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(Endogenous) Growth Slowdowns

Miguel Leon-Ledesma, Katsuyuki Shibayama

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Miguel León-Ledesma
University of Kent and CEPR

Katsuyuki Shibayama
University of Kent

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Abstract

We develop a model where temporary non-technology shocks can lead to permanent changes in the rate of growth of total factor productivity (TFP). The key ingredient of the model is a matching processes between basic researchers, product developers, and the stock of knowledge of the economy. In this context, search externalities generate vicious and virtuous cycles in R&D. The model has a unique equilibrium path but multiple balanced growth paths (BGPs) with different growth rates. After a deep or long-lived shock, the economy can transit between these BGPs, generating "super-hysteresis" in TFP. We calibrate the model in the context of the Japanese growth slowdown and show that, quantitatively, it can explain well the TFP growth decline after the financial crisis in the 1990s. The simultaneous occurrence of demographic shocks and a persistent but temporary financial crisis gave rise to a "wretched coincidence" resulting in the growth slowdown.

JEL Classification: O40, O49, E32.
Keywords: Growth slowdowns, permanent effects of recessions, research and development, super-hysteresis.

*León-Ledesma: School of Economics and Macroeconomics Growth and History Centre (MaGHiC), University of Kent, and CEPR. Sibson Building, Canterbury, CT2 7FS, UK, email: M.A.Leon-Ledesma@kent.ac.uk. Shibayama: School of Economics and Macroeconomics Growth and History Centre (MaGHiC), University of Kent. Sibson Building, Canterbury, CT2 7FS, UK, email: K.Shibayama@kent.ac.uk. We thank seminar participants at the Universities of Warwick, Portsmouth, Kent, Exeter, Málaga, Cagliari, Alicante, Hitotsubashi, Osaka, and Keio, Rotterdam, and NIESR, Bank of England, The Asian Development Bank, Diego Calderón, Michaela Elfsbacka Schmöller, Antonio Fatás, Ippei Fujiwara, Pablo Guerrón-Quintana, Ryo Horii, Elisa Keller, Zheng Liu, Sebastian Merkel, Julián Neira, Roberto Pancrazi, Tom Schmitz, Rish Shingania, Yuta Takahashi, Chris Tonetti, and Thijs van Rens for useful discussions.

1 Introduction

"One day, Sir, you may tax it."
Michael Faraday's reply to the question about the practical worth of electricity given by William Gladstone, Chancellor of the Exchequer (Mackay, 1977).

A fundamental question in economics concerns the potential long-term impact of temporary shocks on an economy's growth rate. Empirical evidence suggests that temporary financial crises can lead to permanent output *level* losses, a phenomenon known as "hysteresis". Moreover, Blanchard et al. (2015) have shown that many recessions not only result in a permanent fall in GDP *levels* but also lower *growth rates*, which Ball (2014) refers to as "super-hysteresis". The poor productivity performance observed after the 1990s crisis in Japan and the Global Financial Crisis (GFC) in many advanced economies have raised doubts about the traditional separation of growth and business cycle analysis. To illustrate this, Figure 1 displays the evolution of Total Factor Productivity (TFP) in Japan, juxtaposed with the Nikkei Stock Market price index. A clear TFP *growth* slowdown occurred around the time of the financial crisis.[1]

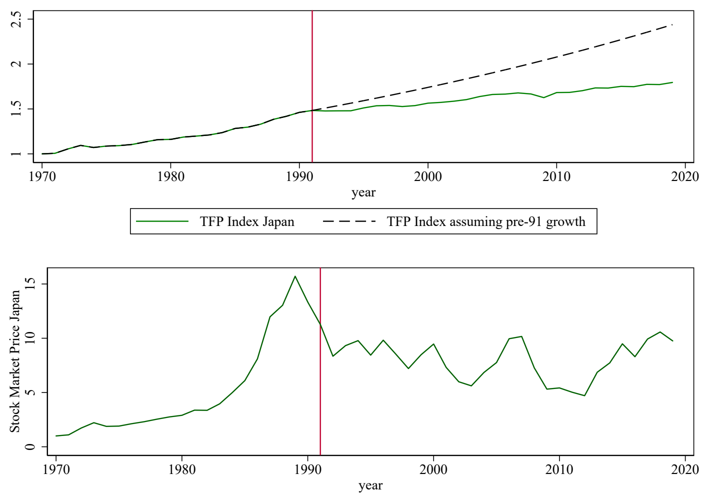

Figure 1: Japan's TFP index and stock market prices.

We propose a model of TFP growth slowdowns. In the model, permanent growth slowdowns can happen after deep or long lasting recessions. Crucially, these *permanent growth slowdowns* are the consequence of *temporary* shocks. The model features a unique equilibrium path for the economy but multiple Balanced Growth Paths (BGPs). In the wake of a sufficiently severe or long-lasting shock, the economy can transit from a high

[1]TFP growth slows down from 1.7% (1970-92) to 0.7% (1993-2019). See Betts (2021) for a detailed account of Japan's TFP growth slowdown from the 1990s.

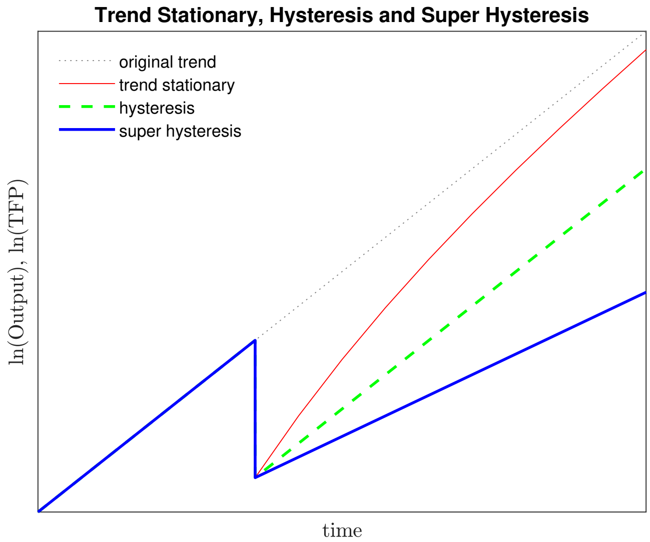

Figure 2: Three examples of different types of recoveries after a crisis.

TFP growth BGP to a low TFP growth BGP.[2] Figure 2 shows a simplified representation of three different views of business cycles. Standard business cycle models presuppose that, after a temporary level shock, the economy returns to the initial growth rate and trend level of output (trend stationarity). If, for instance, there is a temporary shock to the *growth rate* of TFP, even though the economy returns back to the initial growth rate, it will cause a permanent level loss (hysteresis). In our model, after a temporary shock there will be both a permanent level loss *and* a growth rate slowdown (super-hysteresis).

Our model builds on Romer's (1990) variety expansion model. We distinguish between basic research and product development as separate sectors of the R&D process. Basic research generates new ideas that add to the stock of knowledge. Product development (invention) converts existing knowledge into new products. These are produced by new firms whose returns are shared between basic researchers and product developers. In order to produce new research ideas, basic researchers have to be matched with the existing corpus of knowledge. These new basic ideas increase the knowledge stock. In order to develop a new product, developers also have to be matched with the existing stock of knowledge.

In this environment, there are two types of externalities. First, the standard growth or knowledge externalities present in most endogenous growth models: basic researchers and developers do not internalize the spillover effects of their activities. Second, search externalities: both types of R&D researchers do not take into account that their efforts affect the matching efficiency between the stock of knowledge and R&D workers in the other sector. The existence of search externalities generates multiple BGPs.

[2]This is consistent with the evidence in Aikman et al. (2022) who find that deep recessions have long lasting growth depressing effects whereas normal recessions generally do not.

The multiple BGPs can be interpreted as an outcome of the strategic complementarity in the R&D sector caused by the search externalities. In the high BGP, there is a virtuous cycle of high knowledge stock leading to increased matching for basic researchers and developers, resulting in faster generation of new varieties. This, in turn, supports a high level of knowledge stock. In contrast, in the low BGP, a vicious cycle ensues where the lower knowledge stock reduces the probability of successful matches, which lowers the returns to not only new product development, but also basic research. Hence, the stock of knowledge stays low.

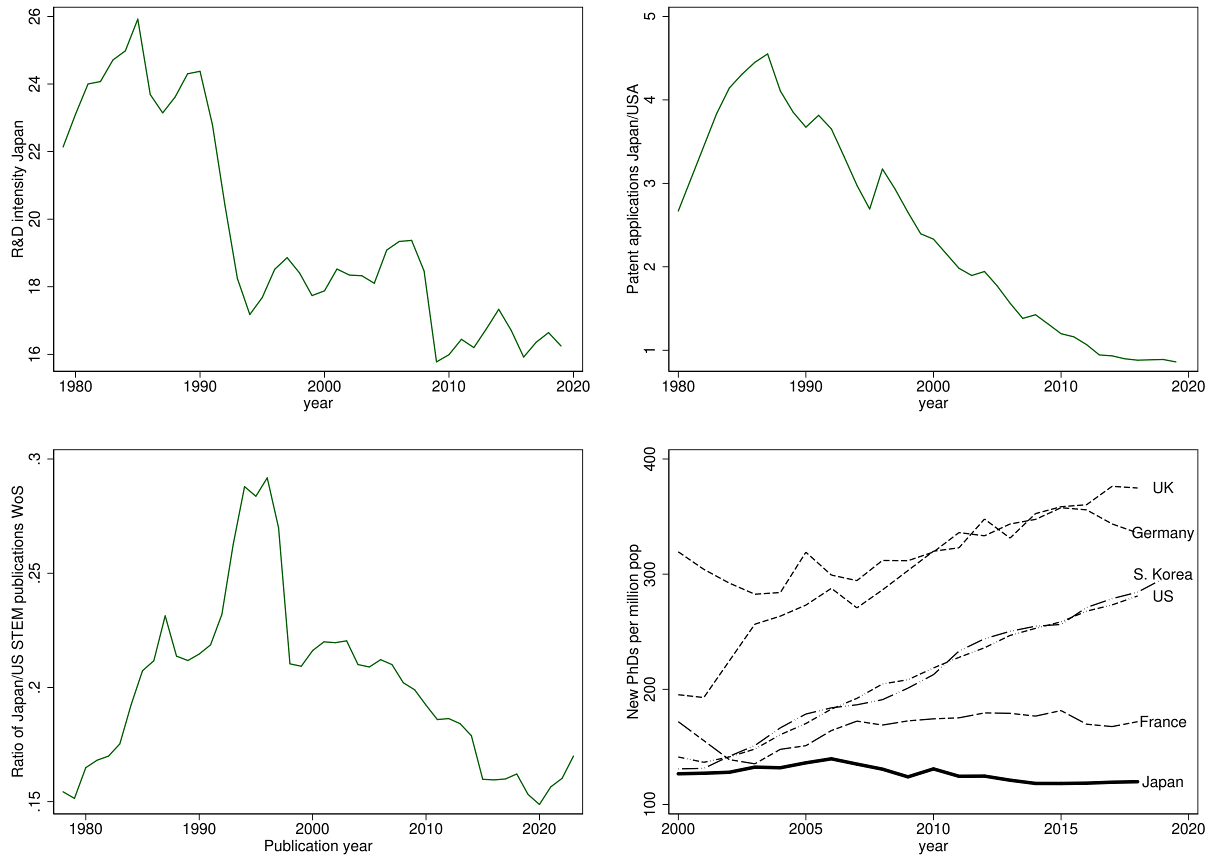

Figure 3: R&D indicators for Japan. R&D intensity (top-left) is calculated as R&D flow relative to R&D stock calculated as in Benigno and Fornaro (2018) with updated data from OECD Science and Technology Indicators data. Patent applications relative to the US (top-right) come from WIPO data as reported by the World Bank. For STEM publications relative to the US (bottom-left), see footnote 4. Number of new PhDs per million population (bottom-right) are obtained from `https://www.nistep.go.jp/sti_indicator/2021/RM311_34.html`.

We analyze the model in deterministic and stochastic environments. In the deterministic case, we show that the model can produce situations whereby initially capital-rich economies may converge to the low growth BGP leading to a "middle income trap". We then introduce stochastic shocks in the model. Particularly, we focus on temporary shocks to financing conditions which we model as reduced form temporary shocks to risk premia for all types of investment, and permanent demographic shocks which we model as shocks to the dependency ratio and the population growth rate of the economy. We analyze under which circumstances these shocks can push the economy towards the low growth BGP.

The stochastic simulations are motivated by the experience of Japan. After the financial crisis in the early 1990s, Japan suffers from a deterioration of R&D performance indicators as extensively reported in Bergeaud and Verluise (2022) and Ito et al. (2023).[3] This can be seen in Figure 3, where we plot R&D intensity, Japanese patent applications relative to the US, publications in STEM subjects relative to the US, and new PhDs per million population.[4] The coincidence of the deterioration in research indicators and R&D activities with the financial slump motivates our focus on the role of basic research, which is often overlooked in the growth literature.[5]

In the stochastic analysis, the calibration aims at matching certain macroeconomic targets for the Japanese economy in the pre-crisis period and is able to explain well the TFP growth slowdown after the early 1990s. The stochastic simulations for the calibrated model show that temporary financial shocks alone can move the economy from the high to the low BGP. In our multiple BGPs environment, the effects of exogenous shocks are discontinuous. If the magnitude of the shocks exceeds a certain threshold their effects can become large. Quantitatively, for financial shocks to have this effect, the crisis has to be either very deep or very long-lasting. We then introduce permanent demographic shocks as a (mild) increase in the dependency ratio and a slowdown in population growth consistent with the data. By themselves, these permanent demographic shocks can lead to permanent but quantitatively only minor growth slowdowns. That is, permanent demographic shocks alone do not cause a change in the BGP. This does not differ from what the standard neoclassical growth model would predict. However, when temporary financial shocks coincide with perverse demographic shocks, their impact is large because they can shift the economy to the low BGP. Our calibration shows that this happens with milder depth and duration of the financial shock. We term this a “wretched coincidence”. Financial and demographic shocks combine to explain why, in the Japanese case, the financial crisis was followed by a TFP growth slowdown.

It is important to highlight that, although we calibrate it to the Japanese economy, the implications of our model extend beyond this case. The model can be used to understand different productivity growth paths after the GFC or why some fast growing countries stagnate after prolonged episodes of financial distress. The latter is important for economies like China where the real-estate crisis and unfavorable demographic trends, when interpreted

[3]This decline, especially in STEM subjects, has also been highlighted by the Japanese Ministry of Economy Trade and Industry in its “2025 Digital Cliff” report. See `https://www.meti.go.jp/shingikai/mono_info_service/digital_transformation/pdf/20180907_03.pdf`.

[4]Data on STEM publications from the National Science Foundation as reported by the World Bank only go back to year 2000. To generate a series for the whole period, we extracted data from Web of Science by address (i.e. Japan, or USA) for papers published in the Science Citation Index Expanded, Conference proceedings Citation Index – Science, and Current Chemical Reactions for the 1978-2022 period. The data from this source tracks well available data from the World Bank since year 2000.

[5]This is also consistent with the evidence provided in Hardy and Sever (2021) who show that financial crises are followed by persistent slumps in innovative activity.

in light of our model, could lead to a permanent productivity slowdown. This scenario resembles the arguments in Zilibotti (2017) about the future of Chinese growth.

We then analyze "big push"fiscal policies to escape from the low BGP. We study government spending, corporate taxes, and subsidies to both basic research and product development. We show that only corporate tax and basic research subsidies can move the economy to the high BGP for reasonable tax/subsidy values and policy support duration. The transition towards the high BGP is slow if the economy starts at the low BGP. That means that the duration of the government support is long and the required fiscal expenditure is substantial. However, the cost of these policies is dwarfed by the benefits in the long run, because these policies essentially trade a temporary, though long, *level* increase in fiscal spending for a permanent *growth rate* increase. We also show that an increase in government spending cannot result in a BGP change because it squeezes the R&D sectors by increasing the returns to labor in the final goods sector. Finally, the sooner the big push policy is implemented the lower the budgetary costs. This is particularly the case for economies that are still in transition towards the low BGP such as Japan in the 2020s.

Related literature. There is by now a well established literature on the long-lasting effects of temporary shocks, i.e. "hysteresis". Both the empirical and theoretical literature are reviewed in Cerra et al. (2023). The empirical literature has highlighted widespread evidence of permanent output losses especially after financial crises.[6] In some cases, this is also accompanied by highly persistent or permanent *growth slowdowns.* Our paper presents a theoretical mechanism generating "super-hysteresis", i.e. a permanent *growth rate* decline after a temporary shock.

The theoretical literature on the long-lasting effects of temporary shocks goes back to the contributions of Stadler (1990), Stiglitz (1993), and Fatás (2000). Comin and Gertler (2006) observe that TFP is procyclical over frequencies longer than business cycles (the medium run). Standard business cycle models that impose exogenous persistence of TFP shocks cannot explain this phenomenon. Motivated by this fact, they build a model integrating business cycles and endogenous productivity growth, in the spirit of Romer (1990). Endogenous technology development and adoption generate highly persistent fluctuations in TFP in response to shocks. Note that these persistent growth fluctuations also lead to permanent TFP *level* effects, but its growth rate converges to a unique BGP. We have a similar approach as we use a variety expansion model of endogenous growth. However, our model can generate multiple BGPs because of the aforementioned search externalities in the R&D process.

Since Comin and Gertler (2006), several papers have built up on the idea that the generation and adoption of knowledge may be influenced by cyclical shocks. These contributions, which Fornaro and Wolf (2023) label "Keynesian Growth Theory", introduce endogenous growth mechanisms in New Keynesian models of fluctuations. Examples of these are Anzoategui et al. (2019), Elfsbacka Schmöller and Spitzer (2021), Guerrón-Quintana and Jinnai (2014), Cozzi et al. (2021), Queraltó (2020), Engler and Tervala (2018), Bianchi et al. (2019), Vinci and Licandro (2020), and Fornaro and Wolf (2023).[7] These papers

[6]For recent evidence, see Furlanetto et al. (2023).

[7]Schmitz (2021) takes a different route and focuses on the role of firm heterogeneity within a Schumpete-

highlight the role of temporary non-technology shocks in driving persistent fluctuations of TFP leading to permanent output level losses. Our paper is similar to this literature as we focus on the role of innovation as an engine of amplification within an endogenous growth model with variety expansion. Unlike them, our model, under certain conditions, generates permanent TFP *growth* slowdowns because the economy can converge to a different BGP after temporary non-technology shocks. It shows that it is possible for an economy to make a transition from a high to a low growth regime after persistent or deep financial crises.[8]

Within this tradition, our paper is closely related to Benigno and Fornaro (2018). They also build on the idea that there is a crucial interaction between temporary shocks and productivity through R&D efforts. They present a New Keynesian model with liquidity traps delivering a high and a low growth steady state. There are multiple equilibrium paths and self-fulfilling expectations and, thus, sunspots determine which equilibrium path the economy takes. Our paper differs in several dimensions. First, and most importantly, in our model there are multiple steady states (BGPs) but the equilibrium path is uniquely determined. Hence, there is no role for self-fulfilling expectations. Secondly, our model does not feature price and wage rigidities common in New Keynesian models. Third, we provide a fairly realistic calibration of the model to the Japanese economy to assess the likelihood of temporary demand shocks leading to permanent slowdowns.

Our work also relates to the idea of "strategic complementarities" in Cooper (1999). These are static models with multiple equilibria. Which of the equilibria is chosen by the economy cannot be determined within the model. Often, coordination is required to achieve a particular equilibrium. In our model, the multiple BGPs can be understood as equilibria of a coordination game between basic researchers and product developers in which, unlike in static coordination games, history (the initial state and the exogenous shocks) determines the unique equilibrium path. Intuitively, because of the existence of search externalities, increased effort in the basic research sector increases the success rate of developers and vice-versa. This strategic complementarity in our model generates virtuous and vicious cycles of R&D and productivity growth. See Appendix C for further discussions.

Technically, our paper is also related to the literature on "poverty traps" and "middle-income traps". A seminal paper in this tradition is Azariadis and Drazen (1990). They present a neoclassical overlapping generations (OLG) growth model where human capital externalities take the form of a threshold beyond which returns to scale allow for growth "take-offs". Their model also features multiple steady states.[9] Their growth differences model version, however, features multiple equilibria. The mechanism behind our model is

rian model of growth. He finds that heterogeneity amplifies the effects of a financial shocks on innovation because small firms are both relatively more innovative than large firms and hit harder by the crisis.

[8]A different strand in the literature studies the growth slowdown as a consequence of slow-moving structural changes in the economy rather than as a consequence of temporary shocks. Examples of these are Eggertsson et al. (2019) for a model of secular stagnation caused by demographic change or exogenous productivity slowdowns, De Ridder (2019) for the role of intangible capital, Akcigit and Ates (2021) for declining imitation rates, Liu et al. (2022) for declining interest rates, and Aghion et al. (2023) for falling firm costs of expanding to multiple markets.

[9]Other important contributions in this tradition include Matsuyama (2002) for learning externalities, Acemoglu and Zilibotti (1997) for financial market incompleteness, and Doepke and Zilibotti (2005) for child labor traps among many others. See Azariadis and Stachurski (2005) for a review.

similar in the sense that search and growth externalities generate regions above or below which the economy will converge to different BGPs. As in their model, a high degree of non-linearity is required to generate multiple BGPs, which we derive from a microfounded R&D model. Our model also emphasises the potential for "big push"policies to lift the economy from the low growth trap.

In the rest of the paper, Section 2 presents the model, Section 3 presents the quantitative analysis and main results, Section 4 analyzes policies, and Section 5 concludes.

2 The model

We extend Romer's (1990) variety expansion model by introducing two research matching processes. As in Romer (1990), the aggregate production function uses labor and capital, while total factor productivity (TFP) is determined by the number of varieties of intermediate goods.[10]

Unlike in Romer (1990), however, we explicitly separate the different roles of basic research and product development. The proportion of labor allocated to product development determines the rate of TFP growth, while the proportion allocated to basic research shapes the success rate of product development. Figure 4 displays the structure of the basic research and development matching process. Basic researchers match with the existing stock of ideas to develop new basic research, and developers/inventors also match with the stock of knowledge to produce new intermediate goods. In our notation, sub/superscript A indicates product development, while Z indicates basic research.

Because basic research activities affect the stock of research, this will impact on the likelihood of a successful match between developers and the stock of research. However, basic researchers do not internalize this effect. Likewise, development effort will affect the success rate of matching between basic researchers and the stock of basic research because it determines the share of profits from new firms allocated to basic researchers. This is consistent with the evidence in Hvide and Jones (2018) who present convincing evidence on the importance of rewards for the productivity of university professors. These matching externalities are key to generating vicious/virtuous cycles and hence multiple BGPs.

To produce a new variety, developers set up new firms whose value depends on the discounted stream of profits from their monopolistic market power. The value of the new firms is the reward to the entire R&D sector, which is split between basic researchers and product developers using Nash bargaining. Because of this, the financial sector plays an important role in our model. Importantly, there is a very small mass of basic researchers that

[10]A criticism of this class of endogenous growth model to explain a country's growth performance is that basic research knowledge also flows across borders and hence the knowledge stock that matters is global. To capture this, our calibration exercises consider the exogenous component of TFP which can be interpreted as imported technology. Importantly, Hausman (2022) argues that innovation activity relies on local research clusters. Local developers rely on interactions with local basic research producers. These basic researchers may either be producing new basic research or absorbing foreign produced research and adapting it to the local productive needs. The model could be reformulated in line with these arguments.

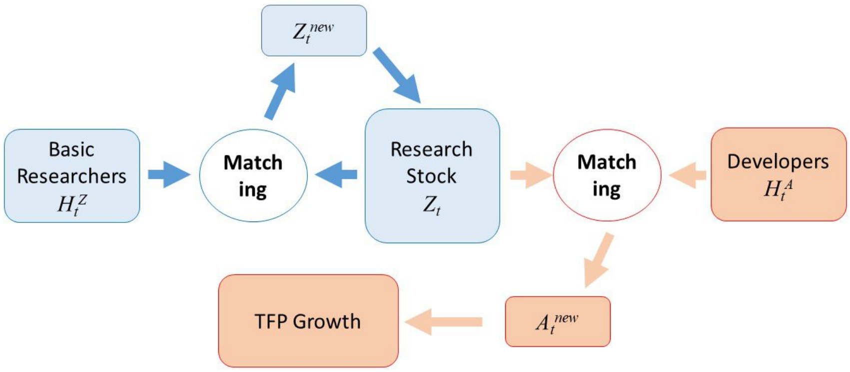

Figure 4: Structure of the basic research and development matching process.

are not pecuniary motivated. We call them Non-Pecuniary Researchers (NPRs hereafter) and they play a crucial role when an economy "takes-off" from the low BGP.

To streamline the exposition, we briefly explain (i) labor supply, and then we focus on (ii) the two research matching functions, (iii) asset pricing equations (financial markets), and (iv) the optimization problems and their first order conditions in the basic research and development sectors. The rest of the model is standard and hence the readers familiar with this class of models may want to skip Sections 2.5, 2.6 and 2.7. The list of equilibrium equations and variables are summarized in Appendix A.

2.1 Labor supply

Workers are endowed with a total number of hours H_t that can be allocated to final production ($H_{Y,t}$), innovation ($H_{A,t}$), basic research ($H_{Z,t}$), or be out of the labor force ($H_{O,t}$):

$$H_t = H_{Y,t} + H_{A,t} + H_{Z,t} + H_{O,t}. \tag{1}$$

The proportion of worker-hours outside the labor force is given exogenously as $H_{O,t} = \underline{h}_O H_t$, which is introduced in order to analyze the effect of demographic changes in the quantitative analysis of the model. Because we assume that labor is inelastially supplied, the rate of growth of H_t is the rate of population growth and is exogenously given by $\gamma_{H,t}$.

We assume that there is a group of researchers in the basic research sector whose motivation is not pecuniary. Their share in the total labor supply is $\underline{h}_Z$. That is, there is a positive mass of researchers who engage in research because of intellectual curiosity, intrinsic motivation, recognition or prestige. We call them "Non-Pecuniary Researchers" (NPR). An alternative interpretation of this group of researchers is that, because of skillspecificity or high mobility costs, they are immobile across sectors. Hence:

$$H_{Z,t} \geq \underline{h}_Z H_t.$$

NPRs are important to prevent the low R&D BGP from becoming an absorbing state from which the economy cannot exit. More specifically, it would be impossible to escape from the low BGP if $\underline{h}_Z = 0$. Also, the low BGP would disappear if $\underline{h}_Z$ is high enough. That is, if the proportion of NPRs is large, the economy *must* always converge to the high BGP. Between extremes, as $\underline{h}_Z$ becomes higher, it becomes easier to escape from the low BGP.

As labor is homogeneous and there is perfect mobility across sectors, the wage rate in all three sectors must be equalized. An exception to this happens when the condition $H_{Z,t} \geq \underline{h}_Z H_t$ is binding. In this case, NPRs are willing to accept a lower wage than in the other two sectors.

2.2 Research and Development matching functions

The matching between *aggregate* research hours $H_{Z,t}$ and existing knowledge stock Z_t generates new research outputs Z_t^{new}. Note that individual R&D researchers take all variables in this and next subsections as given. The functional form is assumed to be Cobb-Douglas:[11]

$$Z_t^{new} = \omega_Z A_t^{\tilde{\kappa}_Z} \left(\left(\frac{H_{Z,t}}{H_t} \right)^{\tilde{\phi}_Z} (Z_t)^{1-\tilde{\phi}_Z} \right)^{\tilde{\eta}_Z}. \tag{2}$$

where ω_Z and $\tilde{\phi}_Z$ are scale and elasticity parameters respectively, while A_t is the stock of existing inventions. There is a growth externality given by $A_t^{\tilde{\kappa}_Z}$ such that the higher the stock of inventions (e.g., Google Scholar and Github), the better the chances of a successful match. This growth externality is common in many endogenous growth models. Unlike in labor matching models, there is no theoretical reason for assuming constant returns to scale (CRS) so that $\tilde{\eta}_Z \lesseqgtr 1$. This subsection mainly focuses on the conditions required to prevent scale effects and some other undesirable model properties. Once these theoretical restrictions are imposed, parameters with tilde (̃) are all eliminated.

The function contains a type of Stepping-on-Toe effect, making the model free from scale effects (Jones and Williams, 2000). That is, successful matching leading to new research output depends on the share of labor hours devoted to research ($\frac{H_{Z,t}}{H_t}$). Although, technically, this formulation is simply a short-cut to prevent scale effects, it does have a simple intuition. Unlike in labor matching, many researchers can be matched with a single research idea. With non-rivalry, most researchers will tend to access the same basic research corpus hence leading to congestion (Stepping-on-Toe effects). E.g., most researchers tend to read outputs published in a selective sub-set of prestigious outlets. In the extreme, all researchers read the same set of papers. In this case, the number of academic papers that

[11] In our notation, Z_t and A_t are the stocks of basic research and invented product designs at the start of period t. Likewise, K_t denotes the capital stock for final production at the start of period t.

are read by researchers is independent from the number of readers. Instead, it depends on the share of hours that each researcher devotes to research. In our simple model setup, $\frac{H_{Z,t}}{H_t}$ can also be interpreted as the share of hours that the representative worker allocates to basic research.

Likewise, in order to produce a new product design A_t^{new}, there is a matching between aggregate development/invention hours $H_{A,t}$ and existing knowledge stock Z_t:

$$A_t^{new} = \omega_A A_t^{\tilde{\kappa}_A} \left(\left(\frac{H_{A,t}}{H_t} \right)^{\tilde{\phi}_A} (Z_t)^{1-\tilde{\phi}_A} \right)^{\tilde{\eta}_A}, \tag{3}$$

where ω_A and $\tilde{\phi}_A$ are scale and elasticity parameters respectively.

With these two formulations, we need to impose a set of parameter restrictions. First of all, to ensure the existence of at least one balanced growth path (BGP), we need to impose a restriction on the growth externality:

$$\tilde{\kappa}_Z = \frac{1 - \left(1 - \tilde{\phi}_Z\right) \tilde{\eta}_Z}{\tilde{\phi}_A \tilde{\eta}_A} (1 - \tilde{\kappa}_A), \tag{4}$$

which implies that we must have growth externalities but these can appear either in the basic research ($\tilde{\kappa}_Z$) or invention sectors ($\tilde{\kappa}_A$) or both as long as this restriction is satisfied.[12]

We also impose a set of restrictions on the degree of returns to scale (RTS) to prevent explosive growth paths. We would have an explosive growth path if RTS in research are too large: $\tilde{\eta}_Z > \frac{1}{\tilde{\phi}_Z}$. On the other hand, if RTS are too weak, the success rate of basic research goes to infinity as the number of scientists goes to zero. That is, $\tilde{\eta}_Z < \frac{1}{\tilde{\phi}_Z}$ implies $\pi_t^{ZH} \to \infty$ as $\bar{H}_{Z,t} \to 0$, where $\pi_t^{ZH} = \frac{Z_t^{new}}{H_{Z,t}}$ is the success rate of basic research. To avoid these, we must impose $\tilde{\eta}_Z = \frac{1}{\tilde{\phi}_Z}$.[13] With this restriction, we regain a Romer-style formulation of knowledge production but without scale effects. Define $\kappa_Z = \frac{\tilde{\kappa}_Z}{1-\left(1-\tilde{\phi}_Z\right)\tilde{\eta}_Z}$. Then, we obtain:

$$Z_t^{new} = \omega_Z A_t^{\kappa_Z \phi_Z} (Z_t)^{1-\phi_Z} \left(\frac{H_{Z,t}}{H_t} \right), \tag{5}$$

where $1 - \phi_Z = \left(1 - \tilde{\phi}_Z\right) \tilde{\eta}_Z$.

Following a similar reasoning, we assume that the matching function in the invention/development sector is constant returns to scale (CRS): $\tilde{\eta}_A = 1$. As above, define

[12]In addition, we impose the following requirement: $0 \leq \tilde{\kappa}_Z, \tilde{\kappa}_A \leq 1$ because the growth externality must be positive. In terms of the quantitative behavior of the model, where the growth externality appears has no effect on either the BGP or transitional dynamics. In practice, we set $\tilde{\kappa}_A = 1$ ($\tilde{\kappa}_Z = 0$). A different choice of $\tilde{\kappa}_Z$ and $\tilde{\kappa}_A$ only leads to a parallel shift of Z_t/A_t both in the BGPs and in the transitional dynamics.

[13]Because $\frac{1}{\phi_Z} > 1$, the basic research matching function is increasing returns to scale.

$\phi_A = \tilde{\phi}_A \tilde{\eta}_A = \tilde{\phi}_A$ and $\kappa_A = \tilde{\kappa}_A \tilde{\eta}_A = \tilde{\kappa}_A$. Then,

$$A_t^{new} = \omega_A A_t^{\kappa_A} (Z_t)^{1-\phi_A} \left(\frac{H_{A,t}}{H_t} \right)^{\phi_A}, \tag{6}$$

with parameter restriction $\kappa_A = 1 - (1 - \phi_A) \kappa_Z$.

Finally, the laws of motion for the stock of intermediate varieties and basic research are given by:

$$A_{t+1} = A_t^{new} + (1 - \delta_A) A_t, \tag{7}$$

$$Z_{t+1} = Z_t^{new} + (1 - \delta_Z) Z_t, \tag{8}$$

where δ_A and δ_Z are the depreciation rates.

2.3 Financial markets

Financial markets play key role in the evolution of R&D because the asset values of product design and research stocks directly affect the reward to successful inventions and basic research, respectively.

We assume that all the monopolistic profits from each product are paid out as dividends D_t:

$$D_t = (1 - \tau_D)(1 - \tilde{\alpha}) \alpha \frac{Y_t}{A_t}, \tag{9}$$

where $\frac{Y_t}{A_t}$ is output per each product design, of which $(1 - \tilde{\alpha})\alpha$ is the share belonging to the firm owner (the successful inventor and the basic researcher who owns the successfully matched research asset).[14] Parameter composite $\tilde{\alpha} = (\theta - 1)/\theta$ is the reciprocal of the markup, which is determined by the elasticity of substitution among varieties.[15] We allow for a corporate tax at a rate τ_D which will later be used in our policy experiments.

The asset value of new firms V_t is given by the present value of dividends D_t:

$$V_t = D_t + E_t \left[\frac{\Lambda_{t,t+1}}{c_v} (1 - \delta_A) V_{t+1} \right], \tag{10}$$

where the stochastic discount factor $\Lambda_{t,t+1}$ is derived from the households' consumption-savings decisions. Parameter c_v captures an exogenous risk-premium reflecting the fact that innovative startups are riskier than bonds. Later, we interpret a financial crisis as an increase in the risk-premia for all types of investment.[16]

[14] Output is distributed as wages $(1 - \alpha)$, capital cost $\tilde{\alpha}\alpha$ and profit $(1 - \tilde{\alpha})\alpha$. See also equations (20) and (21).

[15] Under Romer's (1990) parameterization, $\tilde{\alpha} = \alpha$. Such a coincidence is not compatible with realistic calibration. See Section 3.1.

[16] This type of exogenous risk-premia is common in the macro-finance literature where it is introduced as a wedge. Implicitly, the model assumes that there is a financial intermediary that receives the risk-premium which is paid to households as a lump-sum transfer. This is the case for all the risk-premia that appear in the rest of the model.

Once a developer successfully matches with basic research stock, V_t is split between developers and basic researchers. That is, Nash bargaining determines the shares of product inventors (s_A) and researchers who own the existing research stock $(1 - s_A)$. Hence $s_A V_t$ will appear in the developers' optimization problem as a reward while $(1 - s_A) V_t$ appears in the asset value of new research stock (Z_t^{new}).

The asset value of new research stock then is given by:

$$V_t^Z = (1 - s_A) V_t \pi_t^{AZ} + E_t \left[\frac{\Lambda_{t,t+1}}{c_z} (1 - \delta_Z) V_{t+1}^Z \right]. \tag{11}$$

The value will depend on the success rate of the matching between developers and basic research stock given by $\pi_t^{AZ} = \frac{A_t^{new}}{Z_t}$.[17] That is, the value of new research stock depends on the value of new firms weighted by the success rate of a development match and the bargaining power of basic researchers, plus the continuation value. Equivalent to the risk premium for new firms, c_z is a risk premium for new basic research.

2.4 Optimization problems by researchers and developers

The optimization problem of basic researchers is:

$$\max_{H_{Z,t}} \; V_t^Z \pi_t^{ZH} H_{Z,t} - (1 - \tau_Z) W_t H_{Z,t}, \tag{12}$$

$$\text{s.t. } \pi_t^{ZH} = \frac{Z_t^{new}}{H_{Z,t}}, \tag{13}$$

$$H_{Z,t} \geq \underline{h}_Z H_t. \tag{14}$$

The success rate *per unit labor input* π_t^{ZH} is given to basic researchers. That is, a basic researcher successfully matches with existing research stock with rate π_t^{ZH}. The wage rate W_t and the asset value of basic research stock V_t^Z are also given. Parameter τ_Z is a government subsidy to the research sector. The choice is bounded from below by the existence of NPRs. This condition is binding in the low BGP, in which there are no monetary-motivated researchers. Implicitly, we innocuously assume that the last researcher's bargaining power is 100%.[18]

The first order condition (FOC) of $H_{Z,t}$ is

$$\begin{aligned} V_t^Z \pi_t^{ZH} &= (1 - \tau_Z) W_t \quad \text{if } H_{Z,t} > \underline{h}_Z H_t \\ V_t^Z \pi_t^{ZH} &\leq (1 - \tau_Z) W_t \quad \text{if } H_{Z,t} = \underline{h}_Z H_t \end{aligned} \tag{15}$$

The state space is split into two. In one of the two regions, which includes the high BGP, $V_t^Z \pi_t^{ZH} = (1-\tau_Z) W_t$ holds. In the other region, which includes the low BGP, $H_{Z,t} = \underline{h}_Z H_t$

[17] Note that π_t^{AZ} is not the success *probability* but the success *rate*. Because a single unit of research stock can be matched with multiple inventors, π_t^{AZ} can exceed one.

[18] The bargaining share between H_Z and Z has no effect in our simple model setup. The key reason is that the value of new research stock belongs to basic researchers collectively. That is, regardless of the bargaining shares, all gains are received by the representative basic researcher.

holds.[19] The boundary between these two regions is determined endogenously (a free boundary problem).

Likewise, the optimization program for developers is to choose hours worked in development to maximize:

$$\max_{H_{A,t}} \quad s_A V_t \pi_t^{AH} H_{A,t} - (1-\tau_A) W_t H_{A,t}, \tag{16}$$

$$\text{s.t. } \pi_t^{AH} = \frac{A_t^{new}}{H_{A,t}}, \tag{17}$$

The success rate *per unit labor input* π_t^{AH} is given to developers. An inventor successfully matches with existing research stock at a rate π_t^{AH}. Again, wage rate W_t and the asset value of basic research stock V_t^Z are also given. After a successful invention, they set up a firm of which s_A is their share. Parameter τ_A is a government subsidy to the development sector.

Because of the Inada property of the matching function, unlike in the basic research sector, the negativity constraint $H_{A,t} \geq 0$ never binds. Hence, the FOC for $H_{A,t}$ is much simpler:

$$V_t \pi_t^{AH} = (1-\tau_A) W_t \tag{18}$$

2.5 Aggregate output and TFP

The production sector follows the formulation of Romer (1990). We skip the detailed derivation here. The resultant aggregate production function is given by

$$Y_t = (\eta K_t)^{\alpha} \left(\xi_t A_t^{\varphi_A} H_{Y,t}\right)^{1-\alpha}, \tag{19}$$

where ξ_t is the exogenous component of the labor-augmenting technological progress, while the endogenous component is $A_t^{\varphi_A}$, with φ_A a parameter composite defined as $\varphi_A = \frac{\alpha}{1-\alpha}\frac{1}{\theta-1} = \frac{\alpha}{1-\alpha}\frac{1-\tilde{\alpha}}{\tilde{\alpha}}$.[20]

The FOCs for the production sector are:

$$W_t = (1-\alpha)\frac{Y_t}{H_{Y,t}} \tag{20}$$

$$R_{K,t} = \tilde{\alpha}\alpha\frac{Y_t}{K_t} \tag{21}$$

The logarithm of total factor productivity (TFP) is expressed as a weighted sum of the exogenous and endogenous labor-augmenting technology.

$$\ln TFP_t = (1-\alpha)\left(\ln \xi_t + \varphi_A \ln A_t \right) \tag{22}$$

[19] Romer (1990, pp.95-96) already pointed out the possible the existence of a no-R&D steady state, in which the non-negativity constraint of research labor is binding.

[20] We can safely normalize $\eta = 1$.

Note that ξ_t grows at the same rate $\gamma_{\xi,t}$ in the entire state space. Hence, the growth rate gap between the high and low BGPs is entirely explained by A_t.

2.6 Households

A representative household maximizes intertemporal utility over per-capita consumption where we assume standard CRRA period utility with σ_C being the degree of relative risk aversion. Households own the capital stock of final production firms and use their wage income ($W_t\,(1-\underline{h}_O)\,H_t$) and capital income ($R_{K,t}K_t$) to consume ($C_t$) and invest in new capital (I_t). They receive lump-sum transfers (LST_t) that include the risk-premia and government transfers which, under our calibration described later, are negative (lump-sum taxes).

We assume that the head of a household does not take into account the change of the family size (population) in their decisions. This assumption is quantitatively relevant because population growth affects the *effective* discount factor. Our assumption is motivated by empirical findings. Some authors such as Aksoy et al. (2019) find negative effects of a higher $\underline{h}_O$ on economic growth, while the evidence on the effect of population growth is mixed.[21] If we assume that households take into account future population size, demographic shocks would have larger effects. Because this is particularly important in Section 3.5, we will briefly come back to this point there.

The household problem is given by:

$$\max_{C_t, I_t, K_{t+1}} E_t\left[\sum_{t=0}^{\infty}\beta^t \frac{(C_t/H_t)^{1-\sigma_C}}{1-\sigma_C}\right], \tag{23}$$

$$\text{s.t. } (1+\tau_C)C_t + I_t = (1-\tau_H)W_t\,(1-\underline{h}_O)\,H_t + R_{K,t}K_t + LST_t, \tag{24}$$

$$K_{t+1} = I_t + (1-\delta_K)K_t, \tag{25}$$

where β is the subjective discount factor (time preference parameter) and τ_C and τ_H are consumption and labor income taxes. As long as these taxes are constant, and because labor supply is inelastic, they have no effect on the model dynamics. They are later used to calculate government revenues in the policy experiments.

The first order conditions yield a capital return and a stochastic discount factor conditions:

$$1 = E_t\left[\frac{\Lambda_{t,t+1}}{c_K}\left(R_{K,t+1} + (1-\delta_K)\right)\right], \tag{26}$$

$$\Lambda_{t,t+1} = \frac{\beta}{\gamma_{H,t+1}}\left(\frac{C_{t+1}/H_{t+1}}{C_t/H_t}\right)^{-\sigma_C}. \tag{27}$$

We add a wedge c_K as an exogenous risk-premium on capital in the capital return equation

[21] Pritchett (1996) finds that "There is no correlation, or a weak negative correlation, between measures of total factor productivity growth and population growth. Nearly all of the weak correlation between the growth of output per person and population growth is the result of shifts in participation in the labor force, not of changes in output per worker."

and define $\gamma_{H,t+1} = \frac{H_{t+1}}{H_t}$ as the exogenous population growth rate. This risk premium will be treated as stochastic later on.

2.7 Goods market clearing

We finally close the model with the goods market clearing condition:

$$Y_t = C_t + I_t + G_t. \tag{28}$$

Final output equals expenditure in consumption, capital investment, and government spending (G_t). Government spending in the model does not affect utility or production and is hence treated as "wasteful". It is introduced in the model to study "Big-Push" policies.

2.8 Equilibrium

Since several model variables are growing in steady state, we need to represent the model in stationary form to compute the equilibrium. We first collect the equilibrium equations and non-stationary model variables. We then divide the non-stationary variables by their growth factors. The equilibrium equations are also converted consistently. This process is described in detail in Appendix A in which all de-trended variables will be denoted by a tilde ($\tilde{X}$). Since we have discussed in detail the main building blocks of the model, we provide the full set of (de-trended) equilibrium equations in Appendix A. We can now formally define equilibrium.

Definition 1 *An equilibrium is defined as the set of de-trended endogenous variables listed in Table A.1 in Appendix A such that (i) the initial state ($\tilde{Z}_0$ and $\tilde{K}_0$) is given and the transversality conditions (29) hold, (ii) exogenous variables are also given for all $t \geq 0$, and (iii) the equilibrium equations defined in Table A.3 are all satisfied for all $t \geq 0$.*

In this definition, we intentionally omit the dynamics of the exogenous variables because we consider different versions in the sections below. In the deterministic version, $\gamma_{H,t}$ and $\gamma_{\xi,t}$ (the growth rate of the exogenous TFP growth component ξ_t) are constants. In the stochastic version, the risk premium parameters (c_k, c_z, and c_v), the dependency ratio ($\underline{h}_O$) as well as population growth ($\gamma_{H,t}$) are treated as exogenous shocks.

We define the transversality conditions (TVCs) as follows:

$$\lim_{t\to\infty} E_0 \left[\frac{\tilde{\Lambda}_{0,t+1}}{c_K} (1-\delta_K)^{t+1} \right] = 0, \tag{29a}$$

$$\lim_{t\to\infty} E_0 \left[\frac{\tilde{\Lambda}_{0,t+1}\gamma_{V,t+1}}{c_v} (1-\delta_A)^{t+1} \tilde{V}_{t+1} \right] = 0, \tag{29b}$$

$$\lim_{t\to\infty} E_0 \left[\frac{\tilde{\Lambda}_{0,t+1}\gamma_{VZ,t+1}}{c_Z} (1-\delta_Z)^{t+1} \tilde{V}^Z_{t+1} \right] = 0, \tag{29c}$$

where $\gamma_{V,t}$ and $\gamma_{VZ,t}$ are the trend growth rates of the value of firms and basic research respectively. Intuitively, these conditions state that the present values of the asset prices (capital stock, firm value, and the asset value of the research stock) should not explode.

The equilibrium is computed numerically using the Euler Equation Iteration (EEI), a global solution method. Details of the computation solution method are presented in Appendix B.

2.9 Multiple balanced growth paths: intuition

To garnish intuition for the existence of multiple BGPs in the model, we focus on the labor demand of the three sectors. Assume for now that all variables are at the steady state and we use subscript "ss" to indicate this assumption. In this exercise, we omit one equilibrium condition: $W_{ss}^{R\&D} = W_{ss}^{Y}$.[22] That is, we omit the marginal revenue product equalization (offered wages) between the two R&D sectors and the production sectors. We then treat $H_{A,ss}$ as an exogenous parameter. The crossing points of the W_{ss}^{Y} and $W_{ss}^{R\&D}$ lines are the BGPs.

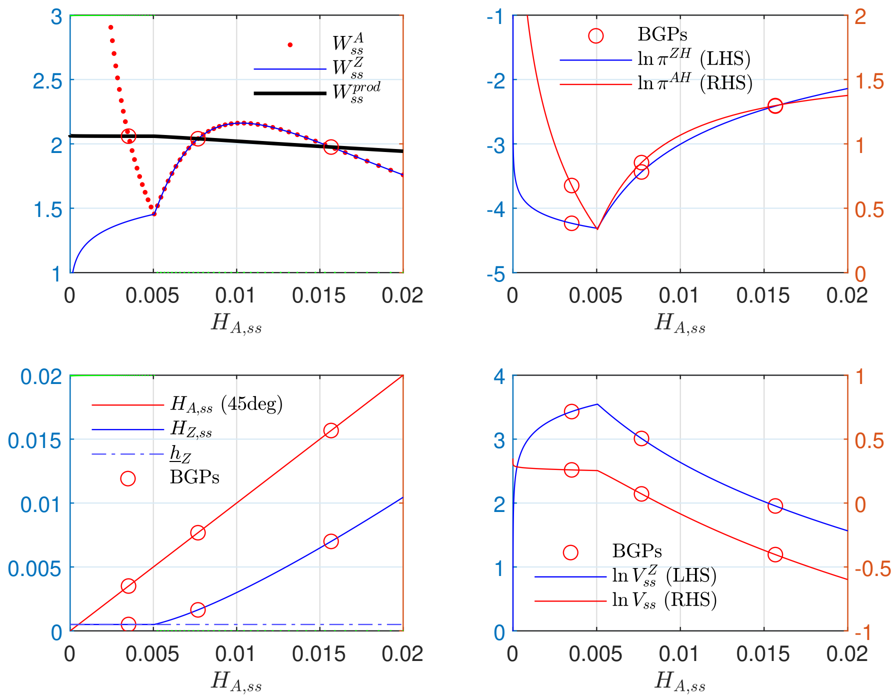

Figure 5: Multiple BGPs in partial equilibrium.

Figure 5 illustrates the results of this exercise under the benchmark parameter values shown in Section 3.1. In the upper-left quadrant, we plot wages offered in the three sectors.

[22] The model implies wage equalization between the two R&D sectors $W_t^{R\&D} = W_{ss}^{A} = W_{ss}^{Z}$ as long as $H_{Z,ss} \geq \underline{h}_Z H_{ss}$ is not binding. If $H_{Z,ss} = \underline{h}_Z H_{ss}$, then $W_{ss}^{R\&D} = W_{ss}^{A} > W_{ss}^{Z}$.

The right two panels show the decomposition of the offered wages.[23] Among three BGPs, the middle one is locally unstable and the outer two are stable (saddle-path stable).

To understand the intuition, note first that, for most of the range of values of $H_{A,ss}$, wages offered in the R&D sectors are downward sloping. This is "normal" in the sense that a lower labor input is accompanied by a higher marginal revenue product of labor.[24] However, due to the search externalities, wages become increasing in $H_{A,ss}$ around the middle BGP. The middle BGP is a threshold point, in the sense that a *vicious cycle* occurs below it while a *virtuous cycle* occurs above it. On the vicious cycle side, if $H_{A,ss}$ falls below the middle BGP, because the success rate of basic research matching decreases sharply (see the top-right panel), basic researchers are discouraged, leading to a shrinkage of the stock of basic research, which, in turn, deteriorates the success rate of product development. This further reduces incentives for basic research. On the virtuous cycle side, in contrast, a higher $H_{A,ss}$ incentivizes further demand for $H_{A,ss}$ because of the search externalities. This continues until the economy arrives at the high BGP.

NPRs prevent a zero R&D world from which the economy cannot escape. As NPRs are not motivated by pecuniary rewards, they still produce basic research even when $H_{A,ss}$ is very low. In the region where $H_{Z,ss} = \underline{h}_Z H_{ss}$, the optimality condition in the research sector is irrelevant and the blue and red lines decouple (i.e., $W^A_{ss} > W^Z_{ss}$ in the top-left panel).

Note that, for reasonable parameter ranges, there are three BGPs even if we assume zero NPRs (i.e., $\underline{h}_Z = 0$). But the existence of multiple BGPs is not always guaranteed. If, for instance, we assume an very low matching efficiency, the high BGP disappears.[25]

3 Quantitative analysis

3.1 Calibration

We calibrate the model to match certain aggregate features of the Japanese economy before the 1990s financial crisis. We take 1970-1992 as the pre-crisis sample, and 1993-2019 as the post crisis sample. With this approach, we are assuming that Japan was in the neighbourhood of the high BGP in the pre-crisis period. Near the end of this period, it is likely that Japan was close to the final stages of the standard neoclassical transitional dynamics driven by capital accumulation. But, in the early part of the pre-crisis period, Japan was still likely experiencing transitional dynamics. Thus, GDP growth and interest rates in the data may exceed those produced by the model in steady state.

Table 1 presents a summary of the parameters' notation, chosen calibration value, their meaning, and a brief explanation of the calibration choice. These values will be used

[23] The decompositions are obtained from FOCs (15) and (18): $\ln W^Z_{ss} = \text{constant} + \ln \pi^{ZH}_{ss} + \ln V^Z_{ss}$; and $\ln W^A_{ss} = \text{constant} + \ln \pi^{AH}_{ss} + \ln V_{ss}$.

[24] In most of the range, the main reason for this "normal" behavior is the downward sloping asset values. See the bottom-right panel. The firm value is decreasing in $H_{A,ss}$ mainly because, as it increases, production labor $H_{Y,ss}$ is squeezed and hence the production size shrinks.

[25] Appendix C presents an alternative intuitive way of viewing multiple BGPs in the spirit of coordination games.

throughout the deterministic and stochastic experiments. For the latter, however, we later discuss the dynamics of the exogenous shocks. The model is rich in parameters and we do not always have a directly observable data counterpart. In some cases, this requires to make informed guesses which we will discuss below. We have four types of parameters: those that are calibrated to standard values in the literature, those directly taken from the data, those taken from previous literature and, finally, those that are estimated or calibrated to match a particular target. Note that one period in the model is one quarter.

	parameter	value	notes
β	subjective discount factor	0.9957	Match risk-free rate
σ_C	relative risk aversion	0.500	Macroeconomics literature
α	capital share	0.360	Capital share
δ_K	capital depreciation	0.026	Annual 10%
G	govt expenditure share	0.238	National Accounts
θ	elast. of subst. varieties	6.00	Markup of 20%
c_k	risk-premium on K	1.005	Match C/Y and I/Y
c_v	risk-premium on new firms	1.0466	Using PER data (see text)
c_z	risk-premium on Z	1.0466	Using PER data (see text)
ξ	exogenous TFP growth	1.0046	World TFP growth
γ_H	population growth	1.0018	Population growth data
$\underline{h}_O$	pop share of non-labor	0.380	Labor market data, OECD
ω_Z	efficiency in research match.	3	Normalization
ω_A	efficiency in develop. match.	4.47	Match pre-92 TFP growth
δ_Z	depreciation of Z	0.0398	Hall, Maireese and Mohnen (2009)
δ_A	depreciation of A	0.0398	Intellectual products deprec., SNA
ϕ_Z	elasticity in research match.	0.2225	Estimated matching function
ϕ_A	elasticity in develop. match.	0.2654	Estimated matching function
s_A	bargaining power	0.80	Match $H_{A,ss}/H_{Z,ss}$ ratio
κ_Z	knowledge externality	1	Irrelevant for model outcomes
$\underline{h}_Z$	% of NPRs	5.0e-04	See text
τ_D	corporate tax	0.40	Corporate tax data
τ_Z	labor subsidy, basic research	0.68	OECD, MITI data
τ_A	labor subsidy, development	0.01	OECD, MITI data
τ_H	labor income tax	0.073	National Accounts
τ_C	VAT tax	0.092	National Accounts

Table 1: Parameter Values. See text for a detailed explanation.

The first group of parameters is standard in the literature. It includes relative risk aversion (σ_c), capital share in final goods production (α), and the depreciation rate of capital (δ_K) which corresponds to a 10% annualized rate. We have chosen the value of the elasticity of substitution among varieties θ so that the mark-up is 20%, which is within the consensus range of New Keynesian models.

The parameters directly taken from the data include the demographic parameters and fiscal policy parameters as well as the depreciation rates of R&D stocks. The growth rate of population ($\gamma_{H,t}$) is calibrated to match the 0.73% growth in the pre-crisis period. To calibrate the share of population outside the labor force, $\underline{h}_O = H_{O,t}/H_t$, we use the share of non-employed population in the pre-crisis period (38%) rather than age dependency ratio. This parameter plays an important role in the analysis of the demographic changes later. Because the model does not consider endogenous participation decisions, our strategy is to treat the increase in female labor force participation, which mitigates the shrinkage of labor supply significantly in the data, as an exogenous change. If we used the age-dependency ratio instead, it would over-estimate the rise in $\underline{h}_O$.

In order to study policy measures in Section 4, we need to set the tax rates and R&D subsidies. The government spending share of output in steady state is taken from National Accounts as the average of the pre-1992 period. Since the model represents a closed economy, government spending is the sum of government consumption, government investment and net exports.[26] In addition, there are five tax rates that are taken from the data. The corporate tax rate is set to 40% to match the typical rate before the 1990s.[27] We set the labor income tax rate to match the ratio of income tax revenue to total income. This is an obvious simplification as the rate is progressive in the data. In our case, however, because labor supply is inelastic, the labor income tax is not distortionary (it is essentially a lump-sum tax). Note that this implies that we ignore taxation of capital income. This is because a significant share of capital income in Japan is tax exempt and, as a result, it is hard to measure it. The effective VAT rate is set to be 9.2% which is lower than the headline rate of 10% because some consumption goods are VAT exempt. Implicitly, other tax revenues are included in the lump-sum transfer (LST_t), because they do not appear to grow in proportion to the economy's size.

For the shares of government subsidies in R&D expenditures, we take the data from OECD and the Japanese Ministry of International Trade and Industry (MITI).[28] Approximately 1% of the private sector R&D expenditure is covered by the government, while 68% is financed publicly for the non-profit sector. Although there is no direct one-to-one mapping between our basic-research and development sectors with the non-profit and private sectors in the data, we believe this is a reasonable approximation.

For the depreciation rates of the basic research stock we resort to Hall et al. (2010) who state that estimates range between 10% and 36% with most researchers settling for a value of 15% in annual terms. For the depreciation rate of A_t we use the "effective depreciation rate" for R&D capital in the Japanese National Accounts.[29] This is also 15%.

[26] With this formulation, we implicitly assume that government investment (8.8% of GDP) does not contribute to the capital formation. Likewise, government consumption (18.0%) does not enter the households' utility function. For the pre-crisis period, the GDP share of net exports is not large (1.3%).

[27] This rate changed almost every year. Subsequently, it has gradually fallen to 30% by 2018.

[28] See NISTEP (2022) and https://stats.oecd.org/Index.aspx?DataSetCode=GERD_SOF

[29] See https://www.esri.cao.go.jp/jp/sna/data/reference1/h27benchmark/pdf/kaisetsu_20211122.pdf (in Japanese).

We turn now to parameters that are calibrated to match certain data moments. The bargaining power between researchers and developers (s_A) is set to match the ratio of business to non-business FTE researchers in the pre-1992 period using research personnel data from OECD. The subjective discount factor (β) is calibrated to match the risk-free rate for the pre-92 period which we calculate using the uncollateralized overnight call rate (the "mutan rate") minus the inflation rate.[30] The risk-premium on capital (c_k) is set to match the private investment to GDP ratio of the pre-1992 SNA data. We set the risk-premia for new firms (c_v) and new basic research (c_Z) both to be 20% in annualized terms. These numbers are within the range of those reported in the literature on the user cost of venture capital. For instance, Kerins et al. (2004) find a cost of capital between 16.7% for a well-diversified investor and 40% for a low diversification investor. They are also consistent with the survival rate of the startup firm data.[31]

We are now left with calibrating ω_Z, ω_A, ϕ_z, ϕ_A, κ_Z, $\underline{h}_Z$, and ξ_{ss}. Two of them have no impact on either the BGP or model dynamics. We set the growth externality parameter $\kappa_Z = 1$, which has no effects as long as condition (4) is satisfied. We set the scale parameter ω_Z to 3. Because ω_Z is subsumed by the measurement units in Z_t, it has no effect either. We calibrate scale parameter ω_A so that the TFP growth rate in the *high* steady state is close to the TFP growth rate in the pre-1992 data. For ξ_{ss}, which we interpret as exogenous or imported technology, we calibrate it to match the World TFP growth estimates in Esfahani et al. (2020). They use World Input-Output tables to calculate the growth rate of world TFP from 1996 to 2016. Because there is no data from the pre-crisis period, we use their estimate of 1.28% World TFP growth for 1996-2014.

Calibrating ϕ_z and ϕ_A is a more complex task and there are no obvious literature parallels to draw from. For this reason, we turn to a data approximation. From the two matching functions (3) and (2) and the decomposition of the TFP (22), given the parameter restrictions discussed in Section 2.2 and the specification for the final output production function, we can derive two equations that would allow us to estimate ϕ_z and ϕ_A:

$$\gamma_{Z^{new},t} - (\gamma_{H_Z,t} - \gamma_{H,t}) = (1 - \phi_Z)\gamma_{Z,t} + \frac{\phi_Z \kappa_Z}{\varphi_A(1-\alpha)}\gamma_{TFP,t} - \frac{\phi_Z \kappa_Z}{\varphi_A}\gamma_{\xi,t}, \tag{30}$$

$$\gamma_{A^{new},t} - (\gamma_{H_A,t} - \gamma_{H,t}) = (1 - \phi_A)(\gamma_{Z,t} - (\gamma_{H_A,t} - \gamma_{H,t})) + \frac{\kappa_A}{\varphi_A(1-\alpha)}\gamma_{TFP,t} - \frac{\kappa_A}{\varphi_A}\gamma_{\xi,t}, \tag{31}$$

where all the $\gamma_{X,t}$'s represent the rate of growth of any variable X. The coefficient on

[30] This money market instrument was launched in July 1985 and we used the average of monthly data for 1985-1992.

[31] Calvino et al. (2015) provide evidence on startup survival rates for a cross-section of countries and find that survival rates are around 40% after 7 years which, if annualized, is 88%. A naïve back-of-envelope calculation implies, if the target portfolio return is 3.7%, the required survivors' earnings yield must be 18% (the salvage value of the exiting firms is assumed to be 0). Note that the historical PERs for the Tokyo Stock Exchange between 1971 and 1985 (excluding the bubble periods) remained between 20 and 30 and take an average value of 27, which implies an earnings yield of 3.7%. Our risk-premum implies that the price earnings ratio (PER) to be 2.5 and the earnings yield to be 22%, which is slightly higher than 18%. However, the value obtained using survival rates is likely to be downward biased because their data includes all firms and not just innovative startups.

the first element of the RHS of the above equations would yield an estimate of the ϕ coefficients. For Z_t^{new}, we used the number of journal publications in all fields by Japan-based researchers. The data come from the October 2022 release of NISTEP. [32] Research stock Z_t is calculated by the perpetual inventory method with the depreciation rate δ_Z above. For A_t^{new}, we use flow of the R&D Intellectual Property Products from Japanese SNA. We eliminate stock A_t using equation (22). The time-varying exogenous component of TFP ξ_t, we used the rate of growth of US TFP from the Penn World Table version 10.01 taking the US as the technology frontier. Finally, for R&D labor inputs, we use the number of researchers in the Higher Education, Government, and Non-Profit sectors for $H_{Z,t}$ and in the corporate sector for $H_{A,t}$ from the NISTEP October 2022 release (full time equivalent). We face severe data limitations as the SNA Intellectual property statistics only go back to 1994. Hence, our estimates are for the 1994-2017 period, which gives us very few degrees of freedom. As a formal econometric estimation, our estimations are far from perfect. However, we only treat these estimates as approximations to pin down the values for ϕ_A and ϕ_Z that are consistent with the evolution of the Japanese TFP and R&D labor inputs. The estimates yield reasonable values of $\phi_Z = 0.2225$ and $\phi_A = 0.2654$.

The proportion of non-pecuniary researchers, $\underline{h}_Z$, is even more difficult to ascertain. In the data, the proportion of researchers in the Higher Education, Government, and Non-Profit sectors for 1994-2017 averages 0.3% of the population. We assume that non-pecuniary researchers are a small number, e.g. 0.05%, of the population which roughly corresponds to 1 out of every 6 workers in basic research. Importantly, although *qualitatively* the existence of NPRs is important, *quantitatively* the actual value of $\underline{h}_Z$ has little impact in most simulations below. This is because the value of $\underline{h}_Z$ only directly affects the position of the low growth BGP, but it has a very minor effect on the location of the middle (unstable) and high BGPs. The exception to this is the policy experiments in Section 4 as they analyze how to escape from the low BGP. For the other simulations, the economy starts from the high BGP and the main concern is whether or not the economy crosses the demarcation line leading to convergence to the low BGP. Most results below are reported using $\underline{h}_Z = 0.05\%$. For the policy experiments, we analyze the sensitivity of the results to more extreme values with $\underline{h}_Z = 0.001\%$ and $\underline{h}_Z = 0.15\%$.

Table 2 reports the values of selected variables from the model in the high, middle (unstable), and low BGPs compared to the data counterpart when available. In bold, we highlight targeted moments. As mentioned above, we chose parameter values by targeting the rate of growth of TFP, C_t/Y_t and I_t/Y_t ratios, and the risk-free rate in the high BGP. The model yields a rate of growth of TFP in the low BGP (a non-targeted moment) of 0.21% against the data counterpart of 0.69%. However, it is likely that the data reflects, rather than the new low BGP, the transitional dynamics where TFP growth is slowly converging to the low steady state. In our stochastic simulations below, under the "wretched coincidence" scenario that pushes the economy towards the low BGP, we can calculate the average rate of growth of TFP for the 30 years following the shock. This yields a value for TFP growth of 0.75%, which is much closer to the data. Given this, the model does a good job at

[32]See `https://www.nistep.go.jp/research/science-and-technology-indicators-and-scientometrics/indicators`.

	Model			Data		
	high	unstable	low	70-92	93-19	notes
γ_{TFP}	1.6879	0.5407	0.2094	**1.690**	0.690	growth TFP annual (%)
$\gamma_{GDP/H}$	2.6498	0.8461	0.3274	3.065	0.564	growth GDP/labor (%)
C_{ss}/Y_{ss}	0.5070	0.5270	0.5332	**0.501**	0.548	consumption/output
I_{ss}/Y_{ss}	0.2550	0.2350	0.2288	**0.264**	0.203	investment/output
$R_{f,ss}$	3.8316	2.9153	2.6503	**3.866**	0.165	risk-free rate annual (%)
$R_{k,ss}$	5.9082	4.9736	4.7033			capital return annual (%)
$R_{eq,ss}$	21.9701	21.8504	21.8081			equity return annual (%)
$R_{eq} - R_f$	18.1385	18.9351	19.1578			equity premium
PER_{ss}	2.5163	3.8323	4.5396			price/earnings ratio
$H_{A,ss}$	1.5674	0.7679	0.3508			hours for development (%)
$H_{Z,ss}$	0.6998	0.1643	0.0500			hours for research(%)
$W^{product}$	1.9755	2.0399	2.0598			wage (MPL) in prod/invention
$W^{research}$	1.9755	2.0399	1.4005			wage (MPL) in basic research

Table 2: Non-Stochastic balanced growth paths in the model and data. In **bold**, targeted moments.

explaining the TFP growth slowdown.[33] Regarding the C_t/Y_t and I_t/Y_t ratios in the low BGP, the model predicts the direction of the change but cannot match the increase in consumption and decline in investment. However, in the table above, we have kept demographics constant whereas the data will reflect the effect of population ageing. Finally, although the model predicts a sizeable risk-free rate decline of almost 1.2pp, it cannot account for the effective zero risk-free rate in the post-92 Japanese data since it abstracts from nominal distortions. Regarding other moments, the equity premia on existing capital and new firms are 2.1% and 18.1%, respectively. The capital return should be safer because it is backed by capital, while our equity-return is the return on the startups, which should be riskier than the listed companies.

3.2 Deterministic dynamics

We first look at the transitional dynamics of the model with no uncertainty. Figure 6 depicts the phase diagram.[34] The isocline curves for (de-trended) $\ln K_t$ and $\ln Z_t$, the two state variables, represent combinations of $\ln K_t$ and $\ln Z_t$ such that $K_{t+1} = K_t$ (blue horizontal line) and $Z_{t+1} = Z_t$ (pink dotted curves), respectively. The isocline curve for Z_t is highly non-linear, reflecting the vicious/virtuous research cycles and the existence of

[33]A comparison of the time-series fit of the model against the data is available in Appendix D.

[34]Figure 6 plots the phase diagram of the de-trended endogenous state variables. Note that, unlike in the simple neoclassical growth model with one endogenous state, our phase diagrams omit the dynamic jump variables such as consumption and asset values. More precisely, Figure 6 is the projection of the full phase diagram, which is 5D, on the state space, which is 2D. In the figure, the low and high BGPs appear as sink points in the $(\ln Z_t, \ln K_t)$ plane, but they are actually saddle points in the full dimension.

the lower bound of $H_{Z,t}$.[35] Their crossing points are the steady states. The steady states in the de-trended model correspond to BGPs in the original model. In the area to the left of the green line, only NPRs engage in basic research. This green line is determined endogenously (a free boundary). Most importantly, the black line demarcates the basins of the low and high BGPs. That is, to the right of it, the economy converges to the high steady state, while to the left, it converges to the low steady state. Because the middle steady state is unstable, the demarcation line must go through it. The grey arrows show the direction and the speed of the transition dynamics.

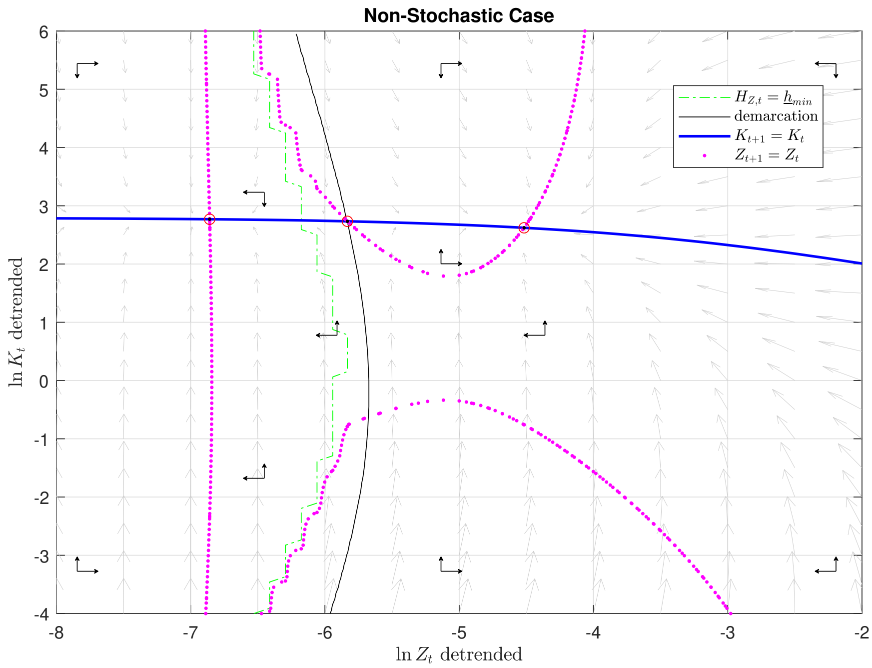

Figure 6: Phase diagram for the deterministic (de-trended) model.

We can look at the transitional dynamics of the model for different initial states. Figure 7 presents the case in which two different economies start with the same level of Z_0 but they have different initial capital stocks K_0. The capital-rich economy starts above the demarcation line, whereas the capital-poor economy starts below it. The upper left panel shows the different transitions taken by these two economies (each dot represents a quarter). The capital-rich economy will converge to the low BGP whereas the capital-poor will converge to the high BGP. This happens because, in the capital-rich economy, the return to working in the production sector is higher and hence more labor is allocated to the production sector in equilibrium to the detriment of the research sectors (lower right panel). This intuition is akin to a "resource curse" where resource rich economies may converge to a low growth equilibrium.

The upper middle panel suggests that both economies can grow fast in the early stages

[35] If there are no NPRs ($\underline{h}_Z = 0$), the vertical part of the Z-isocline lies at $Z_t = 0$.

as long as their initial capital is low. The lower middle panel shows that the early stage of fast growth is not driven by TFP growth. Instead, because the return on capital is high, capital accumulation is the main driver of growth (upper right panel). In later periods, the low growth economy allocates more capital and labor to the production sector, while the high growth one has a lower K_t/Y_t ratio and a larger R&D labor share (right two panels).[36]

Because any economy experiences fast growth in these early stages, there is no "poverty trap" in our model. The model delivers a "middle income trap" as a combination of (i) fast initial growth due to capital accumulation and (ii) failure to switch the growth engine from capital accumulation to R&D. For an economy that is not in a trap, the model exhibits the two growth phases, as in Galor and Weil (2000) and Galor (2005). These middle income traps as a failure to switch between capital accumulation and R&D have been similarly discussed by Zilibotti (2017) for the case of China as mentioned in the introduction.

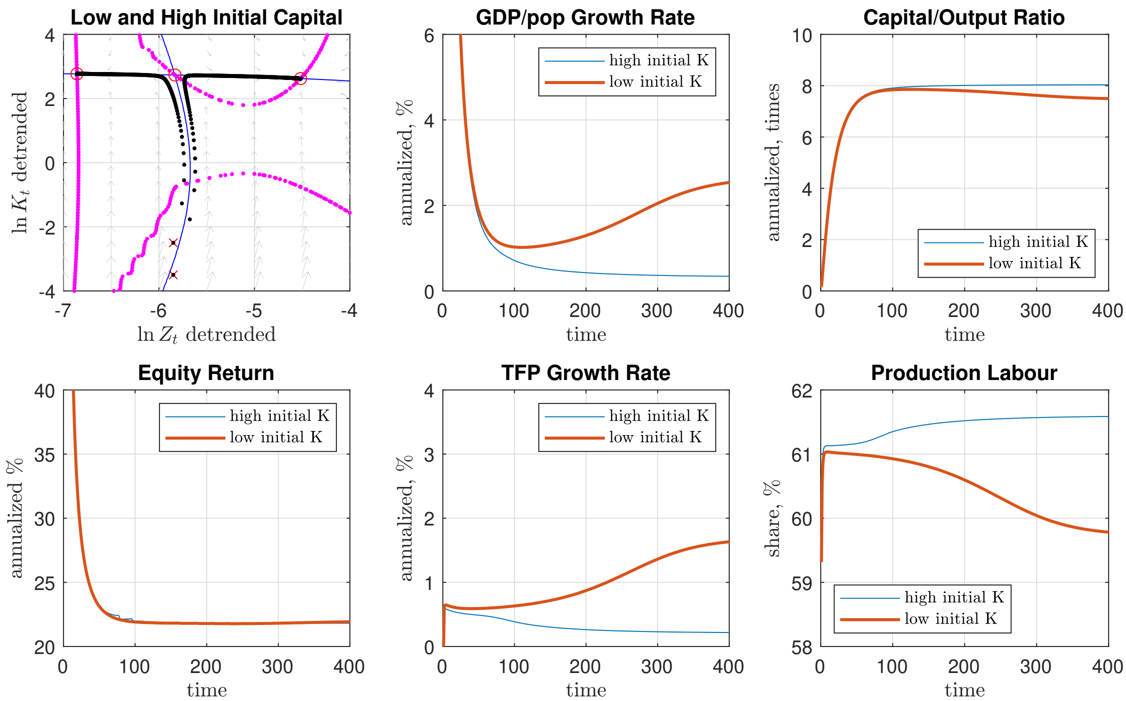

Figure 7: Sample paths for two economies with different initial capital stock (K_0).

3.3 Stochastic dynamics: business cycles and growth

The relevant question from a business cycle perspective is whether temporary shocks can move the economy between regions along the demarcation line. To do so, we introduce stochastic shocks in the model. More specifically, we consider a financial market shock

[36]The convergence speed of the model seems to be slow. As shown in the figure, at 300 quarters, the low initial capital economy is still on its way toward the high BGP. However, the convergence time depends on the initial state. The higher Z_0 the faster convergence will happen.

and a demographic shock, reflecting the events in the early 1990s and the well-known demographic changes that Japan has experienced in the past 30 years. We pay special attention to the depth and duration of these shocks as they will be key determinants of growth regime shifts. Tables 3 and 4 describe the characteristics of these shocks which we explain below.

The financial shock we consider is a two-point Markov process, in which the risk-premium parameters take either normal values or crisis values. Although our model does not have a detailed financial sector, these shocks represent exogenous financial crisis shifters.[37] Relative to the baseline parameter values, we assume that all risk-premia increase by 50% during the crisis periods. We assume that the probability that an economy in the baseline parameter values (normal times) experiences a financial crisis in the next quarter is 1/304 (0.33%), while the probability of exit from the crisis is 1/16 (6.25%). Hence, these shocks are temporary. These numbers imply that the average duration of the normal times and crisis times regimes are 304 and 16 quarters (74 and 4 years), respectively. Also, the share of time of the normal periods is 95% and 5% for financial crises. These numbers roughly match with the results of Tambakis (2021) based on Laeven and Valencia (2020) data. See also Table 1 of Boyd et al. (2019).

For the demographic shock, as discussed in Section 3.1, to measure the change in the share of inactive population in the labor market ($h_O = H_{O,t}/H_t$) we take into account that female labor force participation has increased substantially throughout the period of analysis. We set the demographic surprise consisting of a simultaneous decline in the population growth rate ($\gamma_{H,t}$) from 0.73% p.a. (1970-1992) to 0% p.a. (since 1993) and an increase in the share of non-active workers from 38% to 41%. We will discuss the implications of a more severe demographic shock later.

We treat this demographic shock as a permanent "MIT shock". I.e. agents do not anticipate it in their optimization and, once it happens, it is perceived to be permanent. Thus, the corresponding Markov Transition Matrix for the demographic shock is the identity matrix. These demographic changes are often taken as fully anticipated trends when explaining the Japanese growth slowdown (see Otsu and Shibayama, 2016 among others). However, this is far from what has happened in recent Japanese history. Government statistical offices have consistently failed to predict it and hence we treat it as a surprise.[38]

Combining these two types of shocks, the model contains a four-point Markov shock process. The entire Markov transition matrix is the Kronecker product of the two transition matrices. For convenience, we label these regimes in Table 5. In the baseline regime (regime 1), no shocks affect the economy. In the aging regime (regime 2) the economy is only subject to (permanent) demographic shocks. In the financial shock regime (regime 3)

[37]See, for instance, Cochrane (2011) and Di Tella and Hall (2022).

[38]As Nishimura (2012) remarks: "The popular perception that demographic change is in general predictable is based on this law of large numbers. Unfortunately, this perception is not always true, or to put it bluntly, not true in many instances. Take Japan for example. Between the 1970s and early 2000s, the total fertility rate forecasts regularly turned out to be wrong and were consistently revised down. The government repeatedly published its forecast in which the decline in the fertility rate was declared to be only temporary and the birth rate expected to rise again soon. Similarly, life expectancy forecasts have shown that the actual figures consistently exceeded the forecasts."

Table 3: Financial Shock and Demographic Shock

	Risk-premium Shock	normal regime	crisis regime
c_K	capital	2%	3%
c_v	development	20%	30%
c_Z	basic research	20%	30%

	Demographic Shock	normal regime	aging regime
h_O	$1-$ employment share	38%	41%
γ_H	pop. growth rate	0.73%	0.00%

Table 4: Markov Transition Matrices

Financial Shock				Demographic Shock		
from\to	base	crisis		from\to	base	aging
base	303/304	1/304		base	1	0
crisis	1/16	15/16		aging	0	1

only temporary financial shocks affect the economy. Finally, regime 4 is labelled "wretched coincidence" as both (temporary) financial and (permanent) demographic shocks hit the economy simultaneously. Note that the baseline shock regime is very close to the non-stochastic case above, although it is not the exactly same because agents rationally anticipate the risk of financial shocks in the future.

Table 5: Allocation of panels in the figures below

financial\demo	baseline (pre 1992)	aging
baseline	baseline (regime 1)	aging (regime 2)
crisis	financial shock (regime 3)	wretched coincidence (regime 4)

Corresponding to these four shock regimes, there are four phase diagrams, each of which summarizes the dynamics of the state locus for each shock regime. See Figure 8. Importantly, even if a shock hits the economy, it does not affect the the state of an economy immediately. At the time of the shock, the locus of the endogenous state variables ($\ln Z_t$ and $\ln K_t$) never jumps. After the shock, the state of the economy moves slowly in different directions determined by on the shock regime.

In Figure 8, both shocks expand the distance between the upper and the lower U-shaped parts of the Z-isoclines. They only have a very marginal impact on the K-isocline

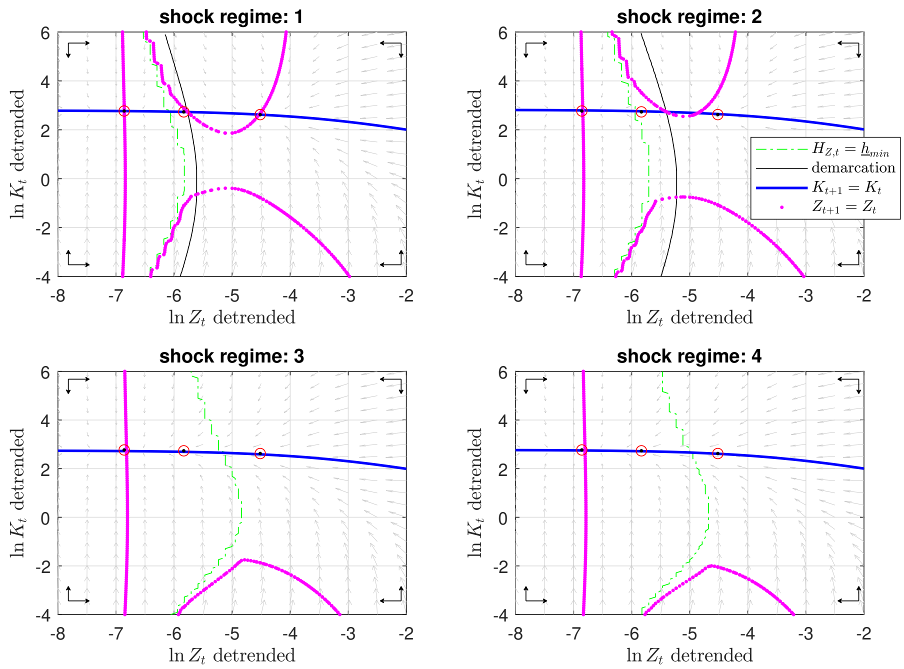

Figure 8: Phase diagrams for the 4 shock regimes. Regime 1: baseline. Regime 2: demographic shock only. Regime 3: financial shock only. Regime 4: financial and demographic shocks. The red circles in all panels show the position of the BGP in the non-stochastic case.

(blue solid line). In all four panels, to make the relative positions of the isoclines clear, we plot three red circles denoting the BGPs in the non-stochastic case. Given our choice of parameters, the only qualitative change in Figure 8 between the four regimes is the number of stationary points. In shock regimes 1 and 2 (upper panels), there are three crossing points though the demarcation line shifts to the right in regime 2. Hence, as long as the economy stays to the right to the demarcation line (solid black line), it will move toward the high BGP. On the other hand, there is only one stationary point in regimes 3 and 4 implying that, starting from any point in the state space, the economy moves toward the low stationary point. In these cases, the demarcation line disappears.

In the following sub-sections, we investigate the dynamic responses of the economy to shocks under these different scenarios.

3.4 Temporary financial shocks only

We now study the dynamic effects of financial shocks. We simulate the model under the following scenario: (i) the economy starts from the high BGP of the baseline regime (top left panel of Figure 8), (ii) a financial shock hits the economy and lasts 40 quarters (bottom left panel) and, (iii) it then returns back to the baseline shock regime (top left panel). Figure

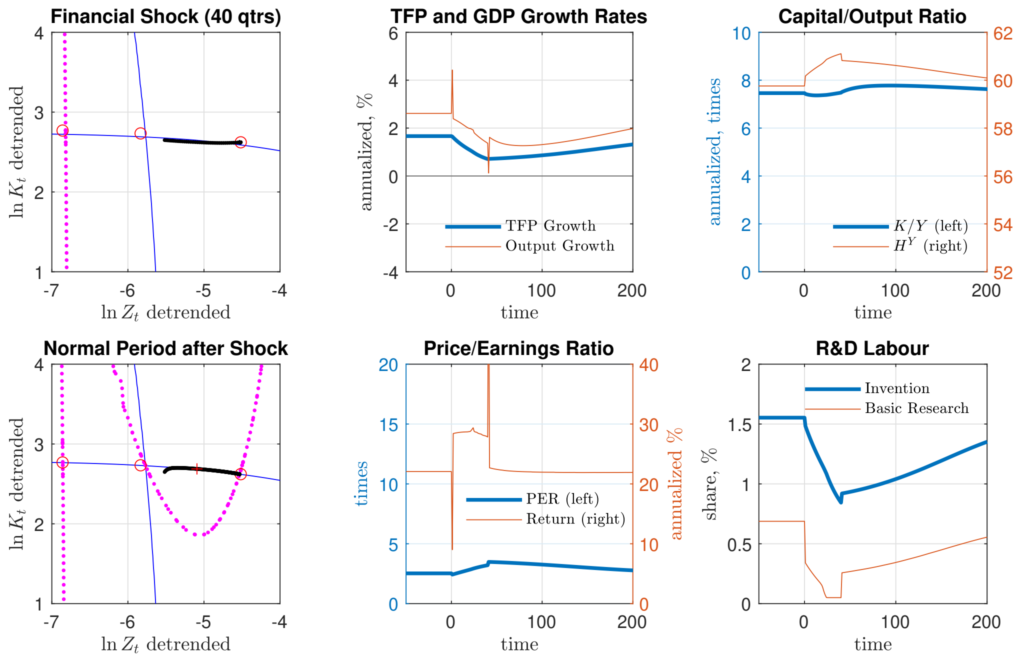

Figure 9: Response to a benchmark (temporary) financial shock of 50% risk premia increase with 40 quarters duration.

9 shows the resulting impact of the shock. In all stochastic simulations throughout this section, GDP growth and the equity market indicators have sharp spikes. They are simply the artifact of fully flexible labor mobility. Under imperfect labor mobility, the effects of a shock should be spread out over several periods.

We have selected a length of 40 quarters, which is 2.5 times longer than the average crisis duration in our benchmark calibration assumption, to effectively capture the characteristics of the Japanese 1990s financial crisis. In Japan, the crisis is generally believed to have lasted from 1991/92 to 2002. The initial stock market crash occurred at the start of 1990, but the crisis did not fully impact the entire financial system until 1992. The crisis reached its peak in 1997/1998 with the failure of four major traditional financial institutions. To address the crisis, the Koizumi Administration introduced the "Financial Revitalization Program" in October 2002, aiming to halve the ratio of non-performing loans in major banks by fiscal year 2004. However, significant corporate failures had already ceased by 2001. See Table E.4 in Appendix E for more details.

Figure 9, under our parameter setup, illustrates that the crisis concludes before the economy crosses the demarcation line, resulting in a return to the high BGP (bottom left panel). The figure also depicts the evolution of TFP and GDP growth rates, the capital/output ratio, price/earnings ratio, and R&D labor allocation during the transition. Following the financial shock, although TFP growth slows down, it eventually returns to its high BGP level, indicating that the slowdown is only temporary. In essence, our benchmark parameterization suggests that the temporary shock does not have any permanent growth effects (although there is a permanent level effect, i.e. hysteresis).

However, it is crucial to consider significant parameter uncertainty. For instance, the actual length of the financial crisis may exceed the assumed 40 quarters (lost decades vs. lost decade), and the depth of the crisis may be greater than our presupposed 50% increase in the risk-premium. The latter is particularly challenging to determine accurately. Thus, we calculated the required duration of the crisis for different depths of the risk-premium shock, as presented in Table 6. The deeper the shock, the shorter the necessary duration of the crisis. For instance, if the shock size reaches 90% (almost doubling risk premia), a reasonable duration of 45 quarters could lead to permanent effects on the rate of growth of TFP. Figure 10 illustrates a scenario with a duration of 60 quarters and a risk-premium shock of 50%. The economy crosses the demarcation line and the growth slowdown becomes permanent as, even after the shock vanishes, it converges to the low growth BGP (super-hysteresis).

Table 6: The Required Length of Financial Shock to Generate Permanent Effects

depth of financial shock	30%	50%	60%	70%	90%
length of financial shock	91 qtrs	60 qtrs	50 qtrs	47 qtrs	45 qtrs

There are two main implications in this subsection. First, a temporary shock can at least potentially have a permanent effect on TFP growth (super-hysteresis) even with reasonable length and depth. Second, for the case of Japan in the 1990s, although we cannot rule out a regime shift, our benchmark calibration indicates a financial shock alone may not be enough to explain the TFP slowdown observed in the data.

3.5 Demographic shock only

We now turn to the demographic shock. Since the shock is permanent, there is no theoretical interest on whether this shock can generate super-hysteresis. Our main interest here is whether demographic shocks alone are enough to trigger a growth slowdown consistent with the data.

As discussed above, we assume that the population growth rate $\gamma_{H,t}$ declines from 0.72% to 0% and the the share of inactive workers $\underline{h}_O$ increases from 38% to 41%. The latter is particularly important, because (i) a smaller labor force simply squeezes R&D labor (as well as production labor), and because (ii) the value of a firm is the reward to successful R&D, a smaller working population share directly implies a smaller reward to R&D.

We analyze the economy starting from the high BGP in regime 1 and then moving to regime 2 (demographic shock only) permanently. Figure 11 shows the results. In this scenario, crucially, the aging shock regime still retains the high BGP though its position slightly moves to the left. To the extent that the high stationary point moves to the left, adverse demographic shocks have a permanent but quantitatively small negative impact

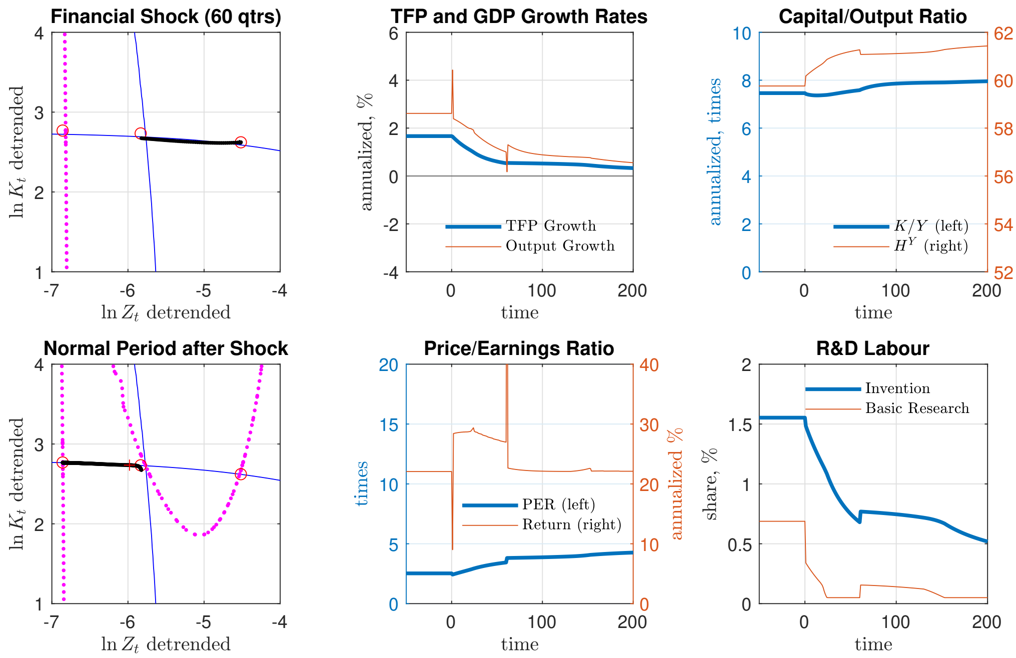

Figure 10: Response to a (temporary) financial shock of 50% risk premia increase with a long 60 quarters duration.

on R&D and TFP growth. In this experiment, R&D labor decreases but not significantly (bottom right panel). It is production labor that is most affected by the demographic shock. This is mitigated to some extent by a higher level of capital stock (top right panel).

However, we do not rule out the possibility that demographic shocks are strong enough to eliminate the high BGP. There are two reasons for this. First, as discussed in Section 2.6, there is model uncertainty. We assume that households do not internalize future population changes. If, instead, households take the future family size into account, the shift of the Z-isocline will be larger, potentially leaving only the low BGP in the state space. Second, there is parameter uncertainty. Our demographic shock assumptions may underestimate the true extent of demographic surprises. If so, demographic shocks alone could be to cause a permanent shift of the BGP.

3.6 Combining financial and demographic shocks

Having analyzed the two shocks separately, we now turn our attention to the case of the "wretched coincidence" regime. Both financial shocks and surprise demographic events affected the Japanese economy from the early 1990s. We thus consider whether a combination of both shocks could explain the observed growth slowdown for reasonable shock depth and duration. In particular, we consider a scenario where the economy starts in the baseline regime (regime 1) and then moves to the "wretched coincidence" regime (regime

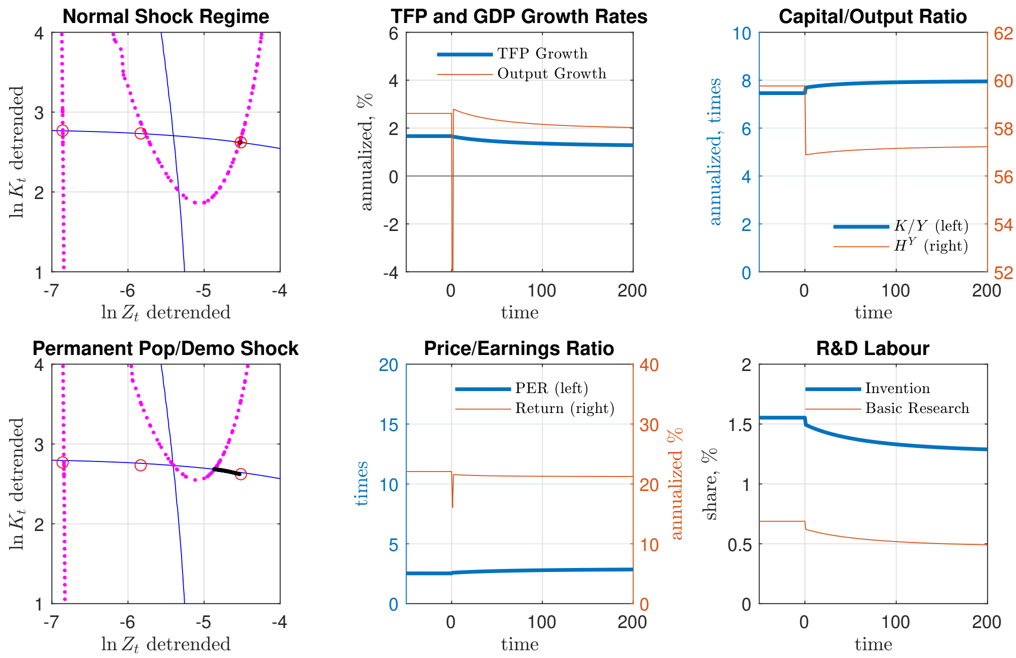

Figure 11: Response to a permanent demographic shock.

4) for 40 quarters, and finally moves to the aging regime (regime 2). That is, it faces a temporary financial shock and a permanent demographic shock with the same characteristics as above. As we will see, in this case, the plausibility of both shocks moving the economy beyond the demarcation line is much less controversial.

Figure 12 shows the evolution of the key variables under consideration. The top left panel is the phase diagram of the wretched coincidence regime. Because the low BGP stationary point is the only attractor, starting from the high BGP, the state locus moves towards it. In this case, the economy moves faster towards the low growth BGP. Note that, in order for the economy to reach the demarcation line, it takes less than 30 quarters (7 years). This is shorter than the consensus duration of the 1990s financial crisis in Japan. After the financial shock disappears, the demographic shock remains. Hence, the phase diagram shifts to shock regime 2 as shown in the bottom left panel (which is the same as regime 2 in Figure 8). Although there are two stable stationary points in the aging regime, because the economy is in the basin of the low stationary point, it never goes back to the high stationary point.

In the bottom left panel, the red "+" shows the state of the economy 120 quarters (30 years) after the initial shock. That is, if the shocks hit in 1992 it would display where the economy is in the transition path in 2022. As discussed in Section 3.1, the average TFP growth rate for these 30 years in this simulation is 0.75%, which is close to the average TFP growth rate for 1992-2019 in the data (0.69%). See also Appendix D for a comparison of this simulation with the data.

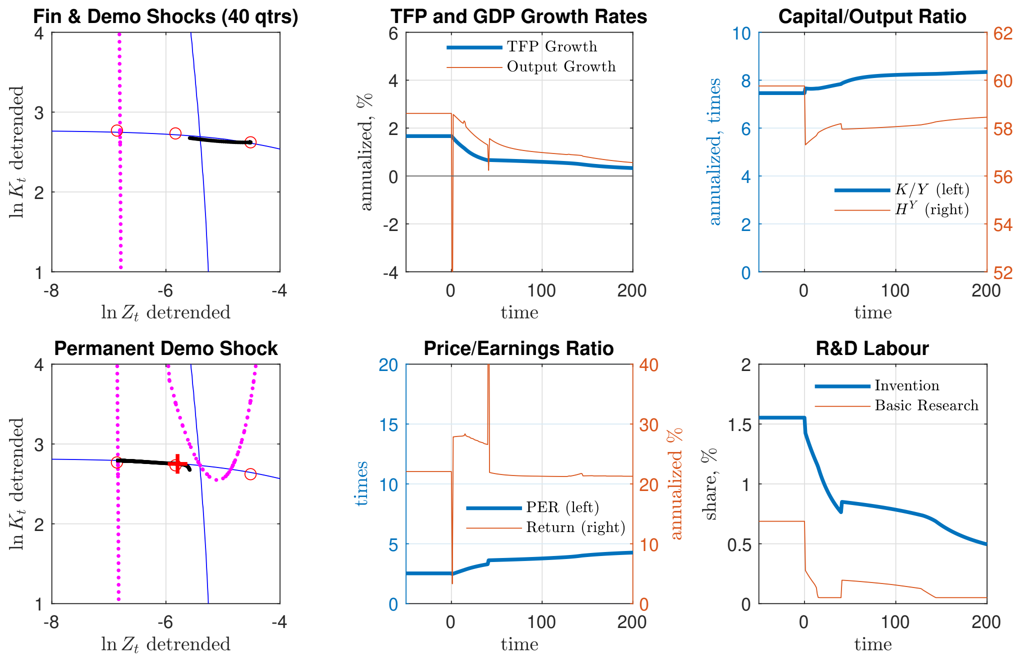

Figure 12: Temporary financial shock of 40 quarters and a 50% increase in risk-premia combined with a permanent demographic shock.

The takeaway from this exercise is that, even though there may be uncertainty about the shock duration and depth, the combination of temporary financial and permanent demographic shocks is far more likely to be enough to move the economy into the low growth trap for a wide range of parameterizations.

4 "Big Push" Policies

We now study the potential for fiscal policies to push the economy from the low to the high BGP. That is, we study which "big push" policies are effective to help the economy escape from the low growth regime. We will consider the following "big push" instruments:

- Changes to the corporate tax rate (τ_D).
- Changes to the labor subsidy in the development sector (τ_A).
- Changes to the labor subsidy in the basic research sector (τ_Z).
- Changes to Government spending (G).

Given the tax rates calibrated in Table 1, we look at the shares of R&D financed by the Government in the data and in the model. Table 7 shows the results. Total

R&D expenditure in Japan in 2021 was 3.2% of GDP of which 2.5% corresponds to the private sector and the rest to non-profit organisations including public research institutions, Higher Education, and other non-profit institutes. The government share of total R&D is 14.8%. However, most of it is concentrated in the non-profit sector. Only 0.8% of private corporations research is funded by the government in contrast to 67.6% for the non-profit sector. In the model, we set 1% and 68% for the government funding shares.

Table 7: Japan R&D Spending 2021 data and model outcomes

Data	Total	Private	Non-Profit	
as % of GDP	3.2%	2.5%	0.7%	
govt funding share	14.8%	**0.8**%	**67.6**%	
govt funds as % of GDP	0.48%	0.02%	0.45%	
Model (baseline)	Low BGP		High BGP	
govt funding	Develop.	Basic R	Develop.	Basic R
govt funding share	1.0%	68.0%	1.0%	68.0%
govt funds as % of GDP	0.004%	0.02%	0.02%	0.51%

Next, we introduce a shock to the tax/subsidy rates one by one with the economy starting from the *low* BGP with a non-active population ratio of 41% as in the post-1992 period. The change in tax/subsidy is perceived to be unexpected and permanent. However, if and when the economy crosses the demarcation line, the government can revert to the original (i.e., current) policy, which is unanticipated by the private sector. As discussed in Section 3.6, even though the policy change is temporary, the economy would still converge to the high BGP. For this analysis, to make the numerical experiments meaningful, the high BGP must exist even after the permanent demographic shock. Recall that if the demographic shock is stronger than what we suppose, the high BGP may disappear. If so, after a temporary support policy, the economy converges to the low BGP. In this case, the government cannot revert the tax/subsidy rates to the original level.

The baseline results are presented in Table 8 under the label $\underline{h}_Z = 5e^{-4}$. We reduce corporate tax, increase subsidies or government spending by different amounts and present the number of quarters required to cross the demarcation line. The picture that emerges is that neither government spending nor subsidies to the product development sector are able to push the economy towards the high BGP. In contrast, both corporate tax cuts or basic research subsidies can be effective. Note, however, that the corporate tax cuts required are very large and, even when zero, it would take more than 13 years of zero corporate tax to achieve a change in regime. Subsidies to basic research shift the economy faster, although they would imply almost all basic research being subsidized by the government. Government spending is the least effective mainly because the positive effect of spending on corporate profits is outweighed by the reallocation of resources toward the final production sector and away from R&D sectors. Corporate tax cuts and the subsidy to product developments both support R&D but the former is better than the latter because subsidies affect directly the development sector, not the basic research sector.[39] Recall that the corporate profits after tax are split between the two R&D sectors.

Table 8: Periods required to escape the low BGP after the policy change

Fiscal Policy	Policy Change/qtrs				Fiscal Policy	Policy Change/qtrs			
tool: τ_D	15%	10%	5%	0%	tool: τ_A	41%	61%	71%	81%
$\underline{h}_Z = 8e^{-4}$	52	41	34	29	$\underline{h}_Z = 8e^{-4}$	∞	∞	∞	253
$\underline{h}_Z = 5e^{-4}$	∞	93	68	54	$\underline{h}_Z = 5e^{-4}$	∞	∞	∞	∞
$\underline{h}_Z = 2e^{-4}$	∞	∞	∞	∞	$\underline{h}_Z = 2e^{-4}$	∞	∞	∞	∞
30yrs after	17	14	12	11	30yrs after	∞	104	80	77
tool: τ_Z	78%	83%	88%	93%	tool: G	18%	23.8%	28%	38%
$\underline{h}_Z = 8e^{-4}$	62	28	16	9	$\underline{h}_Z = 8e^{-4}$	∞	∞	∞	∞
$\underline{h}_Z = 5e^{-4}$	∞	53	26	14	$\underline{h}_Z = 5e^{-4}$	∞	∞	∞	∞
$\underline{h}_Z = 2e^{-4}$	∞	∞	80	28	$\underline{h}_Z = 2e^{-4}$	∞	∞	∞	∞
30yrs after	18	10	6	4	30yrs after	∞	∞	∞	∞

Unlike in the case when the economy starts from the high BGP, the proportion of NPRs will now be important to determine the success of different policies. This is because it regulates the location of the low BGP in the state space. The larger the proportion of NPRs, the closer to the demarcation line and hence the more likely are policies to be successful. Given the uncertainty about this proportion in the data, we carry out a sensitivity analysis for different values of $\underline{h}_Z$ in Table 8 which appear under labels $\underline{h}_Z = 2e^{-4}$ and $\underline{h}_Z = 8e^{-4}$. The main conclusion from this exercise is that the ranking of policies remains unchanged as expected.

It is perhaps more important to recognize that the Japanese economy in the early 2020s is likely to still be in transition towards the low BGP (see Figure 12). This implies that the economy may be closer to the demarcation line than what has been assumed above and the simulation results starting from the low BGP may be too pessimistic. Table 8 also shows that, if the subsidy to basic research is increased to 78% in 2022 (30 years after the onset of the financial shock), it would only take 4.5 years for the economy to cross the demarcation line. Interestingly, in this exercise, the corporate tax cut is as effective as the subsidy to the basic research sector. Clearly, an important conclusion from this analysis is that the earlier the policy is implemented, the better.

Finally, we look at the dynamic adjustment of the economy to basic research and corporate tax shocks. We focus on the evolution of GDP and the cost of the policy in terms of net government revenues given the calibrated parameters for tax policies. We analyze two cases. First, an increase in the basic research subsidy from 68% to 83% for

[39]We can view the subsidy to basic research as a direct improvement in the reward to basic research activities as in Hvide and Jones (2018) having a large impact on the stock of knowledge.

60 quarters. Second, a cut in the corporate tax rate from 40% to 5% for 80 quarters. For both scenarios, the starting point is the low BGP. In the long-run, both policies have the same impact on government revenues minus subsidies: 0.54% of GDP per period. Because both policies are successful in pushing the economy to the high BGP, both yield a long-run growth rate gap of approximately 1.6%.[40] Although the cost is not negligible, the return is orders of magnitude larger.

Figure 13 presents the transition of the economy for the research subsidy shock in terms of GDP level (upper left quadrant), R&D expenditure (upper right), GDP growth (lower left), and government budget as a percentage of GDP (lower right). After 50 years, compared with the benchmark with no policy change, GDP is 50% higher. The time variation of the net cost in the lower right panel is almost entirely explained by the increase in R&D subsidies. Even after the support policy ceases, the volume of subsidies remains higher than in the baseline because there is more R&D activity in the high BGP. Importantly, however, most of the incremental subsidy cost is offset by the increase in tax revenues due to the higher growth rate.

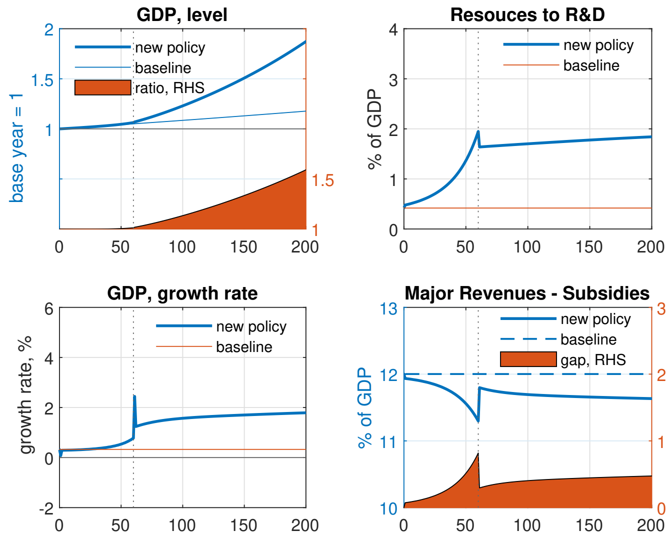

Figure 13: Dynamic adjustment to an increase in τ_Z (subsidy to basic research) to 83% for 60 quarters.

Figure 14 for the 80 quarters cut in corporate taxes has similar results. That is, in the long-run, with relatively moderate budget cost, the successful policy generates a very large increase in GDP. The main difference is the budget cost in transition which is larger in this case.

In summary, there are four main lessons from these policy exercises. First, in order to push the economy back to the high growth BGP, fiscal policies need to be substantial in size and sustained for long periods. However the actual cost is much lower than at face value

[40]This includes the indirect effect of higher capital accumulation. Note also that this number is different from the gap in $\gamma_{GDP/H}$ in Table 2 because, in this section, we start off with adverse demographic conditions.

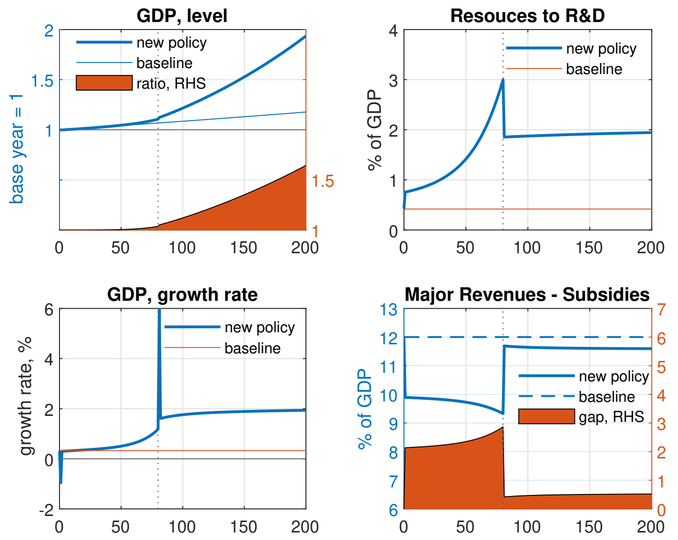

Figure 14: Dynamic adjustment to a reduction in τ_D (corporate tax) to 5% for 80 quarters.

if the increase in tax revenues due to higher GDP growth is taken into account. Second, the sooner the better. That is, as the policy response is delayed, the cost increases as the economy moves away from the threshold above which it converges to the high growth regime. Third, the government must carefully choose the policy instruments. Traditional public expenditure is neutral for R&D activities. The subsidy to basic research appears to be quantitatively most effective. Interestingly, the subsidy to product development is not effective. Fourth and most important, growth rate effects would dwarf almost any level effects. The cost of the big push policy that the model delivers is substantial, but its return is orders of magnitude larger.

5 Conclusions

Standard macroeconomic models assume that, after temporary shocks, the economy either experiences only a temporary loss of output or this loss is permanent but its rate of growth returns back to pre-shock levels. Evidence suggests that, after prolonged or deep financial crises, the rate of growth remains persistently low. This phenomenon has been labelled “super-hysteresis”. We develop a model where temporary financial shocks can lead to permanent changes in the rate of growth of total factor productivity (TFP).

In the model, basic research and product development are treated as separate sectors. In order to generate basic research ideas, researchers have to match with the existing stock of knowledge. Product developers also have to match with this knowledge stock to produce new varieties. This creates search externalities between the two sectors that generate vicious and virtuous R&D cycles. The model has a unique equilibrium path but multiple balanced growth paths (BGPs). There is a threshold below (above) which the economy converges to the low (high) balance growth path (BGP). Starting from the high BGP, if a sufficiently deep or long-lived negative shock pushes the economy beyond this threshold, it will converge to the low growth regime even after the shock vanishes.

We analyze the model in the context of the Japanese growth slowdown where, after the early 1990s crisis, TFP growth declines substantially and R&D indicators deteriorate. Quantitatively, the model can explain well the TFP growth decline in the post-1990s period. After a financial shock, the Japanese economy can transit between these BGPs, generating "super-hysteresis" in TFP. For these shocks to be sufficient to tip the economy into the low growth equilibrium, they have to either be very persistent or very large. In the Japanese case, however, the financial crisis coincided with perverse demographic dynamics which we term the "wretched coincidence". When we take both demographics and the financial crisis into account, the duration and depth properties of the financial shock required to push the economy to the low growth regime are more realistic.

Finally, we investigate "big push" policies aimed at moving the economy back to the high growth regime. We show that the sooner the policy happens the better and, to be successful, any policy needs to be sustained for a prolonged period. Government spending shocks are unable to achieve a transition to the good regime. Corporate tax and, especially, subsidies to the basic research sector are a effective in the sense that they trade a finite period expenditure increase for a permanent increase in the growth rate of the economy. In transition, the required budget cost may be non-negligible. However, because tax revenues grow with the level of output, the long run cost of supporting basic research is low as *one day the government may tax it.*

References

Acemoglu, D. and Zilibotti, F. (1997). Was Prometheus unbound by chance? Risk, diversification, and growth. *Journal of Political Economy*, 105(4):709–751.

Aghion, P., Bergeaud, A., Boppart, T., Klenow, P. J., and Li, H. (2023). A theory of falling growth and rising rents. *The Review of Economic Studies, forthcoming.*

Aikman, D., Drehmann, M., Juselius, M., and Xing, X. (2022). The scarring effects of deep contractions. *BIS Working Papers*, (1043).

Akcigit, U. and Ates, S. T. (2021). Ten facts on declining business dynamism and lessons from endogenous growth theory. *American Economic Journal: Macroeconomics*, 13(1):257–98.

Aksoy, Y., Basso, H. S., Smith, R. P., and Grasl, T. (2019). Demographic structure and macroeconomic trends. *American Economic Journal: Macroeconomics*, 11(1):193–222.

Anzoategui, D., Comin, D., Gertler, M., and Martinez, J. (2019). Endogenous technology adoption and R&D as sources of business cycle persistence. *American Economic Journal: Macroeconomics*, 11(3):67–110.

Azariadis, C. and Drazen, A. (1990). Threshold externalities in economic development. *The Quarterly Journal of Economics*, 105(2):501–526.

Azariadis, C. and Stachurski, J. (2005). Poverty traps. *Handbook of Economic Growth*, 1:295–384.

Ball, L. (2014). Long-term damage from the great recession in oecd countries. *NBER Working Paper*, (w20185).

Benigno, G. and Fornaro, L. (2018). Stagnation traps. *The Review of Economic Studies*, 85(3):1425–1470.

Bergeaud, A. and Verluise, C. (2022). The rise of China's technological power: the perspective from frontier technologies. CEP Discussion Papers DP1876, Centre for Economic Performance, LSE.

Betts, C. (2021). How many (more) lost decades? The great productivity slowdown in Japan. Mimeo, University of Southern California.

Bianchi, F., Kung, H., and Morales, G. (2019). Growth, slowdowns, and recoveries. *Journal of Monetary Economics*, 101:47–63.

Blanchard, O., Cerutti, E., and Summers, L. (2015). Inflation and activity–two explorations and their monetary policy implications. *IMF Working Papers*, (WP/15/230).

Boyd, J. H., De Nicolo, G., and Rodionova, T. (2019). Banking crises and crisis dating: Disentangling shocks and policy responses. *Journal of Financial Stability*, 41:45–54.

Calvino, F., Criscuolo, C., and Menon, C. (2015). Cross-country evidence on start-up dynamics. *OECD Science, Technology and Industry Working Papers*, (2015/06).

Cerra, V., Fatás, A., and Saxena, S. C. (2023). Hysteresis and business cycles. *Journal of Economic Literature*, 61(1):181–225.

Cochrane, J. H. (2011). Presidential address: Discount rates. *The Journal of Finance*, 66(4):1047–1108.

Comin, D. and Gertler, M. (2006). Medium-term business cycles. *American Economic Review*, 96(3):523–551.

Cooper, R. (1999). *Coordination Games: Complementaries and Macroeconomics*. Cambridge University Press.

Cozzi, G., Pataracchia, B., Pfeiffer, P., and Ratto, M. (2021). How much Keynes and how much Schumpeter? *European Economic Review*, 133:103660.

De Ridder, M. (2019). Market power and innovation in the intangible economy. Mimeo, Faculty of Economics, University of Cambridge.

Di Tella, S. and Hall, R. (2022). Risk premium shocks can create inefficient recessions. *The Review of Economic Studies*, 89(3):1335–1369.

Doepke, M. and Zilibotti, F. (2005). The macroeconomics of child labor regulation. *American Economic Review*, 95(5):1492–1524.

Eggertsson, G. B., Mehrotra, N. R., and Robbins, J. A. (2019). A model of secular stagnation: Theory and quantitative evaluation. *American Economic Journal: Macroeconomics*, 11(1):1–48.

Elfsbacka Schmöller, M. and Spitzer, M. (2021). Deep recessions, slowing productivity and missing (dis-)inflation in the euro area. *European Economic Review*, 134:103708.

Engler, P. and Tervala, J. (2018). Hysteresis and fiscal policy. *Journal of Economic Dynamics and Control*, 93:39–53.

Esfahani, M., Fernald, J. G., and Hobijn, B. (2020). World productivity: 1996-2014. Federal Reserve Bank of San Francisco.

Fatás, A. (2000). Do business cycles cast long shadows? short-run persistence and economic growth. *Journal of Economic Growth*, 5:147–162.

Fornaro, L. and Wolf, M. (2023). The scars of supply shocks: Implications for monetary policy. *Journal of Monetary Economics, forthcoming.*

Furlanetto, F., Lepetit, A., Robstad, Ø., Rubio-Ramírez, J. F., and Ulvedal, P. (2023). Estimating Hysteresis Effects. *American Economic Journal: Macroeconomics, forthcoming.*

Galor, O. (2005). From stagnation to growth: unified growth theory. *Handbook of Economic Growth*, 1:171–293.

Galor, O. and Weil, D. N. (2000). Population, technology, and growth: From Malthusian stagnation to the demographic transition and beyond. *American Economic Review*, 90(4):806–828.

Guerrón-Quintana, P. and Jinnai, R. (2014). Liquidity, trends and the great recession. *Tokyo Center for Economic Research (TCER) Paper No. E-66.*

Hall, B. H., Mairesse, J., and Mohnen, P. (2010). Measuring the returns to r&d. In *Handbook of the Economics of Innovation*, volume 2, pages 1033–1082. Elsevier.

Hardy, B. and Sever, C. (2021). Financial crises and innovation. *European Economic Review*, 138(September):103856.

Hausman, N. (2022). University innovation and local economic growth. *The Review of Economics and Statistics*, 104(4):718–735.

Hvide, H. K. and Jones, B. F. (2018). University innovation and the professor's privilege. *American Economic Review*, 108(7):1860–98.

Ito, K., Ikeuchi, K., Criscuolo, C., Timmis, J., and Bergeaud, A. (2023). Global value chains and domestic innovation. *Research Policy*, 52(3):104699.

Jones, C. I. and Williams, J. C. (2000). Too much of a good thing? The economics of investment in R&D. *Journal of Economic Growth*, 5:65–85.

Kerins, F., Smith, J. K., and Smith, R. (2004). Opportunity cost of capital for venture capital investors and entrepreneurs. *Journal of Financial and Quantitative Analysis*, 39(2):385–405.

Laeven, L. and Valencia, F. (2020). Systemic banking crises database ii. *IMF Economic Review*, 68:307–361.

Liu, E., Mian, A., and Sufi, A. (2022). Low interest rates, market power, and productivity growth. *Econometrica*, 90(1):193–221.

Mackay, A. L. (1977). The harvest of a quiet eye. A selection of scientific quotations. *Bristol: Institute of Physics.*

Matsuyama, K. (2002). The rise of mass consumption societies. *Journal of Political Economy*, 110(5):1035–1070.

Nishimura, K. G. (2012). Ageing, finance and regulations. Keynote Address, the Joint Forum Meeting, Bank of Japan.

NISTEP (2022). Japanese science and technology indicators 2022. Technical Report 318.

Pritchett, L. (1996). *Population growth, factor accumulation, and productivity.* World Bank Publications.

Queraltó, A. (2020). A model of slow recoveries from financial crises. *Journal of Monetary Economics*, 114:1–25.

Romer, P. M. (1990). Endogenous technological change. *Journal of Political Economy*, 98(5, Part 2):S71–S102.

Schmitz, T. (2021). Endogenous growth, firm heterogeneity and the long-run impact of financial crises. *European Economic Review*, 132:103637.

Stadler, G. W. (1990). Business cycle models with endogenous technology. *The American Economic Review*, pages 763–778.

Stiglitz, J. E. (1993). Endogenous growth and cycles. *NBER Working Papers 4286.*

Tambakis, D. (2021). A Markov chain measure of systemic banking crisis frequency. *Applied Economics Letters*, 28(16):1351–1356.

Vinci, F. and Licandro, O. (2020). Switching-track after the great recession. Working paper, University of Nottingham, Centre for Finance, Credit and Macroeconomics (CFCM).

Zilibotti, F. (2017). Growing and Slowing Down Like China. *Journal of the European Economic Association*, 15(5):943–988.

Online Appendix

A De-trending and full set of equilibrium equations

To compute the model solution, we need to express the model in stationary form. To do so, we first collect the equilibrium equations and model variables and identify which of these variables are not stationary. We then divide them by their corresponding *growth factors*. Most quantities share the same growth factor, while most prices are unchanged, but there are many exceptions. To maintain consistency, the equilibrium equations are also converted *consistently*. Table A.3 presents all de-trended equilibrium equations, and Table A.1 presents all the definitions of the de-trended variables. De-trending is, of course, not an arbitrary process, particularly for models with multiple BGPs. If we substitute the de-trended variables into the de-trended equilibrium, we must be able to recover the original model with the raw variables. Finally, to interpret the simulation results, we need to reverse the de-trended variables into the original variables. We call this process *reverse-detrending*. We denote de-trended variables with a tilde "$\tilde{X}$".

In our representation of the model before de-trending (i.e., with the raw variables), excluding the definition of the stochastic discount factor, there are 6 dynamic variables and 12 static variables (we treat the stochastic discount factor, SDF, as a short-hand notation). Among the 6 dynamic variables, three are state variables (K_t, Z_t and A_t). After de-trending, there are only 5 dynamic variables, of which the states are $\tilde{K}_t$ and $\tilde{Z}_t$. In the de-trended system of equations, instead of A_t, we use its growth rate $\gamma_{A,t+1} = A_{t+1}/A_t$ (which is observable at time t). Hence, there are 13 static variables, including $\gamma_{A,t+1}$. There are two exogenous variables in the deterministic version of the model.

Table A.1: Detrended Variables

Short Name	Definition	Growth Factor
capital	$\tilde{K}_t = K_t/\Gamma_{k,t}$	$\Gamma_{k,t} = \xi_{t-1} A_t^{\varphi_A} H_{t-1}$
research stock	$\tilde{Z}_t = Z_t/A_t^{\kappa_Z}$	unchanged ($\kappa_Z = 0$)
product design	(A_t)	(disappears)
asset value, research stock	$\tilde{V}_t^Z = V_t^Z/\Gamma_{VZ,t}$	$\Gamma_{VZ,t} = \xi_t A_t^{\varphi_A - \kappa_Z} H_t$
asset value, product design	$\tilde{V}_t = V_t/\Gamma_{V,t}$	$\Gamma_{V,t} = \xi_t A_t^{\varphi_A - 1} H_t$
consumption	$\tilde{C}_t = C_t/\Gamma_t$	$\Gamma_t = \xi_t A_t^{\varphi_A} H_t$
stochastic discount factor	$\tilde{\Lambda}_{t,t+1} = \Lambda_{t,t+1}$	unchanged
output	$\tilde{Y}_t = Y_t/\Gamma_t$	$\Gamma_t = \xi_t A_t^{\varphi_A} H_t$
wage	$\tilde{W}_t = W_t/\Gamma_{w,t}$	$\Gamma_{w,t} = \xi_t A_t^{\varphi_A}$
return on capital	$\tilde{R}_{K,t} = R_{K,t}$	unchanged
dividends	$\tilde{D}_t = D_t/\Gamma_{V,t}$	$\Gamma_{V,t} = \xi_t A_t^{\varphi_A - 1} H_t$
investment	$\tilde{I}_t = I_t/\Gamma_t$	$\Gamma_t = \xi_t A_t^{\varphi_A} H_t$
labor, production	$\tilde{H}_{Y,t} = H_{Y,t}/H_t$	$\bar{H}_t$
labor, basic research	$\tilde{H}_{Z,t} = H_{Y,t}/H_t$	H_t
labor, product design	$\tilde{H}_{A,t} = H_{Y,t}/H_t$	H_t
research stock, new	$\tilde{Z}_t^{new} = Z_t^{new}/A_t^{\kappa_Z}$	unchanged ($\kappa_Z = 0$)
product design, new	$\tilde{A}_t^{new} = A_t^{new}/A_t$	A_t
success rate, basic research	$\tilde{\pi}_t^{ZH} = \pi_t^{ZH}/\left(A^{\kappa_Z}/H_t\right)$	A^{κ_Z}/H_t
success rate, product design	$\tilde{\pi}_t^{AH} = \pi_t^{ZH}/\left(A/H_t\right)$	A/H_t
growth rate of A_t	$\gamma_{A,t+1} = A_{t+1}/A_t$	(instead of A_t)

Table A.2: Exogenous Variables and Short-Hand Notations

Variable Name	Definition
growth rate of population H_t	$\gamma_{H,t} = \frac{H_t}{H_{t-1}}$
growth rate of exo technology ξ_t	$\gamma_{\xi,t} = \frac{\xi_t}{\xi_{t-1}}$
(short-hand notations)	
exogenous productivity	$\tilde{\zeta}_{Y,t} = \left(\frac{\eta}{\gamma_{\xi,t}\gamma_{H,t}}\right)^{\alpha}$
adj. for research stock value	$\gamma_{VZ,t} = \Gamma_{VZ,t}/\Gamma_{VZ,t+1}$
adj. for prod. design asset value	$\gamma_{V,t} = \Gamma_{V,t}/\Gamma_{V,t+1}$

Table A.3: Detrended Equilibrium Equations

Short Name	Equation	
lom $\tilde{K}$	$\tilde{K}_{t+1}\left(\gamma_{A,t+1}\right)^{\varphi_A} = \tilde{I}_t + \frac{1-\delta_K}{\gamma_{\xi,t}\gamma_{H,t}}\tilde{K}_t$	(25′)
lom $\tilde{Z}$	$\tilde{Z}_{t+1}\gamma_{A,t+1}^{\kappa_Z} = \tilde{Z}_t^{new} + \left(1-\delta_Z\right)\tilde{Z}_t$	(8′)
(lom $\tilde{A}$)	($\gamma_{A,t+1} = \tilde{A}_t^{new} + \left(1-\delta_A\right)$ becomes static)	
capital return	$1 = E_t\left[\frac{\tilde{\Lambda}_{t,t+1}}{c_K}\left\{\tilde{R}_{K,t+1} + \left(1-\delta_K\right)\right\}\right]$	(26′)
stock value of research $\tilde{V}_t^Z$	$\tilde{V}_t^Z = \left(1-s_A\right)\tilde{V}_t\frac{\tilde{A}_t^{new}}{\tilde{Z}_t} + E_t\left[\frac{\Lambda_{t,t+1}\gamma_{VZ,t+1}}{c_Z}\left(1-\delta_Z\right)\tilde{V}_{t+1}^Z\right]$	(11′)
firm value of prod design $\tilde{V}_t$	$\tilde{V}_t = \tilde{D}_t + E_t\left[\frac{\Lambda_{t,t+1}\gamma_{V,t+1}}{c_v}\left(1-\delta_A\right)\tilde{V}_{t+1}\right]$	(10′)
SDF	$\tilde{\Lambda}_{t,t+1} = \frac{1+\tau_{C,t}}{1+\tau_{C,t+1}}\frac{\left(\gamma_{\xi,t+1}\gamma_{A,t+1}^{\varphi_A}\right)^{-\sigma}}{\gamma_{H,t+1}}\beta\left(\frac{\tilde{C}_{t+1}}{\tilde{C}_t}\right)^{-\sigma}$	(27′)
production	$\tilde{Y}_t = \tilde{\zeta}_{Y,t}\tilde{K}_t^{\alpha}\tilde{H}_{Y,t}^{1-\alpha}$	(19′)
marginal product of labor	$\tilde{W}_t = \left(1-\alpha\right)\frac{\tilde{Y}_t}{\tilde{H}_{Y,t}}$	(20′)
marginal product of capital	$R_{K,t} = \tilde{\alpha}\alpha\left(\gamma_{\xi,t}\gamma_{H,t}\right)\frac{\tilde{Y}_t}{\tilde{K}_t}$	(21′)
dividends	$\tilde{D}_t = \left(1-\tau_D\right)\left(1-\tilde{\alpha}\right)\alpha\tilde{Y}_t$	(9′)
goods market clearing	$\tilde{Y}_t = \tilde{C}_t + \tilde{I}_t + \tilde{G}_t$	(28′)
labor market clearing	$1 - \underline{h}_O = \tilde{H}_{Y,t} + \tilde{H}_{A,t} + \tilde{H}_{Z,t}$	(1′)
basic research, FOC	$\tilde{V}_t^Z\tilde{\pi}_t^{ZH} = \left(1-\tau_Z\right)\tilde{W}_t \quad \text{or} \quad \tilde{H}_{Z,t} = \underline{h}_Z$	(15′)
product develop, FOC	$s_A\tilde{V}_t\tilde{\pi}_t^{AH} = \left(1-\tau_A\right)\tilde{W}_t$	(18′)
new research output, flow	$\tilde{Z}_t^{new} = \omega_Z\tilde{Z}_t^{1-\phi_Z}\tilde{H}_{Z,t}$	(5′)
new product design, flow	$\tilde{A}_t^{new} = \omega_A\tilde{Z}_t^{1-\phi_A}\tilde{H}_{A,t}^{\phi_A}$	(6′)
success rate per $H_{Z,t}$	$\tilde{\pi}_t^{ZH} = \frac{\tilde{Z}_t^{new}}{\tilde{H}_{Z,t}}$	(13′)
success rate per $H_{A,t}$	$\tilde{\pi}_t^{AH} = \frac{\tilde{A}_t^{new}}{\tilde{H}_{A,t}}$	(17′)
lom $\tilde{A}$ (now static eqn)	$\gamma_{A,t+1} = \tilde{A}_t^{new} + \left(1-\delta_A\right)$	(7′)

B Solution method detail

We numerically solve the de-trended equations (i.e., express all variables as functions of state variables) using the Euler Equation Iteration (EEI) method, which is a projection method (a global solution method). We update the policy functions with the following algorithm:

0. Set an initial guess for the policy functions of dynamic jump variables (C_t, V_t and V_t^Z) as functions of state variables (K_t and Z_t).

1. Compute C_{t+1}, V_{t+1} and V^Z_{t+1}:
   - First, compute K_{t+1} and Z_{t+1} based on guess C_t, V_t and V^Z_t. To do so, do not use their Euler equations (26′), (11′) and (10′).
   - Then, compute C_{t+1}, V_{t+1} and V^Z_{t+1} based on K_{t+1} and Z_{t+1}. To do so, use the same guess policy functions. Usually, interpolation is required.
2. Update C_t, V_t and V^Z_t.
   - Substitute C_{t+1}, V_{t+1} and V^Z_{t+1} into the Euler equations (26′), (11′) and (10′).
   - If the resulting C_t, V_t and V^Z_t are different from the guess policy functions, update the guess policy functions.

Iterate steps 1 and 2 until the errors in the Euler equations become small enough.

Because our method is a collocation method, we can reduce the Euler equation errors on the state grids to effectively zero. Thus, we fix the state grids and, throughout the iteration, we compute not only C_t, V_t and V^Z_t but also C_{t+1}, V_{t+1}, V^Z_{t+1}, K_{t+1} and Z_{t+1} on the same state grids.

Our solution algorithm is also a time iteration method. We could interpret step 2 (Euler equations) above as a backward computation from C_{t+1}, V_{t+1} and V^Z_{t+1} to C_t, V_t and V^Z_t. Note that the TVCs (29a)-(29c) are defined consistently with Euler equations (26′), (11′) and (10′).

C Multiple BGPs as a coordination game

To enhance the intuition behind the idea of vicious and virtuous cycles, we can regard the steady states (BGPs) as equilibria of a simultaneous move game between basic researchers and product developers. In Figure C.1, the blue dashed line shows $H_{A,ss}$ as a best response function (BRF) for each $H_{Z,ss}$. To draw it, we assume $H_{Z,ss}$ is arbitrarily fixed (i.e., we ignore its FOC) and assume all equilibrium equations hold except for the FOC of $H_{Z,ss}$. Likewise, the black solid line is $H_{Z,ss}$ as a BRF for each $H_{A,ss}$. There are three equilibria, of which the middle one is trembling-hand unstable. This is a typical outcome in coordination games.

The arrows in the figure depend on the direction of the FOC errors. For example, if the marginal revenue product of $H_{A,ss}$ is higher than the wage cost in the FOC for $H_{A,ss}$ at a point on the plane, the arrow is depicted such that $H_{A,ss}$ is increasing. We can do this because, unlike in the textbook coordination games in which choices are discrete, both $H_{A,ss}$ and $H_{Z,ss}$ are continuous choices.

This figure is different from the BRFs of the standard coordination game because the BRF of $H_{Z,ss}$ has an inverted Z-shape (the lines are connected to the right of the graph). This is because there is another coordination game among basic researchers. That is, for very high or very low $H_{A,ss}$, there is only one $H_{Z,ss}$ that satisfies its FOC. However, for

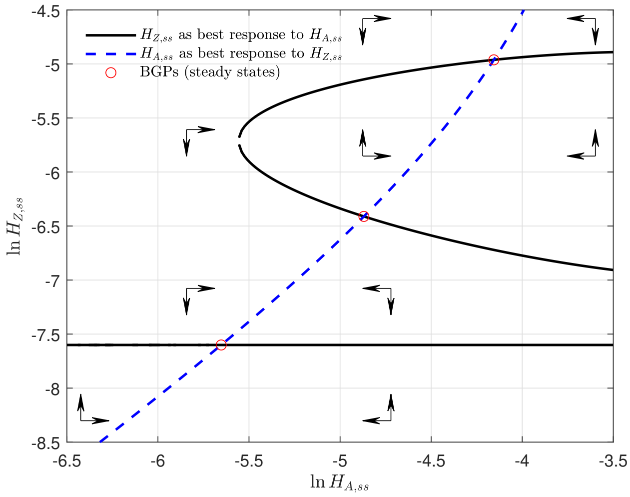

Figure C.1: Steady State Selection as a Simultaneous Move Game

$H_{A,ss}$ in the middle, there are three $H_{Z,ss}$ that satisfy the FOC, of which the middle one (shown as the downward sloping part of the black solid line) is trembling-hand unstable. To understand this, fix $H_{A,ss}$. If all other basic researchers work at the minimum level $\underline{h}_Z$, the resultant research stock Z_{ss} is too low, and hence it is optimal for each basic researcher to work at the minimum level (shown by the horizontal part of the solid line). Likewise, if everybody chooses a high $H_{Z,ss}$, it is best for each researcher to choose a high $H_{Z,ss}$ (shown as the upward-sloping part of the solid line).

To make clear the relationship between the search externality and strategic complementarity, we briefly discuss the Hosios condition, which implies $s_A = \phi_A$ in our case. If this is satisfied, each product developer behaves as if she internalizes the change in the aggregate development workers. However, the Hosios condition does not eliminate multiple BGPs, because the product developers still do not internalize the effect on the matching rate via the change in the research stock Z_t. This is the externality that generates the strategic complementarity between basic researchers and the product developers. Another Hosios condition, $s_Z = 1$ in the basic research matching, is satisfied in our model. But, as discussed in the main text, s_Z does not matter because of the special property of the basic research matching.

D Long-run model simulation

Figure D.2 compares simulated values for TFP growth and GDP per worker growth with their data counterparts assuming the economy is subject to the "wretched coincidence" shocks in the early 1990s. In this figure, we assume that the economy is at the high BGP in 1970. Note that this does not imply that the stock of knowledge in 1970 is at the same level as in the 2020s. Along the high BGP, the reverse-detrended Z_t is increasing at the same rate as A_t.

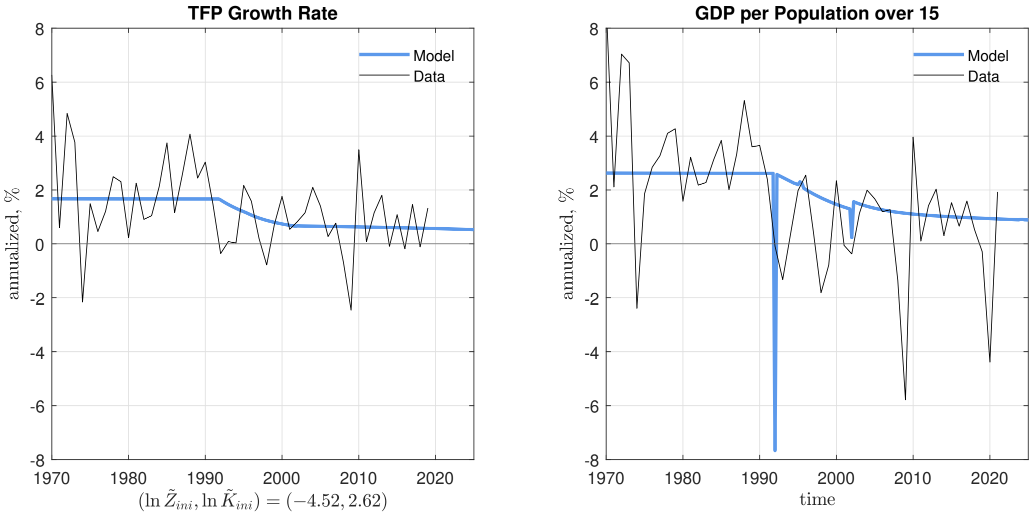

Figure D.2: Comparison with the data ("wretched coincidence" scenario)

The figure shows that the model does a good job at capturing the growth slowdown since early 1990s, even though the parameters are calibrated for 1970-1992. Clearly, in the data, both series are affected by cyclical fluctuations which our model is not designed to capture. The GDP growth rate in the model has two spikes at the start and the end of the financial crisis, which are artefacts of the perfectly flexible labor mobility assumption.

E Japanese major bankruptcies

Table E.4 presents a short history of major Japanese corporate failures between 1989 and 2019. The table displays the year, month, and name of the company as well as its main sector. The last column displays the rank, which is based on the total debt size at the timing of failure. The data is extracted from Teikoku Databank (2019). Note, however, this data does not include some major bankruptcies because several financial institutions were rescued and, hence, they were not technically classified as bankruptcies. Thus, we added some selected failures without ranking. The four bold face cases were the most prominent.

Table E.4: Major Corporate Failure in Heisei Era (1989-2019)

year	mth	Company Name	Type of Business	rank
1996	10	Nichiei Finance	Consumers finance	10
1997	4	Crown Leasing	Leasing	8
1997	4	Nissan Life	Life insurance	-
1997	11	Sanyo Securities	Securities	-
1997	**11**	**Hokkaido Takushoku Bank**	**Bank**	**-**
1997	**11**	**Yamaichi Securities**	**Securities**	**-**
1997	11	Tokuyo City Bank	Bank	-
1997	12	Toshoku Ltd.	Food wholesales	15
1998	9	Japan Leasing and Sales Co.	Leasing	4
1998	**10**	**Long-Term Credit Bank of Japan**	**Bank**	**-**
1998	**12**	**Nippon Credit Bank**	**Bank**	**-**
2000	5	Life Ltd.	Instrument sales	12
2000	7	Sogo	Retailer	13
2000	9	Kyouei Life	Life insurance	1
2000	10	Chiyoda Life Insurance	Life insurance	3
2001	3	Tokyo Life	Life insurance	11
2001	9	Mycal Corp	Retailer	7
2008	9	Lehman Brothers	Investment bank	2
2010	1	Japan Airline	Airline	5
2010	9	Takefuji	Consumers finance	6
2010	9	Incubator Bank of Japan	Bank	14
2017	6	Takata	Auto parts manufacturing	9