a particular level of uncertainty prevailing among investors. Moreover, it also points to the possibility of

---

[1] Chirinko discusses four issues "encountered repeatedly" in the investment literature: the consistency of the theoretical model, the characteristics of the technology, the treatment of expectations and the impact on investment of prices, quantities and shocks. He does not deal with the new "options" literature in his review, concentrating on what he calls implicit dynamics (exogenous adjustment mechanisms), and explicit dynamics (representative agent optimisation theory and Euler equations).

[2] Dixit and Pindyck (1994) has been extensively reviewed elsewhere (see, in particular, Glenn Hubbard, 1994) and it is not the intention here to repeat such an exercise.

threshold effects, that is, rates of return below which investment will not be undertaken. There are also macroeconomic consequences since such effects can generate real rigidities in the capital accumulation process.

Set against this is the conclusion of earlier work by Richard Hartman (1972) and Andrew Abel (1983) that increased uncertainty may raise investment, because of its positive effect on the value of a marginal unit of capital. This result requires that the marginal product of capital (in a competitive firm) is convex in price, so that a mean-preserving increase in the variance of price raises the expected return on a marginal unit of capital and therefore raises the attractiveness of investment.

These insights into investor behaviour have stimulated a considerable body of research at the aggregate, industry and firm level which attempts to subject the relationship between investment and uncertainty, and the effects of irreversibility on investment timing to empirical scrutiny. The purpose of the present paper is to review this growing body of literature and assess what conclusions can be drawn from it. As a precursor to this, it is useful to provide a brief overview of the development of the methodology and the empirical issues that it implies. This is provided in Section 2. The question of the appropriate level of empirical aggregation is discussed in Section 3. Section 4 presents a critical assessment of the various methodologies used to proxy unobservable rate of return uncertainty. Section 5 examines attempts to investigate the presence of non-linearities and threshold effects which follow from investment decisions being irreversible. Section 6 examines the modelling of the time-lag up to the commitment to investment. Section 7 addresses the empirical importance of subsidiary assumptions about market structure. Section 8 draws together a number of the key issues and findings by way of a concluding summary.

2. Irreversibility and uncertainty in investment: methodology and implications for empirical research

There is nothing new in suggesting that investment behaviour will be sensitive to the degree of investor uncertainty about future prices, market conditions and rates of return. Indeed the role of uncertainty is implicit in the early adjustment costs literature (Robert Eisner and Robert Strotz, 1963; Robert Lucas, 1967) in which the role of backward-looking expectations formation is captured through the inclusion of lagged variables. Recent authors, contributing to the growing literature on irreversible investment, criticise the old adjustment costs literature for its essentially *ad hoc* approach to the specification of adjustment cost functions (implicit dynamics in Chirinko, 1993). The role of future expectations was made explicit in *q*-models of investment (James Tobin, 1969) which rely on the assumption that current corporate stock valuation reflects agents' expectations about future conditions as they affect the firm in question. An early attempt to model investment as an irreversible process can be found in Kenneth Arrow (1968), and this, together with subsequent work in the same vein by Stephen Nickell (1974, 1978), demonstrates the importance of expectations in such a context.[3]

As indicated in the introduction a key early result, due to Hartman (1972), and extended in Abel (1983), was the finding that increased uncertainty can raise the marginal profitability of capital, and hence increase investment. In contrast, other authors have argued that irreversibility may lead to the postponement of investment decisions (Ben Bernanke, 1983; Robert McDonald and Daniel Siegel, 1986). As a result of introducing the notion of the "perpetual call option" value of an as-yet-uncommitted investment plan, they show that with increasing uncertainty (more specifically, increasing variance in the distribution of future rates of return from the project), the value of the option to delay increases and thus the investment decision is delayed. Increased uncertainty, other things equal, will therefore *reduce* the current level of investment. In an important paper, Ricardo Caballero (1991) shows that it is, in fact, the assumption of imperfect competition and/or decreasing returns to scale typically made in irreversibility models, and not the assumption of asymmetric adjustment costs embedded in these models, that is critical to the prediction of a negative relationship between investment and uncertainty.

[3] A further strand of more recent literature investigates the separate but related issue of the importance of the firm's financial performance on investment, Richard Blundell *et al.* (1992), Stephen Bond and Costas Meghir (1994).

To illustrate the irreversibility effect[4], consider the case of a monopolist assessing whether to make an investment whose present value is *X*, for which a sunk cost of present value *I* must be incurred. Under a conventional NPV calculation, the firm will make the investment if $X - I \geq 0$. Now assume that *X* is variable over time and evolves following a geometric Brownian motion with drift:

$$dX = \alpha X dt + \sigma X dz \tag{1}$$

where α is the mean of *dX* and σ is the standard deviation of *dX*. *dz* is the random increment of a Wiener process such that:

$$dz = \varepsilon_t \sqrt{dt}$$
$$\text{where } \varepsilon_t \sim N(0,1),\ E(\varepsilon_i \varepsilon_j) = 0\ \forall\ i, j\ i \neq j. \tag{2}$$

i.e. ε_t is a serially uncorrelated standard normal random variate. Equation (1) is a special case of an Ito continuous time stochastic process.[5] Equations (1) and (2) imply that future values of the investment are log-normally distributed with expected value $E(X_t) = X_0 \exp \alpha t$ (where X_0 is the value of *X* today) and a variance that grows exponentially with *t*. The firm will wish to time its investment decision so as to maximise the expected present value of the option to invest, *F(X)*, given by:

$$F(X) = \max E\left[(X_T - I)\exp(-\rho T)\right] \tag{3}$$

where X_T is the value of the investment at the unknown future point in time, *T*, at which the investment decision is made and ρ is the discount rate.[6]

Delaying the investment decision and holding the option is equivalent to holding an asset which pays no dividends but may appreciate as time passes. The fundamental condition

---

[4] This example draws heavily on McDonald and Siegel (1986) and Dixit and Pindyck (1994, Chapters 4 and 5).
[5] The calculus of continuous time stochastic processes is described in Dixit (1993).
[6] Note that ρ must be greater than α, otherwise the firm will hold the option to delay for ever.

for optimality, or Bellman equation, if the firm delays investment and holds the option is given by:[7]

$$\rho F = \frac{E(dF)}{dt} \tag{4}$$

The left hand side of equation (4) is the discounted normal rate of return that an investor would require from holding the option while the right hand side is the expected total return per unit of time from holding the option. If this condition holds, then the firm is equating the expected return from delaying the investment with the opportunity cost of delay. In effect equation (4) describes a no-arbitrage condition.

Using Ito's Lemma to obtain the total differential of a continuous time stochastic process, we can express *dF* as:

$$dF = F'(X)dX + \tfrac{1}{2}F''(X)(dX)^2 \tag{5}$$

Using the expression in equation (1) for *dX* and taking expectations gives:

$$E(dF) = \alpha X F'(X)dt + \tfrac{1}{2}\sigma^2 X^2 F''(X)dt \tag{6}$$

Note that the terms in *dz* disappear since its expectation is zero. Substituting (6) into (4) we obtain the Bellman equation in the case where *dX* is a continuous stochastic process:

$$\rho F = \alpha X F'(X) + \tfrac{1}{2}\sigma^2 X^2 F''(X) \tag{7}$$

If the firm is following the optimal investment rule, the value of the option to wait must satisfy the second order differential equation given in equation (7). In addition it must satisfy three boundary conditions:

$$\begin{aligned} F(0) &= 0 \\ F(X^*) &= X^* - I \\ F'(X^*) &= 1 \end{aligned} \tag{8}$$

[7] See Dixit and Pindyck (1994, Chapter 4) for further details of this derivation.

The first condition simply states that if the value of the investment falls to zero then the value of the option to invest is zero. The second describes the net payoff at the value of *X* at which it is optimal to invest. The third is the "smooth-pasting" condition (Dixit, 1993) which requires the function *F*(.) to be continuous and smooth around the optimal investment timing point.

The solution to (7) subject to the conditions given in (8) is:

$$F(X) = aX^{b} \tag{9}$$

where $a = \frac{X^{*} - I}{X^{*b}}$ is a constant and

$$b = \tfrac{1}{2} - \alpha/\sigma^2 + \sqrt{\left(\alpha/\sigma^2 - \tfrac{1}{2}\right)^2 + 2\rho/\sigma^2} \tag{10}$$

By substituting (9) into the second and third of the boundary conditions given in (8) we obtain the result that the optimal investment timing payoff is given by:

$$X^{*} = \frac{b}{b-1} I \tag{11}$$

Equation (11) defines the "wedge", $\frac{b}{b-1}$, between the payoff necessary to induce the investor to exercise the option to invest, X^{*}, and the present value of the cost of the investment, *I*. Since $b > 1$, then $\frac{b}{b-1} > 1$ and hence $X^{*} > I$. Thus in the presence of irreversibility and uncertainty, the simple NPV principle which equates X^{*} with *I* is no longer applicable. Simple calculus reveals that the size of the wedge between X^{*} and *I* increases as uncertainty about future returns, σ, rises. It is also increasing in the discount rate, ρ, and in the drift term in the evolution of the expected rate of return, α.[8]

---

[8] An analogous interpretation of the wedge defined by equation (11) is in terms of Tobin's *q*, which, as conventionally measured, is the ratio of the expected returns from a project to its construction cost (and hence ignores the opportunity cost of having exercised the option). In the notation used here, the threshold *q* necessary for investment is therefore $q^{*} = \frac{X^{*}}{I} = \frac{b}{b-1} > 1$ whereas the standard literature suggests that firms will invest whenever $q \geq 1$ (see also Dixit and Pindyck, 1994, Chapters 11 and 12).

An important question concerns the source of the uncertainty in the investment payoff function. Indeed, a large proportion of the empirical literature is primarily concerned, firstly, with establishing uncertainty proxies and, secondly, with testing for their effect on investment. One obvious source of uncertainty which might manifest itself in product price fluctuations derives from future demand uncertainty. If demand is subject to a stochastic shock then, through the inverse demand function, product price *P* will also be stochastic. Thus the value of the project *X(P)* becomes a function of a stochastic variable. Equations (3) to (11) can then be reformulated to provide a solution for the optimal price level, P^*, at which the firm becomes indifferent between holding the option to invest and actually making the investment. Figure 1 shows how the conventionally calculated NPV of an investment project, *X*(*P*) - *I*, will increase as *P* rises, and shows its relationship to the value of exercising the "option to wait", *F*(*X*(*P*)). Where the two curves meet (the point of smooth pasting) is the point of indifference, and the price level at that point is P^*. This is clearly higher than the level of *P* at which the NPV of the investment project becomes positive.

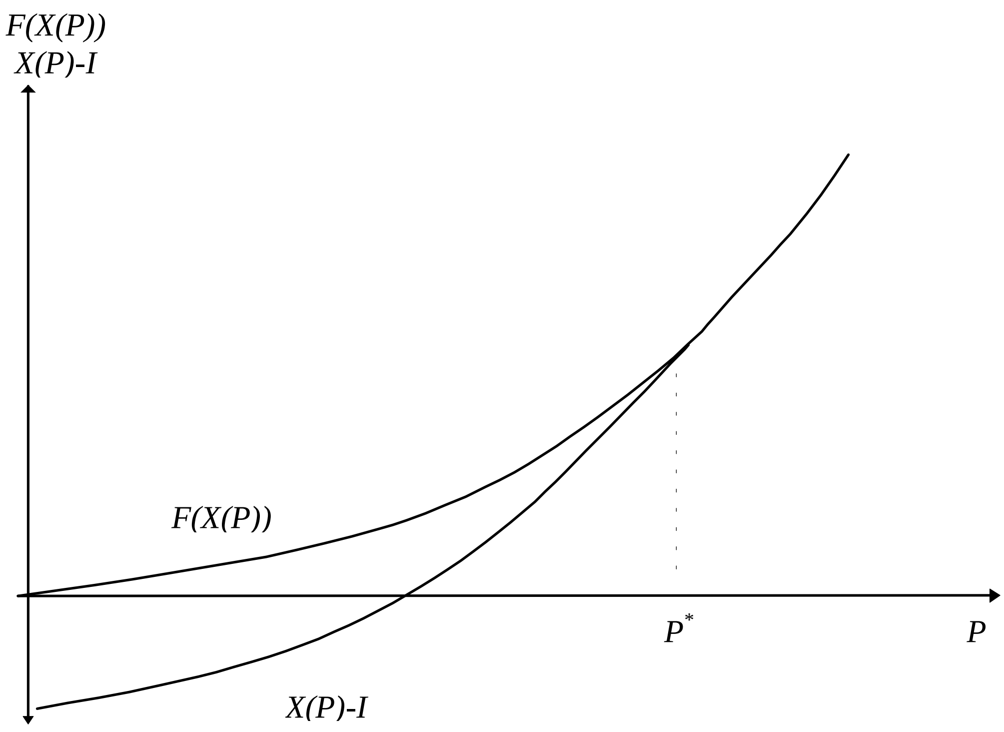

Figure 1:

Relationship between option value, *F(X(P))*, and conventional NPV, *X(P) - I*

Pindyck (1991) reports the results from a number of simulations for different values of σ and α. The greater the volatility in price (σ) the greater the value of the option to wait and thus increased uncertainty reduces investment. The greater the rate of growth in price (α), the lower is the opportunity cost of waiting and thus the incentive to exercise the option today decreases. On the other hand, a higher α leads to a lower break-even price at which the investment opportunity should be realised. These two effects oppose but the second dominates and hence the greater the rate of growth in price, the lower the price P^* at which it is optimal to invest.

What empirical implications follow from these option-based models of irreversible investment under uncertainty? The crucial point to note is that such models do not describe the level of investment *per se*, but simply identify those factors which may affect the threshold at which investment should occur. In particular, an increase in the volatility of the stochastic process which determines the returns from investment will raise the trigger point. However, this result must be set against the Hartman-Abel result that an increase in uncertainty may increase the value of a marginal unit of capital and, *ceteris paribus*, increase the incentive to invest. Thus, as Dixit and Pindyck (1994) note, the theory does not determine the long-run equilibrium level of investment, nor the optimal capital-output ratio. Increased volatility may raise the required rate of return before investment will be undertaken, but it does not necessarily affect the rate of return that will be realised once (if) the decision to invest is made. Though it does not necessarily follow that countries which are less stable will have lower rates of investment or lower growth, it is a likely outcome, particularly if irreversibility results in the permanent postponement of some investment decisions. More importantly it may be the timing of investment decisions in such countries that may differ from those in which economic conditions are more stable.

Despite the ambiguity concerning the models' implications for the level of investment, there are a number of ramifications for empirical modelling of investment behaviour which arise from the previous discussion. The first is that the *timing* of investment depends on how long it takes a critical variable to reach its threshold level, and thus on the extent to which that threshold exceeds the conventional NPV break-even point. As we have seen above, the magnitude of the wedge between the necessary returns before investment is undertaken and the sunk costs depends on a number of factors, the most important of which will be the variance in expected returns (and therefore the key variables which determine those returns) and the rate of "drift" in the process that determines the growth in those returns. This potentially introduces significant non-linearities in the underlying investment process. A second important issue concerns how the drift effect and, in particular, the variance effect within the stochastic process might be formulated empirically and a wide variety of representations can be found in the extant literature. Thirdly, as noted above, since the irreversible investment literature deals with the timing of investment rather than the level of investment activity, a complete model needs to incorporate an independent explanation of the "certainty-equivalent" determinants of investment for a particular firm, industry or economy. Related to this is the extent to which the specification and identification of irreversibility effects are contingent on the underlying model structure. This is an issue which has not yet been addressed empirically.

The vast literature on the empirical determinants of aggregate investment surveyed by Jorgensen (1971) largely grew out of the pioneering work on the flexible accelerator by Hollis Chenery (1952) and Leen Koyck (1954). Jorgensen, concluding his survey, was rather up-beat about the results of this work. Although he notes that the implications of the new dynamic approaches to econometrics to the study of investment were far from being exhausted, for Jorgensen,

> "the most important open question in the study of investment is the integration of uncertainty into the theory and econometrics of investment." (p.1142)

Some twenty-three years later, and having provided a major contribution to the omission that Jorgensen noted, Dixit and Pindyck (1994) are far less optimistic:

> "The explanation of aggregate and sectoral investment spending has been one of the less successful endeavours in empirical economics. For the most part, econometric models have not been very useful for explaining and predicting investment spending. The problem is not just that these models have been unable to explain and predict more than a small portion of the movements of investment. In addition, constructed quantities that in theory should have strong explanatory power - such as Tobin's q, or various measures of the cost of capital - in practice do not." (p.419)

The key issue for Dixit and Pindyck is the abandonment of the standard NPV rule in the face of irreversibility and the resulting option value of delay. The problem for the applied econometrician is the empirical implementation of the irreversibility concept and its implications. The problem is an acute one because the irreversibility effect makes the dynamic structure of investment behaviour dependent on the degree of volatility in returns. Moreover. under irreversibility and uncertainty, the underlying model describing optimal investment is non-linear. Furthermore, as Dixit and Pindyck (1994, p.421) argue, since option-based models focus on the threshold at which investment should occur rather than on the long-run average rate of investment, then the models themselves cannot be directly tested by investigating simple equilibrium relationships between rates of investment and measures of risk or uncertainty. Any test will be a joint test of the option-based approach together with the underlying assumed specification for the capital accumulation process. In practice many studies appear to gloss over this point, preferring to investigate simple correlations of rates of investment with proxies for uncertainty – a strategy which is highly questionable since any observed significant relationship may be an artefact of underlying model misspecification.

By the 1990s, innovations in both the theory of capital accumulation and in econometric methodology have increased the sophistication of empirical best-practice. In his 1993 review Chirinko organises empirical work on investment into that which treats the dynamics of capital accumulation implicitly, through the essentially *ad hoc* use of distributed lag formulations and that which takes explicit account of the dynamic nature of optimisation of capital stock (quadratic adjustment costs and Euler equation frameworks). The majority of recent empirical studies of the relationship between investment and uncertainty under conditions of irreversibility have adopted the former implicit approach to model dynamics. This is potentially a serious limitation, but in mitigation the theoretical literature here is underdeveloped. The challenge of introducing highly non-linear adjustment dynamics into an Eisner and Strotz (1963) adjustment costs framework, or of deriving an intertemporal optimisation condition which takes account of irreversibilities is a daunting one. Such a model is likely to be so rich in structure as to be potentially empirically intractable. Nevertheless, an important limitation of much empirical work on this topic is the unspecified nature of the underlying capital accumulation process. An alternative approach taken by a smaller number of studies is to investigate the implications of non-linearity in adjustment costs for model specification.

3. Aggregate or disaggregate empirical analysis?

For aggregate studies of the investment-uncertainty relationship the pre-eminent problem concerns the additional assumptions necessary to translate the representative-firm model of investment irreversibility described in section 2 into aggregate analysis. If firm or industry fluctuations in uncertainty are not coincident, then these fluctuations will simply cancel each other out at the aggregate level. Bernanke (1983) discusses two possible reasons why the effects of uncertainty may not simply average out at the aggregate level. Firstly, macroeconomic factors, such as uncertainty about future interest, exchange and inflation rates or shocks in monetary, fiscal or regulatory policy, may be important in determining micro-level decision making. Secondly, aggregate uncertainty may be generated or propagated by individual decision makers. If an individual firm is uncertain about whether an aggregate demand shock is transitory or permanent, then the decision to invest may be delayed in order to learn about its degree of permanence. Similarly, if firms are uncertain about the impact of an aggregate demand shock on their individual demand levels, they may delay decisions. Of course, there is in here the elements of a mechanism through which (genuinely) transitory shocks can become converted into a more permanent fluctuation. As Bernanke (1983) argues, the propagating mechanism is the irreversibility of investment.

The existence of a threshold effect in the investment process may lead to episodes of investment hysteresis, during which rates of investment remain persistently sluggish as firms exercise the option to wait (Dixit 1989, 1992). If individual firm uncertainty can in turn generate a self-fulfilling degree of aggregate uncertainty then that hysteresis could be aggregate in nature. Nevertheless the relationship between uncertainty and investment ought to be at least as strongly observed at a disaggregated level, particularly where the unit of observation is the firm. If idiosyncratic factors are more important in affecting the timing of investments then, if anything, any observed empirical relationship should be stronger than that detected in the aggregate analysis. Hysteresis or persistence in investment ought to reveal itself as autoregression of the time series of investment aggregates. However such empirical observation will also be consistent with other explanations, such as the existence of adjustment costs.

An extensive investigation of the properties of investment in aggregate, under conditions of microeconomic irreversibility, has been undertaken by Giuseppe Bertola and Ricardo Caballero (1994). The starting point for Bertola and Caballero is the criticism that early models of investment based on, *inter alia*, smooth adjustment costs, do not provide a persuasive interpretation of the empirical evidence on investment expenditures; typically, firms face lumpy adjustment costs. The existence of irreversibilities can furnish a convincing rationale for the observation that investment appears to be forward looking. As in Bernanke (1983), they argue that microeconomic irreversibilities in the presence of idiosyncratic uncertainty are also relevant to aggregate investment dynamics. Their theoretical analysis demonstrates that, in a frictionless (i.e. completely reversible) but uncertain world, the rate of investment for a revenue-maximising firm at any point in time will depend on the inherited capital stock and the user cost of capital, augmented by terms to capture the stochastic effects of uncertainty on revenue. Aggregation, under suitable functional form assumptions, leads to an aggregate investment process in which firm-specific uncertainty averages out in the dynamics of aggregate capital. However, investment functions derived from the aggregation of representative firms tend to produce serially correlated errors when subjected to actual data. It is this finding that many previous empirical researchers have rationalised through assuming *ad hoc* adjustment costs. Bertola and Caballero demonstrate that the introduction of binding irreversibility constraints undermines the linearity embedded in such models and can generate "inertial" or hysteresis effects, consistent with the observed serial correlation in actual aggregate data.

Bertola and Caballero construct the (hypothetical) desired aggregate investment-capital stock ratio for the US for 1954 to 1986 under the assumption of frictionless (reversible) investment and compare this to the observed investment ratio. Their key relationship linking frictionless and actual investment rates is:

$$\left(\frac{I}{K}\right)_t^* = \frac{1}{1-\alpha}(\Delta \ln Y_t - \Delta \ln r_t) - \frac{\alpha}{1-\alpha}\left(\frac{I}{K}\right)_t + \frac{1}{1-\alpha}\delta \qquad (12)$$

where $\left(\frac{I}{K}\right)_t^*$ and $\left(\frac{I}{K}\right)_t$ are the frictionless and actual investment rates respectively, Y is revenue (proportional to GNP), r is a neoclassical user cost of capital series, δ is the depreciation rate and α is a parameter capturing the elasticity of output with respect to capital. Bertola and Caballero show that for reasonable values of α and δ, the calculated frictionless rate of investment displays much greater cyclical volatility than does the actual series (the standard deviation of the former is 0.046 compared to 0.017 for the latter). Moreover, the actual series displays much greater first order serial correlation than the frictionless series (0.68 compared to 0.25). Thus the smooth, persistent, nature of US aggregate investment can be explained by the presence of irreversibility constraints acting on firms' investment decisions. Indeed, to explain the smoothness and persistence of actual investment without irreversibilities would require unrealistically high levels of volatility in firms' *desired* levels of capital stock.

The work by Bertola and Caballero addresses the question of the importance of irreversibility for aggregate investment by performing a counter-factual empirical experiment, and this experiment suggests that aggregate effects ought to be present. The majority of the other aggregate studies have taken a more traditional approach, namely to quantify and assess the significance of the impact of uncertainty or volatility on investment levels. Evidence for the investment-uncertainty relationship in aggregate has been found using a variety of empirical approaches, and the broad conclusion is that the negative "option" effect outweighs the positive "Hartman-Abel" effect of uncertainty on the marginal value of capital.[9] The options-based approach also yields the prediction that investment expenditure may be subject to the existence of threshold effects, above which investment is triggered and below which the option to delay is exercised. This has implications for the dynamics of investment activity (which may or may not spill over into the long-run), and a small number of studies have focused on this particular aspect of the option-based models.

Table 1 summarises the key features and results of the various aggregate empirical studies which attempt to correlate investment with a proxy measure of uncertainty. The broad

---

[9] However, according to Caballero (1991), the argument that increased uncertainty reduces investment in risk neutral firms depends on two conditions; first there are asymmetric adjustment costs in reaching the optimal capital stock - firms regret having too much capital (irreversibility) more than they regret having too little; and second firms operate in markets characterised by imperfect competition, so the marginal profitability of capital is sensitive to the level of capital chosen.

consensus is that the relationship is negative, although, as can been seen in the Table, this consensus emerges from a wide range of model-types and alternative methods for proxying uncertainty. We shall return to this shortly. Table 2 summarises the key features and results of the smaller number of disaggregated studies. By contrast, and somewhat surprisingly in the light of the earlier discussion, the results obtained are far less conclusive.

Some authors perform both aggregate and disaggregate analysis using the same methodology for constructing an uncertainty proxy. For example, Linda Goldberg (1993) investigates the impact of exchange rate volatility on quarterly US real investment aggregates over the period 1970-1990.[10] There would appear to be no overall effect of exchange rate volatility on investment at the aggregate (all industries) level, although a weakly significant (at 10%) positive impact is found for manufactured durables. In general, the investment function estimates are rather disappointing (for example, none of the exchange rate or cost of capital coefficients are significant at 10% or better over the whole period). When the broad industrial sectors are disaggregated into their 2-digit constituents, it is apparent that the absence of any strong relationship between exchange rate volatility and investment at the aggregate level may conceal some effect at a more disaggregated level. However the results are tentative - less than a fifth of the sectors reveal a (5% level) significant uncertainty effect. Moreover the level of aggregation here is still too high to reveal much about any possible impact derived from more idiosyncratic risk. A second companion paper using the same data source (José Campa and Linda Goldberg 1995) is also rather inconclusive. A third paper examining exchange rate volatility (Campa 1993) examines its impact on foreign direct investment entry into the US, and does find a depressant effect on capital expenditures, particularly for Japanese investors.

A similar contradiction between the aggregate and the less aggregate is, to some extent, apparent in John Huizinga (1993). Increased uncertainty about real wages (proxied by a conditional variance estimate, see below) is found to lead to a sizeable and immediate negative impact on aggregate investment; a one standard deviation increase in real wage volatility will lead to a fall of 0.007 in the investment-sales ratio. The effect of an increase in

---

[10] The five investment aggregates she considers are: all industries, manufacturing, manufactured durables, manufactured nondurables and nonmanufacturing.

price uncertainty is less immediate but indicates a longer-lasting depressant effect on capital expenditures. 4-digit industry-level cross-section regressions investigating the correlation between the rate of investment and the same uncertainty proxies reveal less conclusive results. Investment appears to be negatively correlated with real wage and real materials price volatility, and positively with uncertainty about real output prices. Huizinga's own rationalisation for this concerns the possibility of simultaneity bias between investment rates and his measures of uncertainty in cross-section (whereas in his time-series results, the uncertainty measures are backward looking and thus based on information predetermined at time *t*). However, since this argument would apply equally to uncertainty over real wages and real material prices as well as real output prices, it would therefore appear to invalidate his cross-section analysis in its entirety. One possible solution would be to exploit the panel nature of the data that he utilises.

These apparent inconsistencies in the results reveal the crucial importance of disaggregation when attempting to identify the impact of uncertainty and also highlight the need for appropriate econometric techniques that can integrate both time-series and cross-section information. Moreover, it is apparent that there is a degree of heterogeneity across industries which may potentially bias the results from any aggregate-level study. Given these conclusions, it is clear that the use of company-level panel data, with its even higher level of disaggregation coupled with its greater data variability, is likely to be advantageous. It is therefore encouraging that a small number of very recent studies have utilised firm-level longitudinal data.

The use of company-level data offers a number of potential advantages. Firstly, it enables the focus to be on measures of uncertainty that are more closely related to the idiosyncratic factors which affect an individual firm. According to the irreversibility literature, investment will be more sensitive to variations in idiosyncratic uncertainty than to increases in uncertainty which affect all firms in general. A second advantage concerns the capacity to control for the possible simultaneity between investment and uncertainty where panel data are available. Effects on (aggregate) uncertainty generated *by* changes in investment should be greatly attenuated if data are observed at the company or plant level, and, in any case, any such aggregate effects can be controlled through the use of time-specific dummy variables.

Finally, the use of panel data allows the researcher to control for other unrelated sources of firm-specific heterogeneity in the determination of investment levels.

This approach to controlling for heterogeneity is taken by John Leahy and Toni Whited (1996) who use data on 600 US manufacturing firms for the period 1981 to 1987, proxying uncertainty as the variance in the daily share return for each company and its covariance with a market portfolio (see discussion below). Their estimates show some evidence of a weak negative correlation between the variance measure of uncertainty and investment but no significant correlation for the covariance measure.[11] A 10 percent increase in the variance of a company's returns leads to a 1.7 percent fall in the investment rate. However, the variance coefficient becomes statistically insignificant (and only one tenth as large) once Tobin's *q* is added to the specification. One interpretation of this result is that the effect of uncertainty on investment operates through *q*, and this hypothesis is given support by the strong negative correlation between the variance of returns and Tobin's *q*. Even when traditional explanatory covariates for investment such as output and cash flow are included in their estimating equation, the variance of returns is never significant if *q* is also included, and the covariance with market returns is never significant in any specification. Leahy and Whited therefore conclude that there is a negative relationship between investment and uncertainty which operates primarily through its effect on *q*, and they reject both the "Hartman-Abel" theory, and any effect through a risk-based capital asset pricing model (CAPM). Hence, their preferred explanation of this relationship is the importance of irreversibilities in investment.

Further limited evidence for a negative relationship, using a US plant-level database for twelve highly concentrated 3-digit industries is presented by Ciaran Driver, Paul Yip and Nazera Dakhil (1996). A significant negative effect of market share volatility on investment is found in five of the twelve industries examined, with uncertainty-investment elasticities of the order of –0.05 to –0.15. Rather more robust evidence at this level of disaggregation is presented for a cross-section of Italian firms by Luigi Guiso and Giuseppe Paragi (1996),

[11] Note that their regression results do not appear to take into account the fact that, since the uncertainty terms $\Delta\sigma^{*}_{i,t+n}$ in equation (5), p. 71, 1996, are derived variables, the standard errors should be appropriately adjusted.

using an uncertainty proxy derived from firms' own perceptions, see section 4 below. If the variance of expected demand rises from its sample mean to the 95^{th} percentile, investment falls by 3.1 per cent. An important conclusion that emerges from this disaggregated work, and one which calls into question the interpretation of conclusions drawn from aggregate analysis, is that there is substantial variation in the impact of uncertainty across groups of firms and between different industries. Such heterogeniety may, as Guiso and Parigi's results suggest, be tied to differences in the size of secondary markets for capital equipment. It may also be tied to the importance of market structural conditions, and we shall return to this in section 7.

4. How should uncertainty by captured empirically?

We can identify a number of broad approaches within the literature although, in most cases, some form of uncertainty proxy is incorporated into a fairly conventional investment model specification. One approach is to attempt to estimate the marginal profitability of capital and infer from this a proxy for the threshold at which investment will be triggered. A second approach is to compute the unconditional variance of a particular price or macroeconomic aggregate which influences returns and about which investors are presumed to be uncertain and to use this as a proxy for risk. A third approach is to estimate a statistical model of the process (such as ARCH or GARCH) determining the conditional variance of the price level or other aggregate and use this as a proxy for uncertainty. Such ARCH or GARCH methods are popular in the finance literature as a vehicle for modelling volatility (Terence Mills, 1993). A final approach is to incorporate some more direct measure of risk, such as the risk premium embedded in the term structure of interest rates. As we indicate below, none of these approaches is without its particular problems and criticisms.

Tables 1 and 2 include information on the type of uncertainty proxy used in the various investment-uncertainty studies. As can be seen there is no consensus about the appropriate way to proxy uncertainty in an empirical formulation. Pindyck (1986), as discussed in Peter Ferderer (1996), demonstrates a negative correlation between the variance of lagged stock market returns and aggregate investment spending in the US. One problem with such an approach is that, while lagged stock market returns may reveal something about cash-flow uncertainty, they may reveal rather less about uncertainty over future economic shocks and policy changes. There is also the added criticism that the choice of volatility in stock returns may be unsuitable as a proxy variable because work in the finance literature, such as Robert Shiller (1989), suggests that the volatility in stock market returns may be driven by speculative “bubbles” as much as by movements in “fundamentals”.

As noted above, one advantage of using disaggregated data is that the uncertainty proxy can be constructed as a firm- or industry-specific covariate. So for example the two company/plant level studies discussed above make use of firm specific information. Driver *et al*. construct a simple market share volatility measure to proxy demand uncertainty. Leahy and Whited perform a considerably more complex preliminary modelling process to construct measures of rate of return volatility in order to obtain *ex ante* measures of uncertainty. They derive forecasts of return variance from a vector-autoregression (VAR) model. Since the required rate of return on an investment will be positively related to the level of investment risk through the CAPM (Roger Craine, 1988), a similarly constructed forecast of the covariance of the daily share return with a weighted average of various US stock market indices is also used as an additional and/or alternative uncertainty proxy.[12]

However work at the aggregate level, subsequent to the Pindyck (1986) study, has eschewed the use of stock market index volatility as an uncertainty proxy, preferring to incorporate some measure of macroeconomic volatility as a proxy for uncertainty about investment profitability at the margin. Typically these measures of volatility are (lagged) moving variances of aggregate price, output or exchange rate indices. In the majority of studies these are derived as predictions from some form of univariate time series representation. For example, the series of papers by Goldberg and Campa (Goldberg 1993, Campa 1993 and Campa and Goldberg 1995) perform a prior estimation of an autoregressive moving average (ARMA) model for dollar exchange rates of the following form:

$$ER_t = \alpha_1 ER_{t-1} + \varepsilon_t + \beta_1 \varepsilon_{t-1} \qquad (13)$$

---

[12] The relationship between investment and risk (as measured by the covariance of the firms’ returns with the market return) is investigated by William Brainard *et al* (1980) using data for a small sample of US companies. Their results which examine the direct impact of this CAPM-based measure of risk on Tobin’s *q* are, however, inconclusive.

They construct as an uncertainty proxy a rolling standard error from the model residuals, but this measure of uncertainty provides little additional explanation of aggregate investment. Ciaran Driver and David Moreton (1991, 1992) adopt a similar approach using British data and find that both output and inflation variance terms attract negative and significant coefficients for a (rather short) sample period of 1978 to 1987. A doubling in output variance lowers short-run investment by 8 per cent, and a doubling in inflation variance lowers short-run investment by 5 per cent.

A number of studies (Huizinga 1993, Athanasios Episcopos 1995, Simon Price 1995, 1996) have implemented an ARCH approach to the estimation of price or output volatility which is then used as an uncertainty proxy to include in a model of aggregate investment. A basic univariate ARCH model takes the form of a univariate AR process in the variable in question, together with a moving average process in the residual innovations from this AR process:

$$\begin{aligned} p_t &= \alpha_0 + \alpha_1 p_{t-1} + \ldots + \alpha_j p_{t-j} + \varepsilon_t \qquad \varepsilon_t \mid \varepsilon_{t-1} \sim N(0, \sigma^2) \\ \sigma^2 &= \beta_0 + \beta_1 \varepsilon_{t-1}^2 + \ldots + \beta_q \varepsilon_{t-q}^2 \end{aligned} \tag{14}$$

The variance process can also be generalised to have an ARMA representation yielding a GARCH model as follows:

$$\sigma^2 = \beta_0 + \beta_1 \varepsilon_{t-1}^2 + \ldots + \beta_q \varepsilon_{t-q}^2 + \delta_1 \sigma_{t-1}^2 + \ldots + \delta_p \sigma_{t-p}^2 \tag{15}$$

It is clear from Huizinga’s results that the computed conditional variances for different macroeconomic series are not coincident over time. For example, while volatility is generally greater after 1970, a short period of high volatility can be seen in real output prices and real wages around 1960 while the volatility in profitability is considerably higher in the late 1980s than at any previous time. As already discussed, the choice of proxy here appears to influence the sign of the investment-uncertainty relationship. In some estimations Huizinga “hedges his bets” by including more than one volatility measure as an explanatory variable for investment, raising the question of whether a single macroeconomic volatility measure can adequately capture the various dimensions of uncertainty faced by a firm. By contrast a measure of return variance, such as used by Leahy and Whited, ought better to capture all relevant uncertainty sources (or at least all those known to the stock market in question).

In much the same vein Price (1995) uses a GARCH-M estimate of the conditional variance of GDP as an uncertainty proxy in a study of aggregate UK investment. In the preferred dynamic specification the uncertainty effect is negative and statistically significant and, on average, investment is found to be 5 per cent lower than would have been the case with zero GDP variance. However the solved long-run relationship suggests effects that are arguably implausibly large. The depressant effect of uncertainty on real manufacturing investment was as high as 48 per cent in the immediate aftermath of the first 1974 OPEC shock and 38 per cent following the second shock in 1979. Issues of model mis-specification arise here, in that ARCH-based measures of uncertainty are crucially dependent on the (mis)specification of the empirical model. Engle (1983) argues that,

> “the strength of the ARCH technique is that the conditional means and variances can be estimated jointly using conventionally specified models for economic variables. The weakness of the procedure is that if the model is mispecified, the estimates of the conditional variances will be biased. This points out the importance of carrying out various specification tests” (p.287)

It is quite common to use a univariate VAR on output or inflation which displays an ARCH error test failure as the basis for estimating the conditional variance of these variables. This is not an adequate way of proxying uncertainty because a more appropriate (perhaps multivariate) model of the output or inflation process may not exhibit ARCH error failures. This criticism has important repercussions. Firstly, the measurement of the conditional variance may be very sensitive to model specification. Secondly, the discovery of a better approximation to the underlying data generation process without an ARCH error test failure does not then mean that uncertainty is irrelevant to economic behaviour. Furthermore, Adrian Pagan and Aman Ullah (1988) demonstrate that for measures of uncertainty based on moving averages of past values to estimate the true variance of some underlying fundamental, stationarity in the sample series is necessary. In the work reviewed here, stationarity is often implicitly assumed rather than pre-tested. Pagan and Ullah also show that there is an equally serious econometric issue which appears to have been largely ignored in the literature – namely that proxies for risk are measured with considerable error. The implications of this "errors in variables" problem is that uncertainty effects on investment will only be estimated consistently by an instrumental variables technique.

An additional problem with unconditional or conditional variance calculation methods is that they produce, in essence, backward-looking measures of uncertainty. A rather different approach, and one which attempts to address the criticism that moving-average or ARCH methods can only produce backward-looking uncertainty measures, is adopted by Ferderer (1993) and Ferderer and David Zalewski (1994). Rather than attempting to estimate or compute a variance measure for some economic indicator, Ferderer (1993) uses the implicit risk premium embedded in the term structure of interest rates. The risk premium is calculated, following Robert Shiller and Huston McCulloch (1987), as the linearised expected excess holding premium associated with buying a bond with a term of *k* months at time *t* and selling it after *j* months:

$$RP_t^{j,k} = \left[D_k r_t^k - (D_k - D_j) e_{t+j}^{k-j}\right] / D_j - r_t^j \tag{16}$$

where r_t^k and r_t^j are the spot market yields at time *t* on *k* and *j* period bonds and e_{t+j}^{k-j} is the expectation at time *t* of the yield on a (*k*-*j*) month bond *j* months ahead. D_k and D_j are the duration of *k* and *j* month bonds priced at par. The interest rate expectation is derived from commercial forecast survey reports. The holding premium is calculated for a 6 month holding of a 9-month Treasury Bill and for a 6 month holding of a 20-year bond, thus providing estimates of the risk premia at both the short and long ends of the yield curve. For the Treasury Bill, the risk premium varies between a peak of almost 2.5 per cent in the early 1980s to marginally negative in 1971 (although it is predominantly between zero and one per cent) while for the long bond, it varies from over 18 per cent in 1981 to around minus 9 per cent in 1972, 1984 and 1988 (and there are long periods for which the calculated premium is negative).[13] Lagged terms in these holding premia are included as additional explanatory variables in both neo-classical and *q* models of producer investment in durables and in plant and machinery using a first-order autoregressive least squares estimation method for quarterly

[13] The positive relationship between the risk premium embedded in the interest rate term structure and ARCH estimates of the conditional variance of interest rates has been shown by Engle *et al* (1987).

data from 1969 to 1989. In almost all cases, significant negative coefficients are found for the two holding premium variables. Quantitatively, the magnitudes of the deleterious effect of both the long and short term risk premia on investment are similar and quite large. On average, a one standard deviation increase in the long bond risk premium reduces investment expenditures by 0.254 standard deviations. The equivalent figure for an identical increase in the Treasury Bill risk premium is an investment reduction of 0.241 standard deviations.

Ferderer and Zalewski (1994) perform a similar exercise on a historical data set for the US Great Depression between 1929 and 1940. They find comparable effects of the risk premium on investment for this period. A 1 per cent rise in the risk premium of holding a 15-year bond for three months into the future reduces producer durables investment by 0.34 per cent. Ferderer and Zalewski also show that their calculated risk premium corresponds closely to gold price and gold imports volatility, during a time of great instability in the gold market due to Britain’s departure from the Gold Standard in 1931.

Information concerning investor uncertainty contained within movements in the international gold price is also exploited by Alan Carruth, Andy Dickerson and Andrew Henley (1997) to explain recent movements in UK aggregate corporate fixed investment. There is some evidence for a negative relationship between investment and this indicator of uncertainty in both the long-run and the short-run. A short-run dynamic model is also estimated which incorporates a measure of the unconditional variance from the abnormal return to holding gold.[14] The volatility in the excess returns to gold is found to have a negative short-run impact on investment. Only one study so far has attempted to use “attitudinal” data to proxy uncertainty, and this is the Italian study of Guiso and Parigi (1996). They use information on each firm’s subjective assessment of the evolution of its product demand one and three years ahead.

Published studies to date reveal a number of very different approaches to the construction of proxy measures for uncertainty, and a lack of consensus about best practice. A

---

[14] Abnormal returns to gold are modelled in a similar fashion to that utilised in the finance literature (Norman Strong, 1992).

high proportion of studies make use of some form of derived regressor, which raises concern over the adequacy of model specification and estimation. A major implication of these concerns is that correlation of investment with a volatility measure, particularly in aggregate empirical analyses, may reveal more about correlation with some omitted underlying “fundamental”. One further issue that warrants comparative assessment is that of backward- versus forward-looking measures of uncertainty. A reasonable prior might be that forward-looking measures will reveal rather more about both the range and the magnitude of uncertainty facing decision makers.

5. Non-linearities and threshold effects

If the relationship between investment and other fundamentals is non-linear in the presence of irreversibilities, then traditional linear investment specifications ought to be misspecified. Moreover, such non-linearities ought to be identifiable through appropriate empirical representation.[15] Andrew Abel and Janice Eberly (1994) integrate irreversibilities in investment with adjustment costs in a generalised model of investment under uncertainty. In contrast to traditional models with quadratic adjustment costs (which generate a linear relationship between investment and (marginal) q), they show that in the presence of general adjustment costs and irreversibilities, the relationship between investment and q will be non-linear. Under adjustment costs that are fixed, linear or convex, they establish that the relationship between q and the rate of investment will follow the pattern shown in Figure 2.

---

[15] In support of this proposition, some recent highly disaggregated empirical work using US plant level data has shown that rates of adjustment to desired levels of investment are neither smooth nor constant (Caballero *et al*, 1995; Cooper *et al*, 1995).

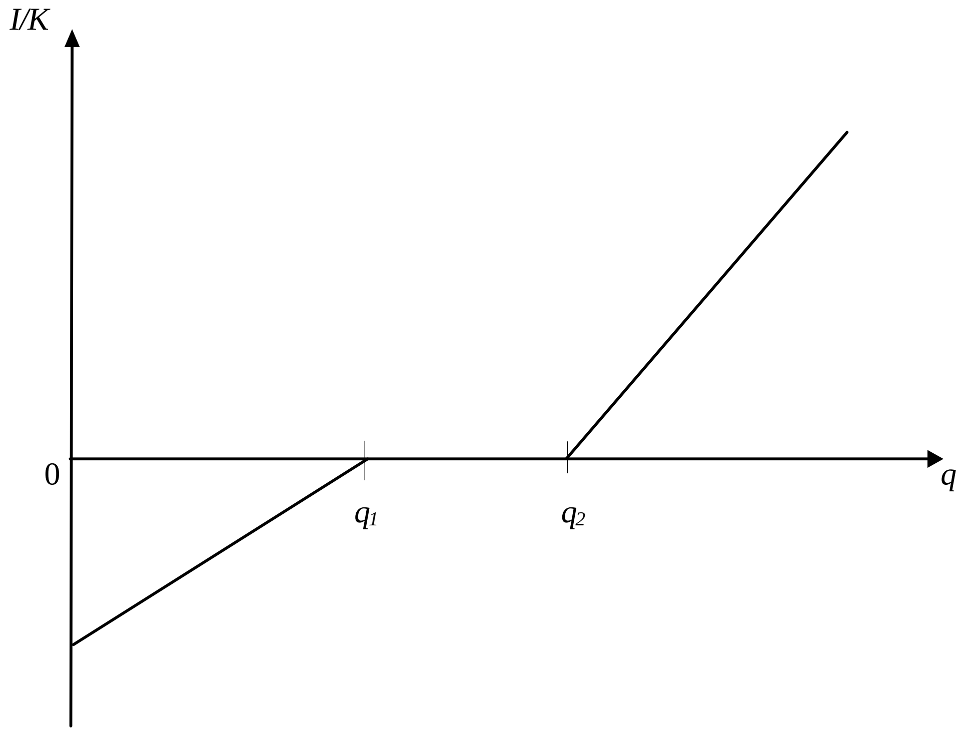

Figure 2:

Relationship between rate of investment and *q* under non-quadratic adjustment costs

Thus, there are potentially three regimes for investment; if $q < q_1$, gross investment is negative (ruled out by assumption in all of the irreversibility literature); if $q_1 < q < q_2$, investment is zero (this is the innovation which the irreversibility literature has introduced); if $q > q_2$, then investment is positive and increasing in q (which is the standard result in the literature). Note that in the regions where investment does respond to q, the relationship need not necessarily be linear. The important feature is that, while investment is a non-decreasing function of q, the relationship now contains a region (between q_1 and q_2) in which the optimal rate of investment is zero and is insensitive to movements in q. In effect, within this range, returns are neither high enough to justify investing, nor low enough to warrant disinvesting. These ideas are illustrated in Abel and Eberly (1997) in which they derive a closed-form solution for optimal investment in the special case of a competitive firm with constant returns production and a particular parametric specification for adjustment costs.[16]

Such misspecification is investigated empirically for the US using COMPUSTAT data by Abel and Eberly (1995) and by Eberly (1997) using a company-level international comparative panel dataset (the Global Vantage data). Eberly's (1997) data relate to publicly traded companies between 1981 and 1994 in 11 countries (Belgium, Canada, France, Germany, Japan, Netherlands, Spain, Sweden, Switzerland, UK and US). She estimates and compares the results of both linear and non-linear investment functions, where the latter permits fixed, linear and convex adjustment costs. The non-linear specification (under the assumption of homogenous capital) takes the following form:

$$\ln(I/K)_{i,t} = \beta_1 \ln(\beta_0 + q_{i,t} + \alpha p_{i,t}) + \gamma_i + \varepsilon_{i,t} \qquad (17)$$

where p is the (tax adjusted) implicit price deflator for non-residential investment relative to the producer price index, β_0 captures the linear component of adjustment costs, β_1 reflects any non-linearity, and γ_i is a firm-specific fixed effect. Estimation of equation (17) is performed separately for each country (since accounting standards differ between countries) using non-linear least squares, with differencing to remove the firm-specific fixed effect. A further extension to incorporate heterogeneous capital types is also specified, which introduces an additional term in (17) for the different thresholds of each capital type. A significant role for non-linearities in adjustment costs is found for all countries except the Netherlands and Spain and in eight countries, linearity is rejected at the 95% level of confidence or above.[17] Moreover, the predictive performance of the non-linear models is superior to that of the linear specification, particularly where there is evidence of significant

---

[16] Of course, following Fumio Hayashi (1982), it is typically assumed that production and costs of investment are linearly homogeneous in K thus implying that marginal and average q are equal.

[17] One concern is that the degree of non-linearity estimated for the US by Eberly (1997) indicates that adjustment costs are less convex than quadratic ($\beta_1 > 1$) using the Global Vantage Data, while Abel and Eberly (1995) find that adjustment costs are more convex than quadratic ($\beta_1 < 1$) using COMPUSTAT data. The discrepancy may be due to the rather different sample coverage - see Eberly (1997, pp.1075-1076).

non-linearities. In addition Eberly (1997) performs some aggregation simulations in which she compares the predictive performance of a traditional aggregate linear investment function with the corresponding aggregate equivalents of the non-linear specifications, pooling the data across countries. Her results indicate that higher moments of q add significantly to the performance of the aggregate investment function. Thus she concludes that there are important fixed or other non-quadratic costs of adjusting investment rates, consistent with the presence of irreversibilities.

This theme is also taken up by Pasquale Scaramozzino (1997) who, using a British company panel, investigates the effects of irreversibility on the relationship between investment and q in the context of binding constraints on the firm's financial policy. Irreversibility is incorporated in a theoretical model of corporate investment as a non-negativity constraint on the level of gross investment. This is somewhat arbitrary given that a corporate investment flow during a typical 12 month accounting period may be an aggregation of some reversible and other irreversible projects but such aggregation issues are ignored by Scaramozzino. The constraint raises the shadow cost of capital above the level derived from the standard first-order conditions of a profit-maximising firm. If, in addition, the firm faces a minimum dividend payout constraint, then irreversibility exacerbates the opportunity cost of this constraint. This means that, for firms with a binding irreversibility constraint, the standard relationship between investment and marginal q will no longer hold (even though adjustment costs are assumed to be quadratic). Any attempt to estimate such a relationship will therefore result in model misspecification. If a dividend constraint is also present, then potential misspecification is doubly likely. Scaramozzino specifies an econometric relationship between the rate of investment to capital and (empirically measurable) average q, incorporating some model dynamics. Partitioning of the sample into firms with below median rates of investment and below median rates of dividend payout is used to identify those firms which are likely to be constrained, and, thus, those firms for which the standard q model is expected to be misspecified. Estimates of the investment function do indeed suggest that the q model performs poorly for firms where the irreversibility constraint and/or the use-of-funds constraint may be expected to bind, while for firms which are probably unconstrained (those with both higher than median investment and higher than median dividend payout), the standard (linear) q model performs satisfactorily.

An investigation of non-linearity, in the spirit of Chirinko's implicit dynamics, is provided by Price (1996) who extends his earlier aggregate British work by introducing non-linear dynamic adjustment. His hypothesis is that, during times of greater uncertainty, the speed at which firm's adjust to their desired steady-state level of investment will be slower, and may depend on whether the degree of uncertainty is above or below some critical threshold, τ. The resulting specification is of the following form:

$$\Delta i_t = a_0 + a_1 \Delta i_{t-1} - (c_0 + c_1 \sigma^2_{t-1} d)[i_{t-1} - b_1(y_{t-1} - y^*_{t-1}) - b_2 \sigma^2_{t-1}] + u_t \qquad (18)$$

where a_i, b_i and c_i are parameters, y^* is (log) capacity output obtained from using the Hodrick-Prescott filter technique applied to manufacturing output and d is a dummy variable defined as:

$$\begin{aligned} d &= 1 \text{ if } \sigma^2_{t-1} \geq \tau \\ &= 0 \text{ otherwise} \end{aligned} \qquad (19)$$

The estimating equation, (18), is heavily parameterised and the non-linearity necessitates the use of maximum-likelihood estimation conditional on a pre-set value for the threshold parameter τ, and with a confidence interval for τ obtained through a bootstrapping technique. Uncertainty is found to have an even larger long-run impact (b_2) on investment than in his earlier study. With zero output variance, investment would be 60 per cent higher in the long-run. However, the impact of uncertainty on the speed of adjustment (c_1), while statistically significant, is quantitatively small.

An alternative empirical strategy for verifying the implications of irreversibility theory is to investigate the existence of threshold effects. However a difficulty with this approach is the need to impose a great deal of model structure. One attempt at such an exercise is performed by Pindyck and Andrés Solimano (1993). They calculate a measure of the marginal profitability of capital and use the volatility in this series as a proxy for uncertainty together with its extreme values as an indicator of the threshold at which investment will be triggered. They begin by assuming that the economy at time t comprises a continuum of small firms, N_t, each producing a (possibly) differentiated product. N_t fluctuates over time with the entry and exit of firms. Q_t is an index of aggregate consumption equal to the aggregation of individual firms' outputs, $A_{i,t}$, which derive from a CES production function:

$$Q_t = \left[\int_0^{N_t} (A_{i,t})^\rho \, di \right]^{1/\rho}, \quad 0 < \rho < 1 \tag{20}$$

The firms' outputs, $A_{i,t}$, are assumed to be decomposable into the product of an average or aggregate component, A_t, and a firm-specific or idiosyncratic component, a_i, such that:

$$A_{i,t} = A_t a_i \quad \text{with} \quad \int_0^{N_t} a_i \, di = N_t \tag{21}$$

Thus $A_t = \frac{Q_t}{N_t}$ is average firm output and a_i is the output of firm i relative to the average (and is assumed to be time-invariant). Both A_t and a_i are assumed to follow a stochastic process. The aggregate price level which all firms expect to face is determined by an iso-elastic aggregate demand relationship

$$P_t = Z_t (Q_t)^{-1/\eta} \tag{22}$$

where Z_t is an exogenous stochastic aggregate demand shock. The exogenous rate of firm failure or exit is δ, such that, with no entry, $\frac{dN_t}{dt} = -\delta N_t$. Firms only discover their relative productivity, and thus a_i, after entry and so, *ex ante*, all expect to face the same price, even if *ex post* prices may vary between firms.

Irreversibility is introduced through the existence of a sunk entry cost, F. For entry to occur requires that the expected future marginal productivity of capital exceeds the sunk cost:

$$F \le E_0 \left[\int_0^{\infty} P_t A_t e^{-(r+\delta)t} \, dt \right] \tag{23}$$

where r is the discount rate and E_0 denotes expectations formed at time 0. For a firm considering entry, its expected[18] contemporaneous marginal profitability of capital is given by the average value of output, B_t, where:

$$B_t = P_t A_t = Z_t A_t^{(\eta-1)/\eta} N_t^{-1/\eta} \tag{24}$$

---

[18] Recall that a_i is unknown *ex ante*.

Pindyck and Solimano assume that both A_t and Z_t follow (uncorrelated) geometric Brownian motions which allows them to derive a “trigger” level of the marginal profitability of capital at which the firm becomes indifferent between entry and holding the option to delay entry.

For their empirical implementation, Pindyck and Solimano calculate directly the marginal profitability of capital by assuming a competitive economy in which output, *Y*, (GDP plus imported materials) is determined by a constant returns to scale Cobb-Douglas technology:

$$Y = AK^{\alpha_K} L^{\alpha_L} M^{\alpha_M} \quad \text{with} \quad \alpha_K + \alpha_L + \alpha_M = 1 \tag{25}$$

where *K* is a capital stock estimate, *L* is a measure of aggregate labour input and *M* are imported raw materials. Denoting P_L and P_M as the real price of labour and imported materials respectively, then the marginal profitability of capital is given by:

$$\Pi_K = \alpha_K \alpha_L^{\alpha_L/\alpha_K} \alpha_M^{\alpha_M/\alpha_K} A^{1/\alpha_K} P_L^{-\alpha_L/\alpha_K} P_M^{-\alpha_M/\alpha_K} \tag{26}$$

Taking logs, and using lower case to denote logs of upper case variables:

$$\pi_{K,t} = \ln(\alpha_K \alpha_L^{\alpha_L/\alpha_K} \alpha_M^{\alpha_M/\alpha_K}) + \frac{a_t}{\alpha_K} - \frac{\alpha_L}{\alpha_K} p_{L,t} - \frac{\alpha_M}{\alpha_K} p_{M,t} \tag{27}$$

where $a_t = y_t - \alpha_K k_t - \alpha_L l_t - \alpha_M m_t$ is the Solow residual.

Using information on output, factor inputs, factor prices, and factor shares for 29 countries (16 from the OECD and 13 LDCs), Pindyck and Solimano calculate $\pi_{K,t}$ for 1962-1989 and use it to investigate the relationship between the volatility in returns and investment. Table 3, taken from Pindyck and Solimano (1993), shows the mean and standard deviation of the annual log change in the marginal profitability of capital, $\Pi_{K,t}$, together with mean private sector investment as a share of GDP for the period 1981 to 1989. There is some evidence in the Table that higher volatility in $\Pi_{K,t}$ is associated with lower investment ratios.[19] Pindyck

[19] The correlation between the standard deviation and the investment ratio is -0.4104, which is significant at the 5% level.

and Solimano confirm this impression by estimating pooled time-series cross-section regressions of investment share on the standard deviation and means of $\Delta\pi_{K,t}$ across the 29 countries and three time periods (1962-71, 1972-80 and 1981-89). A significant negative relationship is found between the investment ratio and volatility for the LDCs although not for the OECD countries. An increase in volatility in the LDCs of 0.05 is associated with a 2 per cent drop in the investment ratio, but across all countries this effect is only half as large and only significant at around the 10 per cent level.

As a proxy for the height of the investment "trigger", Pindyck and Solimano compute the average of the largest three of their 28 time-series observations on $\pi_{K,t}$ for each country, together with the average of the three values of $\pi_{K,t}$ corresponding to the years when the change in real capital stock was greatest. This exercise is also repeated for the largest six values in each case too. They use these indicators of the threshold to examine whether countries with more volatile marginal capital profitability do indeed require higher investment trigger thresholds as the theory predicts. In cross-sectional regressions of these extreme values of π_K on the standard deviations and means of $\Delta\pi_{K,t}$, a significant positive coefficient on the standard deviation is found, with the interpretation that an increase of 0.05 in the standard deviation of $\Delta\pi_{K,t}$ is associated with a 5-15% increase in the threshold. As the authors explain, the size of this effect is "qualitatively important (but not overwhelming)" (p. 281).[20] Further investigation is undertaken to identify what economic and political factors correlate with higher volatility in profitability. The only consistently significant variable (with a positive coefficient) is the mean inflation rate. This is confirmed in pooled time-series cross-section regressions of the investment ratio on a set of variables including the level and standard deviation of inflation (across the 12 months of each observation year). There would indeed appear to be some evidence for a negative impact of inflation as an indicator of uncertainty on the investment ratio for both low inflation and high inflation countries,

[20] However, as Hubbard (1994, p.1828) notes, the proxies for the threshold used by Pindyck and Solimano (i.e. the extreme values of $\pi_{K,t}$) will be correlated with the variance even if there is no causal relationship between investment and uncertainty. See also Eberly (1993).

although for the latter, the impact of inflation on the private sector investment ratio is not significantly different from zero.

Caballero and Pindyck (1996) apply the same methodology for calculating the marginal profitability of capital as used in Pindyck and Solimano (1993) to both 2-digit and 4-digit US industrial data. Shocks at the more disaggregated 4-digit level should have a larger idiosyncratic component, and so the volatility in the marginal profitability of capital (equation 27) should be greater at the higher level of disaggregation. Consequently, the implied premium that investors require to make an investment commitment over and above the neoclassical cost of capital will also be higher at the 4-digit level. Table 4 reports results presented in Caballero and Pindyck which indicate that this would indeed appear to be the case. The first column reports the standard deviation of the time series of observations on the marginal profitability of capital for each 2-digit sector. The second column reports the mean standard deviation calculated across the 4-digit industries that make up each 2-digit sector. The estimates in the second column are always higher, and typically by a factor of two or three. The implication of this result is that any relationship between uncertainty and investment is likely to be more evident in disaggregated data.

Pindyck and Solimano's work is a thorough and comprehensive analysis of the relationship between aggregate investment and volatility. They incorporate a good deal of international comparative material, and use an innovative technique to estimate directly the volatility in the marginal profitability of capital and the investment threshold. However, this comes at the expense of having to assume a considerable amount of structure (e.g. Cobb-Douglas, constant returns technology within a perfectly competitive economy with free entry, etc.) which cannot be directly or separately tested. Reservations concerning the appropriateness of the assumed structure for all of the 29 countries considered, together with the sensitivity of the calculation of Π_K to measurement errors in factor prices (Robert Hall, 1993) imply that one should perhaps be cautious of placing too much emphasis on their results. Furthermore, their findings are, at best, only weakly supportive of the predictions of the theory since, while they find that increased volatility in the returns to investment does appear to lead to a higher "required" rate of return, the size of this effect is quite small, and, in general, is not well-determined.

While such work does not represent a direct test of the impact of increased uncertainty on investment, if that investment is irreversible, it does nevertheless offer a promising avenue for addressing the problem of inconsistency in the traditional approaches to modelling investment. So far it has established that *q*-models of investment are more likely to break down when irreversibilities are present. However the problem for empirical research in this direction is that a full model of investment threshold behaviour will be very complex in structure and consequently empirically intractable.

6. Modelling the impact of uncertainty on investment lags

A very different and novel econometric approach to examining the implication of irreversibility for investment is to investigate investment lags in a hazard or duration modelling framework. Cooper *et al* (1995) examine this issue using a very large plant-level longitudinal data set for the United States on 6,900 plants over the period 1972-1991. Investment rates at the plant level are typically observed to be lumpy, with only a fraction of plants engaged in a large investment in any year. A duration modelling approach is therefore taken to model the duration between investment spikes, defined variously as a relative or absolute threshold. The results obtained suggest the duration between investment spikes is reduced during times of higher economic activity.

The duration modelling approach is taken in the context of a very specific form of investment, namely offshore oil production, in two very similar papers by Carlo Favero *et al.* (1994) and Stan Hurn and Robert Wright (1994). Both papers exploit longitudinal information about the length of time between oil field discovery and development for production in the North Sea, and explicitly incorporate a proxy for economic uncertainty. The discovery of an oil field can be regarded as providing an option to invest. The investment in the production facility can be regarded as an irreversible commitment. The empirical question is then whether the development lag between discovery and production is related to uncertainty over the real price of oil. The conditional probability that an oil field development begins at time $t+\Delta t$ given that it has not started at time t (the "hazard" rate) can be expressed as:

$$h(t, X, \alpha, \beta) = h_0(t, \alpha)\exp(X\beta) \tag{27}$$

where $h_0(t,\alpha)$ is the baseline hazard function, X is a vector of (possibly time-varying) covariates and β is the vector of estimated coefficients capturing the effect on the hazard rate of these covariates. Favero *et al.* estimate both Cox and Weibull specifications for the baseline hazard h_0. Included among the covariates is a measure of real post-tax oil price volatility to proxy uncertainty. This volatility measure is computed as the squared regression standard error obtained from recursive estimates of a price expectations model. Expectations are assumed to be formed either rationally (and thus oil price follows a random walk) or adaptively (with the degree of persistence in the adaptive expectations model obtained through a grid search), and the recursive estimation runs from 1960, when oil field exploration was first licensed on the UK Continental Shelf, up to the time when investment activity began in the particular field.[21] Heterogeneity between units of observation is a common source of bias in hazard models (see, for example, Tony Lancaster, 1990) and thus Favero *et al.* also include a number of other covariates including the size of the oil field, the operating company size, the difficulty of exploitation (sea depth) and the size of potentially jointly extracted gas reserves alongside the expected price and its associated volatility. Their sample consists of 53 fields that were on stream in 1989. As in previous work using this data (Hashem Pesaran, 1990; Favero, 1992), adaptive expectations are (marginally) preferred over rational expectations for the formation of real oil price expectations. There is some evidence that the relationship between price volatility and the development lag is non-linear: if the expected oil price is low, then increased price volatility reduces the development lag, but if the expected oil price is high, then increased price volatility increases the length of the lag. This suggests that, at high oil prices, greater uncertainty will increase the tendency for oil companies to exercise the option to delay oil field development, but at lower expected price levels, the positive effect of uncertainty on the marginal value of capital dominates the option effect. However, these results are not consistent across different specifications of the underlying hazard function, nor between the two different models of price expectations formation, and, somewhat surprisingly, none of the physical characteristics of the oil field nor the size of the operating company appear to be important in determining the development lag.

---

[21] Or, more precisely, the date of the government awarding Annex B approval, which signals the start of the technical development of the field.

Hurn and Wright (1994) additionally incorporate a vector of operating company fixed-effects to account for any unobserved heterogeneity across oil field operators (for example, in their differing perception of risk), and also extend the vector of geological and other characteristics of the oil field. Their sample comprises 108 wells observed in 1991. Assuming rational expectations, real oil price volatility is proxied by the squared residuals from a random walk specification. In their results, they obtain a negative but statistically insignificant relationship between the length of the development lag and the measure of price volatility. However, in contrast to the results of Favero *et al.*, the various geological covariates provide a more important explanation for the length of this lag.

One possible explanation for the fact that neither of these two studies can find any strong or robust evidence for the importance of uncertainty as proxied by oil price volatility on the timing of large scale investments is that the short-term oil price volatility (calculated using monthly data in the two studies discussed above) may not be an appropriate measure of uncertainty for long-term investment decisions such as the development of an oil field (Favero *et al.*, 1994, p. S107). Empirical modelling of the interval duration between discovery of an investment opportunity and the decision to commit funds to that opportunity, and the effect of uncertainty on this interval, gets much closer to the nub of investment irreversibility than attempts to correlate investment expenditures with uncertainty proxies. This said, empirical investigation along these lines makes for very specific data requirements which may only be observable in specialised cases such as oil exploration. The absence of any strong conclusions to emerge from work undertaken so far suggests that further evaluation, and the identification of suitable data sources, is needed.

7. Are irreversibility effects conditional on market structure?

Caballero’s (1991) demonstration that the existence of imperfect competition is a necessary condition for any irreversibility-driven negative relationship between investment and uncertainty ought to introduce an important secondary consideration to empirical work, particularly of a disaggregated nature. Is the sign of the uncertainty-investment relationship conditional on the degree of market power? Very little evaluation of this has been undertaken so far.

At the industry level of aggregation Vivek Ghosal and Prakash Loungani (1996) use the 4-digit US Annual Survey of Manufactures/Census of Manufactures database with an industrial price volatility measure constructed as the standard error from a second order autoregression for a 14 year rolling sample (i.e. for 1958-1971, 1959-1972, …, 1975-1988) for each of 254 industries.[22] The estimated coefficient on the price uncertainty proxy is small and insignificantly different from zero for the full sample of industries. However, for "competitive" industries (those with a 4-firm seller concentration level (CR4) below 40%), it is significantly negative and implies an elasticity of the investment rate with respect to price uncertainty of -0.055. For "highly competitive" industries (CR4 ≤ 20%), the elasticity is -0.116. In contrast, in "non-competitive" industries (those with a high degree of concentration), price uncertainty would appear to have no discernible impact on investment. This conclusion is rather surprising in the light of the theory and appears robust to various alternative constructions of price uncertainty. It is also robust to different dynamic specifications of the estimating equation, and to the inclusion of variables to capture credit-rationing. On the other hand, Guiso and Parigi (1996) present results of a sample partition by the size of the firm's price-cost margin for their Italian company cross-section, and report results which are consistent with the theoretical prior. Uncertainty has over twice as large a negative effect on investment for firms with above average market power. Clearly this is an avenue for further work.

8. Conclusions and Assessment

This paper has reviewed the recent developments in modelling investment under uncertainty in the presence of irreversibilities and has surveyed the growing literature that has endeavoured to examine empirically the insights that this new theory has provided. While irreversibilities can be regarded as another form of adjustment costs (whereby the cost for negative investment is assumed to be infinite), the recent innovations in the literature, as

---

[22] The sample of industries is rather smaller than the 450 used by Huizinga (1993), since Ghosal and Loungani subsequently use information on seller concentration which is only consistently available for a subset of 4-digit industries.

expounded by Dixit and Pindyck (1994) in particular, reveal a number of important implications when irreversibilities are considered in terms of their impact on the value of the option to delay an investment commitment. In particular, the consequence for the timing of investment decisions and the existence of thresholds in excess of the standard NPV criterion have extended economists understanding of the investment process at both the micro and macro level.

The renewed attention on the processes underlying investment decisions has given rise to more general models of adjustment costs. These have highlighted the fact that non-quadratic adjustment costs will imply that investment will not be linearly related to *q*, or other fundamentals correlated with *q*. As a consequence, we have perhaps enhanced our understanding of why standard *q*-models of investment (which are linear in variables having been derived from the assumption of quadratic adjustment costs) typically perform so poorly. Once we allow for irreversibilities, then adjustment costs cannot be quadratic, and hence standard models of investment will be fundamentally misspecified.

Empirically implementing the ideas generated by this new approach has, however, been problematic. The fundamental difficulty is that while irreversibilities will influence the timing of investment decisions and the threshold necessary before investment will take place, the theory says nothing about the level of investment *per se*. Thus any test of the impact of irreversibilities on investment can only be implemented as a joint test of irreversibilities together with the validity of the underlying specification chosen for the level of investment under uncertainty. In many cases the specification for this underlying capital accumulation process is rather *ad hoc*, leading the reader to ponder whether the sign and size of any observed uncertainty effect is really an artefact of model misspecification. Additionally, there is considerable debate about how best to proxy uncertainty - and indeed, what precisely we might mean by uncertainty in the context of investment decisions - with a myriad of possible measures constructed in a number of different ways being utilised in the extant literature. Issues are further complicated by the fact that the direction of the impact of uncertainty on investment is theoretically ambiguous.

While the aggregate and sectoral literature has yielded some important findings, further progress in empirically validating these new models would appear to be most likely to be derived from firm- or plant-level longitudinal data. Not only is there evidence of considerable heterogeneity which more aggregate studies mask, but investment decisions are predominantly made at the micro level, and thus it is at this level of disaggregation that any measurable impact of irreversibilities on investment seems most likely to be detected. Furthermore, it is at this level that idiosyncratic uncertainties are most prevalent. Methodologically, the more direct methods of investigating the impact of irreversibilities under uncertainty, such as those that attempt to measure the threshold at which investment is triggered and those which explicitly model the delay before investment is undertaken, appear to offer the greatest likelihood of successfully testing the key predictions of the theory. Such methods should typically be less sensitive to alternative specifications of the underlying investment functions (particularly where these are encompassing in their approach), and their focus on the primary conjectures of the theory should enable the applied economist to confront theory with data in a positive manner.

While the development and implementation of models of irreversible investment under uncertainty is still in its infancy, and much remains to be done, recent progress has been rapid and extremely productive. What we now know about investment in an uncertain world has improved in recent years, and looks certain to improve further in the years to come.

BIBLIOGRAPHY

Abel, Andrew B. "Optimal investment under uncertainty", *American Economic Review*, 1983, 73, 228-33.

________ and Eberly, Janice C. "A unified model of investment under uncertainty", *American Economic Review*, 1994, 84, 1369-1384.

________________________ "Investment and *q* with fixed costs: an empirical analysis", *mimeo*, Wharton School, University of Pennsylvania, 1995 [cited in Eberly (1997)].

________________________ "An Exact Solution for the Investment and Value of a Firm facing Uncertainty, Adjustment Costs and Irreversibility", *Journal of Economic Dynamics and Control*, 1997, 21, 831-852.

Arrow, Kenneth J. "Optimal capital policy with irreversible investment", in J.N. Wolfe (ed.), *Value, Capital and Growth: Papers in Honour of Sir John Hicks*, Edinburgh: Edinburgh University Press. 1968

Bernanke, Ben S. "Irreversibility, uncertainty and cyclical investment", *Quarterly Journal of Economics*, 1983, 98, 85-106.

Bertola, Giuseppe and Caballero, Ricardo J. "Irreversibility and aggregate investment", *Review of Economic Studies*, 1994, 61, 223-246.

Blundell, Richard; Bond, Stephen; Devereux, Michael and Schiantarelli, Fabio. "Investment and Tobin's Q: Evidence from company panel data", *Journal of Econometrics*, 1992, 51, 233-257.

Bond, Stephen and Meghir Costas. "Dynamic investment models and the firm's financial policy", *Review of Economic Studies*, 1994, 61, 197-222.

Brainard, William C.; Shoven, John B. and Weiss Laurence. "The financial valuation of the return to capital", *Brookings Papers on Economic Activity*, 1980, 2, 453-502.

Caballero, Ricardo J. "On the sign of the investment-uncertainty relationship", *American Economic Review*, 1991, 81, 279-88

________________; Engel, Eduardo M. R. A. and Haltiwanger, John C. "Plant-level adjustment and aggregate investment dynamics", *Brookings Papers on Economic Activity*, 2, 1995, 1-39.

________________ and Pindyck, Robert S. "Uncertainty, investment and industry evolution", *International Economic Review*, 1996, 37(3): 641-662.

Campa, Jose M. "Entry by foreign firms in the United States under exchange rate uncertainty", *Review of Economics and Statistics*, 1993, 75(4): 614-622.

_______________ and Goldberg, Linda S. "Investment in manufacturing, exchange rates and external exposure", *Journal of International Economics*, 1995, 38, 297-320.

Carruth, Alan; Dickerson, Andrew and Henley, Andrew. "Econometric modelling of UK aggregate investment: the role of profits and uncertainty", *Aberystwyth Economic Research Paper*, 1997, No. 97-06, University of Wales Aberystwyth.

Chenery, Hollis B. "Overcapacity and the acceleration principle", *Econometrica*, 1952, 20(1), 1-28.

Chirinko, Robert S. "Business fixed investment spending: modeling strategies, empirical results and policy implications", *Journal of Economic Literature*, 1993, 31, 1875-1911.

Cooper, Russell; Haltiwanger, John and Power Laura. "Machine replacement and the business cycle: lumps and bumps", *NBER Working Paper*, 1995, No. 5260.

Craine, Roger. "Risky business: the allocation of capital", *Journal of Monetary Economics*, 1988, 23, 201-18.

Dixit, Avinash K. "Entry and exit decisions under uncertainty", *Journal of Political Economy*, 1989, 97, 620-638.

__________"Investment and hysteresis", *Journal of Economic Perspectives*, 1992, 6, 107-32.

__________ *The Art of Smooth Pasting*, Switzerland: Harwood Academic Publishers, 1993.

__________ and Pindyck, Robert S. *Investment Under Uncertainty*, 1994, Princeton: Princeton University Press.

Driver, Ciaran and Moreton, David. "The influence of uncertainty on aggregate spending: an empirical analysis", *Economic Journal*, 1991, 101, 1452-59.

_____________________________ *Investment, Expectations and Uncertainty*, 1992, Oxford: Basil Blackwell.

Driver, Ciaran; Yip, Paul and Dakhil, Nazera. "Large company capital formation and effects of market share turbulence: micro-data evidence from the PIMS database", *Applied Economics*, 1996, 28, 641-651.

Eberly, Janice C. "Comment on "Economic Instability and Aggregate Investment", by R. Pindyck and A. Solimano", *NBER Macroeconomics Annual*, 1993, MIT Press.

_____________ "International evidence on investment and fundamentals", *European Economic Review*, 1997, 41, 1055-1078.

Eisner, Robert and Strotz, Robert H. "Determinants of business investment", in Commission on Money and Credit, *Impacts of Monetary Policy*, 1993, Englewood Cliffs, NJ: Prentice-Hall, 59-233.

Engle, Robert F. "Estimates of the variance of US inflation based upon the ARCH model", *Journal of Money Credit and Banking*, 1983, 15, 286-301.

________________ and Granger, Clive W. J. "Cointegration and error correction: representation, estimation and testing", *Econometrica*, 1987, 55, 251-76.

________________ ; Lillien, David and Robbins Russell P. "Estimating time varying risk premia in the term structure: the ARCH-M model", *Econometrica*, 1987, 55, 391-407.

Episcopos, Athanasios. "Evidence on the relationship between uncertainty and irreversible investment", *Quarterly Review of Economics and Finance*, 1995, 35(1), 41-52.

Favero, Carlo, A. "Taxation and the Optimisation of Oil Exploration and Production: the U.K. Continental Shelf", *Oxford Economic Papers*, 1992, 44, 187-208.

_______________; Pesaran, M. Hashem and Sharma, Sunil. "A duration model of irreversible investment: theory and empirical evidence", *Journal of Applied Econometrics*, 1994, 9, S95-S112.

Ferderer, J. Peter. "The impact of uncertainty on aggregate investment spending: an empirical analysis", *Journal of Money, Credit and Banking*, 1993, 25, 30-48.

_______________ "Oil price volatility and the macroeconomy", *Journal of Macroeconomics*, 1996, 18(1), 1-26.

_______________ and Zalewski, David A. "Uncertainty as a propagating force in the Great Depression", *Journal of Economic History*, 1994, 54(4), 825-849

Ghosal, Vivek and Loungani, Prakash. "Product market competition and the impact of price uncertainty on investment: some evidence from US manufacturing industries", *Journal of Industrial Economics*, 1996, 44(2), 217-228.

Goldberg, Linda S. "Exchange rates and investment in United States industry", *Review of Economics and Statistics*, 1993, 75(4), 575-589.

Guiso, Luigi and Parigi Giuseppe. "Investment and demand uncertainty", *Discussion Paper* No. 1497, 1996, Centre for Economic Policy Research, London.

Hall, Robert E. "Comment on "Economic Instability and Aggregate Investment", by R. Pindyck and A. Solimano", *NBER Macroeconomics Annual*, 1993, MIT Press.

Hartman, Richard. "The effects of price and cost uncertainty on investment", *Journal of Economic Theory*, 1972, 5, 258-66.

Hayashi, Fumio. "Tobin's marginal *q* and average *q*: a neoclassical interpretation", *Econometrica*, 1982, 50, 213-24.

Hubbard, R. Glenn. "*Investment Under Uncertainty*: keeping one's options open", *Journal of Economic Literature*, 1994, 32, 1816-31.

Huizinga, John. "Inflation uncertainty, relative price uncertainty, and investment in US manufacturing", *Journal of Money, Credit and Banking*, 1993, 25, 521-54.

Hurn, A. Stan and Wright, Robert, E. "Geology or economics? Testing models of irreversible investment using North Sea Oil Data", *Economic Journal*, 1994, 104, 363-71.

Jorgenson, Dale W. "Econometric studies of investment behaviour: a survey", *Journal of Economic Literature*, 1971, 9, 1111-47.

Koyck, Leen M. *Distributed Lags and Investment Analysis*, 1954, Amsterdam: North Holland.

Lancaster, Tony. *The Econometric Analysis of Transition Data*, 1990, Cambridge: Cambridge University Press.

Leahy, John and Whited, Toni M. "The effect of uncertainty on investment: some stylized facts", *Journal of Money, Credit and Banking*, 1996, 28(1), 64-83.

Lucas, Robert E. "Adjustment costs and the theory of supply", *Journal of Political Economy*, 1967, 75(4), 321-34.

McDonald, Robert and Siegel, Daniel R. "The Value of Waiting to Invest", *Quarterly Journal of Economics*, 1986, 101(4), 707-27.

Mills, Terence C. *The Econometric Modelling of Financial Time Series*, 1993, Cambridge: Cambridge University Press.

Nickell, Stephen J. "On the role of expectations in the pure theory of investment", *Review of Economic Studies*, 1974, 41(1), 1-19.

________________ *The Investment Decisions of Firms*, 1978, Cambridge: Cambridge University Press.

Pagan, Adrian and Ullah, Aman. "The econometric analysis of models with risk terms", *Journal of Applied Econometrics*, 1988, 3, 87-105.

Pesaran, M. Hashem. "An econometric analysis of exploration and extraction of oil in the U.K. continental shelf", *Economic Journal*, 1990, 100, 367-391.

Pindyck, Robert S. (1986), "Capital risk and models of investment behaviour", 1986, *mimeo*, Sloan School of Management, MIT.

________________ "Irreversibility, uncertainty and investment", *Journal of Economic Literature*, 1991, 29, 1110-48.

________________ and Solimano, Andrés. "Economic instability and aggregate investment", *NBER Macroeconomics Annual*, 1993, 259-303.

Price, Simon. "Aggregate uncertainty, capacity utilisation and manufacturing investment", *Applied Economics*, 1995, 27, 147-154.

____________"Aggregate uncertainty, investment and asymmetric adjustment in the UK manufacturing sector", *Applied Economics*, 1996, 28, 1369-79.

Scaramozzino, Pasquale. "Investment, irreversibility and finance constraints", *Oxford Bulletin of Economics and Statistics*, 1997, 59(1), 89-108.

Shiller, Robert J. *Market Volatility*, 1989, Cambridge MA: MIT Press.

____________ and McCulloch, Huston J. "The term structure of interest rates", *National Bureau of Economic Research Working Paper*, 1987, No. 2341, August.

Strong, Norman. "Modelling Abnormal Returns: A Review Article", *Journal of Business Finance and Accounting*, 1992, 19, 533-553.

Tobin, James. "A general equilibrium approach to monetary policy", *Journal of Money, Credit and Banking*, 1969, 1, 15-29.

Table 1

Aggregate Studies of Investment-Uncertainty Relationship

	Country	Model Fundamentals	Uncertainty Proxy	Uncertainty Effect
Pindyck (1986)	US	not available	lagged stock market returns	-ve
Driver and Moreton (1991, 1992)	UK	Investment-output ECM	unconditional variance of output and inflation	-ve
Goldberg (1993)	US	Investment = f(output, cost of capital)	exchange rate volatility	None / weak -ve
Huizinga (1993)	US	Investment = f(sales, profits, factor prices)	ARCH estimates of conditional variances of inflation, real wages and real profits	-ve (but +ve for profit uncertainty)
Episcopos (1995)	US	Growth in investment = f(growth in GDP, growth in real interest rate)	ARCH estimates of conditional variances of interest rates, stock mkt index, consumer spending, GDP deflator	-ve
Price (1995, 1996)	UK	Investment-output ECM	GARCH estimates of conditional variance of GDP	-ve
Ferderer (1993)	US	Jorgensen and q models	Risk premium computed from interest rate term structure	-ve
Ferderer and Zalewski (1994)	US (1930s)	Accelerator and q models	Risk premium computed from interest rate term structure	-ve
Carruth, Dickerson and Henley (1997)	UK	Investment, output, real interest rate ECM	Gold price and abnormal return to holding gold	-ve

Table 2

Disaggregate Studies of Investment-Uncertainty Relationship

	Country	**Level of disaggregation**	**Uncertainty Proxy**	**Uncertainty Effect**
Goldberg (1993)	US	2-digit industrial time series	Exchange rate volatility from ARMA model residuals	None/-ve
Campa and Goldberg (1995)	US	2-digit pooled time series/cross section	Exchange rate volatility from ARMA model residuals	None
Campa (1993)	US	4-digit panel of foreign direct investment data	Exchange rate volatility from ARMA model residuals	-ve especially for Japanese inward investment
Huizinga (1993)	US	4-digit cross-sections	Volatility of real wages, material prices and output prices from AR model residuals	-ve for wages and material prices/+ve for output prices
Ghosal and Lougani (1996)	US	4-digit panel	Volatility of output price from AR model residuals	-ve for low concentration industries
Leahy and Whited (1996)	US	Panel of manufacturing firms	Forecast share return volatility	Weak –ve /none
Driver, Yip and Dahkil (1996)	US	Panel of manufacturing plants	Market share volatility	Weak –ve /none
Guiso and Parigi (1996)	Italy	Cross-section of manufacturing firms	Firm’s perception about future product demand	-ve

Table 3

International comparisons of the marginal profitability of capital and investment shares, 1981-1989

	Std. Dev. of $\Delta\pi_{K,t}$	Mean of $\Delta\pi_{K,t}$	Investment/GDP
OECD:			
Austria	0.040	0.018	0.196
Belgium	0.140	0.039	0.146
Canada	0.118	0.039	0.186
Denmark	0.100	0.070	0.156
Finland	0.065	0.057	0.212
France	0.084	0.002	0.172
Germany	0.059	0.009	0.178
Ireland	0.144	0.047	0.178
Italy	0.060	-0.006	0.176
Japan	0.060	0.028	0.237
Netherlands	0.087	0.016	0.167
Norway	0.128	-0.025	0.224
Portugal	0.134	0.064	0.242
Spain	0.127	0.036	0.182
UK	0.074	0.014	0.154
USA	0.101	0.029	0.162
LDCs:			
Argentina	0.221	-0.069	0.056
Brazil	0.111	-0.026	0.133
Chile	0.150	-0.002	0.096
Colombia	0.043	-0.033	0.180
Hong Kong	0.102	0.022	0.227
Isreal	0.086	0.012	0.185
Korea	0.036	0.002	0.252
Malaysia	0.067	-0.034	0.324
Mexico	0.048	-0.020	0.129
Taiwan	0.080	0.002	0.126
Singapore	0.071	0.043	0.401
Thailand	0.033	-0.022	0.178
Venezuela	0.113	-0.057	0.082

Source: Pindyck and Solimano (1993), Table 1, p. 279.

Table 4

Effects of Aggregation on the Estimated Volatility in the Marginal Profitability of Capital

SIC/Industrial Sector	Standard deviation at 2-digit level	Mean standard deviation of constituent 4-digit industries
20 Food products	0.058	0.246
21 Tobacco	0.104	0.451
22 Textile mill products	0.118	0.366
23 Apparel and other products	0.076	0.304
24 Lumber and wood	0.168	0.327
25 Furniture and fixtures	0.125	0.258
26 Paper and allied products	0.113	0.217
27 Printing and publishing	0.061	0.192
28 Chemicals and allied products	0.088	0.224
29 Petroleum and coal products	0.201	0.256
30 Rubber and plastics	0.127	0.242
31 Leather and leather products	0.093	0.231
32 Stone clay and glass	0.099	0.236
33 Primary metal products	0.250	0.506
34 Fabricated metal products	0.123	0.277
35 Non-electrical machinery	0.160	0.301
36 Electrical machinery	0.147	0.264
37 Transportation equipment	0.184	0.403
38 Instruments and related	0.105	0.220
39 Miscellaneous manufacturing	0.109	0.255

Source: Caballero and Pindyck (1996), Table 1, p. 655.

Table 1

Aggregate Studies of Investment-Uncertainty Relationship

Study	Country	Model Fundamentals	Uncertainty Proxy	Uncertainty Effect
Pindyck (1986)	US	not available	lagged stock market returns	Negative
Driver and Moreton (1991, 1992)	UK	Investment-output ECM	Unconditional variance of output and inflation	Negative
Goldberg (1993)	US	Investment = f(output, cost of capital)	Exchange rate volatility	None / weak negative
Huizinga (1993)	US	Investment = f(sales, profits, factor prices)	ARCH estimates of conditional variances of inflation, real wages and real profits	Negative (but positive for profit uncertainty)
Episcopos (1995)	US	Growth in investment = f(growth in GDP, growth in real interest rate)	ARCH estimates of conditional variances of interest rates, stock market index, consumer spending, GDP deflator	Negative
Price (1995, 1996)	UK	Investment-output ECM	GARCH estimates of conditional variance of GDP	Negative
Ferderer (1993)	US	Jorgensen and *q* models	Risk premium computed from interest rate term structure	Negative
Ferderer and Zalewski (1994)	US (1930s)	Accelerator and *q* models	Risk premium computed from interest rate term structure	Negative
Carruth, Dickerson and Henley (1997)	UK	Investment, output, real interest rate ECM	Gold price and abnormal return to holding gold	Negative

Table 2

Disaggregate Studies of Investment-Uncertainty Relationship

Study	**Country**	**Level of disaggregation**	**Uncertainty Proxy**	**Uncertainty Effect**
Goldberg (1993)	US	2-digit industrial time series	Exchange rate volatility from ARMA model residuals	None / negative
Campa and Goldberg (1995)	US	2-digit pooled time-series cross-section	Exchange rate volatility from ARMA model residuals	None
Campa (1993)	US	4 digit panel of foreign direct investment data	Exchange rate volatility from ARMA model residuals	Negative especially for Japanese inward investment
Huizinga (1993)	US	4-digit cross-sections	Volatility of real wages, material prices and output prices from AR model residuals	Negative for wages and material prices; Positive for output prices
Ghosal and Lougani (1996)	US	4-digit panel	Volatility of output price from AR model residuals	Negative for low concentration industries
Leahy and Whited (1996)	US	Panel of manufacturing firms	Forecast share return volatility	Weak negative / none
Driver, Yip and Dahkil (1996)	US	Panel of manufacturing plants	Market share volatility	Weak negative / none
Guiso and Parigi (1996)	Italy	Cross-section of manufacturing firms	Firm’s perception about future product demand	Negative