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Firm Dynamics, Dynamic Reallocation, Variable Markups, and Productivity Behaviour

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Abstract

I analyze two opposing effects of firm dynamics on productivity over the business cycle. Consider net exit, on the one hand it reallocates resources to incumbents whose productivity improves through scale economies, on the other hand it reduces the competitive pressure incumbents face which depresses productivity. Contrarily net entry strengthens competition, thus increasing productivity, but worsens incumbents' scale economies, thus decreasing productivity. I outline a theory that focuses on two industrial features (1) slow firm entry/exit and (2) firm pricing that depends on the number of competitors. In this environment a negative shock strikes incumbents due to slow exit responses. This weakens their scale thus worsening productivity but the effect recedes as exit occurs which reallocates resources to incumbents. However, the remaining firms face fewer competitors and thus charge higher markups which damages productivity. I analyze this trade-off between productivity improving resource reallocation and productivity degrading market power, by developing a continuous time, analytically tractable DGE model of endogenous firm entry/exit and endogenous markups.

JEL: E32, D21, D43, L13, C62, Endogenous markups, Entry, Endogenous Productivity, Imperfect product markets, dynamical systems

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Non-technical Summary

Traditionally macroeconomists assume that the number of firms in an economy adjusts instantaneously to arbitrage profits. This assumption ignores *'slow'* fluctuations in firm entry and exit over the business cycle. This paper develops a model of firm dynamics in the macroeconomy with sunk costs that cause firms to respond slowly to economic shocks, hence entry and exit decisions are non-instantaneous. The resulting firm adjustment towards zero profit causes endogenous fluctuations in profits, competition and business allocation that helps to explain business cycle productivity dynamics.

The paper studies how firm entry determines macroeconomic productivity through division of resources and competitive pressure on markups. Recent research examines the importance of firm entry for macroeconomic productivity, but the arguments focus on instantaneous firm entry. I argue that this overlooks the changing allocation of business as firms adjust intertemporally. My contribution is to combine this dynamic firm entry with endogenous markups (markups determined by competition). The result is a new trade-off: an exiter reduces industry competition which reduces incumbents' productivity, but exit reallocates business to incumbents who improve productivity through better returns to scale (vice-versa for entry which raises competition, but steals business reducing scale). The mechanism helps to clarify productivity puzzles. It explains that economic shocks cause exacerbated productivity responses that weaken as firms adjust, and entry/exit effects on competition prevent reversion to previous long-run outcomes.

The main finding is that firm competition from entry ameliorates short-run productivity volatility but in the long run productivity effects persist because of structural changes to competition.

1 Introduction

The paper proposes a business cycle theory in which firm entry and exit cause endogenous short-run and long-run productivity movements. Interest in endogenous productivity over the business cycle is high in light of Great Recession *productivity puzzles*[1]. The puzzles describe exacerbated productivity falls with weak recovery, and are prominent in several European countries as shown in figure 1. The problem is especially pronounced in the UK, and empirical studies (Barnett et al. 2014) find that up to half of the shortfall in UK labour productivity relative to pre-crisis trend arose because of impaired resource allocation and unusually high firm survival rates[2]. This evidence emphasizes the importance of firm dynamics in explaining macroeconomic productivity, but traditional macroeconomic theory nullifies entry by assuming that the number of firms in an economy adjusts instantaneously to arbitrage profits. If entry is instantaneous, it can only affect productivity through an immediate change in the number of competitors which affects pricing markups, but it ignores the short-run effect of sluggish entry reallocating resources as firms adjust to arbitrage profit. In this paper I analyze the new trade-off that emerges when noninstantaneous entry is combined with competitive endogenous markups.

The main result is a theory to explain that shocks initially exacerbate productivity movements but the exacerbation relinquishes as firm entry/exit adjusts. Crucially productivity never regains long-run underlying productivity because of structural changes in competition due to long-run changes in the number of incumbents. To be clear, 'entry' is net entry, so when negative it is exit. Therefore entry and exit the same process–they cannot arise together. A contractionary shock will solely cause exit (negative net entry); an expansionary shock will solely cause entry (positive net entry). Hence the theory is a general explanation of endogenous productivity over the business cycle, with initially exacerbated and persistent positive productivity effects associated with entry in expansion and exacerbated and persistent negative productivity effects associated with exit in contraction. Although general, Great Recession productivity puzzles provide a contemporary view of the theory since they depict a short-run exacerbated fall in productivity followed by some persistence due to structural factors. Therefore my results provide a theory that mimics dynamics observed by many European economies, see

[1] The term has been used extensively in the media and academia e.g. The Productivity Puzzle Under the Bonnet, The Economist, May 30, 2015; Budget 2015: How do you solve the 'productivity puzzle'?, BBC News, July 8, 2015

[2] Goodridge, Haskel, and Wallis 2014 show that accounting for labour and capital still leaves a TFP puzzle. I focus on TFP.

figure 2. I demonstrate that a negative shock to the economy, modeled as a supply-side technology shock, is first absorbed by incumbent firms because exit cannot arise initially. Therefore productivity falls drastically as the incumbents output falls and they suffer worse returns to scale (hypothesis I in figure 2). Lower output per firms causes negative profits which leads to exit. As exit occurs productivity improves because resources are reallocated among incumbents and better returns to scale improves productivity, as shown by the hypothesis I reversion. However, this consolidation of resources among fewer firms reduces the competitive pressure on those who remain, as shown by the strengthening market power in 3, allowing them to charge higher markups. Higher markups mean each unit sold generates more revenue so that in a long-run zero profit equilibrium firms can produce less to cover their fixed cost of production. By choosing to produce less their scale suffers which creates an offsetting negative productivity effect that persists in equilibrium and hence links to hypothesis II in figure 2.

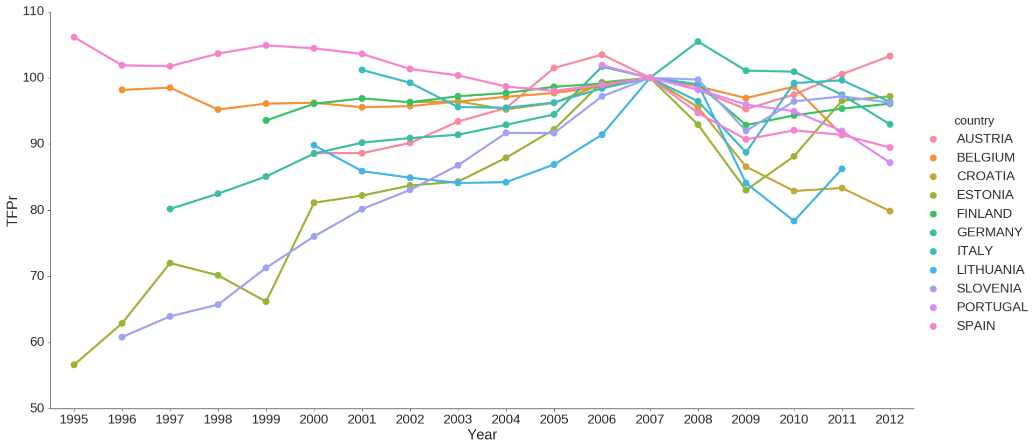

Figure 1: Cross-country TFPR

I develop a tractable model of dynamic (endogenous) firm entry in the macroeconomy with imperfectly competitive product markets that cause endogenous markups. Dynamic entry means that firms slowly adjust to arbitrage profits, so short-run profits are nonzero. This entry friction arises because a *congestion effect* raises sunk entry costs as entry increases[3]. Imperfect competition creates a markup of factor prices above their marginal products, and the markup is endogenous because it depends on the number of firms. The relationship is negative and occurs because firms are large in their industry so they strategically interact under Cournot competition.

[3] A familiar notion in industrial organization (IO) theory e.g. Ericson and Pakes 1995

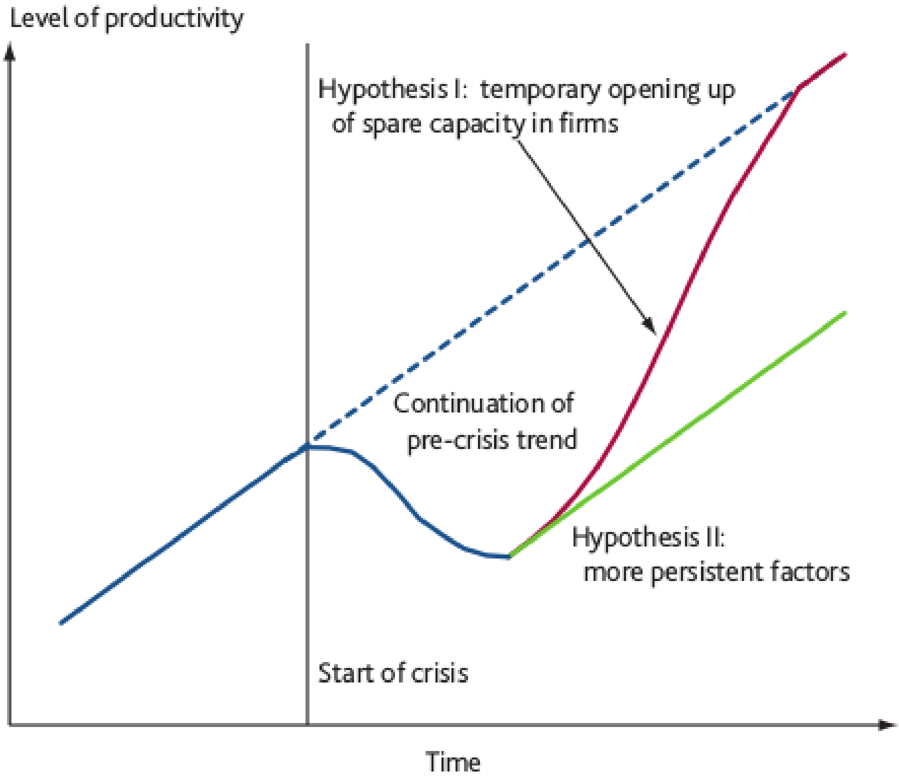

Figure 2: Productivity Puzzle Hypotheses (Source Barnett et al. 2014)

With this model setup, I analyze the trade-off between *endogenous markups* and *dynamic reallocation*. Endogenous markups cause entry to increase productivity and exit to decrease productivity. Dynamic reallocation causes entry to decrease productivity and exit to increase productivity. For example, with endogenous markups exit (entry) weakens (strengthens) competition which raises (lowers) markups, thus decreases (increases) productivity. In opposition, dynamic reallocation means exit (entry) concentrates (dissipates) resources thus increasing (decreasing) incumbents' scale and therefore productivity. Dynamic reallocation emphasises not the amount of resources, but their division among firms. And entry determines this division. I analyze measured productivity, which is an adjusted measure of total factor productivity (TFP).

The model demonstrates procyclical profits, entry, employment and productivity, whereas markups are countercyclical. For example, a positive shock to technology is initially borne by incumbents who raise their output whilst entry is inert in the short-run. Through greater scale incumbents' produc-

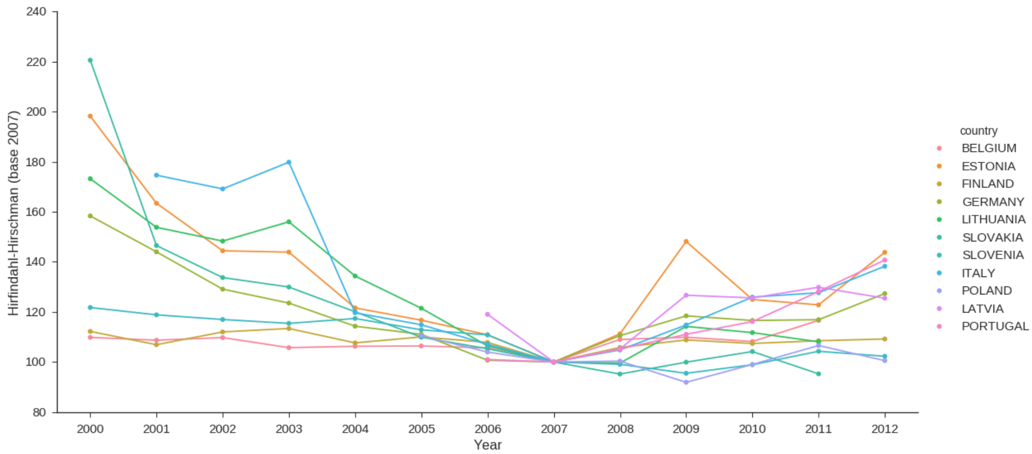

Figure 3: Market Power

tivity increases. However, by raising output incumbents accrue monopoly profits, these non-zero profits incentivise potential firms to begin entering. Entry reallocates resources and reduces output per firm which diminishes scale and therefore productivity. The influx in entry diminishes profits and through congestion raises the sunk entry cost which slows the rate of entry. Eventually the profits from incumbency arbitrage to zero so entry ceases and zero incumbency profits are balanced with zero sunk entry costs because there is no congestion. The long-run effect of a rise in the number of incumbents is that competition in the market is fiercer, so firms charge a lower markup. In order to cover fixed costs, firms with lower markups must raise revenue by increasing output, therefore in zero-profit (free-entry) equilibrium firm scale is increased which means there is a long-run permanent effect on productivity. In summary, the positive shock increases output, profit, employment (an input) and entry, whereas markups decrease because entry increases competition.

Formally the model follows a Ramsey-Cass-Koopmans setup. There is endogenous labour and capital, and the novel additions are firm entry and endogenous markups. A representative household chooses its consumption and labour exertion, but the household is limited by a budget which consists of labour income and investment income. Investment income consists of returns on capital and returns on firm ownership (firm profit). The return on capital is the economy's risk-free rate, which consequently determines the opportunity cost of investing in a firm. This balance between paying a cost to setup a firm and investing that cost at the market rate binds firm entry. It is a dynamic condition because sunk costs depend on the number of entering firms (congestion effect). Hence in free entry equilibrium[4] profits are zero, so a household is indifferent between creating a firm or investing that sunk cost at the risk free rate. For example, if the value of incumbency exceeds the risk free alternative, then there will be entry. Consequently congestion will stifle start-ups and entry will slow. On the firm side of the economy there is imperfect competition and generalized returns to scale (U-shaped cost curves). All firms produce with the same production function (firms are symmetric) which has a fixed overhead cost and nondecreasing marginal cost. The fixed overhead allows for imperfect competition which causes pricing markups. Firms are aggregated across two levels. The lowest level of aggregation is the firm level, and aggregating firms gives the industry level. The macroeconomy is the aggregate across all industries. I focus on symmetric equilibria so an industry is representative of the whole economy. Firms have price setting power within their industry, but are small in the aggregate economy. The influence of a firm on industry price causes endogenous markups[5]. Within an industry firms strategically interact with Cournot competition, so they maximise profits by choosing output to produce. This output choice is influenced by the number of competitors in the industry. When there are more competitors demand functions reflect a higher elasticity of market demand and therefore weaker markup setting power.

The model economy includes three core assumptions 1) endogenous entry 2) returns to scale 3) endogenous markups. The counterfactual of each assumption emphasizes its importance. First, in the absence of endogenous entry there is instantaneous free entry[6]. This counterfactual implies that there is no short-run productivity effect as incumbent firms bear shocks.Second, in the absence of increasing returns to scale, returns to scale are constant. This counterfactual makes entry impotent because firms produce at the same productivity regardless of size. Third, in the absence of endogenous markups, markups are fixed. This counterfactual implies there is no persistent effect on productivity because firms do not alter their markups.

Related Literature This paper links Etro and Colciago 2010[7] to Jaimovich and Floetotto 2008. The first paper includes sluggish firm entry and endogenous markups, but does not discuss productivity. Their contribution is to improve business cycle moment-matching using Cournot and Bertrand

[4] The long-run equilibrium when firms have freely entered to arbitrage positive profits.

[5] If a firm were small in its industry, markups would be fixed as in the status-quo Dixit and Stiglitz 1977 case.

[6] This is a limiting case of my model, as is the other extreme a fixed number of firms.

[7] Etro and Colciago 2010 also note that Cournot competition causes inefficiency through excess entry. Etro 2009 provides an excellent survey of macroeconomic models with endogenous entry and endogenous market structures.

strategic interactions; I use Cournot which the authors advocate. The second paper has endogenous markups and analyzes productivity, but firm entry is instantaneous. Their contribution is to explain the productivity effect of instantaneous entry on markups. This is equivalent to the long-run effect that causes productivity persistence in my paper. My link combines endogenous entry with endogenous markups to explain productivity over the business cycle. The result is that endogenous entry distinguishes short-run productivity dynamics from long-run productivity dynamics.

The endogenous entry setup of this paper follows Datta and Dixon 2002 which is close to industrial organization literature by Das and Das 1997. Importantly this differs from most recent endogenous entry literature that uses Bilbiie, Ghironi, and Melitz 2012 (BGM)[8]. However, the interpretation of the two approaches is analogous. Both endogenous entry formulation reduce to an arbitrage condition that equates sunk entry costs to incumbency profits. A strength of the Datta and Dixon 2002 formulation is that dynamics stem from endogenous sunk costs, rather than fixed sunk costs in BGM. These endogenous sunk costs are called congestion effects since they increase as number of entrants (congestion) increases. It is then a lemma that sunk entry costs equate to profit from incumbency, rather than in BGM where this is assumed. The BGM setup is influential in discrete time, simulation exercises[9], whereas the model in this paper is continuous time and analytically tractable. BGM distinguish entry from exit (exit is exogenous), whereas this paper treats them as symmetric. Entry measures the change in the number of incumbent firms, so negative entry is exit. Lewis 2009, Lewis and Poilly 2012, Lewis and Stevens 2015 and Berentsen and Waller 2009 all recognise the importance of congestion effects in macroeconomic models with entry.

An important distinction of this paper is its focus on qualitative dynamical systems, rather than quantitative simulations in the afforementioned works. This follows Brito and Dixon 2013. Rather than productivity, their focus is on theorems to show that firm entry is sufficient for nonmonotone responses to fiscal shocks. Excluding imperfect competition removes the vital mechanism for generating increasing returns to scale that are necessary for productivity dynamics. This mechanism is present in Aloi and Dixon 2003 who use firm entry to explain productivity in an open economy without capital or endogenous markups. This mechanism between imperfect competition, increasing returns to scale and productivity is an established explanation for procyclical productivity over the business cycle (Hall 1989, Hall 1987, Ca-

[8]For example, Lewis and Poilly 2012 and Etro and Colciago 2010.

[9]And not limited to macroeconomics. Examples include Loualiche 2014 in finance, Peters 2013 in growth and Hamano and Zanetti 2014 in macroeconomics.

ballero and Lyons 1992).

There are two competing formulations of endogenous markups. There is a supply-side approach, used in this paper, Etro and Colciago 2010, Jaimovich 2007, and Jaimovich and Floetotto 2008. There is a demand-side approach used by Bilbiie, Ghironi, and Melitz 2012. The supply-side approach relies on firms strategically interacting which affects market demand. The demand-side approach relies on consumers' elasticity of substitution varying as product variety changes with entry. Lewis and Poilly 2012 compare the methods. Empirical business cycle literature shows many examples of countercylical markups. A cornerstone work is Bils 1987, and there are many contributions by Rotemberg and Woodford surveyed in Rotemberg and Woodford 1999. These traditional explanations of countercyclical markups rely on price stickiness. Whereas, the study of entry provides a new factor to enrichen markup countercylicality. The idea stems from the ubiquity of the relationship in empirical IO. For example Campbell and Hopenhayn 2005 find a negative correlation between markups and entry in many sectors of the US economy. In macroeconomics, Portier 1995 shows that entry is procylical and markups countercyclical over the French business cycle. Other empirical features that relate to this paper are procyclical productivity Rotemberg and Summers 1990, and procyclical net business formation Bergin and Corsetti 2008. Lastly Brito, Costa, and Dixon 2013 model an economy with endogenous markups that embeds both traditional Dixit-Stiglitz monopolistic competition and entry-driven supply-side markups. This shows that monopolistic competition is a special case of the endogenous markups framework when there is only one firm per industry. They explore the critical bifurcation that arises as an economy moves from a continuum of 1-firm industries competing under monopolistic competition to a continuum of multi-firm industries competing under Cournot.

Roadmap – Section 2 explains the intuition behind the model. Section 3 outlines a model of firm entry in the macroeconomy where firms compete with strategic interactions. Section 4 begins analysis by explaining how the competition effect of entrants reducing markups affects factor prices and profits. Section 5 investigates static outcomes showing that long-run output and productivity are endogenous since they depend on the number of operating firms. Section 6 is the main result which presents a theorem to explain productivity puzzles, where productivity overshoots on the impact of a shock, then relinquishes but leaving some persistence.

2 Intuition of Excess Capacity with Short-run and Long-run Capacity Utilization

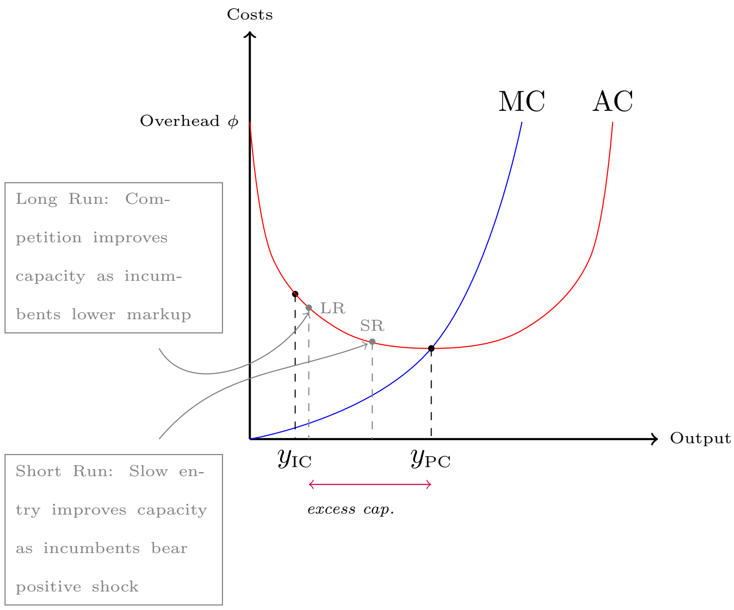

Figure 4: Excess Capacity, Short-run and Long-run Utilization

Before developing a complex dynamic model with endogenous entry, imperfect competition and endogenous markups, a simple diagram can explain the intuition of how entry causes endogenous and persistent productivity dynamics. Figure 4 shows the cost curves and equilibria of a firm with increasing marginal costs and a U-shaped average cost due to a fixed overhead cost ϕ. Under imperfect competition a firm produces y_{IC}[10] which is less than the perfect competition outcome y_{PC}, which is also the efficient outcome as it minimizes costs. The difference between y_{IC} and y_{PC} is excess capacity[11] (labelled), and utilizing excess capacity lowers costs which improves firm productivity and in turn aggregate productivity. With an entry mechanism the underproduction of each firm in imperfect competition corresponds to excess entry. This means ther are 'too many' firms each underproducing, so a more efficient outcome is fewer firms but each producing more, hence with y_{IC} closer to minimium cost y_{PC}.

[10] This is the long-run Chamberlin-Robinson equilibrium in which marginal revenue equals marginal cost and profits are zero. I omit the MR and MC curves for clarity, and I assume the curves have fully shifted following any shock.

[11] Macroeconomists should note this definition of excess capacity which follows Vives 1999. It is distinct from *capital utilization* or any form of input intensity.

If there is a positive shock to the economy, and entry is slow (endogenous), then that shock is initially borne by the incumbents so they utilize capacity and costs lower. A move from y_{IC} to point SR in the diagram. This indicates an immediate increase in productivity in aggregate. But in this new position, incumbents earn monpoly profits which attracts entrants. Over time entrants move into the market, gradually reducing the capacity of incumbents so excess capacity rises back towards the initial level, finally halting at a position like LR where profits are zero again. This mechanism corresponds to the gradual amelioration of the initial boost in productivity as firms adjust. The final part of the story is most important, because it explains why although the initial drastic effect subsides there is still some long-run effect on productivity, so output per firm returns to LR rather than initial y_{IC}. As firms enter to arbitrage profit to zero they must now each produce more $y_{LR} > y_{IC}$ because the positive shock encouraged entry (raised the number of competitors) which put downward pressure on markups. Therefore in long-run zero profit equilibrium firms charge lower markups than in the initial pre-shock position. Consequently each incumbent must raise revenue by increasing output to cover the costs of production ϕ and attain zero profit. So overall there is a small fall in costs per firm due to capacity utilization, and thus a small but persistent improvement in productivity after the initial positive shock.

3 Endogenous Entry Model with Imperfect Competition and Endogenous Markups

The model follows a Ramsey-Cass-Koopmans setup. Additions are imperfect competition, firm entry, endogenous markups and capital accumulation. The model is deterministic, and labour is endogenous. There are two state variables: capital and number of firms $(K, n) \in \mathbb{M} \subseteq \mathbb{R}^2$, where $\mathbb{M}$ is the state space of the control problem that later forms a subset of the general dynamical system state (or phase) space. I solve the model as a decentralised equilibrium because imperfect competition distorts the optimising behaviour of the firm.

Definition 1 (Notation and Terminology)**.** Y_x denotes the derivative of Y with respect to X, except when $X = t$ which denotes time dependence. For clarity I usually omit the (t) notation that denotes time dependence in ordinary differential equations (ODEs). To be clear, the primitive endogeneous model variables are $C(t), e(t), K(t), n(t)$, defined later. They are the state variables of the four dimensional dynamical system which forms the model

economy. The four states depend on time and therefore so do functions of them
$L(t), r(t), w(t), \pi(t), Y(t), y(t), \mu(t), \mathcal{P}(t), \Pi(t), Z(t)$. Time dependence is irrelevant in steady state, which I denote with an asterix Y^*. Also for clarity, I often suppress function domains. For example, after first introduction $F : K \times L \to \mathbb{R}$ is written F rather than $F(K, L)$.

3.1 Firm

In the economy there is a continuum of sectors of measure one. In each industry, there is a finite number of intermediate firms that each produce a homogenenous good. Since the goods are homogeneous, they are perfectly substitutable in the production of an industry good. However, at the next level of aggregation, industry goods are imperfect substitutes for eachother when aggregated into a final good. Entry and exit of firms into existing sectors occurs until profits are zero. This does not happen immediately but occurs in the long run. This is known as the free entry equilibrium. In the short-run profits will diverge from zero as they are arbitraged by entrants. Perfect factor markets mean that each firm faces the same price w for labour and r for capital, and the result is that aggregate capital and labour are divided equally among firms $k = \frac{K}{n}$ and $l = \frac{L}{n}$. A lowercase letter denotes *per firm*, so output per firm $y = \frac{Y}{n}$.

A fixed cost in production allows firms to compete under imperfect competition in the product market. Strategic interactions occur under imperfect competiton because firms are large in their industry so can influence industry price. This is why markups are endogenous (depend on number of incumbent firms), rather than fixed in the traditional Dixit-Stiglitz monopolistic competition case where firms are small therefore do not affect industry price level (in fact this is a special case of my endogenous markup model where there is one firm per industry). I focus on Cournot competition so firms strategically interact through their choice of output to maximise profits given the behaviour of others. The form of strategic interaction determines the markup of factor price above marginal cost. Specifically I focus on the level of factor price markup above the factor marginal product. That is the markup of wage above marginal product of labour and interest rate above marginal product of capital.

Final output Y is produced by a competitive firm using the output of a continuum of industries (*aka* intermediate goods or sectors) Q_j for $j \in [0, 1]$ as inputs in a CES production function with constant elasticity of substitution $\theta_I \in (0, \infty)$.

$$Y(t) = \left(\int_0^1 Q_j(t)^{\frac{\theta_I - 1}{\theta_I}} dj \right)^{\frac{\theta_I}{\theta_I - 1}}, \quad \theta_I \in (0, 1) \tag{1}$$

Cost minimization leads to conditional demand for industry j

$$Q_j(t) = \left(\frac{P_j}{P} \right)^{-\theta_I} Y \tag{2}$$

Thus the inverse demand function is $P_j = \left(\frac{Q_j}{Y} \right)^{-\frac{1}{\theta_I}} P$. Substituting the conditional industry demand (2) into the aggregate production function (1) gives the aggregate price index

$$P = \left(\int_0^1 P_j^{1-\theta_I} dj \right)^{\frac{1}{1-\theta_I}} \tag{3}$$

Notice that perfect competition in the final goods market requires equality of price and marginal cost P.

Assumption 1 (Firm Production with U-shaped Average Cost Curve)**.** *Firms are symmetric, so each has the same production technology. The ith firm in the jth industry produces output:*

$$y_{j,i}(t) := \max\{AF(k_{j,i}(t), l_{j,i}(t)) - \phi, 0\} \tag{4}$$

where $F : \mathbb{R}_+^2 \supseteq (k, l) \to \mathbb{R}_+$ is a firm production function with continuous partial derivatives which is homogenous of degree $\nu \in (0, 1)$ (hod-ν) on the open cone $\mathbb{R}_+^2$, and $\phi \in \mathbb{R}_+$ a fixed cost denominated in output. The Hessian matrix of F has a symmetric main diagonal (Young's theorem), negative mixed derivatives (off-diagonal), and its determinant is positive so the concavity properties are

$$F_{kl} = F_{lk} > 0, \qquad F_{kk}, F_{ll} < 0, \qquad F_{kk}F_{ll} - F_{kl}^2 > 0$$

Inada's conditions hold so that marginal products of capital and labour are strictly positive which rules out corner solutions.

$$F_k, F_l > 0$$

Although we shall focus on the case of U-shaped average costs many of our calculations hold without loss of generality for several cases. Appendix F.1 solves the firms static cost minimization problem from which these conclusions are deducible.

- $\phi > 0, \nu \in (0,1)$ U-Shaped average cost and increasing marginal cost curve compatible with imperfect and perfect competition.

- $\phi = 0, \nu = 1$ Constant returns and no fixed cost so globally constant returns to scale. Average cost and marginal cost are equivalent.

- $\phi > 0, \nu = 1$ A fixed cost with constant marginal cost leads to globally decreasing average cost.

- $\phi \geqslant 0$ and $\nu \in (1, \infty)$ Both average and marginal costs are increasing, so there are globally increasing returns to scale. The extent to which ν exceeds 1 is bounded.

Notice that we view number of firms as a factor of production $F(k,l) = F\left(\frac{K}{n}, \frac{L}{n}\right) = n^{-\nu} F(K,L)$. It is essentially a measure of organization which captures how resources are divided. The production function with a fixed cost and decreasing returns to scale cause a U-shaped average cost curve. Decreasing returns to scale arise because the variable production function $F : \mathbb{R}^2_+ \supseteq (k,l) \to \mathbb{R}_+$ is convex, $\nu \in (0,1)$, in capital and labour which causes increasing marginal cost. The fixed cost ϕ creates a nonconvexity which prevents some firms producing because an active firm must sell at least enough to cover the fixed cost. The fixed cost occurs each period, and is different to the sunk entry cost which is paid once to enter (see Entry Section 3.1.3)[12]. $A \in [1, \infty)$ is a scale parameter reflecting the productivity level. It may be interpreted as total factor productivity (TFP). In Section 5.1.1 we derive *measured productivity* which is a function of TFP that captures the fixed cost and returns to scale effect. Since the average cost curve is U-shaped, there is an efficient level of production at minimum average cost, where average cost and marginal cost intersect. This is the Walrasian outcome that would arise under perfect competition.

3.1.1 Strategic Interactions and Endogenous Markups

Within each industry j there is Cournot monopolisitc competition among a set $\mathcal{I}(j)$ of $n(j) \in (1, \infty)$ firms. So the representative ith firm in industry j chooses output to maximise profits subject to the inverse demand function implicit in (2) and the quantities $y_{j,i'}$ supplied by other firms $i' \in \mathcal{I}(j) \setminus i$. It takes as given the quantitiy of final output Y produced by the competitive

[12]As in Jaimovich 2007 and Rotemberg and Woodford 1996 the role of this parameter is to reproduce the apparent absence of pure profits despite market power. It allows zero profits in the presence of market power.

sector, the aggregate price level P of the intermediate sector (it cannot influence this price level) and the factor market prices w and r. Therefore it solves

$$\max_{(y(j,i),l_i,k_i)} P\left(\frac{(y_i + \sum_{i'} y_{i'})}{Y}\right)^{-\frac{1}{\theta_I}} y_i - rk_i - wl_i \tag{5}$$

$$\text{s.t.} \quad y_{j,i}(t) \leqslant AF(k_{j,i}(t), l_{j,i}(t)) - \phi \tag{6}$$

Each firm's technology is symmetric with respect to intermediate inputs that are shared equally due to perfect factor markets $k = \frac{K}{n}$ and $l = \frac{L}{n}$. The result is a symmetric equilibrium outcome, so we can drop i, j indexes and focus on a single representative industry as the whole economy.

Proposition 1 (Markups are Endogenous)**.** *Under symmetric equilibrium the first order conditions of the firms profit maximising problem lead to a markup $\mu(n(t)) \in (1, \infty)$ of price above marginal cost.*

$$\mu(n(t)) = \frac{\theta_I n(t)}{\theta_I n(t) - 1} \tag{7}$$

Where $\theta_I \in (1, \infty)$ is intersectoral substitutability

Lemma 1 (Markup Decreasing in Entry)**.** *The markup is endogenous and decreasing in the number of firms $\mu_n = -\frac{\theta_I}{(\theta_I n - 1)^2} < 0$.*

The negativity of the derivative of the markup with respect to number of firms captures the competition effect of entry lowering markup. When there are many firms in the industry $n \to \infty$, the markup disappears $\mu \to 1$ so price equals marginal cost which is the perfect competition outcome. The opposing limit $n = 1$ is the monopolistic competition special case.

Corollary 1 (Fixed Monopolistic Competition Markup Special Case)**.** *If $n(t) = 1$ then the economy is populated by a continuum of one firm industries each producing a differentiated product and the resulting fixed markup is the well-known monopolistic competition case (Dixit and Stiglitz 1977).*

$$\bar{\mu} = \frac{\theta_I}{\theta_I - 1} \tag{8}$$

It is clear that the Dixit-Stiglitz case is an upper bound on the markup. Therefore endogenous markups will always be lower than the fixed markup case.

$$\mu(n) \leqslant \bar{\mu}$$

In this paper, the most important feature of the endogenous markup is that it is decreasing in n. However, there are a number of ways to make the markup more complicated by assuming substitutability in industry rather than the homogeneous goods I have, or changing Cournot competition to Bertrand. This leads to various forms of markup that rely on both intra- and inter- sectoral substitutability and therefore provide useful extra degrees of freedom in numerical exercises like Jaimovich 2007 and Etro and Colciago 2010. However, despite possible additions all these papers' markups embody the key feature of a competition effect. In fact, like other theory papers Dos Santos Ferreira and Dufourt 2006 I shall later set $\theta_I = 1$, so the markup is only in terms of n. This is equivalent to a Cobb-Douglas aggregator of industry level goods.

An optimizing firm's choice of labour and capital correspond to an imperfectly competitive factor market equilibrium such that the price of a factor does not reflect its marginal product.

Proposition 2 (Factor Market Equilibrium)**.** *Under symmetric inter and intra-industrial equilibrium the optimal price setting rules are a markup of firms' marginal products.*

$$AF_k(k(t), l(t)) = \mu(n)r(t) \tag{9}$$

$$AF_l(k(t), l(t)) = \mu(n)w(t) \tag{10}$$

The marginal revenue product of capital (MRPK) $\frac{AF_k}{\mu(n)}$ equates to the price of capital and the MRPL $\frac{AF_l}{\mu(n)}$ equals to the price of labour. As markups increase the marginal revenue from an additional unit of production is less. Because the MRPs are nonmonotone functions of n_t there is the possibility of multiple equilibria. Different numbers of firm cause the factor market relationship to hold. I do not investigate these implications, instead I assume a unique solution.

3.1.2 Profit

Operating profit $\pi(t) : (K, L, n) \to \mathbb{R}$ is the profit of an incumbent firm in a given period. Operating profits exclude the one-time sunk entry cost that is included in aggregate profits, discussed after we cover the entry process.. Therefore operating profit of a firm is $\pi(t) := y(t) - r(t)k(t) + w(t)l(t)$ and by substituting in factor prices and using Euler's homogeneous function theorem we get profit under imperfect competition[13]

$$\pi(L, K, n; A, \phi) = \left(1 - \frac{\nu}{\mu(n(t))}\right) AF(k, l) - \phi \tag{11}$$

Profit is increasing in the markup and is greater than the perfect competition case of $\mu \to 1$. Profits are nonzero in the short run, but in long-run steady state we shall see they are zero (Section 5).

3.1.3 Firm Entry

I use the entry setup developed in Datta and Dixon 2002. The process of entry determines the number of firms $n(t)$ and the amount of entry $e(t)$ in a period. It is important to emphasize that 'entry' is 'net entry', so it measures the change in the stock of firms. If the stock of firms increases then net entry is positive so there has been entry, whereas if the stock of firms decreases then net entry is negative so there has been exit. This emphasizes that entry is a single symmetric process incorporating both entry and exit, and they cannot occur together. This is unlike papers that treat entry and exit as different processes. For example recent macroeconomics literature models a process of firm creation (entry), but treats exit as a fixed exogenous process (analogous to depreciation of capital). The importance of this point is that a positive shock to the economy will always cause solely entry and a negative shock solely exit. I shall focus on negative exit-inducing shocks, but the inverse argument would hold for positive shocks.

An endogenous sunk entry cost and an entry arbitrage condition determine the number of firms operating at time t. The sunk entry cost increases with the the number of entrants, and the arbitrage condition equates sunk cost with incumbency profits. Das and Das 1997 term the endogenous sunk cost an entry adjustment cost; in macroeconomics, Lewis 2009 and Berentsen and Waller 2009 use the term *congestion effect*, since more entrants cause congestion in entry that increases the sunk cost. The justification for congestion effects is that resources used to setup a firm are in inelastic supply, so that more entrants raises competition for the resources and therefore increases sunk cost. For example, when introducing a new product, if more firms are entering there is a negative entry externality because it is more costly to differentiate a product. Additionally to evidence for entry externalities and

[13] Rearranging the profit function gives the income identity which makes it clearer how markups enter output per firm and is equivalent to the production approach as follows $y(t) := r(t)k(t) + w(t)l(t) + \pi(t) = \frac{AF_l}{\mu(n(t))}l + \frac{AF_k}{\mu(n(t))}k + \left(1 - \frac{\nu}{\mu(n(t))}\right) AF - \phi = \frac{\nu AF}{\mu(n(t))} + \left(1 - \frac{\nu}{\mu(n(t))}\right) AF - \phi = AF - \phi = y(t)$.

their prevalence in industrial organization literature, the assumption provides an analytical framework to study short-run dynamics away from steady state. It is the sunk entry cost that prevents instantaneous adjustment of firms to steady state[14].

Assumption 2 (Sunk Entry Cost (congestion effect))**.** *Sunk entry cost $q \in \mathbb{R}$ increases with the number of entrants $\dot{n}$ in t.*

$$q(t) = \gamma \dot{n}, \qquad \gamma \in (0, \infty) \tag{12}$$

Entry and exit are symmetric for simplicity. A prospective firm pays sunk cost q to enter, and an incumbent firm pays $-q$ to exit. When firms are exiting $\dot{n} < 0 \implies q < 0$, hence $-q > 0$ so the cost of exit is positive. The congestion parameter γ is the marginal cost of entry, and its bounds are the two well-known cases: less sensitivity to congestion $\lim_{\gamma \to 0} q(t)$ implies instantaneous free entry, and more congestion sensitivity $\lim_{\gamma \to \infty} q(t)$ implies fixed number of firms. An extension of the sunk cost assumption to have a fixed cost and the congestion effect, where the fixed cost is paid regardless of the number of entering firms. This setup is closer to Das and Das 1997, and captures the classic case of fixed sunk costs as in Hopenhayn 1992 and Jovanovic 1982, but leads to multiple equilibria in our setup.

The congestion effect assumption is common in industrial organization literature, and has growing support in macroeconomics. Mata and Portugal 1994 show empirically that firm failure and industry entry rates are positively correlated, and theoretically Das and Das 1997 and Ericson and Pakes 1995 both assume sunk entry costs that rise with number of entrants[15]. The intuition for congestion is that there is more competition for a fixed resource needed to setup. For example, many firms entrants raise initial advertising costs to make consumers aware of the product. If many firms are entering there will be many startup advertising campaigns vying for attention. Congestion effects are also called "entry adjustment costs". In macroeconomics, Lewis 2009 uses a VAR analysis to show that congestion effects in entry weaken the volatility of entry responses which can improve model fit. Both Lewis and Poilly 2012 and Berentsen and Waller 2009 model congestion effects in a DSGE model. They differ slightly to our setup because entry

[14] The entry adjustment costs theory is analogous to capital adjustment cost models which recognise that investment (deinvestment) in capital is more costly for larger investment (deinvestment). The cost of investment depends on level of investment which is the flow of capital; analogously, the cost of entry depends of the level of entry which is the flow of no. firms. See Stokey 2008 for a modern account of capital adjustment costs.

[15] Ericson and Pakes 1995 assume the sunk cost is non-decreasing in number of entrants. The assumption includes the simple case of fixed cost not responding to entrants, which they assume in the numerical exercise.

reduces the probability of survival, for example by reducing the likelihood of a sale.

Similarly to Bilbiie, Ghironi, and Melitz 2012 (BGM) the crucial equation that binds entry is an arbitrage condition between returns to entry and the opportunity cost. Despite the literature's differing approaches to attaining endogenous entry (such as congestion sunk cost), all models of this theme ultimately reduce to a condition which equates profits from incumbency to the outside option.

Assumption 3 (Entry Arbitrage (intertemporal zero profit))**.** *The return to paying a sunk costs q to enter and receiving profits equals the return from investing the cost of entry at the market rate $r(t)$.*

$$\dot{q}(t) + \pi(t) = r(t)q(t) \tag{13}$$

Therefore there is an *intertemporal zero profit condition* that implies expected profits of an entrant are always zero; if they were ever non-zero, a firm would revise entry to a more profitable time. The zero-profit condition is dynamic rather than static. In the static case current profits, rather than expected future profits, are instantaneously zero (e.g. Jaimovich and Floetotto 2008), so the value of the firm equates to current profits.

Together the congestion effect assumption 2 and arbitrage assumption 3 form a second-order ODE in number of firms

$$\gamma\ddot{n}(t) - r(t)\gamma\dot{n}(t) + \pi(t) = 0 \tag{14}$$

The equation states that if profits are high, then to maintain zero the cost of entry is also high because there will be many entering firms. By defining entry, this second-order ODE is separable into two first-order ODEs

Definition 2 (Net Entry and Exit)**.** Entry (or exit) is measured by the change in the stock of firms, therefore it is net entry, which if negative is called exit.

$$e(t) = \dot{n} \tag{15}$$

Therefore the model of industry dynamics which determines the number of firms is two ODEs

$$\dot{n}(t) = e(t) \tag{16}$$

$$\dot{e}(t) = -\frac{\pi(t)}{\gamma} + r(t)e(t), \quad \gamma \in (0, \infty) \tag{17}$$

With entry defined as the change in number of firms (15), the arbitrage condition's (17) interpretation depends on the rate of change of entry $\dot{e}(t)$, which is acceleration in number of firms $\ddot{n}(t)$. For example the rate of entry is increasing $\dot{e} > 0$ if the outside option $r(t)e(t)$ exceeds the profit from entering $\frac{\pi(t)}{\gamma}$. This is because when households invest in the more attractive outside option, as opposed to setting up firms, the cost of setting up a firm falls because there is less congestion. The result is an increase in the amount of entry. Initially, it is counterintuitive that the rate of entry decreases with profits, but this captures that when profits are high entry is high, so via congestion the cost of entry is high, and thus the rate of entry slows. The dynamic sunk cost causes firms to respond overtime rather than immediately. Intuitively a firm cannot instantaneously know its cost of entry. A propspective entrant must wait an instance in order to observe the amount of entry and therefore its sunk cost. Consider the contradiction that entry cost is fixed so observable in an instance, $q(t) = \gamma \ \forall t$. In which case the second-order ODE that dictate industry dynamics becomes static $\pi = r\gamma$, so there is no dynamic entry. Rather than the intertemporal zero-profit condition, there is an instantaneous alignment of current profit and opportunity cost. As shown in Datta and Dixon 2002 an implication of the model is that net present value of the firm (stock market value) equates to the sunk entry costs. In this sense the model is equivalent to BGM's approach, except the advantage here is that efficient stock market value is a corollary whereas in BGM it is assumed and then firms dynamics follow.

The aggregation of the sunk costs paid by entering firms leads to a deadweight loss that is not accounted for in operating profits $\pi(t)$ (which are period by period profits). Therefore aggregate profits must account for each firm's operating profits, less the aggregate sunk cost of entry. Based on the congestion effect assumption 2, if net entry is 0 then cost of entry is 0, for the next firm entering in that instance the cost now rises by an increment γ, and so on for each additional entrant up to the final e^{th} entrant in that time instance. Therefore the aggregate deadweight loss of entry $Z(t) \in \mathbb{R}$ is

$$Z(t) = \gamma \int_0^{e(t)} i \, di = \gamma \frac{e(t)^2}{2} \tag{18}$$

Therefore aggregating all n firms operating profits and deducting the deadweight loss gives

$$\Pi(t) = n(t)\pi(t) - Z(t) \tag{19}$$

$$\Pi = n \left[AF(k,l) \left(1 - \frac{\nu}{\mu(n)} \right) - \phi \right] - \gamma \frac{e(t)^2}{2} \tag{20}$$

Aggregate profits are an important factor driving capital investment $\dot{K}$. Ceteris paribus entry reduces aggregate profits. It increases the aggregate sunk costs of entry, and diminishes supernormal operating profits through the competition effect lowering markups. Further, this heightened effect of entry on profits will reduce the amount of entry and therefore reduce the size of the aggregate sunk entry cost.

3.2 Household

In the economy there is a continuum of identical households. This identical household chooses its future series of consumption $\{C(t)\}_0^\infty \in \mathbb{R}$ and labour supply $\{L(t)\}_0^\infty \in [0,1]$ to maximise lifetime utility $\mathrm{U} : \mathbb{R}^2 \to \mathbb{R}$. We assume $u : \mathbb{R}\times[0,1] \to \mathbb{R}$ is jointly concave and differentiable in both of its arguments. It is strictly increasing in C and strictly decreasing in L. A household's choice of consumption and labour is contrained by a its budget constraint which accrues capital income, labour income and profit income. The household owns capital $K \in \mathbb{R}$ and takes equilibrium rental rate and wage $r, w \in \mathbb{R}$ as given. Similarly they own firms and take profits as given $\Pi \in \mathbb{R}_+$. The household maximizes utility subject to a budget constraint rearranged to the law of motion of capital (22). The budget constraint shows that income is earned from capital income, labour income and profit from owning firms and it is spent on consumption or investment in more capital.

$$\mathrm{U} := \int_0^\infty u(C(t), 1 - L(t))e^{-\rho t}dt \tag{21}$$

$$\text{s.t.} \quad \dot{K}(t) = rK(t) + wL(t) + \Pi(t) - C(t) \tag{22}$$

The optimization conditions from the problem reduce to three[16]: an intertemporal consumption Euler equation (23), an intratemporal labour-consumption trade-off (24) and the resource constraint (22).

$$\dot{C}(t) = \frac{C(t)}{\sigma(C(t))}(r(t) - \rho), \tag{23}$$

$$w(t) = -\frac{u_L(L(t))}{u_C(C(t))} \tag{24}$$

where σ represents risk aversion $\sigma(C(t)) = -C(t)\frac{u_{CC}(C(t))}{u_C(C(t))}$. To complete the solution for the boundary value problem, we impose two transversality condi-

[16] Appendix A derives the Hamiltonian and 6 associated Pontryagin conditions.

tions on the upper boundary and an initial condition on the lower boundary.

$$\lim_{t\to\infty} K(t)e^{-\rho t} \geqslant 0, \quad \lim_{t\to\infty} K(t)\lambda(t)e^{-\rho t} = 0, \quad K_0 = K(0) \tag{25}$$

This completes the unique solution for the boundary value problem that characterizes the optimal path of consumption and labour: three variables (C, K, n), three equations (22)-(24), three boundary conditions (25).

In general equilibrium these equations hold and boundary conditions hold, with factor prices and profit determined endogenously from factor market equilibrium $r, w, \Pi : C \times K \times n \to \mathbb{R}_+$.

3.3 Canonical Model in General Equilibrium

Combining the equilibrium conditions from the household and the firm side of the economy defines the model economy as a four dimensional dynamical system that determines consumption, entry, capital and number of firms (C, e, K, n). Importantly labour supply L does not enter the system as an independent variable because it can be defined in terms of C, K, n by combining household intratemporal equilibrium condition with the factor market equilibrium from the firm problem. By understanding the trajectories of labour, we can trace how the *competition effect* of entry reducing markups affects the model through factor price equilibrium and consequently profits.

Proposition 3 (General Equilibrium labour Supply)**.** *Consumption reduces labour supply, whereas capital and number of firms increase labour supply* $L(\underset{-}{C}, \underset{+}{K}, \underset{+}{n})$

$$L_C(C, K, n) < 0, \quad L_K(C, K, n) > 0, \quad L_n(C, K, n) > 0$$

Proof. The effects arise through combining factor market equilibrium (10), which determines wage, with the intratemporal condition (24), which determines consumption-labour choice. Then by the implicit function theorem differentiate the intratemporal condition with labour defined implicitly by $L(C, K, n)$. Derivations in appendix B. □

For capital the important determinant of the sign of labour response is labour marginal product which influences wage. Capital complements labour, so a rise in capital improves the marginal product of labour which consequently raises wage and labour supply. Consumption decreases in labour supply because additional consumption reduces the marginal utility of consumption so the value of consumption declines, thus reducing labour to support consumption (in other words leisure–inverse labour–becomes more attractive.)

The effect of entry on labour supply is more complex because it has an effect on markups too. The effect will feed through to the wage response to entry in section 4.

Corollary 2 (Endogenous Markups Increase Labour Response to Entry)**.** *Endogenous markups strengthen the labour response to entry relative to fixed markup ($\bar{\mu}$) case*

$$L_n \geqslant L_n^{\bar{\mu}} \tag{26}$$

Proof. ($\mu(n)$ domain supressed.)

$$L_n = \frac{u_C A F_l \mu^{-1}(\mu_n \mu^{-1} - (1-\nu)n^{-1})}{u_{LL} + u_C A n^{-1} F_{ll} \mu^{-1}}$$

Consider the case of a fixed exogenous markup then $\mu_n = 0$ □

Entry (a rise in n) increases labour supply because it raises marginal product of labour and in turn wage (labour fixed). Wage rises because labour per firm falls which increases its marginal product due to decreasing returns $\nu < 0$. With constant returns there is no effect $L_n = 0$ as firms employ labour at equal productivity regardless of size. However with endogenous markups this does not hold, as even with constant returns there is the negative effect, $\mu_n \mu^{-1}$, which captures that a lower markup increases marginal revenue product of labour. Since this effect of entry diminishing markup brings wage inline with marginal product, it increases labour. Later we shall term this the *competition effect*, and the first effect due to returns to scale will be the *allocation effect*. When we analyse wage behaviour we shall see this extra markup effect will strengthen the labour effect on wage which creates downward pressure, but is offset by the direct effect of lower markup bringing wage closer to marginal product.

An interesting implication of Corollary 2 is that entry and labour supply are positively related. Vice-versa, exit leads to a fall in labour supply. In this paper, we investigate the influence of entry on measured productivity, whereas much empirical discussion is based on labour productivity. The result shows that our model encapsulates labour productivity arguments as a specific case. For example, if there is a negative shock to the economy, to which firm exit does not respond instantaneously, labour supply will be buoyed which worsens labour productivity. Only when firms exit will employment begin to fall which will raise the productivity of remaining labour.

Definition 3 (General Equilibrium)**.** Competitive equilibrium is the equilibrium paths of aggregate quantities and prices $\{C(t), L(t), K(t), n(t), e(t), w(t), r(t)\}_{t=0}^{\infty}$, with prices strictly positive, such that $\{C(t), L(t)\}_{t=0}^{\infty}$ solve the household problem. $\{K(t)\}_{t=0}^{\infty}$ satisfies the law of motion for capital. Labour and capital $\{L(t), K(t)\}_{t=0}^{\infty}$ maximise firm profits given factor prices. The flow of entry causes the arbitrage condition on entry to hold (price of entry equals net present value of incumbency). State variables $\{K(t), n(t)\}_{t=0}^{\infty}$ satisfy transversality. Factor prices are set according to factor market equilibrium (10) and ensure goods and factor markets clear.

The dynamic equilibrium conditions from the previous section are the capital accumulation equation, the number of firms definition, the consumption Euler, and the entry arbitrage condition.

Definition 4 (Nonlinear System)**.** The dynamical system defines at a point in time $t \in \mathbb{R}$ the state of the system $(C(t), e(t), K(t), n(t)) = x(t) \in \mathbb{X} \subseteq \mathbb{R}^4$ described by a C^1 vector valued transition map $g : \mathbb{R}^{5+\mathbb{P}} \supseteq \mathbb{X} \times \mathbb{R} \times \Omega \longrightarrow \mathbb{R}^4$. The parameterization $(\phi, \nu, \gamma, \rho)$ is defined on an open set $\Omega \in \mathbb{R}^{\mathbb{P}}$

The system is

$$\dot{K} = Y(t) - \frac{\gamma}{2}e(t)^2 - C(t), \qquad Y = n(F(k,l) - \phi) \tag{27}$$

$$\dot{n} = e(t) \tag{28}$$

$$\dot{C} = \frac{C(t)}{\sigma(C)}(r(t) - \rho), \qquad \sigma(C) = -\frac{Cu_{CC}}{u_C} \tag{29}$$

$$\dot{e} = r(t)e(t) - \frac{\pi(t)}{\gamma}, \qquad \pi = AF(k,l)(1 - \frac{\nu}{\mu}) - \phi \tag{30}$$

where factor prices

$$r = \frac{AF_k(k,l)}{\mu(n)}, \qquad w = \frac{AF_l(k,l)}{\mu(n)} \tag{31}$$

Substituting in factor prices, profits and output which are all in terms of (C, K, n) and noting that by the intratemporal condition and wage equilibrium L is implicitly defined as $L(C, K, n)$ the model economy is a system of four ODEs in consumption, entry, capital, number of firms (C, e, K, n). Also by Euler's homogeneous function theorem note $F(k,l) = n^{-\nu}F(K,L)$ and $F_k(k,l) = n^{1-\nu}F_K(K,L)$. This gives a more primitive description of the system, with less economic intuition, but easier to understand the underlying dynamics.

$$\dot{K} = n(An^{-\nu}F(K, L(C, K, n)) - \phi) - \frac{\gamma}{2}e^2 - C \tag{32}$$
$$\dot{n} = e \tag{33}$$
$$\dot{C} = -\frac{u_C}{u_{CC}}\left(\frac{An^{1-\nu}F_K(K, L(C, K, n))}{\mu(n)} - \rho\right) \tag{34}$$
$$\dot{e} = \frac{An^{1-\nu}F_K(K, L(C, K, n))e}{\mu(n)} - \frac{An^{-\nu}F(K, L(C, K, n))(1 - \frac{\nu}{\mu(n)}) - \phi}{\gamma} \tag{35}$$

4 Competition Effect on Factor Prices and Profit

The competition effect enters the model through factor market equilibrium affecting factor prices r and w and in turn affecting profit π. After outlining these mechanisms in this section, the following section on steady state analysis shows that the mechanism propagates to long-run outcomes, where it raises output per firm and productivity.

Definition 5 (Competition Effect of Entry/Exit)**.** The competition effect is caused by entry's effect on markups. That is, the markup $\mu(n(t))$ decreases in the number of firms competing $\mu_n < 0$. This was shown in lemma 1. The competition effect is zero with exogenous markups $\bar{\mu}_n = 0$.

Definition 6 (Allocation Effect of Entry/Exit)**.** The allocation effect is that entry and exit alter the allocation of resources (capital and labour) among firms. This affects scale of production, which is important due to decreasing returns to scale in production. Entry causes 'business stealing' reducing inputs per firm, whereas exit causes 'business consolidation' raising inputs per firm.

There are three effects of an entering firm on factor prices, and analogous three effects on operating profits which are a function of factor prices and form a key result (proposition 4).

$$w_n = \frac{A}{\mu(n)}\left[(1-\nu)n^{-\nu}F_L + n^{1-\nu}F_{LL}L_n - \frac{n^{1-\nu}F_L}{\mu}\mu_n\right] \lesseqgtr 0 \tag{36}$$
$$r_n = \frac{A}{\mu(n)}\left[(1-\nu)n^{-\nu}F_K + n^{1-\nu}F_{KL}L_n - \frac{n^{1-\nu}F_K}{\mu}\mu_n\right] > 0 \tag{37}$$

The three effects are a positive allocation effect, an ambiguous labour effect and a positive competition effect. The allocation effect captures that an extra firm reduces per firm allocation of inputs, but raises aggregate number of firms. The reduction in labour or capital per firm raises their marginal product and therefore price due to returns to scale. Hence with constant returns $\nu = 0$ the effect is not present. The labour effect captures that entry increases labour supply and therefore lowers wage or raises interest rate. Lastly the competition effect that arises from endogenous markups $\mu_n < 0$ captures that an extra firm increases competition and lowers markups which raises the marginal revenue product of labour (capital) and so wage (interest rate) must increase to maintain equilibrium. The interest rate response (37) is unambiguous because the labour effect is positive since labour complements capital, so it raises marginal product of capital. However in the wage result (36) this same labour effect is negative which creates an ambiguity because extra labour (caused by entry) reduces the marginal product of labour so depresses wages. Despite us showing in corollary 2 that the labour effect is stronger with endogenous markups, there is the offsetting positive wage effect that endogenous markups reduce the disparity between wage an marginal product (competition effect), which is reasonable to assume dominates the second-order effect on labour supply. Furthermore labour marginal product is buoyed by entry dividing resources among more firms (allocation effect). I merely acknowledge that entry and the surge in labour it creates can detriment marginal product of labour to the extent that wage indeed falls.

Profits are increasing in markups, which adds an extra effect of a firm entering the market. The result is that profits diminish faster, than if markups were fixed.

Proposition 4 (Entry Effect on Profits)**.** *Entry has two negative effects on operating profit.*

$$\pi_n = \frac{A}{\mu}\left[(\mu - \nu)\left(-\nu n^{-\nu-1}F + n^{-\nu}F_L L_n\right) + \frac{\nu n^{-\nu}F}{\mu}\mu_n\right] < 0 \qquad (38)$$

Proof. By substituting in L_n it can be shown that the negative scale effect dominates the positive labour effect in the second component of π_n. See appendix C □

The three effects of an entering firm on factor prices feed through to profits. The allocation effect decreases profit, the labour effect increases profit, the competition effect decreases profit. The competition (markup effect) and allocation (business stealing) effect reinforce eachother. Making the negative effect of entry on profits larger than the case with fixed markups. However, there is a positive effect on profits from labour, since an entrant raises the market wage leading to a higher supply of labour which raises per firm output and revenues. The effect can be shown to be dominated hence the inequality.

Corollary 3 (Profit More Responsive to Entry Under Endogenous Markup)**.** *The responsiveness of profit to firm entry is absolutely larger in the case of endogenous markups.*

$$|\pi(\mu(n))_n| > |\pi(\bar{\mu})_n|$$

In sum the competition effect of entry depresses markups which raises wage and interest rate in factor market equilibrium. This effect of higher factor prices (prices closer to their marginal product) causes profits to fall more from each entrant. The result is that zero profit arises when fewer firms have entered and so each firm has a larger market share. Conversely a negative shock that leads to negative profits and exit means each exiter raises incumbents profits less regaining zero profits requires more exit and then remaining firms can produce less. So the mechanisms in this section are responsible for the results in the next section that show output per firm and productivity are increasing in the number of firms.

5 Efficiency and Steady State Outcomes

This section first derives the efficient outcomes that correspond to minimum average cost, or the number of firms that maximises output. It then analyzes the fixed point of the dynamical system, which corresponds to the zero profit outcome, often assumed instantaneously in other papers.

5.1 Efficient Output and Productivity

In symmetric equilibrium aggregate output is the number of firms in a representative sector multiplied by the amount a firm produces. It is homogeneous of degree 1 in K, L, n. This captures that capital and labour per firm do not change if all factors are changed equally, so output per firm is homogeneous of degree 0, but aggregation across all firms causes a proportional increase because of a proportional change in the number of firms that are being aggregated.

$$Y(t) = n(t)y(t) = n(t)^{(1-\nu)}AF(K(t), L(t)) - n(t)\phi \tag{39}$$

The effect of an entrant on aggregate output is

$$\begin{aligned} Y_n &= (1-\nu)n^{-\nu}AF(K,L) + n^{1-\nu}AF_L(K,L)L_n(C,K,n) - \phi \\ &\approx (1-\nu)n^{-\nu}AF(K,L) - \phi \qquad (40) \\ Y_{nn} &= (1-\nu)n^{-1}[-\nu n^{-\nu}AF(K,L) + n^{1-\nu}AF_L(K,L)L_n(C,K,n)] \\ &\approx (1-\nu)n^{-1}[-\nu n^{-\nu}AF(K,L)] \qquad (41) \end{aligned}$$

with approximations due to small second-order labour effect; alternatively, take labour taken as given at aggregate level. There are three effects of firm entry on aggregate output: an ambiguous allocation (returns to scale) effect, a positive labour effect and a negative fixed cost effect (resource duplication). The scale effect is positive with decreasing returns $\nu \in (0,1)$ and zero with constant returns $\nu = 0$. The effect captures that business reallocation among more firms improves aggregate output when there are decreasing returns.

At the point where the positive returns to scale and labour effect equate the fixed cost effect, there is an optimal efficient number of firms $Y_n = 0|_{\nu<1}$. These outcomes are those that would arise under Walrasian perfect competition, and if there were no markups in our model ($\mu = 1$). In the AC-MC diagram this is where they intersect at minimum average cost. To ensure the outcomes are defined we assume rising marginal cost $\nu \in (0,1)$ and to ensure production is nonnegative we assume fixed cost effect exceeds positive labour effect $\nu\phi \geqslant An^{1-\nu}F_L(K,L)L_n$ so there are initially decreasing returns as costs decrease toward the minimum. Generally we assume the second-order labour effect is small.

Proposition 5. *When output is maximised with respect to number of firms in the economy the efficient levels of output are*

$$F(k^e,l^e) = \frac{1}{(1-\nu)}\left(\frac{\phi}{A} - F_l(k,l)L_n\right) \approx \frac{\phi}{A(1-\nu)} \qquad (42)$$

$$\begin{aligned} y^e &= AF(k^e,l^e) - \phi \\ &= \frac{1}{1-\nu}(\phi\nu - AF_l(k,l)L_n) \approx \frac{\phi\nu}{1-\nu} \qquad (43) \end{aligned}$$

It is notable that most papers dicussing entry focus on constant returns to scale. As this section has shown this implies there is no optimal firm size; analogously there is no perfect competition equilibrium because the market tends to a natural monopoly due to the fixed cost teamed with constant returns to scale. Firm size, in terms of factors it employs, is unimportant because all firms produce at the same efficiency. This limits the role of entry, so that productivity results arise solely from the competition effect of more firms reducing markups. How output is divided among firms does not matter. We shall see in corollary 5 that when entry is high (the market is competitive), imperfect competition outcomes converge on this section's efficient outcomes ((43) and later (45)).

5.1.1 Homogeneous Degree Zero Productivity

I call productivity at a point in time *measured productivity.* The measure is equivalent to TFPR (R for revenue) in Peters 2013[17]. Corresponding to the efficient levels of output is a definition of productivity that is also maximised at these efficient output levels, taking labour as given.

Definition 7 (Measured Productivity)**.** Measured productivity $\mathcal{P} : K, L, n \to \mathbb{R}_+$ is the amount of output an economy produces for a given technology, with technology normalized to be homogeneous of degree 1 to remove scale effects

$$\mathcal{P}(t) = \frac{Y(t)}{F(K(t), L(t))^{\frac{1}{\nu}}} \tag{44}$$

This aggregate measure is the same as the per firm measure $\mathcal{P} = \frac{n(t)y(t)}{F(K(t),L(t))^{\frac{1}{\nu}}} = \frac{y(t)}{F(k(t),l(t))^{\frac{1}{\nu}}}$.

A more productive economy has larger measured productivity because it combines inputs more efficiently and produces more output with the same technology as another economy. An outcome of this definition of measured productivity is that when it is maximised with respect to number of firms $\mathcal{P}_n = 0$ the corresponding levels of output are the efficient outcomes y^e. Therefore the maximum attainable productivity $\mathcal{P}^e$ that arises at the efficient level of production is

$$\begin{aligned}
\mathcal{P}^e &= \frac{y^e}{F(k^e, l^e)^{\frac{1}{\nu}}} \\
&= \frac{(\phi\nu - AF_l L_n) A^{\frac{1}{\nu}} (1-\nu)^{\frac{1}{\nu}-1}}{(\phi\nu - AF_l L_n)^{\frac{1}{\nu}}}, \quad \phi\nu > AF_l L_n,\ \nu \in (0,1) \\
&\approx \left(A\nu^{\nu} \left(\frac{\phi}{1-\nu} \right)^{\nu-1} \right)^{\frac{1}{\nu}}
\end{aligned} \tag{45}$$

And in the constant returns limit the maximum attainable measured productivity is equivalent to TFP $\lim_{\nu \to 1} \mathcal{P}^e = A$.

[17]Based on Foster, Haltiwanger, and Syverson 2008.

Since production technology in the denominator is $hod - \nu$ we need to normalize it to be $hod - 1$. Then productivity will be $hod - 0$ in inputs. That means that the scale of inputs K, L, n does not affect productivity. Whereas with a typical non-normalized measure an economy with more inputs would always appear less productive. Hence we capture changes in efficiency of technology use, how effectively the inputs are combined with a given technology, rather than how many inputs there are. Consider an example of two economies $\mathcal{A}$ and $\mathcal{B}$. They are identical in every sense, except economy $\mathcal{B}$ is endowed with $\lambda \in (1, \infty)$ times more factors K, L, n. Since the economies are identical, except for scale of factors, then a good productivity measure should reflect that both economies have the same productivity: they combine factors with the same efficiency to produce output. Now assume the contradiction that we do not normalize technology and use a standard, non-normalized, TFP measure $\hat{\mathcal{P}}$. Then $\hat{\mathcal{P}}^{\mathcal{A}} = \frac{Y(t)}{F(K(t),L(t))}$ and since Y is $hod - 1$ and F is $hod - \nu$ then $\hat{\mathcal{P}}^{\mathcal{B}} = \frac{\lambda Y(t)}{\lambda^{\nu} F(K(t),L(t))} = \lambda^{1-\nu} \hat{\mathcal{P}}^{\mathcal{A}}$. So $\hat{\mathcal{P}}^{\mathcal{A}} < \hat{\mathcal{P}}^{\mathcal{B}}$ we conclude erroneously that economy $\mathcal{B}$ is more productive simply because it has more factors, not because it combines those factors more efficiently. Under our normalized measure $\mathcal{P}^{\mathcal{B}} = \frac{\lambda Y(t)}{(\lambda^{\nu} F(K(t),L(t)))^{\frac{1}{\nu}}} = \mathcal{P}^{\mathcal{A}}$. With constant returns $\nu = 1$ there are no scale effects, so our measure collapses to the common definition.

5.2 Steady State

Now I shall show that the steady state of our economy corresponds to zero profits. And leads to levels of output and productivity that depend endogenously on the number of firms, and these levels are strictly less than the efficient levels that would arise under perfect competition defined in section 5.1.

Assume that a solution of the system converges to a unique steady state $(K, n, C, e) \to (K^*, n^*, C^*, e^*)$ as $t \to +\infty$[18]. In steady state $\dot{K} = \dot{n} = \dot{C} = \dot{e} = 0$, which immediately implies entry is zero via (47), which in turn, via 49, implies profits are zero.

$$\dot{K} = 0 \Leftrightarrow Y^*(C^*, K^*, n^*) = C^* \tag{46}$$

$$\dot{n} = 0 \Leftrightarrow e^* = 0 \tag{47}$$

$$\dot{C} = 0 \Leftrightarrow r^*(C^*, K^*, n^*) = \rho \tag{48}$$

$$\dot{e} = 0 \Leftrightarrow \pi^*(C^*, K^*, n^*) = 0 \tag{49}$$

[18]Ignore the trivial steady state that arises when the state vector is the zero vector. It is possible that with endogenous markups there are multiple values of n^* that allow steady-state to hold. This is an investigation for future research.

In steady state aggregate output equates to consumption; entry is zero; the interest rate equals the discount factor and profits are zero. Intuitively when profits are zero entry ceases as there is no entry incentive, and when the discount factor and interest rate are equated there is indifference between consumption and saving so all output is consumed. Rewriting the system in terms of underlying variables (C, e, K, n), again with labour defined implicitly $L(C, K, n)$, shows that ouput per firm and therefore measured productivity depend endogenously on the number of firms.

$$n^* \left[An^{-\nu} F(K^*, L^*) - \phi\right] = C^* \tag{50}$$
$$e^* = 0 \tag{51}$$
$$\frac{A{n^*}^{1-\nu} F_K(K^*, L^*)}{\mu(n^*)} = \rho \tag{52}$$
$${n^*}^{-\nu} F(K^*, L^*) = \frac{\phi}{A\left(1 - \frac{\nu}{\mu(n^*)}\right)} \tag{53}$$

5.3 Steady State Existence

In this section I provide a condition under which a steady state solution always exists, and in appendix D I calculate a specific solution numerically. For a steady state to exist the following system must be solvable for (C^*, K^*, n^*) (where I have substituted in the trivial $\dot{n}$ condition that $e = 0$).

$$\dot{C}: \quad 0 = r(C^*, K^*, n^*) - \rho \tag{54}$$
$$\dot{e}: \quad 0 = \pi(C^*, K^*, n^*) \tag{55}$$
$$\dot{K}: \quad 0 = Y(C^*, K^*, n^*) - C^* \tag{56}$$

Therefore the determinant of the Jacobian of this three dimensional system with respect to (C, K, n) is[19]

$$\left.\begin{bmatrix} r_C & r_K & r_n \\ -\pi_C & -\pi_K & -\pi_n \\ Y_C - 1 & Y_K & Y_n \end{bmatrix}\right|_{x=x^*} \implies \begin{bmatrix} - & - & + \\ + & - & + \\ - & + & \pm \end{bmatrix} \tag{57}$$

where determining the signs $r_K < 0$ and $\pi_n < 0$ requires extra work[20], and the sign structure is determinable before evaluating at steady state. I assume

[19]This proof of existence follows the approach of Caputo 2005, pp.419.

[20]See Appendix C the results involve substituting in labour effects and showing they are dominated by using the second partial derivative test for concavity which the production function is assumed to satisfy.

$Y_n > 0$. Each element is evaluated in a neighborhood of the conjectured steady state, although the sign structures hold regardless[21].

The determinant is

$$\overbrace{-\pi_K r_C Y_n}^{+} + \overbrace{-\pi_n r_K (Y_C - 1)}^{+} + \overbrace{-\pi_C r_n Y_K}^{+} \\ \overbrace{- - \pi_K r_n (Y_C - 1)}^{-} \overbrace{- - \pi_C r_K Y_n}^{+} \overbrace{- - \pi_n r_C Y_K}^{+} \quad (58)$$

Unfortunately the determinant is not clearly nonzero due to the $\overbrace{- - \pi_K r_n (Y_C - 1)}^{-}$ term even though all other terms are positive. However if this negative term can be shown to be dominated by one of the positive terms, it provides a sufficient condition for determinacy I.e. the determinant is strictly positive ensuring that a solution C^*, K^*, n^* exists by the implicit function theorem. The positive term $\overbrace{-\pi_n r_K (Y_C - 1)}^{+}$ proves a good candidate to dominate the negative $\overbrace{- - \pi_K r_n (Y_C - 1)}^{-}$ so their sum is positive. That is, $\overbrace{-\pi_K r_n (Y_C - 1)}^{+} - \overbrace{-\pi_n r_K (Y_C - 1)}^{+} > 0$, or simply

Lemma 2 (Steady State Existence)**.** *A steady state solution $\{C^*, K^*, n^*\}$ of the the system (54)-(56) exists if the following sufficiency condition holds*

$$r_n \pi_K - r_K \pi_n < 0$$

This is a sufficient condition for determinacy, and it has an intuitive interpretation to support it. The condition states that the combined effect of $r_K \pi_n$ dominates the combined effect of $r_n \pi_K$. Since r_K is a direct effect (i.e. how capital effects its own price) it is sensible to believe it outweighs r_n, and similarly since π_n is the direct effect of number of firms on value of firms it is sensible to believe it is stronger than the π_K effect. Hence we should believe that the combined positive effect of two second-order effects $r_n \pi_K$ is weaker than the combined positive effect of two first-order effects $r_K \pi_n$. And thus the former minus the latter will be negative. Indeed Appendix D shows that for a standard numerical example with Cobb-Douglas production and Isoelastic utility a solution to the system exists.

[21]Notice it is important to distinguish between the derivative of a variable x with respect to z when x is in steady state x_z^*, as opposed to the derivative of a variable evaluated at steady state $x_z|_{x=x^*}$.

Theorem 1 (Endogenous Steady State Output and Productivity)**.** *Steady state output per firm y^* and measured productivity $\mathcal{P}^*$ are endogenous because they depend on the markup which depends on the endogenous variable $n(t)$, the number of active firms.*

$$y^*(\underset{+}{n^*}, \underset{-}{\mu(n^*)}) = \frac{\phi\nu}{\mu(n^*) - \nu} \tag{59}$$

$$\mathcal{P}^*(\underset{+}{n^*}, \underset{-}{\mu(n^*)}) := \frac{y^*}{F(k^*, l^*)^{\frac{1}{\nu}}} = \nu \left[\frac{A}{\mu(n^*)} \left(\frac{\mu(n^*) - \nu}{\phi} \right)^{1-\nu} \right]^{\frac{1}{\nu}} \tag{60}$$

Proof. From the zero profit condition variable production becomes $AF(k^*, l^*) = \frac{\phi\mu(n^*)}{(\mu(n^*)-\nu)}$, and then $y^* = AF(k^*, l^*) - \phi$. Then substitute into the expression for productivity. □

Corollary 4. *Steady state output per firm and measured productivity are increasing in the number of firms in the economy.*

$$y^*_n > 0, \qquad \mathcal{P}^*_n > 0$$

Proof. The result for output y^* is clear. For productivity consider

$$\frac{\partial \mathcal{P}^*}{\partial n} = -\frac{A\nu}{\mu^2} \frac{(\mu - 1)}{(\mu - \nu)^\nu \phi^{1-\nu}} \cdot \frac{\partial \mu(n^*)}{\partial n} > 0$$

□

Since markups are decreasing in number of firms, output is increasing in number of firms, and similarly productivity is increasing in number of firms. The simpler case of constant marginal cost gives a similar outcome $\mathcal{P}^*|_{\nu=1} = \frac{A}{\mu}$ as in Jaimovich and Floetotto 2008 and Peters 2013. Given markups are negatively related to the number of firms, a single firm needs to sell more output to cover its fixed cost and break even in a free entry equilibrium. Hence with more firms, output per firm rises.

From (60) the fixed cost (ϕ) and decreasing returns to scale ($\nu < 1$) cause a dampening effect that captures that productivity is less sensitive to markups when the fixed cost is high. This is because fixed costs induce higher output per firm and therefore closer to constant returns to scale (nearer minimum AC), thus variations in output around this point caused by the changing markup has less of a productivity effect. This component falls out when there are constant returns to scale $\phi^{1-\nu}|_{\nu\to 1} \to 1$, because the fixed cost is used at equal efficiency regardless of scale.

The markup causes extra profit that helps us to understand the mechanism through which entry is affected. Profits offer entry incentives, and incentives rise when markups are higher which encourages more entry than would arise under perfect competition. However since markups are decreasing in number of firms the excessive amount of profit will diminish faster as firms enter, and zero profit will arise when fewer firms have entered so the allocation effect (business stealing effect) is dampened and larger firms remain. Thus each firm produces more and benefits from returns to scale, which fosters endogenous productivity. That is, long-run underlying productivity is a function of number of firms whereas with fixed markups firms entry always returns the economy to a position with the same productivity. The extra mechanism is important, since a prospective firm now considers how fierce competition in the market is, whereas with fixed markup it took for granted that it could enter and charge a given markup, thus produce a given amount in the long run. This leads to an important corollary

Corollary 5 (Efficiency of Imperfect Competition)**.** *When there is a high degree of competition (a large amount of entry) the imperfect competition outcomes ((59) and (60)) converge upon the efficient outcomes ((43) and (45)) because the markup is suppressed.*

5.4 Aggregate Output in Steady State

Using the endogenous steady state output expression (59) we can infer the behavior of number of firms in steady state.

Lemma 3 (Firm Procyclicality)**.** *Given aggregate output at steady-state level Y^*, the number of active firms is procyclical.*

$$n^* = \left[AF(K^*, L^*(C^*, K^*, n^*)) \left(\frac{\mu(n^*) - \nu}{\mu(n^*)\phi} \right) \right]^{\frac{1}{\nu}} = \left(\frac{\mu(n^*) - \nu}{\phi\nu} \right) Y^* \qquad (61)$$

Proof. For a given steady-state level of output Y^*, a change in this level on n^* causes $n^*_{Y^*} = \left(\frac{\mu(n^*)-\nu}{\phi\nu}\right) > 0$. The sign can be determined since the markup $\mu(n) > 1$ and returns to scale are decreasing (increasing marginal costs) $\nu \in (0, 1)$, therefore the numerator is positive.

See appendix D.2 for extended proof treating Y^* endogenously via implicit function theorem. □

When there is a rise in steady state aggregate output Y^* the number of firms increases since $\left(\frac{\mu(n^*)-\nu}{\phi\nu}\right) > 0$. Closer to constant returns to scale $\nu \to 1$ and higher fixed costs ϕ weaken the procyclicality effect, whereas greater imperfect competition $\mu(n^*)$ strengthens the effect[22]. Furthermore the direct effect of an increase in technology A is to raise number of firms $n^*_A > 0$ through its positive effect on Y^*. I validate this result numerically in section 6.2.

With fixed markups output per firm always returns to a constant level that is the amount of sales required to cover the fixed cost. Now μ is endogenous, if n rises then more sales are needed to cover the fixed cost and therefore output per firm in equilibrium depends on the number of firms in the market. Therefore in aggregate there is an increase in number of firms, and there is an increase in output per firm

$$Y^* = n^*y^* = (AF(K^*, L^*(C^*, K^*, n^*)))^{\frac{1}{\nu}} \left(\frac{\mu(n^*)\phi}{\mu(n^*) - \nu}\right)^{1-\frac{1}{\nu}} \frac{\nu}{\mu(n^*)} \tag{62}$$

Aggregate output is much simpler with constant returns $Y^*|_{\nu=1} = \frac{AF(K^*,L^*)}{\mu(n^*)}$. There is productive inefficiency from the markup, which reduces Y^*, but the fixed cost component $\left(\frac{\mu(n^*)\phi}{\mu(n^*)-\nu}\right)^{1-\frac{1}{\nu}}\nu$ is unimportant as all firms use the fixed cost with the equal efficiency. It is useful to compare the endogenous markup case, to the better-known case of fixed markups in steady state. With a fixed markup number of firms does not affect aggregate output through y^* which is always fixed exogenously as a function of given parameters, so an extra firm simply contributes this fixed extra amount to output. With endogenous markup an extra firm alters per firm output y^* since a firm needs to produce more to cover fixed costs due to fiercer markup competition. $\frac{\partial Y^*}{\partial n} = y^* + n^*y^*_n$. With fixed markups only the first effect is present (the contribution of an entrant is to add y^* to aggregate output), but with endogenous markup there is also the competition effect which raises output per firm of every incubent because they face more competition $n^*y^*_n$.

The conclusions from the static analysis are that output per firm increases with number of firms, and productivity increases with number of firms. These results arise because number of firms degrade monopoly power, and this effect will always prevail over the dynamic business reallocation effect that causes output per firm to decrease as each firm enters because any shock rises productivity and output per firm too much on impact. From the long-run perspective more firms is better in the sense it raises output per firm, so more aggregate output can be produced from fewer firms.

[22]This result generalizes eq. 19 Jaimovich and Floetotto 2008, pp. 1245 who show an analogous outcome under constant returns and instantaneous entry.

6 Main Result: Productivity Dynamics

The main result shows that the impact effect of a TFP shock causes an exacerbated response in short-run productivity that relinquishes over time but leaves some long-run persistence due to the competition effect. This means that the difference between measured productivity on impact and measured productivity in the long-run is dampened because of a persistent change in productivity draws it closer to the initial level.

Theorem 2 (Permanent Change in TFP)**.** *On impact of a shock productivity overshoots the long-run effect, but there is no reversion to underlying productivity due to a persistent change in degree of competition.*

$$\mathcal{P}(0)_A - \mathcal{P}^*_A = \mathcal{P}(0)_A - \mathcal{P}^{*\bar{\mu}}_A - \mathcal{P}^*_\mu \mu(n^*)_A \tag{63}$$

$$\mathcal{P}(0)_A|_{x(0)=x^*} - \mathcal{P}^*_A = (\mu^* - 1)\frac{\mathcal{P}^*}{\nu}\left[\underbrace{\left(\frac{1}{A} + \frac{F^*_L L^*_A}{F^*}\right)}_{\textit{Alloc. Effect (+)}} + \underbrace{\frac{\nu}{\mu^*(\mu^*-\nu)}\mu_n n^*_A}_{\textit{Comp. Effect (-)}}\right] \tag{64}$$

where $\mathcal{P}^* = \nu\left[\frac{A}{\mu(n^*)}\left(\frac{\mu(n^*)-\nu}{\phi}\right)^{1-\nu}\right]^{\frac{1}{\nu}}$

Proof. Details in appendix E. From lemma 3, equation (61), $n^*_A > 0$ improved TFP raises the number of firms, see also numerical result in section 6.2. □

where $\mathcal{P}(0)_A|_{x(0)=x^*}$ is the response of productivity on impact (at $t = 0$), with all variables x beginning at steady state $x(0) = x^*$.

The positive allocation effect captures that only incumbents bear the change in TFP due to entry/exit inertia thus on impact there is a direct effect on incumbents' productivity from having a different TFP and there is a reinforcing labour effect that also responds immediately. The negative competition effect captures that the long-run level of productivity moves in the same direction as the initial effect which closes the gap between initial impact and long-run productivity. In the absence of a competition effect $\mu_n = 0$, there is no persistent effect on productivity.

Corollary 6. *Entry reduces the size of productivity overshooting, such that as* $n \to \infty$

$$\mathcal{P}(0)_A|_{x(0)=x^*} = \mathcal{P}^*_A \tag{65}$$

The result only exists with rising marginal cost $\nu \in (0, 1)$*.*

Proof. Since the markup disappears as firms increase $\lim_{n\to\infty} \mu(n_t) = 1$, see appendix G, then

$$\lim_{n\to\infty} \left(\mathcal{P}(0)_A|_{x(0)=x^*} - \mathcal{P}^*_A \right) = 0$$

□

This corollary can be interpreted as more competitive economies have less productivity volatility. The result implies that productivity puzzles are weakened when there is more competition. Conversly overshooting is greater when there are few firms per sector. This strengthens imperfect competition and therefore markups are higher. It also means that the long-run structural change to competition will be greater. Consequently, the impact of a technology shock causes a large change in measured productivity initially but it then reverts to a similar but weaker level of productivity in the long-run. Contrarily, if the sector is very competitive, there are many firms in the sector and the initial effect on productivity is small, likewise there is little structural change to competition from more firms entering because there are still many firms competing. The implication is that more competition, which is synonymous to more firms, implies less volatile productivity and less persistence in productivity shocks[23]. Importantly this result does not hold under constant returns to scale $\nu \to 1$ because as firms drive markup to unitary no equilibrium exists as there is no cost minimizing level of output.

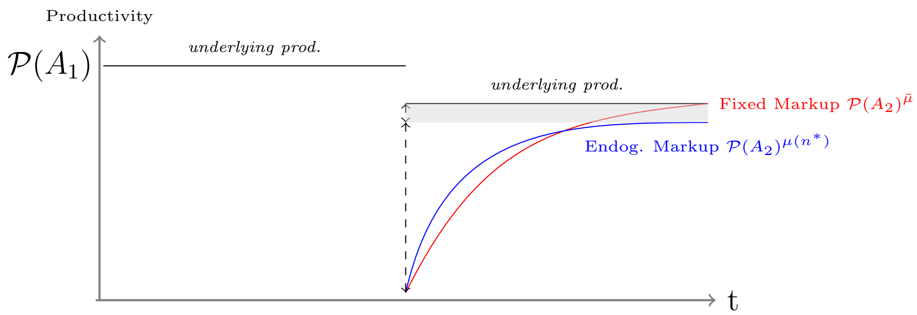

Figure 5: Exacerbated Productivity Followed by Long-run Persistence

Figure 5 shows that the long-run competition effect tightens the gap between impact and long-run effect. A negative shock to technology from A_1 to A_2 causes an initially big fall in measured productivity (dashed arrow), but it recovers as firms begin to exit. However, the *"Fixed Markup $\mathcal{P}(A_2)^{\bar{\mu}}$"*

[23] This is an interesting testable implication to expand upon empirically. As number of firms in the economy gets large then the long-run effect arises immediately $\lim_{n\to\infty} \mu(n) = 1$ so $\mathcal{P}(0)_A = \mathcal{P}^*_A$

curve shows that with fixed markups $\bar{\mu}$ productivity recovers to regain the underlying level that incorporates the new worse technology A_2, whereas the *"Endog. Markup $\mathcal{P}(A_2)^{\mu(n^*)}$"* time path shows that despite some recovery there is always persistently worse productivity in the long run (shown by the gray box), and this is because the markup $\mu(n^*)$ rises due to less long-run competition from firms exiting.

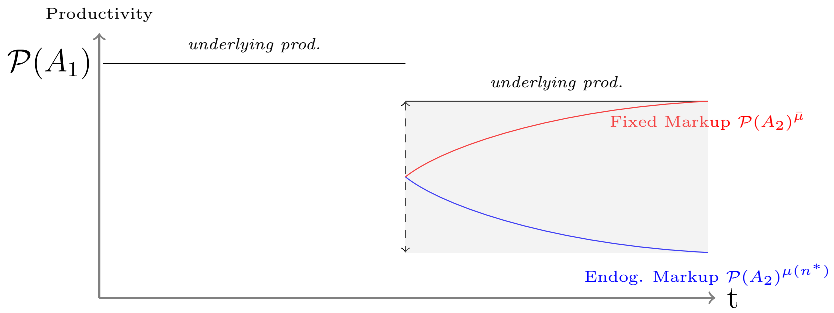

Figure 6: Short-run Productivity Undershooting

6.1 Competition Effect Strengthens Misallocation Effect

A special case that may arise is if the negative competition effect is larger than the positive allocation effect. The previous discussion assumed that $\mathcal{P}(0)_A - \mathcal{P}_A^* > 0$. However, if the competition effect is large then $\mathcal{P}(0)_A - \mathcal{P}_A^* < 0$, so the initial movement in measured productivity is less than the long-run change in productivity. In terms of a positive shock to TFP this would mean an increase in measured productivity on impact as incumbent firms benefit from the improved technology, but then as firms begin to enter their negative effect of reallocating business is less than their positive effect reducing markups, so as they enter productivity continues to improve. If there is a negative TFP shock as in figure 5 the result is an initiall fall in productivity, followed by further worsening of productivity to a long-run level below the initial movement. After the initial fall in productivity the further worsening occurs as firms exit and weaker competition reduces productivity more than the reallocation of resources among incumbents.

Although I suggest this result is unlikely to arise Jaimovich and Floetotto 2008 have an appendix exercise where this arises. They find that TFP measure bias is weaker with dynamic entry versus static entry, but they do not explain that this is because their competition effect is much bigger than their allocation effect. In other words on impact of a positive shock the incumbents who benefit from better technology and improved returns to scale may improve productivity even further as they face more competition whereas I have assumed as each entrant comes in it strengthen competition positive affecting productivity but also divides resources more which negatively affects competition. I assumed the latter effect more important! Clear JF assume the greater competition is more important which explains why on impact the overshooting is less in their case than with a static entry setup where firms jump immediately.

6.2 Supplementary Numerical Exercise

The theory of the previous section demonstrates the main result of the paper, but a numerical exercise is useful to gauge the two effects to gain an intuition for whether undershooting or overshooting in productivity arises. The baseline RBC model assumes isoelastic (constant elasticity) separable subutilties and a Cobb-Douglas production function.

6.2.1 Utility

$$U(C,L) = \frac{C^{1-\sigma}-1}{1-\sigma} - \xi\frac{L^{1+\eta}}{1+\eta} \tag{66}$$

The derivatives are

$$U_C = C^{-\sigma}, \quad U_{CC} = -\sigma C^{-\sigma-1}, \quad U_L = -\xi L^{\eta} \tag{67}$$

The degree of relative risk aversion is constant $\sigma(C) = -C\frac{U_{CC}}{U_C} = \sigma$. Isoelastic utility implies there is constant elasticity of utility with respect to each good. $\sigma \neq 1$ is the constant coefficient of relative risk aversion. $\sigma \to \infty$ implies infinite risk aversion, so consumption has little effect on utility. η is Frisch elasticity of labour supply.

6.2.2 Production

$$F(k,l) = k^{\alpha}l^{\beta} = K^{\alpha}L^{\beta}n^{-(\alpha+\beta)} = F(K,L)n^{-(\alpha+\beta)} \tag{68}$$

Cobb-Douglas production conforms to our assumptions on the production function derivatives,

$$F_k = \alpha k^{\alpha-1}l^{\beta} = \alpha K^{\alpha-1}L^{\beta}n^{1-(\alpha+\beta)}, \tag{69}$$

$$F_l = k^{\alpha}\beta l^{\beta-1} = K^{\alpha}\beta L^{\beta-1}n^{1-(\alpha+\beta)} \tag{70}$$

and it is homogeneous of degree $\alpha+\beta$, so $\nu = \alpha+\beta$ in our general notation. α and β are capital and labour shares respectively. This implies increasing marginal costs if $\alpha+\beta < 1$. Thus the impact versus long-run effect of a change in technology becomes[24]

$$\mathcal{P}(0)_A|_{x(0)=x^*} - \mathcal{P}^*_A = \frac{\mathcal{P}^*}{\nu(n^*-1)}\left[\frac{2}{A} + \beta\frac{n^*_A}{n^*} - \frac{\nu}{\mu(n^*)\left(\mu(n^*)-\nu\right)}\frac{n^*_A}{n^{*2}}\right] \tag{71}$$

If we assume that intersector substitutability is $\theta_I = 1$ then the markup purely depends on number of firms $\mu(n) = \frac{n}{n-1}$

$$\mathcal{P}(0)_A|_{x(0)=x^*} - \mathcal{P}^*_A = \frac{\mathcal{P}^*}{(\alpha+\beta)(n^*-1)}\left[\frac{2}{A} + \left(\beta - \frac{(\alpha+\beta)}{\frac{n^{*2}}{n^*-1}\left(\frac{n^*}{n^*-1}-(\alpha+\beta)\right)}\right)\frac{n^*_A}{n^*}\right] \tag{72}$$

Given this parameterization a sufficient condition for undershooting $\mathcal{P}(0)_A - \mathcal{P}^*_A < 0$ is

$$\frac{2}{A} + \left(\beta - \frac{(\alpha+\beta)}{\frac{n^{*2}}{n^*-1}\left(\frac{n^*}{n^*-1}-(\alpha+\beta)\right)}\right)\frac{n^*_A}{n^*} < 0 \tag{73}$$

therefore a necessary condition is

$$\beta < \frac{(\alpha+\beta)}{\frac{n^{*2}}{n^*-1}\left(\frac{n^*}{n^*-1}-(\alpha+\beta)\right)} \tag{74}$$

where the right-hand side is the competition effect that arises from endogenous markups[25]. The competition effect is zero with exogenous (fixed) markups ($n^* \to 1$), therefore the necessary condition is always violated $\beta \not< 0$ and undershooting cannot arise. This formalizes the logic that if there is no persistent effect on productivity then undershooting of the long-run level cannot arise. Similarly, any effect that weakens the long-run persistent effect of competition (decreases $\mathcal{P}^*_\mu$) will reduce the likelihood of undershooting. For example, the competition effect is increasing in returns to scale $\nu := \alpha+\beta \to 1$, leading to undershooting.

Table 1 summarizes the parameter values used for simulation exercises. They are benchmark for this simple model, and are replicated from Brito and Dixon 2013. For comparison, Jaimovich 2007 and Jaimovich and Floetotto

[24] Details in appendix F.

[25] I label it the competition effect since it determines the magnitude of the negative component.

Market Power	ζ	$(0,1)$
Capital Share	α	0.3
Labour Share	β	0.5
Fixed Cost	ϕ	0.3
Entry Cost	γ	3.0
Technology	A	1.0
Risk Aversion	σ	1.0
Discount Rate	ρ	0.025
Labour Weight	ξ	0.01
Labour Elast. (Frisch)	η	0.5

Table 1: Parameter Values for Numerical Exercises

2008 calibrate a similar model and parameters taking a formal calibration approach. In Jaimovich 2007 the markup (price over marginal cost definition) is switched between $\mu = 1.05$ and $\mu = 1.10$, so in terms of Lerner Index ($\zeta = 1 - \frac{1}{\mu}$) then $\zeta = 0.047$ and $\zeta = 0.09$. In Jaimovich and Floetotto 2008 it is $\mu = 1.3$ so $\zeta = 0.231$. Jaimovich and Floetotto 2008 calibrate the fixed cost ϕ to be a percentage of sales $\frac{n\phi}{Y} = \frac{\phi}{y} = 0.127$, and use slightly larger proportion of 15% in the appendix. In this paper we assume no intersector substitutability $\theta_I = 1$ (different products across industries i.e. no substitution across industry) and infinite intrasector substitutability $\theta_F = \infty$ (homogeneous goods within industry i.e. perfect substitution within industry). Whereas Jaimovich and Floetotto 2008 emphasize keeping these parameters free, but it gives similar outcomes to our assumptions $\theta_I = 1.001$ and $\theta_F = 19.6$[26]. They assume indivisible labour $\eta = 0$.

In general equilibrium using previous parameter values (table 1), numerical simulation[27] makes the right-hand side of the necessary condition 0.07 where number of firms is determined endogenously in steady state as $n^* = 44.156$. Clearly the right-hand side value does not exceed the labour share $\beta = 0.5 \nless 0.07$, thus undershooting does not arise in general equilibrium. The conclusion is that the competition effect is small relative to the allocation effect, an important result given most literature focuses on endogenous markups (competition effect) rather than business allocation. So let us ask under what partial equilibrium conditions could undershooting arise, and

[26]Specifically they use ω and τ which since their aggregators are written as p-norms and Holder conjugates, rather than elasticities as is common in economics $\theta_I = \frac{1}{1-\omega} = \frac{1}{1-0.001} = 1.001$ and $\theta_F = \frac{1}{1-\tau} = \frac{1}{1-0.949} = 19.6$.

[27]This is a purely numerical calculation because the added nonlinearity of endogenous markups precludes an analytic derivation of n^*. The value of firms is high. The purpose here is intuition to aid the analytics.

interpret the plausibility of the economic narrative.

Figure 7 shows a calibration that strengthens the necessary condition by choosing close to CRTS with a low β. The negative region is quantitiatively small, so is unlikely to offset the initial effect required to meet the sufficient condition, unless n_A^* is unrealistically large. For example, we can see there is a region $n^* \in (3, 15)$ when the necessary condition is met, thus take $n^* = 5$ and $\nu = \alpha + \beta = 0.6 + 0.2 = 0.8$ meets the necessary condition for negativity, so the sufficient condition, where assuming $A = 1$, is $2 - 2\frac{n_A^*}{5} < 0$. Therefore the condition is $n_A^* > 5 = n^*$, so there must be a 100% change in market size if the competition effect from entry is to exceed the initial misallocation effect.

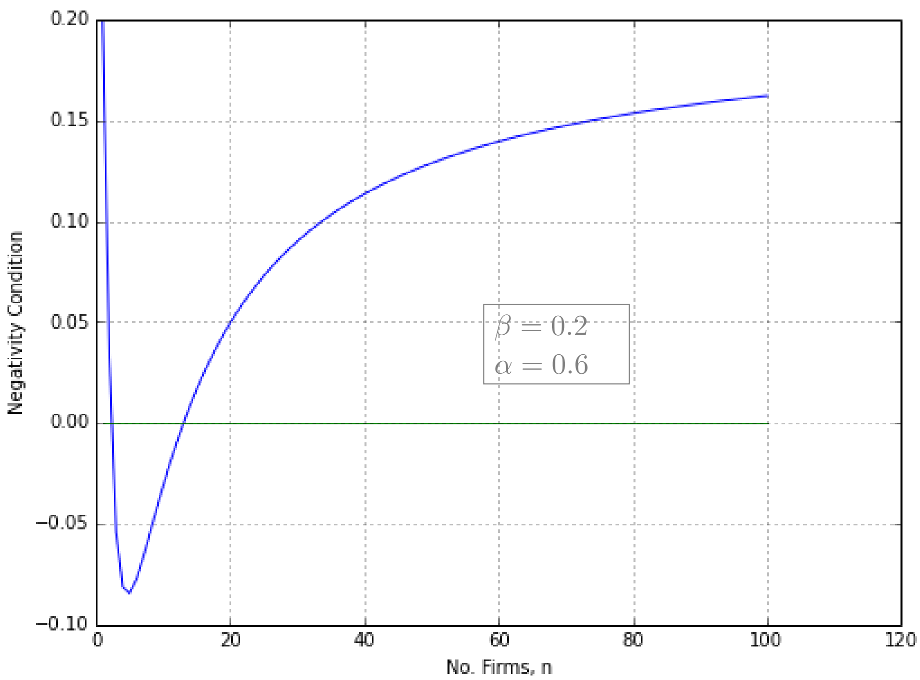

Figure 7: Initial Overshooting Versus Undershooting

Productivity overshooting can be seen in several scenarios. The baseline scenario in figure 8 shows the effect after a fall in the scale technology parameter $A = 1$ to $A = 0.8$ with the calibration in table 1. It behaves as we expect from the analytical results, with initial amplification followed by greater persistence in the endogenous markup case.

Figure 9 shows the near perfect competition outcome when there is a large number of firms in the market $n \to \infty$ due to a low fixed cost $\phi \to 0$

Conversely figure 10 a very high fixed cost $\phi \to \infty$ limits entry $n \to 1$

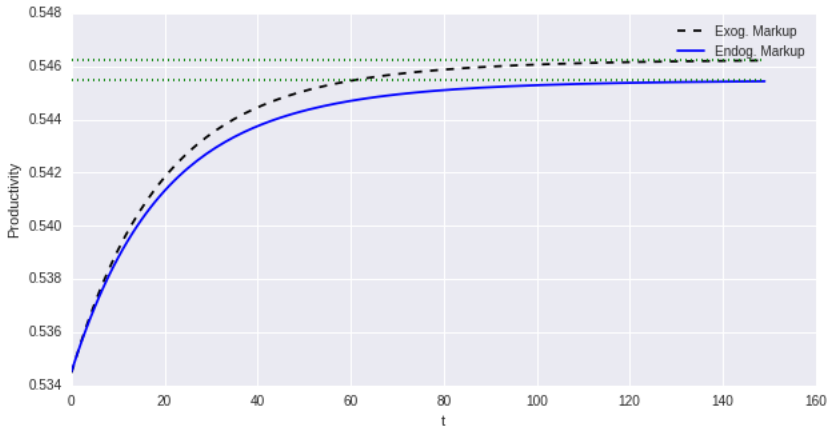

Figure 8: Technology Fall Amplified and Persistent Relative to Exogenous Markup

which is the Dixit-Stiglitz case of one firm industries. The effect of this very strong imperfect competition is to strengthen the amplifcation and propagation relative to the shift in steady state.

We can also see how markups behave. In 12 $\phi = 2.5$ and the markups effect is large due to a small number of firms. Whereas in 11 the markup is small.

7 Summary

The paper investigates the effect of firm entry on measured productivity over the business cycle. I consider that entry is noninstantaneous and entry affects the price markups that incumbents charge. Together these mechanisms can explain short-run procylical productivity and weaker long-run persistence. Contemporary productivity puzzles provide a lens to view the theory through. In relation to productivity puzzles, the theory explains that productivity is exacerbated on impact, since firms cannot adjust immediately so incumbents bear shocks, and in the long run underlying productivity is not regained because subsequent adjustment of firms causes structural changes in competition. The structural changes in competition reflect that entry strengthens competition which improves productivity in the long run (inversely, exit weakens competition, decreases productivity). Furthermore I show that in highly competitive industries the distinction between short-run and long-run productivity is small, so measured productivity quickly and accurately reflects underlying productivity.

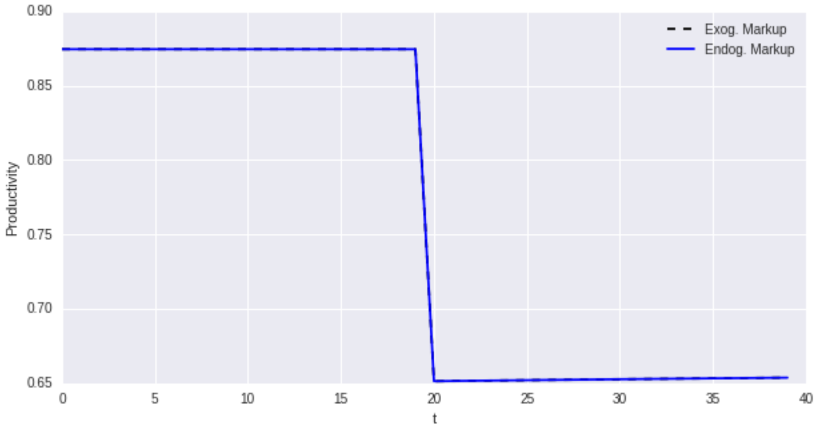

Figure 9: Perfect Competition Dampens Amplification and Propagation

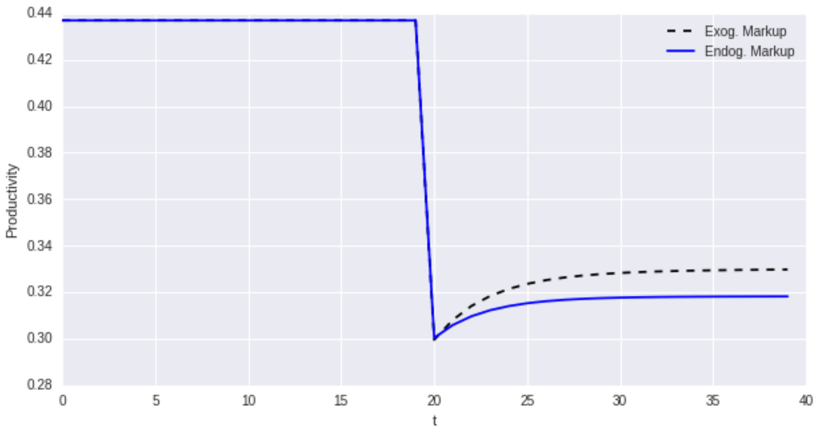

Figure 10: Imperfect Competition Strenthens Amplification and Propagation

A growing number of DSGE papers show promising quantitative simulation results from adding a firm entry process. Despite these appealing data matching properties, little research has reduced models to minimal state variables to understand the analytical effect of entry. This paper allows economists to understand how the entry variables interact with the model in a general setup before specifying functional forms or numerical calibrations. We learn that firm entry dynamics can explain short-run dynamic changes in productivity over the business cycle and long-run static changes that persist. The two explanations arise from two different effects of entry, a dynamic reallocation effect that redistributes resources as firms adjust and a static competition effect that alters firms' pricing markup decisions in response to competition from entry. A simple quantitative exercise emphasizes

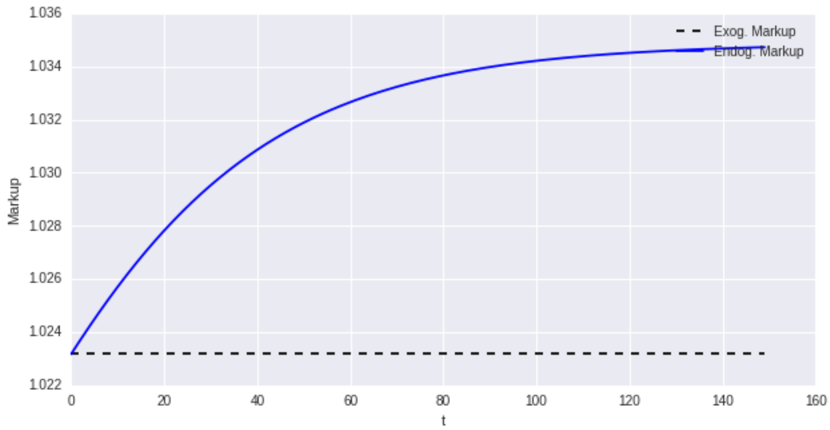

Figure 11: Markup Behavior in Competitive Economy

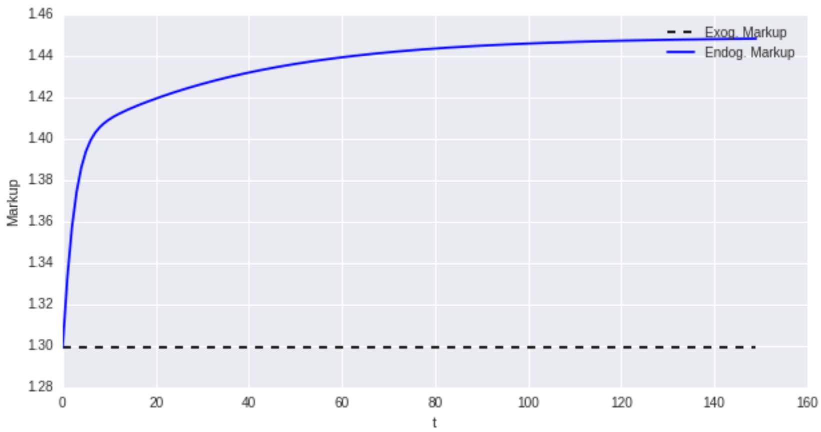

Figure 12: Markup Behavior in Uncompetitive Economy

the dominance of the allocation effect over the competition effect, which is an important lesson for researchers who have tended to focus on firm dynamics' effects on markups rather than allocation.

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A Household Optimization Problem

Use the Maxmimum Principle to obtain the necessary conditions for a solution to the household’s utility maximisation problem. The current value Hamiltonian is

$$\hat{\mathcal{H}}(t) = u(C(t), L(t)) + \lambda(t)(w(t)L(t) + r(t)K(t) + \Pi(t) - C(t)) \tag{75}$$

The costate variable λ_t is the shadow price of wealth in utility units. The Pontryagin necessary conditions are

$$\hat{\mathcal{H}}_C(K, L, C, \lambda) = 0 \implies u_C - \lambda = 0 \tag{76}$$

$$\hat{\mathcal{H}}_L(K, L, C, \lambda) = 0 \implies u_L + \lambda w = 0 \tag{77}$$

$$\hat{\mathcal{H}}_K(K, L, C, \lambda) = \rho\lambda - \dot{\lambda} \implies \lambda r = \rho\lambda - \dot{\lambda} \implies \frac{\dot{\lambda}}{\lambda} = -(r - \rho) \tag{78}$$

$$\hat{\mathcal{H}}_\lambda := \dot{K}_t \implies \dot{K} = rK + wL + \Pi - C \tag{79}$$

The four Pontryagin conditions (76)-(78) reduce to two equations: a differential equation in consumption (*consumption Euler equation* or *intertemporal condition*), and a static injective mapping between labour and consumption (*intratemporal condition*).

B Optimal Labour Derivatives

Partially differentiate the intratemporal Euler with respect to each variable treating labour as an implicit function, and with wage set at the imperfect competition market rate $w(K,L,n) = \frac{An^{1-\nu}F_L(K,L)}{\mu(n)}$.

$$u_L(L) + u_C(C)w(K,L,n) = 0 \tag{80}$$

$$u_L(L) + u_C(C)\frac{An^{1-\nu}F_L(K,L)}{\mu(n)} = 0 \tag{81}$$

Recall the utility and production function assumptions:

$$F_{LL}(K,L),\ u_{CC}(C),\ u_{LL}(L) < 0$$
$$u_C(C),\ F_L(K,L),\ F_{LK}(K,L) = F_{KL}(K,L) > 0$$

These can be used to sign the behaviour of labour

$$u_{LL}L_C + u_C\frac{An^{1-\nu}F_{LL}L_C}{\mu(n)} + \frac{u_{CC}An^{1-\nu}F_L}{\mu(n)} = 0 \tag{82}$$

$$L_C = \frac{-u_{CC}An^{1-\nu}F_L\mu(n)^{-1}}{u_{LL} + u_C An^{1-\nu}F_{LL}\mu(n)^{-1}} < 0 \tag{83}$$

$$u_{LL}L_K + u_C\frac{An^{1-\nu}F_{LL}L_K}{\mu(n)} + \frac{u_C An^{1-\nu}F_{LK}}{\mu(n)} = 0 \tag{84}$$

$$L_K = \frac{-u_C An^{1-\nu}F_{LK}\mu(n)^{-1}}{u_{LL} + u_C An^{1-\nu}F_{LL}\mu(n)^{-1}} > 0 \tag{85}$$

$$u_{LL}L_n + \frac{u_C A(1-\nu)n^{-\nu}F_L}{\mu(n)} + \frac{u_C An^{1-\nu}F_{LL}L_n}{\mu(n)} + u_C An^{1-\nu}F_L\frac{-\mu(n)_n}{\mu(n)^2} \tag{86}$$

$$L_n = \frac{u_C An^{1-\nu}F_L\mu_n\mu^{-2} - u_C A(1-\nu)n^{-\nu}F_L\mu^{-1}}{u_{LL} + u_C An^{1-\nu}F_{LL}\mu(n)^{-1}} \tag{87}$$

Therefore if we suppress notation and simplify (e.g. $n^{1-\nu}F_{LL}(K,L) = n^{-1}n^{2-\nu}F_{LL}(K,L) = n^{-1}F_{ll}(\frac{K}{n},\frac{L}{n})$ by Euler's homogeneous function theorem) we get

$$L_n = \frac{Au_C F_l(\mu^{-1}\mu_n - (1-\nu)n^{-1})}{\mu u_{LL} + U_C An^{-1}F_{ll}} > 0, \quad \nu \in (0,1) \tag{88}$$

In all cases the denominator $u_{LL}+u_C An^{1-\nu}F_{LL}\mu(n)^{-1} < 0$ is the intratemporal condition differentiated with respect to labour, and it is negative. Therefore the numerator distinguishes signs. Concavity of the production and utility functions, assumptions above, are sufficient to determine the signs of the numerator except for L_n which depends on returns to scale of the technology ν. With decreasing returns $\nu < 1$ labour increases; with increasing returns labour decreases and with constant returns $\nu = 1$ labour would be irresponsive to entry if there were fixed markups $\mu_n = 0$, but the endogenous markup $\mu_n < 0$ means labour increases with entry even with constant returns. This is because although the marginal product of labour does not change because of constant returns, the fall in markups reduces the wedge between marginal product of labour and wage, so wage increases.

The economic intuition is easier to understand in terms of wages, where $w^{\bar{L}}$ is wage with labour fixed.

$$L_C = \frac{-u_{CC}w}{u_{LL}+u_C w_L} < 0, \quad L_K = \frac{-u_C w_K^{\bar{L}}}{u_{LL}+u_C w_L} > 0, \tag{89}$$

$$L_n = \frac{w\frac{\mu_n}{\mu} - u_C w_n^{\bar{L}}}{u_{LL}+u_C w_L} > 0 \tag{90}$$

C Optimal interest rate, profit, output

Given optimal labour choice $L(C,K,n)$ we can evaluate how interest rate, wage, profit and output respond. The markup μ is a function of number of firms $\mu(n)$, but I suppress the domain for clarity.

C.1 Output

$$Y(L(C,K,n),K,n) = n^{1-\nu}[AF(K,L(C,K,n)) - \phi] \tag{91}$$

$$Y_C = An^{1-\nu}F_L(K,L)L_C(C,K,n) < 0 \tag{92}$$

$$Y_K = An^{1-\nu}[F_K(K,L) + F_L(K,L)L_K(C,K,n)] > 0 \tag{93}$$

$$Y_n = (1-\nu)AF - \phi + AF_l L_n \lesseqgtr 0 \tag{94}$$

Furthermore in steady state when $F(\frac{K}{n},\frac{L}{n})^* = \frac{\phi}{A(1-\frac{\nu}{\mu})}$ then $Y_n|_{x*} = \frac{-\phi\mu\nu}{1-\frac{\nu}{\mu}} + AF_l L_n$ which is positive or negative depending whether the negative component outweighs the positive labour effect.

C.2 Wage

$$w = \frac{1}{\mu} A F_l \tag{95}$$

$$w_C = \frac{1}{\mu}\frac{A}{n} F_{ll} L_C > 0 \tag{96}$$

$$w_K = \frac{1}{\mu}\frac{A}{n}[F_{lk} + F_{ll} L_K] = \frac{1}{\mu}\frac{A}{n}\left[\frac{\mu F_{lk} u_{LL}}{\mu u_{LL} + u_C A F_{ll}}\right] > 0 \tag{97}$$

$$w_n = \frac{1}{\mu}\frac{A}{n}[(1-\nu)F_l + F_{ll} L_n] - \frac{1}{\mu^2}\mu_n A F_l \gtreqless 0 \tag{98}$$

C.3 Rents

$$r = \frac{1}{\mu} A F_k \tag{99}$$

$$r_C = \frac{1}{\mu}\frac{A}{n} F_{kl} L_C < 0 \tag{100}$$

$$r_K = \frac{1}{\mu}\frac{A}{n}[F_{kk} + F_{kl} L_K] = \frac{1}{\mu}\frac{A}{n}\left[\frac{\mu F_{kk} u_{LL} + A u_C (F_{kk} F_{ll} - F_{kl}^2)}{\mu u_{LL} + u_C A F_{ll}}\right] < 0 \tag{101}$$

$$r_n = \frac{1}{\mu}\frac{A}{n}[(1-\nu)F_k + F_{kl} L_n] - \frac{1}{\mu^2}\mu_n A F_k > 0 \tag{102}$$

Both r_K and w_K require extra work to derive the signs. They are found by substituting in L_K. Then r_K can can be rearranged into a form including $F_{KK}F_{LL} - F_{KL}^2$ which is positive by the second partial derivative test for concavity assumption.

C.4 Profit

$$\pi = AF(k,l)(1 - \frac{\nu}{\mu}) - \phi \tag{103}$$

$$\pi_C = AF_l \frac{L_C}{n}(1 - \frac{\nu}{\mu}) < 0 \tag{104}$$

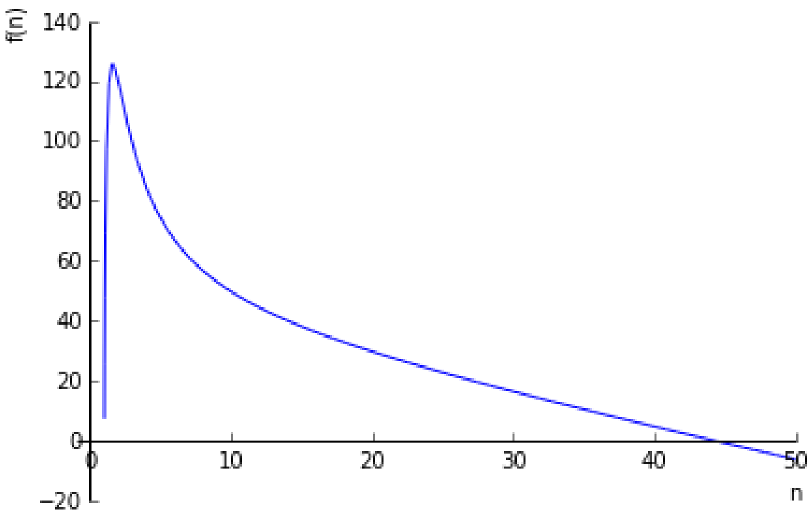
$$\pi_K = \frac{A}{n}(F_k + F_l L_K)(1 - \frac{\nu}{\mu}) > 0 \tag{105}$$

$$\pi_n = \frac{A}{n}(-\nu F + F_l L_n)(1 - \frac{\nu}{\mu}) + AF(k,l)\frac{\nu}{\mu^2}\mu_n < 0 \tag{106}$$

For any K, L, n profit is higher when imperfect competition μ increases, but not necessarily higher for any given K, n. This explains that even if imperfect competition increases and therefore higher profits are available, the number of firms can (counterintuitively) decrease. The offsetting factor is the the indirect labour effect: labour supply is discouraged by the rise in imperfect competition and as there is a bigger wedge between wage and marginal product of labour. Therefore labour supply falls such that profits are lower for a given K, n. I typically assume these secondary labour effects to be too small to offset the primary mechanisms. So operating profit increases with imperfect competition, even as L is allowed to adjust.

D Steady State Results

D.1 Existence with Functional Forms Numerically



(a) Steady State Numerical Solution

Since we have shown an analytical condition for existence, we can move on from existence to ask what that solution is for a set of numerical parameter values? Solving the highly nonlinear number of firms in steady-state equation yields $n^* = 44.156$. The function for number of firms in steady state

$$n^* = \left[\frac{\beta}{\xi \nu^\sigma} \left\{ \left(A \left(\frac{\alpha}{\rho} \right)^\alpha \right)^{1+\eta} \left(\frac{1}{\mu(n^*)} \right)^{\alpha(1+\eta)+\beta(1-\sigma)} \left(\frac{1 - \frac{\nu}{\mu(n^*)}}{\phi} \right)^{1-\nu+\eta(1-\alpha)+\sigma\beta} \right\}^{\frac{1}{\beta}} \right]^{\frac{1}{\eta+\sigma}} \tag{107}$$

is highly nonlinear as shown in figure 13a, where it intersects the x-axis at the solution $n^* = 44.156$. The markup is $\mu = \frac{n}{n-1}$ hence the graph is undefined in the $n = 1$ region. The other solutions are $n^* = 44.156,\ K^* = 713.685,\ C^* = 47.486$.

D.2 Procyclical Firms

From (59) we have

$$Y^* = \frac{n^* \phi \nu}{\mu(n^*) - \nu} \tag{108}$$

Then by the product rule and the implicit function theorem

$$Y^*_n = \frac{(\mu(n^*) - \nu)\phi\nu - n^*\phi\nu\mu_n}{(\mu(n^*) - \nu)^2} > 0 \tag{109}$$

The quadratic denominator is positive. The first component of the numerator is positive because $\mu(n^*) > 1$ and decreasing returns $\nu \in (0, 1)$ so $\mu(n) - \nu > 0$ and the second component is positive due to the double negative which occurs from endogenous markups decreasing in number of firms $\mu_n < 0$.

E Productivity Dynamics

Throughout the derivations remember that the markup is a function of number of firms $\mu(n)$, but for simplicity I write μ.

$$\mathcal{P}(t)_A = \frac{n^{1-\nu} F + y n_A + (n^{1-\nu} A - n y \frac{1}{\nu} F^{-1})(F_K K_A + F_L L_A)}{F^{\frac{1}{\nu}}} \tag{110}$$

The crucial step with dynamic firms and capital is that state variables do not move on impact $K_A = 0$ and $n_A = 0$. This is what causes the distinction between short-run and long-run productivity that is not present with instantaneous free entry. Therefore at $t = 0$ the change in productivity depends on the direct effect of better technology, and its indirect effect on labour, which increases labour supply.

$$\mathcal{P}(0)_A = \frac{n^{1-\nu}F + (n^{1-\nu}A - ny\frac{1}{\nu}F^{-1})F_L L_A}{F^{\frac{1}{\nu}}} \tag{111}$$

$$\mathcal{P}(0)_A = n^{1-\nu}F^{1-\frac{1}{\nu}} + \left(F^{-\frac{1}{\nu}}n^{1-\nu}A - \mathcal{P}\frac{1}{\nu}F^{-1}\right)F_L L_A \tag{112}$$

Assuming that the economy is initially in steady state when the shock occurs, evaluate the expression with all variables x at steady state $x(0) = x^*$. From $\pi = y - rK - wL$ then $y^* = rK^* + wL^*$ so $y^* = \frac{A\nu n^{*-\nu}F}{\mu^*}$ and thus $\mathcal{P}^* = \frac{n^* y^*}{F^{*\frac{1}{\nu}}} = \frac{A\nu n^{*1-\nu}F^{1-\frac{1}{\nu}}}{\mu}$. This expression for productivity makes it easier to represent the impact effect of a TFP shock in terms of steady state productivity $\mathcal{P}^*$ as follows

$$\mathcal{P}(0)_A|_{x(0)=x^*} = \frac{\mu^*\mathcal{P}^*}{A\nu} + \left(\frac{\mu^*\mathcal{P}^*}{\nu F^*} - \frac{\mathcal{P}^*}{\nu F^*}\right)F^*_L L^*_A \tag{113}$$

$$\mathcal{P}(0)_A|_{x(0)=x^*} = \frac{\mu^*\mathcal{P}^*}{A\nu} + (\mu^* - 1)\frac{\mathcal{P}^*}{\nu F^*}F^*_L L^*_A \tag{114}$$

Comparing the short-run impact effect to the long-run steady state effect $\mathcal{P}^*_A = \mathcal{P}^{*\bar{\mu}}_A + \mathcal{P}^*_\mu \mu_A = \frac{\mathcal{P}^*}{A\nu} + \mathcal{P}^*_\mu \mu_n n^*_A$ shows that the endogenous productivity effect dampens the difference between short-run and long-run effects

$$\begin{aligned}\mathcal{P}(0)_A - \mathcal{P}^*_A|_{x(0)=x^*} &= \frac{\mu^*\mathcal{P}^*}{A\nu} + (\mu^* - 1)\frac{\mathcal{P}^*}{\nu F^*}F^*_L L^*_A \\ &\quad - (\mathcal{P}^{*\bar{\mu}}_A + \mathcal{P}^*_\mu \mu_A)\end{aligned} \tag{115}$$

$$\begin{aligned}\mathcal{P}(0)_A - \mathcal{P}^*_A|_{x(0)=x^*} &= \frac{\mu^*\mathcal{P}^*}{A\nu} + (\mu^* - 1)\frac{\mathcal{P}^*}{\nu F^*}F^*_L L^*_A \\ &\quad - \left(\frac{\mathcal{P}^*}{A\nu} + \mathcal{P}^*_\mu \mu_n n^*_A\right)\end{aligned} \tag{116}$$

$$\begin{aligned}\mathcal{P}(0)_A - \mathcal{P}^*_A|_{x(0)=x^*} &= (\mu^* - 1)\frac{\mathcal{P}^*}{A\nu} + (\mu^* - 1)\frac{\mathcal{P}^*}{\nu F^*}F^*_L L^*_A \\ &\quad - \mathcal{P}^*_\mu \mu_n n^*_A\end{aligned} \tag{117}$$

$$\mathcal{P}(0)_A - \mathcal{P}^*_A|_{x(0)=x^*} = (\mu^* - 1)\frac{\mathcal{P}^*}{\nu}\left(\frac{1}{A} + \frac{F^*_L L^*_A}{F^*}\right) - \mathcal{P}^*_\mu \mu_n n^*_A \tag{118}$$

The expression for $\mathcal{P}^*_\mu$ can simplyify the expression further

Lemma 4.

$$\mathcal{P}^*_\mu = -\frac{\mathcal{P}^*(\mu - 1)}{\mu(\mu - \nu)} < 0 \tag{119}$$

Proof.

$$\mathcal{P}^*_\mu = \left[\frac{A}{\mu}\left(\frac{\mu-\nu}{\phi}\right)^{1-\nu}\right]^{\frac{1}{\nu}-1} \times$$
$$\left(\frac{-A}{\mu^2}\left(\frac{\mu-\nu}{\phi}\right)^{1-\nu} + \frac{A}{\mu}(1-\nu)\left(\frac{\mu-\nu}{\phi}\right)^{-\nu}\frac{1}{\phi}\right) \tag{120}$$
$$= \left[\frac{A}{\mu}\left(\frac{\mu-\nu}{\phi}\right)^{1-\nu}\right]^{\frac{1}{\nu}}\left(-\frac{1}{\mu}+\frac{1-\nu}{\mu-\nu}\right) \tag{121}$$
$$= \left[\frac{A}{\mu}\left(\frac{\mu-\nu}{\phi}\right)^{1-\nu}\right]^{\frac{1}{\nu}}\left(\frac{\nu(1-\mu)}{\mu(\mu-\nu)}\right) = -\frac{\mathcal{P}^*(\mu-1)}{\mu(\mu-\nu)} < 0 \tag{122}$$

□

Notice that with constant returns the expression is simply

$$\mathcal{P}^*_\mu\big|_{\nu\to 1} = -\frac{A}{\mu^2}$$

and with many firms the markup tends to unity, therefore long-run underlying productivity reflects true TFP.

$$\mathcal{P}^*_\mu = -\frac{\mathcal{P}^*(\mu^*-1)}{\mu^*(\mu^*-\nu)} \tag{123}$$
$$\mathcal{P}(0)_A - \mathcal{P}^*_A|_{x(0)=x^*} =$$
$$(\mu^*-1)\frac{\mathcal{P}^*}{\nu}\left(\frac{1}{A}+\frac{F^*_L L^*_A}{F^*}\right) + \frac{\mathcal{P}^*(\mu^*-1)}{\mu^*(\mu^*-\nu)}\mu_n n^*_A \tag{124}$$
$$\mathcal{P}(0)_A - \mathcal{P}^*_A|_{x(0)=x^*} =$$
$$(\mu^*-1)\mathcal{P}^*\left[\frac{1}{\nu}\left(\frac{1}{A}+\frac{F^*_L L^*_A}{F^*}\right) + \frac{1}{\mu^*(\mu^*-\nu)}\mu_n n^*_A\right] \tag{125}$$

F Productivity Dynamics with Functional Forms

First let us restate the short-run versus long-run productivity effect, and focus attention on the square bracketed component Γ that represents allocation versus competition effect.

$$\mathcal{P}(0)_A\big|_{x(0)=x^*} - \mathcal{P}_A^* =$$
$$(\mu^* - 1)\frac{\mathcal{P}^*}{\nu}\left[\left(\frac{1}{A} + \frac{F_L^* L_A^*}{F^*}\right) + \frac{\nu}{\mu^*(\mu^* - \nu)}\mu_n n_A^*\right] \tag{126}$$
$$\text{Define } \Gamma = \frac{1}{A} + \frac{F_L^* L_A^*}{F^*} + \frac{\nu}{\mu^*(\mu^* - \nu)}\mu_n n_A^* \tag{127}$$

From the state-state labour per firm l^* we have

$$L^* = n^* \left[\frac{1}{A}\left(\frac{\rho}{\alpha}\right)^{\alpha}\left(\frac{1-\frac{\nu}{\mu}}{\frac{\phi}{\mu}}\right)^{1-\alpha}\right]^{\frac{1}{\beta}} \text{ thus } L_A^* = L^*\left(\frac{n_A^*}{n^*} + \frac{1}{\beta A}\right) \tag{128}$$

We can also substitute out the following simplifications

$$\frac{F_L^*}{F^*} = \frac{\beta K^{\alpha} L^{\beta-1} n^{-(\alpha+\beta)}}{K^{\alpha} L^{\beta} n^{-(\alpha+\beta)}} = \beta L^{*-1} \tag{129}$$
$$\mu = \frac{n}{n-1} \text{ therefore } \mu_n = -\frac{1}{n^2} \tag{130}$$

Therefore the allocation versus competition effect component becomes

$$\Gamma = \frac{2}{A} + \left(\beta - \frac{\nu}{\left(\frac{n^*}{n^*-1}\right)^2 (n^* - (n^* - 1)\nu)}\right)\frac{n_A^*}{n^*} \tag{131}$$

And short-run versus long-run productivity dynamics simplify to

$$\mathcal{P}(0)_A\big|_{x(0)=x^*} - \mathcal{P}_A^* =$$
$$\left(\frac{1}{n^* - 1}\right)\frac{\mathcal{P}^*}{\nu}\left[\frac{2}{A} + \left(\beta - \frac{\nu}{\left(\frac{n^*}{n^*-1}\right)^2 (n^* - (n^* - 1)\nu)}\right)\frac{n_A^*}{n^*}\right] \tag{132}$$
$$\mathcal{P}^* = \nu\left[\frac{A(n^* - 1)^{\nu}}{n^*}\left(\frac{n^*(1-\nu) + \nu}{\phi}\right)^{1-\nu}\right]^{\frac{1}{\nu}} \tag{133}$$

F.1 Cost function

Static optimization problem so drop time subscripts

$$C(r, w, y) = \min_{l,k} wl + rk + \phi \quad \text{s.t.} y \leqslant Ak^{\alpha} l^{\beta} - \phi \tag{134}$$

With Cobb-Douglas production the total cost function from substituting Lagrangean obtained conditional input demands $k(r,w,y) = \left[\left(\frac{w\alpha}{r\beta}\right)^{\beta}\left(\frac{y+\phi}{A}\right)\right]^{\frac{1}{\alpha+\beta}}$ and $l(r,w,y) = \left[\left(\frac{r\beta}{w\alpha}\right)^{\alpha}\left(\frac{y+\phi}{A}\right)\right]^{\frac{1}{\alpha+\beta}}$ into the cost function is

$$C(r,w,y) = (\alpha+\beta)\left(\frac{y+\phi}{A}\right)^{\frac{1}{\alpha+\beta}}\left(\frac{r}{\alpha}\right)^{\frac{\alpha}{\alpha+\beta}}\left(\frac{w}{\beta}\right)^{\frac{\beta}{\alpha+\beta}} + \phi \tag{135}$$

Where the firm takes factor prices as given. The average cost $AC := \frac{C}{y}$ is U-shaped and the marginal cost $MC := \frac{dC}{dy}$ is increasing in output with $\alpha + \beta < 1$.

$$MC = \frac{\partial C(r,w,y)}{\partial y} = \frac{(y+\phi)^{\frac{1}{\alpha+\beta}-1}}{A^{\frac{1}{\alpha+\beta}}}\left(\frac{r}{\alpha}\right)^{\frac{\alpha}{\alpha+\beta}}\left(\frac{w}{\beta}\right)^{\frac{\beta}{\alpha+\beta}} \tag{136}$$

$$\frac{\partial MC}{y} = \left(\frac{1}{\alpha+\beta} - 1\right)\frac{(y+\phi)^{\frac{1}{\alpha+\beta}-2}}{A^{\frac{1}{\alpha+\beta}}}\left(\frac{r}{\alpha}\right)^{\frac{\alpha}{\alpha+\beta}}\left(\frac{w}{\beta}\right)^{\frac{\beta}{\alpha+\beta}} \tag{137}$$

The leading multiplier $\frac{1}{\alpha+\beta} - 1$ determines how marginal cost responds to changing output. This shows that it is increasing when $\alpha+\beta < 1$ but is zero with constrant returns to scale $\alpha + \beta = 1$ which reflects a flat marginal cost curve.

G Markup Properties

If $\theta_I = 1$ industry goods are imperfectly substitutable, and the aggregate good is a Cobb-Douglas composite of industry goods. Thus the markup is a common asymptotic function[28].

Remark 1 (Endogenous markup)**.** *With many firms per industry the markup is 1*

Proof.

$$\lim_{n\to+\infty} \mu(n(t)) = \lim_{n\to+\infty} \frac{n(t)}{n(t)-1} \tag{138}$$

$$= \lim_{n\to+\infty}\left(\frac{1}{n(t)-1}\right) + 1 = 1, \quad n(t) \in (0,\infty] \tag{139}$$

□

[28] See Wolfram Alpha for eloquent properties.

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