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Appropriate Technology and Balanced Growth

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Abstract

We provide a general theoretical characterization of how firms' technology choice on a technology frontier determines the long-run elasticity of substitution between capital and labor. We show that the shape of the frontier determines factor shares and the elasticity of substitution between capital and labor. If there are adjustment costs to technology choice, the short- and long-run elasticities differ, with the long-run always higher. If the technology frontier is log-linear, the production function becomes Cobb-Douglas in the long run but, consistent with empirical evidence, short-run dynamics are characterized by gross complementarity. The approach is easily implementable and yields a powerful way to introduce CES-type production functions in macroeconomic models. We provide an illustration within an estimated dynamic general equilibrium model and show that the use of our production technology provides a good match for the short- and medium-run behavior of the US labor share.

JEL Classification: E25, O33, O40.

Keywords: Balanced growth, appropriate technology, elasticity of substitution.

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Non-technical summary

In macroeconomics, we typically model production by specifying a 'production function,' which tells us how much output is produced with given quantities of the 'factors of production,' often taken simply as capital and labour. Factor shares refer to the proportion of the income earned by production that goes to each factor, so the labour share is the proportion of this income that is earned by workers through supplying labour. There are various issues with how we measure factor shares, but a key aspect of what is known as 'balanced growth' is the idea that as income grows over long periods of time, the labour share remains approximately constant.

Some researchers dispute the idea of balanced growth, arguing for example that the labour share is currently declining. What there is no disagreement about is the fact that these factor shares are far more stable in the long run than they are in the short run. This creates a problem in the way we specify production functions. For example, the assumption of balanced growth has led Cobb-Douglas production functions to become standard in macroeconomic models because they imply constant factor shares with perfectly competitive markets. This, however, makes it more difficult to capture short- and medium-run fluctuations in factor shares. Market failures such as wage and price rigidities allow us to explain some of these fluctuations, but it is unlikely that they account for all of the fluctuations we see in factor shares, particularly in the medium run.

When the relative price of capital (to labour) rises, firms hire relatively less capital and more labour. The elasticity of substitution between capital and labour quantifies this effect; it tells us by how many percent the capital-labour ratio declines when the relative price of capital goes up by 1%. With Cobb-Douglas production functions, this elasticity is always one. However, much of the empirical evidence finds support for an elasticity below one. Indeed, production functions with an elasticity below 1 typically capture short-run fluctuations in factor shares significantly better than Cobb-Douglas. However, they have very important long-run consequences for income distribution. If the elasticity is different to one, productivity changes can cause the labour share to change. Since we have observed permanent changes in the productivity of investment goods in the last 30 years, an elasticity below one would lead to unbalanced growth with an increasing labour share, whereas typically researchers think that it is either constant or declining.

In this paper we propose a solution to this problem, using the idea of "appropriate technology." This is the idea that firms not only choose the quantities of capital and labour to employ, but also make a technology choice – how labour- or capital-intensive they want production methods to be. This trade-off is expressed graphically by a technology frontier: technologies that are more efficient in using labour are less efficient in using capital and vice-versa. Given a change in factor prices, firms change their position on the frontier. We show how the shape of the frontier determines the long-run elasticity of substitution and long-run factor shares. Importantly, if firms face adjustment costs when changing their choice of technology, the short-run elasticity will be lower than the long-run elasticity. This provides a way of modelling production that is very easy to implement in macroeconomic models but that is flexible enough to be compatible with both short- and long-run data. The short-run elasticity can be calibrated to capture short-run fluctuations in factor shares in line with the evidence, while the shape of the frontier captures the properties of long-run growth. There is a specific shape of frontier that implies balanced growth. Here elasticity of substitution is below one in the short-run but adjusts towards one in the long run. We use this to provide a quantitative example for the US economy. The results support the use of this new production function because it improves the model's ability to explain the business cycle and medium-run behaviour of the labour share.

1 Introduction

As well known among economists, Uzawa's (1961) Steady State Growth Theorem (henceforth *SSGT*) states that balanced growth[1] requires either all technical progress to be labor-augmenting or the elasticity of substitution between capital and labor to equal one in the long run (see Jones and Scrimgeour, 2008, for a useful proof). Evidence that factor shares are approximately constant in the long run, such as shown in figure 1, has led to balanced growth being a standard baseline description of long-run data and also a standard constraint for most solution methods in dynamic macro models. However, as discussed below, the assumption that technical progress is indeed *purely* labor augmenting is difficult to justify. On the other hand, Cobb-Douglas, which imposes a unitary capital-labor elasticity of substitution, sits at odds both with the substantial *cyclical* fluctuations observed in factor shares and the weight of evidence (reviewed in e.g. Chirinko, 2008, and León-Ledesma *et al.*, 2010) which supports a value of this elasticity significantly below unity at standard frequencies. Specifying an elasticity less than unity might be beneficial in modelling fluctuations, but doing so precludes long-run balanced growth unless all technical progress is labor augmenting. It is in this sense that the theorem constrains modelling practice.

We propose a method of relaxing this constraint by deriving a general production function in which the elasticity of substitution between capital and labor is lower in the short run than in the long run. The special case where the long-run elasticity $\sigma_{LR} = 1$ allows flexibility in modelling short-run dynamics (since the modeller can choose any short-run elasticity σ_{SR} such that $0 < \sigma_{SR} < 1$) while retaining general compatibility with balanced growth. A particular focus here is in providing a tractable method to achieve this flexibility. In comparison to alternative approaches, this method can be more easily applied in a wide range of macroeconomic models commonly used for policy making, where the nature of technical progress is not the primary research question.

The framework is based on considering technology choice among firms. The appropriate technology literature (see Atkinson and Stiglitz, 1969, Caselli and Coleman, 2006 and Jones, 2005) analyses this type of technology choice among firms and forms the basis for the current paper. Here we particularly build on Caselli and Coleman (2006; henceforth CC). Following an important idea in CC, firms face a standard constant elasticity of substitution production function where output Y is derived from a CES production function

$$Y = [A^{\rho} K^{\rho} + B^{\rho} L^{\rho}]^{1/\rho}, \tag{1}$$

where, in addition to choosing capital K and labor L, the firm also chooses its technology $A \geq 0$ and $B \geq 0$ subject to (A, B) lying within a given technology frontier.[2]

[1]Here, balanced growth refers to a long-run growth path consistent with Kaldor's facts. In particular, with constant factors shares, constant great ratios, as well as constant real interest rate.

[2]CC draw a world technology frontier. In CC, there are three inputs, capital, unskilled labor and

Since optimal technology choices will vary with factor prices, and technology choice influences the quantities of factors employed, technology choice also alters the elasticity of substitution (provided $\rho \neq 0$ as assumed throughout).

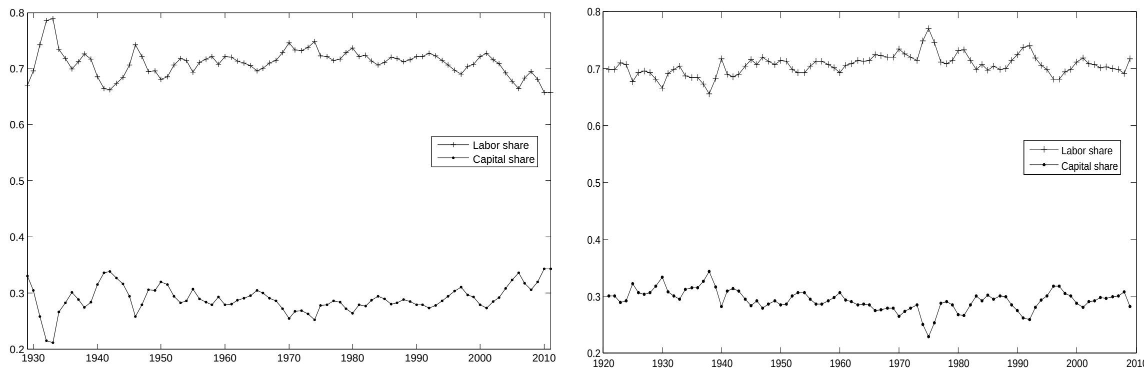
Importantly, here we assume that it is costly for firms to adjust their choice of technology. The expressions 'short run' and 'long run' in the paper refer respectively to the situations where (following a shock say) no adjustment has occurred and where adjustment is complete. In (1) the short-run elasticity σ_{SR} is therefore simply $\sigma_{SR} = \frac{1}{1-\rho}$. Given σ_{SR}, we characterise how the nature of technological choice (that is the shape of the frontier) determines the long-run elasticity σ_{LR}. The slope of the frontier in the space of the log efficiencies determines the capital share. A straightforward intuitive explanation of this is given in the paper: the capital share is directly related to the slope of the iso-quants in this space. Given an interior solution, we show that we always have $\sigma_{LR} > \sigma_{SR}$ and it is the *curvature* of the frontier, in conjunction with σ_{SR} , that determines σ_{LR}. The paper provides sufficient conditions for the existence of such an interior solution, the principal one being that the curvature of the frontier is not 'too pronounced.'

It follows from the above that a log-linear frontier implies a constant long-run capital share – effectively long-run Cobb-Douglas – and therefore balanced growth. We also provide functional forms for the shape of the frontier that result in a more general long-run CES production function. Thus one can write down a general production function which is CES in both the long- and the short-run limits but has different elasticities at these horizons, with $\sigma_{LR} > 1 > \sigma_{SR}$. Finally, using the key case with $\sigma_{LR} = 1$ for compatibility with balanced growth, we present an illustrative dynamic macro model that, despite its simplicity, captures well the empirical short- and medium-run behavior of the labor share.

Empirical Context. Suppose we take compatibility with long-run balanced growth as a model requirement.[3] An alternative approach to reconcile evidence of cyclicality

skilled labor and the firm chooses the efficiencies (constrained by a technology frontier) of skilled and unskilled labor (which empirical evidence suggests are gross substitutes) rather of capital and labor (which have short-run gross complementarity) as in our case. Since it is hard to argue that balanced growth applies to these two inputs, this issue (and more generally the time variation of the elasticity of substitution) is not of great relevance in CC.

[3]Some recent literature (see e.g. Piketty, 2013 and Karabarbounis and Nieman, 2014) has argued that current trends suggest that the capital share is increasing over time, and so growth is not balanced but best described by $\sigma_{LR} > 1$. This argument is far from settled, since observations on the capital share and the capital-output ratio are disputed due to measurement issues (see e.g. Elsby et al., 2013, Bonnet et al, 2014, and Bridgman, 2014). It is important to note that since short-run evidence favors $\sigma_{SR} < 1$, either of these two views of long-run growth suggest $\sigma_{LR} > \sigma_{SR}$. We can argue in fact that the Piketty view point actually strengthens the argument for modelling technological choice. With a conventional CES production function, the assumption that technical progress is purely labor-augmenting may (at least theoretically) reconcile evidence that $\sigma_{SR} < 1$ with evidence that long-run labor share is constant. A falling long-run labor share with constant real interest rates would however require some form of capital-augmenting regress. The approach outlined in this paper can reconcile either view.



(a) US 1929-2011 (b) UK 1921-2012

Figure 1: Historical capital and labor shares for the US and UK. Sources: US, Piketty and Saez (2003), UK, Mitchell (1988) updated by Bank of England.

in labor shares with the requirements of balanced growth is to introduce wage and/or price rigidity in a model with a Cobb-Douglas production function. This may generate short-run variations in factor shares if they produce cyclical fluctuations in firm markups. While such rigidities are likely to affect the labor share at business cycle frequencies, at medium-run frequencies we might expect wages and prices to adjust. Following the Comin and Gertler (2006) methodology, we do in fact find significant medium-run fluctuations in factor shares which rigidities are less likely to explain. As indicated by Beaudry (2005), technical change might play a role in explaining such medium-run phenomena. Furthermore, estimates of capital income built by directly calculating the real user cost as in Klump et al. (2007) display considerable business cycle fluctuations that cannot be attributed to changing markups. Bentolila and Saint-Paul (2003) also find evidence that changes in the labor share are significantly driven by technological shifts unrelated to labor market rigidities.

As discussed above, the primary alternative offered by the $SSGT$ is the assumption that technical progress is purely labor-augmenting in the long run. If $\sigma_{SR} = \sigma_{LR} \neq 1$, permanent labor-augmenting technology shocks will produce short-run fluctuations in factor shares, potentially allowing models to match the data while satisfying balanced growth.[4] However, it is difficult to make a clear theoretical case as to why any permanent technical progress should be purely labor augmenting.[5] Investment-specific

[4]Note also that the joint assumptions of a CES production function and purely labor-augmenting technical progress imply long-run cointegration of the log of the capital share and the log of the user cost of capital. See figure A1 in appendix B, where this issue is discussed at length.

[5]Theoretical reasons for why technical progress may be purely labor-augmenting are examined in the "induced innovation" strand of the literature, going back to Hicks (1932), Kennedy (1964), Samuelson (1965), Drandakis and Phelps (1966) and Kamien and Schwarz (1968) and including more recently Acemoglu (2002, 2003, 2007) and Zeira (1998) amongst others. An adequate survey is beyond the scope of this paper, but the question of whether the induced innovation literature as a

technical change (henceforth IST) has also taken an empirically important role in macroeconomics in the past two decades (see e.g. Greenwood et al. 1997 and 2000, and Fisher, 2006, who find IST to be one of the key drivers of macroeconomic fluctuations in the US economy). IST is clearly not temporary, and, moreover, relative price data for investment goods, a proxy for IST, are clearly trended. Since IST has similar implications for balanced growth as capital-augmenting technical progress, trends in IST will not result in balanced growth in conventional models with a CES production function. Nonetheless, our approach provides a justification for the use of CES production functions in modelling short-run dynamics, since any such model can potentially be made compatible with balanced growth by the introduction of technology choice.[6]

Related Theory. This paper is most closely related to that 'appropriate technology' literature that describes models of technological choice. Prominent examples are CC, and Jones (2005), in turn extended in various interesting ways by Growiec (2008 and 2013). In these approaches, the firm typically makes a technological choice by selecting e.g. a pair (A, B) that represents the efficiency of two inputs of production. The space of available technologies might take the form of a deterministic frontier in (A, B) space (CC), or represent an accumulated stock of arrived individual technologies (A_i, B_i) drawn stochastically from given distributions (Jones, 2005 and Growiec, 2008 and 2013).

In Jones (2005), firms choose the most appropriate of the technologies that have arrived, each of which is Leontief (or CES with a low elasticity of substitution). In the short run, while the firm remains on the current technology, the elasticity of substitution is zero (or low), but in the long run switching to new technologies will cause the elasticity of substitution to increase. The intuition for this is shown in figure 2, which is very similar to figure 1 of Jones (2005). The isoquants of the 'global' production function that incorporates endogenous technology choice are the convex hull of the isoquants of the individual technologies, and therefore the former should have less curvature than the latter. Jones (2005) shows that if A and B are drawn from independent Pareto distributions, the elasticity of substitution is unity in the long run. Like our present approach, it produces Cobb-Douglas at the firm level in the long run, rather than as a result of aggregation.[7] Growiec (2013), in turn,

whole produces the outcome of balanced growth without overly-restrictive assumptions on the nature of innovation is not clear (see Acemoglu, 2003, for a useful discussion). One purpose of this paper is also to allow researchers to relax the SSGT constraint without formally modelling innovation when this is not essential to the research question.

[6]The introduction of CES production technologies in business cycle analysis has gained relevance in recent years due to an increasing interest in the drivers of factor income shares (see Choi and Ríos-Rull, 2009). Cantore et. al (2014), for instance, show that the effect of technology shocks on hours worked can solve the technology-hours correlation puzzle when the elasticity of factor substitution differs from one and there are biased technology shocks.

[7]The aggregation approach is taken by Houthakker (1955-56). Jones (2005) and Lagos (2006) provide useful discussions of this classic paper. Lagos (2006), in the spirit of Houthakker, derives

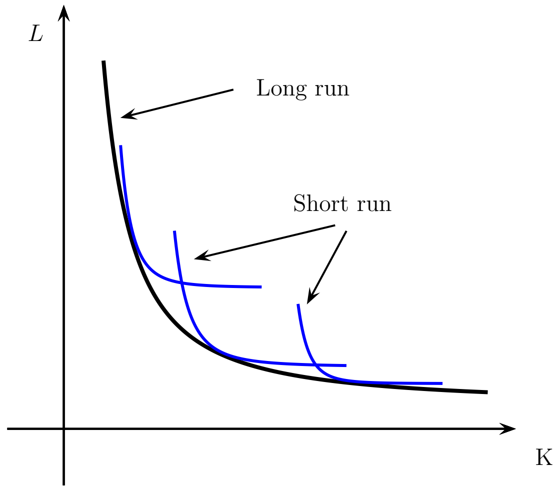


Figure 2: Isoquants of the long- and short-run production functions

shows how the use of Weibull distributions can lead to long-run CES.

While these papers provide a rich and elegant description of technical progress, applying these ideas in a conventional macroeconomic modelling framework is difficult. For instance, due to the fact that firms switch to the best available technology, the dynamics in Jones (2005) have an extreme value property that makes simulation difficult in a conventional forward-looking macroeconomic setting such as a DSGE using the usual solution techniques.[8] CC (see foonote 2) is not explicitly related to balanced growth. However, as we show here, extending the CC framework to address this and the more general question of creating a production function with a time-varying elasticity of substitution results in a formulation that is very straightforward to introduce into conventional macroeconomic models.

The rest of the paper is organized as follows. The next section contains the key theoretical results. It presents the production technology and its core characteristics, dynamics, and relationship to balanced growth. Section 3 presents an application of this approach to modelling the behavior of the labor share of income. Section 4 concludes.

a Cobb-Douglas form for the aggregate production function by aggregating Leontief production technologies at the firm level using a model with search frictions (assuming an exogenous rental on capital). Since we principally aim at providing a production function, the aims are very different from those here and are primarily directed at accounting for the determinants of observed TFP.

[8]The extreme value property will also have a significant impact on the short-run dynamics of factor shares, increasing the likelihood of sharp adjustments. Here, a standard adjustment cost mechanism results in smoother changes in factor shares.

2 The Production Technology

Though firm heterogeneity is likely to be of importance when considering technology choice, we leave this for future research in order to focus on the simplest possible setting. Since constant-returns-to-scale is the most important assumption for many of our results, we begin with a generalisation of (1). Suppose for all firms output Y is given by the following production function

$$Y = F(AK, BL) \equiv F(e^a K, e^b L) \tag{2}$$

where $F(.,.)$ is a standard twice continuously differentiable constant returns to scale production function with F_K, $F_L > 0$ and F_{KK}, $F_{LL} < 0$.

The efficiencies of capital and labor are A and B respectively, and a and b their logs. As well as choosing K and L, the firm now chooses these efficiencies constrained by a given technology frontier. This choice represents the choice of relative efficiency of both inputs of production. For instance, capital might make a relatively greater marginal contribution to output in a firm providing web-based customer support service compared to a telephone-based one. A firm may decide whether to change to a new factory design where the organisation of machinery and operators changes their relative marginal contribution to output by changing to web-based provision. This choice of factory, however, is limited by the available designs for new factories determined by the state of knowledge of the economy. In contrast to Jones (2005), we use a continuous technology frontier as in CC. In CC, the technology frontier takes a specific functional form (see again footnote 2). In order to examine balanced growth and other outcomes, we are interested in exploring different possible shapes of the frontier.

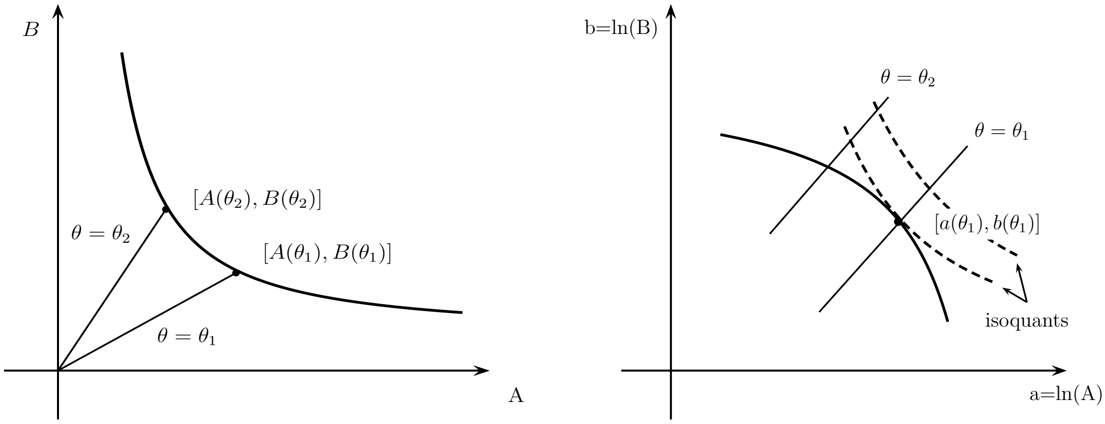
In this section, we proceed as follows. We first briefly discuss the frontier and introduce notation and two definitions of equilibrium corresponding to the short and long run (section 2.1). We then present the main results regarding balanced growth (section 2.2). Section 2.3 presents more general results in addition to providing the second order conditions necessary for the balanced growth results. In section 2.4, we consider the specific case where production takes the CES form in both the short- and long-run limits but with differing elasticities of subsitution. Finally, in section 2.5, we formally introduce adjustment costs in technology choice.

2.1 The technology frontier

We can draw the frontier in the space of the efficiencies A and B, or their logs a and b. When we do the latter, we refer to “log-efficiencies” or the “log-frontier.” Just as the capital-labor ratio $k \equiv \frac{K}{L}$ is clearly fundamental to the concept of the elasticity of substitution, with technological choice, the ratio of the efficiencies, here denoted $\theta \equiv {}^{B}\!/_{A} \equiv e^{b-a}$, also plays an important role. The quantity $\frac{k}{\theta}$ is then the ratio of capital to labor in efficiency units.

The frontier can only intersect a given ray $B = \theta A$ at one point[9] which we call $(A(\theta; x), B(\theta; x))$, introducing a shift parameter x that represents the 'level' of technology. Hence we can label any point on the frontier by the ratio of efficiencies θ at that point. The log-efficiencies are $a(\theta; x) \equiv \ln A(\theta)$ and $b(\theta; x) \equiv \ln B(\theta) = \ln \theta + a(\theta; x)$. Note that the function $a(\theta; x)$ defines the shape of the frontier. We assume throughout that that $a(\theta; x)$ is twice continously differentiable, so the frontier is continuous and "smooth." We also assume the frontier is strictly downward-sloping but not vertical (i.e. $a(\theta)$ is strictly decreasing and $\theta a(\theta)$ is strictly increasing in θ) so its slope is always defined; we denote this slope $s(\theta; x) < 0$.

The left panel of figure 3 shows an example of a technology frontier in (A, B) space together with two rays through the origin $B = \theta_1 A$ and $B = \theta_2 A$; the right panel shows the same frontier and rays in the (a, b) space.[10]



(a) The frontier in efficiency space

(b) The frontier in log-efficiency space

Figure 3: The technology frontier. Left panel, the technology frontier in (A,B) space. Right panel, the frontier in log space.

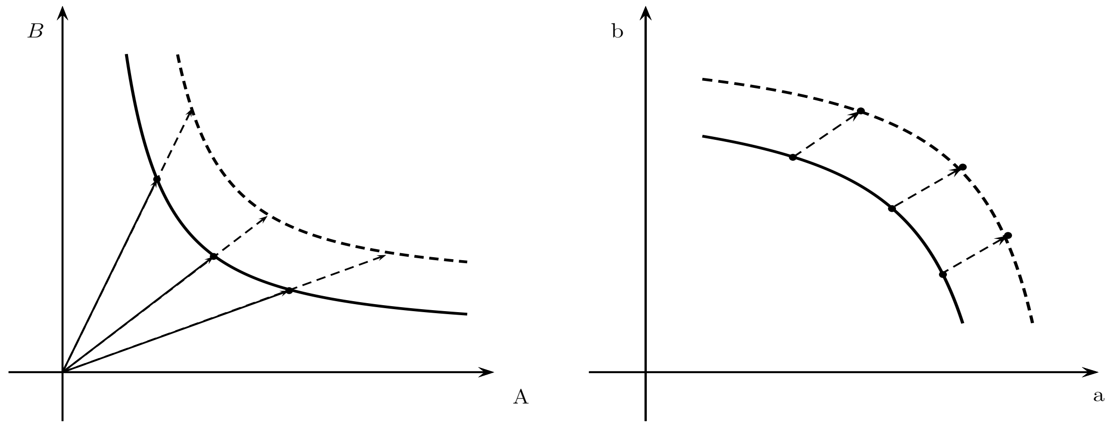
We assume that technical progress takes the following form: an increase in x, Δx, represents an equi-proportionate shift from each point (A, B) on the frontier to the point $\left(Ae^{\Delta x}, Be^{\Delta x}\right)$. We will see below that this implies that firms will not change their optimal long-run capital-labor ratio or choice of technology if factor price ratios stay constant. So effectively we are assuming Hicks-neutral technical progress. In log-efficiency space, this corresponds to a parallel shift of the log-frontier along 45°

[9]By contradiction: if it intersects the ray at two points, the technology represented by the point closest to the origin cannot lie on the frontier since it is dominated by the one further from it.

[10]In the paper we use the word "ray" to mean lines of constant θ that go though the origin when drawn in (A, B) space. Movement along a ray represents a proportionately equal change in the efficiencies of both inputs. When these lines are redrawn in (a, b) space, they will not in general go through the origin but we will still refer to them as "rays" as they retain the same economic meaning.

lines. An example is shown in figure 4. Under this type of expansion, the slope of the frontier will not change as we move along a ray. Hence $s(\theta; x) \equiv s(\theta)$ is independent of x as are in fact $a'(\theta)$ and $b'(\theta)$ (mathematically, this is what implies Hicks-neutrality). Note that

$$s(\theta) \equiv \frac{b'(\theta)}{a'(\theta)} = \frac{1}{\theta a'(\theta)} + 1. \tag{3}$$



(a) In efficiency space

(b) In log-efficiency space

Figure 4: Technological progress: an increase in x. Left panel represents the shift in (A,B) space. Right panel represents the shift in log space.

We also assume perfect competition throughout. The firm's problem is to choose the quantities of its inputs and its technology subject to the factor prices of labor and capital, w and $r + \delta$ respectively, to maximise its profits $Y - (r + \delta)K - wL$. Due to constant returns to scale, the solution to this problem determines the capital-labor ratio k in terms of the factor price ratio $\Lambda \equiv \frac{w}{r+\delta}$. For the moment, we do not introduce explicitly adjustment costs in technology choice. However, given that we will assume adjustment is costly, it is useful to have two definitions of equilibrium that correspond to the cases of complete adjustment in technology choice (the long-run equilibrium) and to that of no adjustment (the short-run equilibrium). In the short run, the firm only chooses the optimal input mix within a given factory. In the long run, however, the firm can also choose the type of factory from a design on the technology frontier, which can be indexed by θ.

Definition: *Given factor prices $r + \delta$ and w and a frontier $a(\theta; x)$, an **interior long-run equilibrium** is a pair (k^*, θ^*), that satisfies the standard first order conditions $Y_K = r + \delta$, $Y_L = w$ and $Y_\theta = 0$ and which, if all firms choose $\theta = \theta^*$ and $k = k^*$, is such that no firm can increase its profits by deviating from this choice.*

Given factor prices $r+\delta$ and w ***and*** *current technology (a,b), an* ***interior short-run equilibrium*** *is a capital-labor ratio k^*, that satisfies the standard first order conditions $Y_K = r+\delta$ and $Y_L = w$ and which, if all firms choose $k = k^*$, is such that no firm can increase its profits by deviating from $k = k^*$ holding (a,b) fixed.*

2.2 Balanced growth

Let us now return to figure 3. Holding K and L fixed, we can draw "efficiency" isoquants in (a,b) space (shown by the dotted lines in the right panel). Let $s^I(a,b)$ represent the the slope of the isoquant through the point (a,b). Note that

$$-s^I(a,b) = \frac{Y_a}{Y_b} = \frac{Ke^aF(e^aK,e^bL)}{Le^bF_2(e^aK,e^bL)} = \frac{KY_K}{LY_L}. \tag{4}$$

Equation (4) applies in and out of equilibrium. The ratio of the marginal gains from increasing capital efficiency to those from increasing labor efficiency must depend on the ratio of capital to labor employed. Beyond that, it depends on the marginal rate of technical substitution between capital and labor, since both efficiency gains and factor quantity increases raise production via efficiency units of the relevant factor. In (an interior long-run) equilibrium, however, we know that the firm must choose a technology on the frontier at which the slope of the frontier must equal the slope of the technology isoquants (i.e. $Y_\theta = 0$). The expression on the right-hand side of (4) must also equal the ratio of capital to labor income. Hence we must have

$$-s(\theta) = \frac{k}{\Lambda} = \frac{\alpha}{1-\alpha}, \tag{5}$$

where α is the capital share. Despite its simplicity, equation (5) is a very useful one. If the log-frontier is linear, the capital share will remain constant in the long run and thus we will have balanced growth – provided we remain at an interior long-run equilibrium.

From equation (3), for the capital share α to remain constant in (5), we require the following functional form for $a(\theta)$, upto a constant:

$$a(\theta) = x + (\alpha - 1)\ln\theta. \tag{6}$$

Substituting in (2), then gives,

$$Y = F(e^x\theta^{\alpha-1}K, e^x\theta^\alpha L) \tag{7}$$

We summarise the above results in the following lemma:

Lemma 1 *Suppose output is given by equation (7), where $0<\alpha<1$ is constant, x represents the level of technology, and $F(.,.)$ is a standard constant returns to scale production function. If θ is a choice variable of the firm, it can be interpreted as* *representing technology choice on a log-linear technology frontier. If the solutions to the first-order conditions $Y_K = r + \delta$, $Y_L = w$ and $Y_\theta = 0$ maximise firm profits for all x in a given interval, then the capital share is constant and equal to α along the growth path as x increases in that interval.*

Lemma 1 is potentially powerful because it shows that long-run balanced growth does not depend on the shape of $F(.,.)$. If changing technologies is costly in the short and medium run, however, the shape of $F(.,.)$ will influence dynamics at these time horizons. Hence, we are be able to choose the shape of $F(.,.)$ to best model these short and medium run dynamics while knowing that technology choice will ensure balanced growth in the long run. Thus for example one can choose a CES production function to model short- and medium-run phenomena, without the restrictive assumptions imposed by Uzawa's (1961) BGP theorem.

However, we are yet to establish any restrictions on $F(.,.)$ that imply that the solution to the first order conditions does indeed maximize the profits of the firm. This is discussed further below and in appendix A.2. If $F(.,.)$ takes the form (1), we show that a sufficient condition is that $\rho < 0$. Since gross complementarity between capital and labor is the empirically preferred assumption for modelling short- and medium-run fluctuations in the labor share, this is unlikely to prove too restrictive. More generally, if, for any given θ, capital and labor are always gross complements in $F(.,.)$ then there is a unique solution to the first order conditions. This globally maximises firm profits if an additional regularity condition holds (closely related to the concept of *strict essentiality* in production functions; see appendix A.2).

2.3 Some general results

Without technology choice, if Y has constant returns to scale in K and L, and is 'well-behaved,' the standard condition $Y_{KK} < 0$ is necessary and sufficient to ensure that appropriate second order conditions are satisfied in the firm's problem. Our aim is thus to develop an equivalent condition with technology choice for a general (non-linear) log-frontier.

Suppose we are at a solution (k^*, θ^*) to the first order conditions $Y_K = r + \delta$, $Y_L = w$ and $Y_\theta = 0$. Since we assume technology is sticky in the short-run, the short-run elasticity $\sigma_{SR}(k^*, \theta^*)$ between capital and labor is determined purely by the shape of $F(.,.)$ at (k^*, θ^*): it describes how k responds to the factor price ratio $\Lambda \equiv \frac{w}{r+\delta}$ holding technology (i.e. θ) fixed,

$$\sigma_{SR}(k^*, \theta^*) = \left.\frac{\partial \ln k^*}{\partial \ln \Lambda}\right|_{\theta=\theta^*} = \frac{Y_K Y_L}{Y_{KL} Y}. \tag{8}$$

The standard expression for the elasticity of substitution on the far right-hand side of equation (8) only applies because we hold θ constant. This short-run elasticity determines the curvature or convexity of the efficiency isoquants in figure 3b. In contrast, the long-run elasticity $\sigma_{LR}(k^*, \theta^*)$ gives the response of k^* to a change in Λ when the firm can also optimally choose θ in response to this change.

A measure of the the convexity of the frontier is given by the elasticity $\eta(\theta)$ of the slope of the log-frontier $s(\theta)$ with respect to θ:

$$\eta(\theta) = \frac{\theta s'(\theta)}{s(\theta)}. \tag{9}$$

It is straightforward to show that the log-frontier is strictly convex [concave] wherever $\eta(\theta) > 0$ [$\eta(\theta) < 0$]. We can see from figure 3b that there is a natural upper bound on the curvature of the log-frontier for the first-order-conditions solution to locally maximise firm profits: the log-frontier must at least be less convex than the efficiency isoquants (i.e. $Y_{\theta\theta} < 0$). If the firm only chose technology, this would also be a sufficient condition. However, since the firm has an additional dimension of optimisation in the choice of the capital-labor ratio, this in general is not the case.[11] However, the necessary and sufficient condition given below takes a similar form:

Proposition 2 *The solution to the first order conditions* (k^*, θ^*) *locally maximises firm profits if and only if*

$$\eta(\theta^*) < 1 - \sigma_{SR}(k^*, \theta^*). \tag{10}$$

Proof. See appendix A.1. ■

This is the second order condition for technology choice. The appendix also provides a set of sufficient conditions for a solution to the first order conditions to be unique and globally maximise firm profits, and therefore constitute an interior long-run equlibrium.[12] Assuming condition (10) holds, we now derive an expression for the long-run elasticity of substitution, $\sigma_{LR}(k^*, \theta^*)$. Note that since $F(.,.)$ has constant returns to scale, the partial derivatives $F_1(.,.)$ and $F_2(.,.)$ are homogeneous of degree zero, so we can write

$$\Lambda = \frac{Y_L}{Y_K} = \frac{\theta^* F_2(e^{a(\theta^*)}K, \theta^* e^{a(\theta^*)}L)}{F_1(e^{a(\theta^*)}K, \theta^* e^{a(\theta^*)}L)} = \frac{\theta^* F_2(k^*/\theta^*, 1)}{F_1(k^*/\theta^*, 1)} \equiv \theta^* g\left(\frac{k^*}{\theta^*}\right) \tag{11}$$

[11] It can be shown that the log-frontier is less convex than the efficiency isoquants at (k^*, θ^*) if and only if:

$$\eta(\theta^*) < \frac{1 - \sigma_{SR}(k^*, \theta^*)}{\sigma_{SR}(k^*, \theta^*)}.$$

This is always implied by condition (10).

[12] For example, if condition (10) holds for all k and θ, then it can be shown that the marginal rate of technical substitution between labor and capital is strictly monotonic in k (given an optimal choice of θ conditional on k) and therefore any solution to the first order conditions is unique. It remains therefore to exclude corner solutions, and a variety of conditions might allow this depending on the shape of the log-frontier and $F(.,.)$. One such set is given in the appendix A.2.

for some differentiable function $g(.)$. Let ψ be the elasticity of $g(.)$ with respect to k/θ at k^*/θ^*. Let us now take logs and partial derivatives of equation (11) with respect to $\ln\Lambda$, holding θ^*constant:

$$1=\psi\frac{\partial\ln k^*}{\partial\ln\Lambda}. \tag{12}$$

It follows from this (and equation 8) that $\psi = 1/\sigma_{SR}$ (dropping arguments for convenience). Noting this, and now taking logs and total derivatives of both first order conditions (11) and (5), we have respectively

$$d\ln\theta^* + \frac{d\ln k^* - d\ln\theta^*}{\sigma_{SR}} = d\ln\Lambda \tag{13}$$

and

$$\eta(\theta^*)d\ln\theta^* = d\ln k - d\ln\Lambda. \tag{14}$$

Therefore, assuming condition (10) is satisfied, straightforward algebra gives

$$\sigma_{LR} = \frac{d\ln k^*}{d\ln\Lambda} = \frac{1-\sigma_{SR}-\eta\sigma_{SR}}{1-\sigma_{SR}-\eta} = \sigma_{SR} + \frac{(1-\sigma_{SR})^2}{1-\sigma_{SR}-\eta} \geqslant \sigma_{SR}. \tag{15}$$

We can see that in the specific case of a log-linear frontier where $\eta = 0$, as discussed in section 2.2, the long-run elasticity will be one, ensuring balanced growth with a constant capital share of income. If the short-run production function is Cobb-Douglas, then so is the long-run one since then technology choice only amounts to choosing total factor productivity (there being an interior solution if the log-frontier is concave). Otherwise, (15) provides a general confirmation of the intuition discussed in the introduction: whenever we have an interior solution with technology choice, the long-run capital-labor elasticity of substitution will always exceed or equal the short-run elasticity. Because technology choice allows the firm to choose a factory design more appropriate to a new capital-labor ratio, it augments the response of the capital-labor ratio to a change in factor price ratios. The more convex the log-frontier is, the greater is the impact of technology choice on the optimal capital-labor ratio. In fact, the restriction that capital and labor can be no more than perfect substitutes places an upper bound on the convexity of the frontier;[13] this is the same upper bound given in the second order condition (10) necessary for an interior solution. We cannot have an interior solution at a point where the frontier is too convex since the incentive for the firm to deviate from this point will be too strong.

[13] This is perhaps clearer if (15) is rearranged as $\eta = (1-\sigma_{SR})\frac{\sigma_{LR}-1}{\sigma_{LR}-\sigma_{SR}}$. The upper bound reached as $\sigma_{LR}\to\infty$ is $1-\sigma_{SR}$.

2.4 Long- and short-run CES production functions

The previous results were obtained for any generic twice continuously differentiable production function. We now assume that the short-run production function takes the form given in equation (1) which, given the above notation, is equivalent to

$$Y = \left[\mathrm{e}^{\rho a(\theta;x)} K^{\rho} + \theta^{\rho} \mathrm{e}^{\rho a(\theta;x)} L^{\rho} \right]^{1/\rho}, \tag{16}$$

Hence the short-run elasticity of substitution, when the firm chooses only K and L, is given by $\sigma_{SR} = {}^{1}/_{1-\rho}$. Given short-run gross complementarity between capital and labor, we also assume that $\sigma_{SR} < 1$. In section 2.2 we showed that if the log-frontier was linear, then the long-run production function that emerges from technology choice is Cobb-Douglas. However, suppose we prefer to construct a model where the long-run production function takes a CES form with an elasticity $\sigma_{LR} > \sigma_{SR}$ not necessarily equal to one. For example, we might wish to have a model with $\sigma_{SR} < 1$, in line with evidence on short-run dynamics, but where $\sigma_{LR} > 1$ for reasons described in Piketty (2013) and Karabarbounis and Neiman (2014).

In order to achieve this, we are interested in the following question given factor prices $r + \delta$ and w: for what shape of frontier $a(\theta)$ does the firm choosing K, L, and θ in production function (16) always choose the same capital-labor ratio as it would if, instead, it only chose K and L with Y given by a standard CES production technology:

$$Y = \begin{cases} e^{x} \left(\alpha K^{\mathcal{R}} + (1-\alpha) L^{\mathcal{R}} \right)^{\frac{1}{\mathcal{R}}} & \text{when } \mathcal{R} \neq 0 \\ e^{x} K^{\alpha} L^{1-\alpha} & \text{when } \mathcal{R} = 0. \end{cases} \tag{17}$$

Thus the long-run elasticity of substitution is $\sigma_{LR} = \frac{1}{1-\mathcal{R}}$ where $\rho < \mathcal{R} < 1$. We call the former problem with technology choice P_1 and the latter (standard) problem without it P_2. We would like to know when they have the same solutions. Note that in P_2, if $\mathcal{R} > 0$ (i.e. $\sigma_{LR} > 1$) neither input is essential to production and a firm choosing either $L = 0$ or $K = 0$ would obtain output $Y = e^{x} \alpha K$ or $Y = e^{x}(1-\alpha)^{\frac{1}{\mathcal{R}}} L$ respectively. Hence, if $\sigma_{LR} > 1$, we can only have a symmetric equilibrium where all firms choose the same capital-labor ratio if the following conditions, as we assume, are both satisfied:[14]

[14]See Akerlof and Nordhaus, 1967, and La Grandville, 2012; see also the conditions for a symmetric equilibrium in the main proposition of CC. We can see that if X grows over time and $r+\delta$ is constant, then condition (18) must at some point in the growth path be violated and a symmetric equilibrium can no longer prevail. Related to this, La Grandville (2012) shows that if K and L are gross substitutes and there is permanent capital-augmenting technical progress, the long-run growth path must have the property that $r+\delta$ is unbounded over time. La Grandville (2012) therefore argues that a CES production function with $\sigma > 1$ is incompatible with competitive equilibrium in the presence of any sustained capital-augmenting progress. Note, however, that this is a general property of CES production functions, rather than being related to the derivation of CES via a model of technology

$$e^x \alpha^{\frac{1}{\mathcal{R}}} \leq r + \delta \tag{18}$$

$$e^x (1-\alpha)^{\frac{1}{\mathcal{R}}} \leq w. \tag{19}$$

We can then prove the following proposition.

Proposition 3 *Consider the following function form for the shape of the frontier $a(\theta; x)$:*

$$a(\theta; x, \mathcal{R}) = \begin{cases} x + \frac{1}{\mathcal{R}\zeta} \ln\left(\alpha^{\zeta} + (1-\alpha)^{\zeta}\theta^{-\mathcal{R}\zeta}\right) & \textit{when } \mathcal{R} \neq 0 \\ x + \frac{1}{\rho}\left[\alpha \ln \alpha + (1-\alpha)\ln(1-\alpha)\right] - (1-\alpha)\ln\theta & \textit{when } \mathcal{R} = 0 \end{cases} \tag{20}$$

where the constant $\zeta \equiv \frac{\rho}{\rho - \mathcal{R}}$.[15] *The functions for the slope and elasticity of the frontier implied by (20) are:*

$$s(\theta; \mathcal{R}) = -\left(\frac{\alpha}{1-\alpha}\right)^{\zeta} \theta^{\mathcal{R}\zeta} \; ; \; \eta(\theta; \mathcal{R}) = \mathcal{R}\zeta = (1 - \sigma_{SR})\frac{\sigma_{LR} - 1}{\sigma_{LR} - \sigma_{SR}} < 1 - \sigma_{SR}. \tag{21}$$

Then, if P_2 has an interior equilibrium solution, the unique interior solution to P_1 will result in identical outcomes for $y \equiv {}^{Y}/_{L}$ and $k \equiv {}^{K}/_{L}$ if and only if the function takes the form given by equation (20).

Proof. See appendix A.3. ■

Since $\sigma_{LR} > \sigma_{SR}$, equation (21) implies that η is increasing in σ_{LR}, so increasing the long-run elasticity of substitution corresponds to increasing the convexity of the log-frontier.

2.5 Adjustment costs in technology choice

Now consider the explicit introduction of adjustment cost as follows. Suppose a change in θ implies a loss of output $\varphi\left(\frac{\theta_t}{\theta_{t-1}}\right) Y$ where $\varphi \geqslant 0$, $\varphi(1) = \varphi'(1) = 0$ and $\varphi''(.) > 0$. Given factor prices for capital and labor $r_t + \delta$ and w_t respectively, the firm chooses θ_t, K_t and L_t to maximise

$$\sum_{t=0}^{\infty} \left\{ \left[\prod_{s=0}^{t} \left(\frac{1}{1+r_s}\right)\right] \left[Y_t \left(1 - \varphi\left(\frac{\theta_t}{\theta_{t-1}}\right)\right) - (r_t + \delta)K_t - w_t L_t \right] \right\}, \tag{22}$$

choice. In fact technology choice potentially removes some of the reassurance that La Grandville's argument offers: it is possible to have a frontier such that the labor share falls to zero asymptotically in the long-run growth path, but where σ tends to 1 asymptotically from above in such a way as to imply that condition (18) is always satisfied. In fact, many simple piecewise polynomial functional forms for the log-frontier satisfy this property.

[15] Note that, since $\zeta \to 1$ as $\mathcal{R} \to 0$ and that $\frac{\partial \zeta}{\partial \mathcal{R}}|_{\mathcal{R}=0} = \frac{1}{\rho}$, it follows by L'Hôpital's rule that $a(\theta; x, \mathcal{R})$ is continuous in $\mathcal{R}$.

where Y_t is given by (16) and the frontier takes the form (20). The transition between the short- and long-run depends on the speed of adjustment and hence how costly it is to change θ. We now consider an empirical application which corresponds to the balanced growth case where the long-run production function is $Y = e^x K^\alpha L^{1-\alpha}$ implying the frontier has the form $a(\theta) = x + \frac{1}{\rho}\left[\alpha \ln \alpha + (1-\alpha)\ln(1-\alpha)\right] - (1-\alpha)\ln\theta$.

3 The short and medium run dynamics of the labor share

The production technology of section 2 has a wide range of applications in macroeconomics and growth models where the value of the elasticity of substitution matters for both cyclical and long-run phenomena.[16] In this exercise, we examine how the production function (22) performs in the simplest possible model of macroeconomic fluctuations, with a particular view to modelling the labor income share. Business cycle models typically embody a concept of balanced growth in steady state that is consistent with the well known Kaldor stylized facts. Because of this, we limit ourselves to the specific case of a linear log-frontier so that the long run production function is Cobb-Douglas (see, however, footnote 3) and the short-run production function is assumed to take the form given in equation (1). As a result, Y_t in (22) is given by:

$$Y_t = X_t \left((\theta_t^{\alpha-1} K_t)^\rho + (\theta_t^\alpha L_t)^\rho \right)^{\frac{1}{\rho}}. \tag{23}$$

where we have written $X_t = e^{x_t}$. An increase in X_t represents a Hicks-neutral expansion of the technology frontier as described above.

We compare the performance of the model with technology choice (henceforth the TC model) to an equivalent model with the two standard production functions used in modelling macroeconomic fluctuations: Cobb-Douglas and CES. In the CES production function, in order to be compatible with balanced growth, we have to replace the Hicks-neutral shock X_t with a labor-augmenting shock Z_t. Since technology choice is absent, the associated adjustment costs disappear so $(1-\varphi)Y_t$ in (22) is replaced by $Y_t = \left(\alpha K_t^\rho + (1-\alpha)(Z_t L_t)^\rho\right)^{\frac{1}{\rho}}$. For Cobb-Douglas, it is replaced by $Y_t = X_t K_t^\alpha L_t^{1-\alpha}$.

Given the focus in this exercise on the evolution of the labor share, we introduce investment specific technical change (IST) into the model. Due to the observed

[16] For instance, the analysis of factor-bias in cross-country technology differences (Caselli and Coleman, 2006), the production of "appropriate technologies" and their impact on cross-country income gaps (Acemoglu and Zilibotti, 2001 and Basu and Weil, 1998), and the effect of taxes on factor shares (Chirinko, 2002). In business cycle models, Cantore et al, (2014) show the important of capital-labor substitution for the response of hours to technology shocks, and Di Pace and Villa (2016) its relevance to match labor market moments in search and matching models.

decline in the relative price of investment goods, this has been considered an important source of macroeconomic fluctuations (Greenwood et al., 1997, 2000 and Fisher, 2006). Permanent investment-specific shocks, however, traditionally have required the restrictive assumption of a Cobb-Douglas functional form for the production function, preventing any fluctuations in factor shares. The combination of investment-specific permanent shocks with the observed volatility of factor income shares[17] (see Growiec et al., 2015 for an overview) thus presents a puzzle in macroeconomics. To capture IST, capital accumulation in the model is given by

$$K_{t+1} = Q_t \big((1 - \varphi(\Omega_t))Y_t - C_t \big) + (1-\delta)K_t \tag{24}$$

where Q_t represents investment specific technical change.

Though counterfactual, in the CES model, shocks to Q_t have to be made temporary for compatibiliy with balanced growth. We can interpret Q_t as a proxy for technical progress in producing investment goods. In a two sector model with consumption and investment goods, with each sector having an identical production structure, we would obtain an equation like (24) with Q_t representing Hicks-neutral efficiency in the investment goods sector. In the model with technology choice, an increase in Q_t might represent the impact of an expansion of the technology frontier in the investment goods sector (which for simplicity would also be assumed linear with the same slope as the technology frontier in the consumption goods sector).[18]

In order to highlight the role of technology choice, these production technologies are compared in an otherwise standard real business cycle model which abstracts from some of the rigidities and frictions emphasized by the business cycle literature. Because these rigidities form important amplification and persistence mechanisms, such an exercise can have limited success in matching the various aspects of the data that we might analyse. Since the impact of such rigidities might be expected to diminish at medium-run frequencies, we also examine the ability of the models to match medium-run moments in the data following Comin and Gertler (2006). Given these constraints, we show that the behavior of the labor share generated by this simple model with technology choice matches reasonably well its observed countercyclicality in the short run and mild pro-cyclicality in the medium run, as well as the shape of its dynamic response conditional on technology shocks.

3.1 The Data

Table 1 presents the behavior of some key macroeconomic aggregates using quarterly data for the US for the 1948Q1:2013:Q3 period. The way we filter the data

[17] As mentioned earlier, factor income shares fluctuations appear to be driven also by factors unrelated to cyclical fluctuations in mark-ups. As we will see below, even at medium-run frequencies when wage and price rigidities are diminished, we observe substantial fluctuations in the labor share.

[18] Note the short-run dynamics implied by this simple interpretation would in reality be complicated by the adjustment costs in technology choice.

follows Comin and Gertler (2006). They distinguish between short- and medium-run frequencies of the data to capture medium-run cycles. The *medium-run cycle* is obtained using a band pass filter that includes frequencies between 2 and 200 quarters, i.e. it filters the data using a very smooth nonlinear trend. The medium-run cycle is made up of a *short-run component* (frequencies between 2 and 32 quarters) and a *medium-run component* (frequencies between 32 and 200 quarters). Since most of the data are nonstationary in levels, we applied the filters to the growth rate of the series and reconstructed the filtered levels using their cumulative sum.[19] Note that, although the filter includes frequencies of 200 quarters, in the time domain, this translated into cycles of around 10-12 years as in Comin and Gertler (2006). In the results below, we present the standard deviation of the variables relative to output, the correlation with output, and the 95% GMM confidence intervals for this correlation.

The construction of the data follows standard procedures in the literature (see appendix C for details and sources). Output (Y) was measured as output of the non-farm business sector over civilian non-institutionalized population, consumption (C) is real non-durable and services consumption over civilian non-institutionalized population, investment (Inv) is real private fixed investment plus durable consumption over civilian non-institutionalized population, wages (W) are compensation per hour in the non-farm business sector, and hours worked (L) are measured as all hours in the non-farm business sector over civilian non-institutionalized population. Labor productivity ($Prod$) is measured as Y/L. The labor share measure is important for our exercise. However, measuring the labor share of income is complicated by problems related to how certain categories of income should be imputed to labor and capital owners. Supplementary appendix C contains a more thorough discussion of the measures of the labor share we used and their construction. Following Gomme and Rupert (2004), we present three different measures. The first ($LSH1$) is the labor share of income in the non-farm business sector as reported by the Bureau of labor Statistics. The second ($LSH2$) is the labor share of the domestic corporate non-financial business sector, which is calculated as one minus corporate profits and interests net of indirect taxes over value added. The third measure ($LSH3$) also follows Gomme and Rupert (2004) and calculates the labor share as unambiguous labor income over unambiguous capital income plus unambiguous labor income. We also obtained quarterly data for the labor share for Australia, Canada, The Netherlands, Spain, and the UK reported in Table 2. The countries were chosen on the basis of data availability, and supplementary appendix C gives details of the sources and data construction.

The behavior of consumption, investment and labor market variables is standard, and the short- medium-run split displays the same behavior as that reported in Comin and Gertler (2006). The standard deviations of all variables, except for investment,

[19]For stationary variables this procedure and filtering directly the level series yielded virtually the same results.

Table 1: US data moments, 1948Q1:2013:Q3

	Short-run cycle				Medium run cycles (CG filter)			
	Std/Std(Y)	Cor(X,Y)	95% CI		Std/Std(Y)	Corr(X,Y)	95% CI	
C	0.370	0.799	0.746	0.866	0.803	0.837	0.837	0.877
Inv	2.101	0.884	0.835	0.933	2.070	0.918	0.918	0.937
W	0.441	0.166	0.056	0.364	0.750	0.656	0.656	0.708
L	0.853	0.866	0.827	0.904	0.902	0.574	0.574	0.734
Prod	0.490	0.452	0.401	0.654	0.722	0.664	0.664	0.766
LSH1	0.469	-0.243	-0.398	-0.093	0.335	0.203	-0.025	0.432
LSH2	0.590	-0.416	-0.521	-0.247	0.434	0.181	-0.049	0.412
LSH3	0.481	-0.426	-0.533	-0.319	0.410	0.362	0.181	0.543

Table 2: Other countries: labor share moments

		Short run cycle				Medium run cycles (CG filter)			
	Sample (Q)	Std/Std(Y)	Cor(X,Y)	95% CI		Std/Std(Y)	Cor(X,Y)	95% CI	
Australia	1960:Q1-2010:Q4	0.729	-0.382	-0.542	-0.240	0.331	-0.245	-0.432	-0.058
Canada	1981:Q1-2010:Q4	0.894	-0.555	-0.735	-0.374	0.625	-0.430	-0.644	-0.215
Netherlands	1988:Q1-2013:Q3	1.148	-0.668	-0.815	-0.521	0.272	-0.463	-0.713	-0.213
Spain	1995:Q1-2010:Q4	0.672	-0.232	-0.612	0.148	0.308	-0.313	-0.690	0.064
UK	1960:Q1-2010:Q4	0.999	-0.507	-0.663	-0.351	0.539	0.277	0.099	0.454

increase in the medium run relative to output. Correlations with output also increase with the exception of hours worked, which become less pro-cyclical in the medium-run. The labor share displays a clear counter-cyclical behavior at short-run frequencies, with a standard deviation that is about 50% that of output for the US, and even larger for the other countries in the sample. In the long run, if balanced growth holds, factor income shares should display no variation or correlation with output. We observe that, at medium-run frequencies, the standard deviation of the labor share falls, and its correlation with output becomes positive, although not significant for two out of the three measures. For the rest of the countries, the standard deviations of the labor share also fall when compared to short-run frequencies, and the medium-run counter-cyclical behavior becomes milder. In the case of the UK, the medium-run correlation with output is positive, but only marginally significant. Overall, hence, we observe that the business cycle counter-cyclical behavior of the labor share tends to fade at medium run frequencies, as does its volatility, indicating a process of convergence towards balanced growth in the long run. In the medium run, the labor share becomes mildly pro-cyclical in some cases. We use these results as a benchmark for the model developed next.

3.2 The model

We use a standard RBC model with optimizing representative households and firms. Households maximize their lifetime utility defined over their stream of consumption and leisure, and firms maximize profits. We use a decentralized version of the model where households own the capital and rent it to firms. Note that by including a labor-augmenting efficiency term in the production function (23), we get:

$$Y_t = X_t \left((\theta_t^{\alpha-1} K_t)^\rho + (\theta_t^\alpha Z_t L_t)^\rho \right)^{\frac{1}{\rho}} . \tag{25}$$

This gives a general model that nests the three models described at the beginning of section 3 firms. Each specific model is arrived at by appropriately setting the parameters that govern the evolution of the efficiency terms X_t and Z_t and adjustment costs. For example the technology choice model is arrived at by setting $Z_t = 0$. For Cobb-Douglas we set both $Z_t = 0$ and adjustment costs to zero, whereas for the CES model we set $X_t = 0$ and adjustment costs to a very large value. Since this simplifies the exposition, we describe this general model below.[20]

For the sake of brevity, we skip some of the detail for the explanation of the standard parts of the model. Households choose consumption (C_t), hours worked (L_t), capital stock (K_{t+1}) and one-period non-contingent bonds (B_{t+1}) to maximize their expected lifetime utility $U(\cdot)$:

$$\underset{C_t, L_t, K_{t+1}, B_{t+1}}{\text{Max}} E_0 \sum_{t=0}^{\infty} \beta^t U\left(C_t, L_t\right),$$

subject to the budget constraint,

$$C_t + I_t + B_{t+1} = r_t^K K_t + w_t L_t + (1 + r_t) B_t, \tag{26}$$

and the law of motion for capital,

$$K_{t+1} = (1 - \delta) K_t + Q_t I_t. \tag{27}$$

I_t is investment in new capital stock, r_t^K is the rental rate of capital, r_t is the interest rate on one-period bonds, w_t are wages, and δ is the rate of depreciation of capital. Investment-specific technical change enters the capital accumulation equation by increasing the productivity of new investment goods. In this model, Q_t is also the inverse of the price of investment relative to consumption goods. We specify a utility function separable in consumption and labor:

$$U_t = \log C_t - \upsilon \frac{L_t^{1+\mu}}{1+\mu}, \tag{28}$$

where μ is the inverse of the Frisch elasticity, and υ is a shift parameter.

[20] In appendix D, we compare the performance of several other models nested by this general model featuring different combinations of shocks. In two of them, we introduce labor augmenting-shocks in our technology choice model. There, Z_t is not seen as a shock to the frontier, but as a shock that affects labor productivity independently of how the firm organises production. This is just implemented to check the robustness of alternative specifications of the shocks.

The firm's problem is to choose K_t, L_t and θ_t to maximize (22) subject to the technology constraint given by the production function (25) and the adjustment costs to a change in technology, $\varphi\left(\frac{\theta_t}{\theta_{t-1}}\right)$. The law of motion for technology shocks is given by:

$$d\log Z_t = (1-\kappa_Z)\nu_Z + \kappa_Z d\log Z_{t-1} + (1-\kappa_Z)\epsilon_Z, \tag{29}$$

$$d\log X_t = (1-\kappa_X)\nu_X + \kappa_X d\log X_{t-1} + (1-\kappa_X)\epsilon_X, \tag{30}$$

$$d\log Q_t = (1-\kappa_Q)\nu_Q + \kappa_Q d\log Q_{t-1} + (1-\kappa_Q)\epsilon_Q, \tag{31}$$

so that technological progress is specified as (permanent) rate of growth shocks with drifts ν_i and persistence parameters κ_i for $i = Z, X, Q$. The innovations ϵ_i are zero mean normally distributed with covariance matrix Σ. This specification nests the pure random walk when $\nu_i = 0$, $\kappa_i = 0$, and all the off-diagonal elements of Σ are zero.

Defining $\Omega_t = \frac{\theta_t}{\theta_{t-1}}$ and dropping the expectations operator from forward-looking variables for notation convenience, the first order conditions for households and firms yield:

$$\frac{C_{t+1}}{C_t} = \beta(1+r_{t+1}), \tag{32}$$

$$w_t = \upsilon h_t^{\mu} C_t, \tag{33}$$

$$1 + r_{t+1} = \frac{(1-\delta)Q_t}{Q_{t+1}} + r_{t+1}^K Q_t, \tag{34}$$

$$\left\{1-\varphi\left(\Omega_t\right)\right\}\left(\theta_t^{\alpha-1}X_t\right)^{\rho}\left(\frac{Y_t}{K_t}\right)^{1-\rho} = r_t^K, \tag{35}$$

$$\left\{1-\varphi\left(\Omega_t\right)\right\}\left(\theta_t^{\alpha}X_t Z_t\right)^{\rho}\left(\frac{Y_t}{L_t}\right)^{1-\rho} = w_t, \tag{36}$$

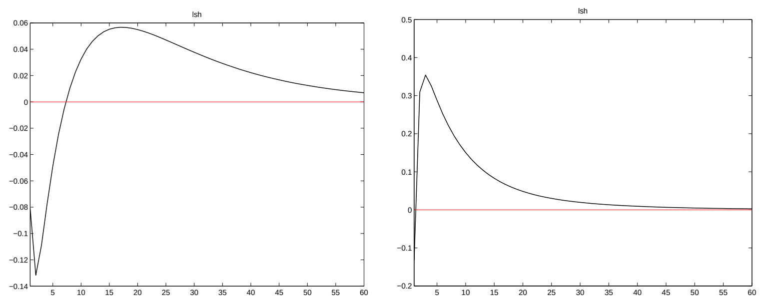
$$\alpha\left\{1-\varphi\left(\Omega_t\right)\right\} - \frac{r_t^K K_t}{Y_t} - \left\{\Omega_t\varphi'\left(\Omega_t\right) - \frac{\Omega_{t+1}}{1+r_t}\varphi'(\Omega_{t+1})\,\frac{Y_{t+1}}{Y_t}\right\} = 0. \tag{37}$$

The capital accumulation equation is given by (24) introduced earlier. Equations (32)-(36) are standard in RBC models: a consumption Euler equation (32), labor supply (33), an arbitrage condition in capital markets (34), and the two factor demand equations (35)-(36). The final equation (37) is the first order condition for θ_t with adjustment costs. Note that, if there were no adjustment costs, the equation would reduce to $\alpha = \frac{r_t^k K_t}{Y_t}$, i.e. the economy is always in the long-run equilibrium where production is Cobb-Douglas.

An equilibrium in this context is a set of decision rules $\mathcal{D}_t = \mathcal{D}(K_t, Z_t, X_t, Q_t)$ for $\mathcal{D}_t = \{C_t, L_t, K_{t+1}, B_{t+1}, \theta_t\}$ such that (23), (24) and (32)-(37) are satisfied. The model is then appropriately stationarised by dividing all trended variables $K_t, C_t, Y_t, w_t, \theta_t$ by their stochastic trends. The trend for K_t, C_t, Y_t, w_t is defined by $\bar{Y}_t = Z_t X_t^{\frac{1}{1-\alpha}} (Q_{t-1})^{\frac{\alpha}{1-\alpha}}$, whereas the trend for θ_t is given by $\bar{\Theta}_t = (X_t Q_{t-1})^{\frac{1}{1-\alpha}}$.

The functional form for the technology adjustment costs is assumed to be a symmetric exponential function[21] $\varphi(\theta_t/\theta_{t-1}) = 1 - \mathrm{e}^{-\frac{1}{2}\tau(\theta_t/\theta_{t-1}-1)^2}$. Parameter τ determines the speed of adjustment. The model then nests a standard RBC with Cobb-Douglas when $\tau = 0$, and an RBC model with CES production function as $\tau \to \infty$. Note that, in the latter case, only the Z_t process can be allowed to contain stochastic or deterministic trends. Given the observed decline in the relative price of investment, forcing Q_t to be temporary is counterfactual.

Before entering into the parameterisation and estimation details of the model, it is worth analysing the transmission of the two key shocks, Q_t and X_t to the labor share. Figure 5 presents the impulse response of the labor share to a 1% shock to X_t and Q_t. The parameter values used are explained in more detail in the next subsection. They take standard values in the literature (see Table 3 below). We also used a short run σ of 0.2, a rate of growth persistence of shocks of 0.2 and a τ value of 20.[22]



(a) X shock (b) Q shock

Figure 5: Impulse response for the labor share to a 1% X, and Q shocks. $\tau = 20$, $\sigma = 0.2$, $\kappa_i = 0.2$.

We can observe that the X_t shock leads to an initial fall and then an overshooting after around 15 quarters akin to the empirical overshooting to TFP shocks in US data found in Choi and Ríos-Rull (2009) and Ríos-Rull and Santaeulàlia-Llopis (2010). Without choice of technologies, a Hicks neutral shock would lead to an initial fall in the labor share and then an increase as the economy accumulates capital. The labor share would end up higher than the initial level. However, as θ_t adjusts and

[21]Other functional forms (e.g. quadratic) do not change the results quantitatively.

[22]Section 3.5 contains a more detailed analysis of the transmission of shocks in the data and the model.

the elasticity of substitution converges to one, the labor share falls back to its initial level, explaining the overshooting effect. The Q_t shock leads to an initial fall on impact, but the labor share immediately increases and then falls monotonically. This is because, on impact, investment experiences a large increase and consumption falls, leading to an increase in labor supply and a fall in wages. Because of the short-run gross complementarity, this leads to a fall in the labor share on impact. Afterwards, the shock acts much like a capital-augmenting process which increases the labor share with $\rho < 0$.

3.3 Calibration and estimation

In order to simulate the model, we obtain parameter values by a combination of calibration and estimation. We calibrate those parameters for which we can obtain an observable steady state condition or use information from previous studies, and estimate the rest of the parameters. Table 3 presents the calibration values in the first eight rows. We used a standard value for the steady state capital share of 0.33. β is set to 0.99 as we are matching quarterly data, whereas the depreciation rate is a standard 2.5% per quarter. Parameter μ is set to 0.33, which is consistent with macroeconomic estimates of a Frisch elasticity between 2 and 4 (see Peterman, 2012 and Chetty, 2009). For the parameters driving the law of motion of investment-specific technical change, we estimated equation (31) using data for the relative price of investment goods for the 1948Q1-2013:Q3 period. The data were obtained using the implicit deflator for fixed investment and durable goods over the price deflator for non-durables and services consumption. All data were obtained from the BEA. We estimated a drift coefficient of 0.0018 per quarter, a persistence of 0.266, and a standard deviation of the residual of 0.0067.

Table 3: Calibrated parameters and priors

α	0.33
μ	0.33
β	0.99
δ	0.025
ρ	-9/-2.3
κ_Q	0.266
ν_Q	0.0018
$stdv(\epsilon_Q)$	0.0067
Priors	
$stdv(\epsilon_i)$	InvGamma(0.001, 1)
ν_i	Normal(0.005, 0.01)
κ_i	Beta(0.1, 0.1)
τ	Gamma(10, 5)

The values used for the *short-run* elasticity of substitution ranged from 0.1 to 0.3 (implying a ρ coefficient between -9 and -2.3). Time series estimates of the elasticity of substitution for the US range from 0.4 to 0.7 (see León-Ledesma et al. 2010 and 2014). In our context, these estimated elasticities would be capturing the average value during the adjustment towards unity in the long run. Hence, the short run elasticity must be below these benchmark values. In order to test this effect, we simulated the model with a short run elasticity of 0.2 and an adjustment speed coefficient $\tau = 20$.[23] We then simulated time-series for the data and estimated the elasticity of substitution using an OLS regression on the log first order condition for labor. The estimated value for the elasticity of substitution was 0.6, which is comfortably in the range of estimates from previous studies. Hence, low values in the range of 0.1-0.3 for the short-run elasticity of substitution are consistent with the estimates offered in the literature.

The rest of the parameters were estimated using Bayesian likelihood methods based on the state-space representation of the model and now standard in the DSGE literature. The bottom part of Table 3 presents the prior distributions used to obtain posterior modes using MCMC methods. The priors are standard in the estimated DSGE literature. The standard deviation of shocks (other than ϵ_Q) follows an Inverse Gamma distribution as they are bounded below by zero and unbounded above, drifts follow a Normal distribution, and persistence coefficients follow Beta distributions as they are restricted to the open unit interval. The prior for the adjustment speed τ is drawn from a Gamma distribution as it excludes negative draws.

We estimated different versions of the model, allowing for a variety of shocks (see section 3.4 below and supplementary appendix D). All the models contained combinations of two technology shocks and hence we only used a maximum of two observables given that non-singularity requires the same number of observables and shocks.[24] The observables we used were the first difference of the log of labor productivity ($d\log(Prod_t)$) and hours per-capita (L_t). We also used alternative variables as observables such as the growth rate of output per capita, consumption growth, investment growth, and the labor share. However, the results remained robust to the choice of observable variables. Given that we estimated a large number of models, we do not report here all the estimation results.[25] It suffices to mention that, in the overwhelming majority of cases, posteriors appeared to be far from the priors, indicating that data are adding relevant information in the estimation process. Standard deviations of shocks vary by specification, but generally match well the volatility of output. Drift parameters are also consistent with the average rate of growth of per capita variables in the economy. Finally, the estimated posterior mode for τ was generally found to be close to 20.[26]

[23] A τ of 20 is a common value obtained from the estimates discussed below.

[24] We do not add ad hoc measurement errors.

[25] A complete set of results, data, and codes are available upon request.

[26] A value of 20 implies that a 1% change in θ_t incurs a reasonable output cost of 0.1%.

For the RBC model with Cobb-Douglas, the estimation follows the same procedure, but τ is restricted to be zero. For the CES model, as discussed above, investment-specific technical change is specified as a stationary AR(1) process with persistence and standard deviations estimated as above. Finally, for the CES model also, we calibrated the elasticity of substitution to 0.5, a higher value than the technology choice model, as there is no adjustment towards Cobb-Douglas.

3.4 Model and data moments

Synthetic data for the macroeconomic variables considered were generated using calibrated values and posterior means of the estimated parameters. We simulated 2,000 data points and kept the last 261 to have the same sample size as in the data. We then applied the same data filters, so we can consistently compare the short and medium run moments with those in the data. We compare a large number of models with Cobb-Douglas, CES, and technology choice with different combinations of shocks. Details of this comparison are available in supplementary appendix D. As mentioned earlier, we focus on three models, each of which has two technology shocks:

1. RBC: is a standard RBC model with Cobb-Douglas and both Hicks-neutral and investment-specific shocks.[27]

2. CES: a model with short and long run CES with permanent labor-augmenting and temporary investment-specific shocks.

3. TC: a model with technology choice and permanent Hicks-neutral and investment-specific shocks.

It is obvious from the outset that the RBC model is unable to generate any dynamics in the labor share. On the other hand, because IST shocks can only be temporary, the CES model is inconsistent with the observed trends in the relative price of investment. These are clear a priori disadvantages of these models. Nevertheless, we can compare them in terms of other data moments to see whether the introduction of technology choice comes at the cost of missing important features of the data relative to other models. We present the results for the short-run moments in Table 4 and for the medium-run moments in Table 5. For ease of comparison, we report again the data moments. For the labor share, we used the simple average of the three measures for the US economy. Nevertheless we will refer back to table 2 for the rest of countries when necessary. For the TC models, we used a short-run elasticity of substitution of 0.2 ($\rho = -4$). Italics denote statistically insignificant correlations using 95% GMM confidence intervals.

[27]With Cobb-Douglas, Hicks-neutral and labor-augmenting shocks are proportional and have the same effects on the dynamics.

In the short-run, the RBC model with Cobb-Douglas reflects well known results in the literature. Consumption appears to be more volatile than in the data, and labor market moments perform poorly, with an excess volatility of wages relative to hours worked. The CES model does a better job at matching the correlation of wages with output. However, the CES model generates twice the volatility of investment relative to the data, and an almost zero consumption correlation. The TC model is able to generate lower consumption volatility and an investment volatility closer to the data. However, it is not able to match well the labor market moments, just as in standard business cycle models without rigidities.[28] Finally, regarding the short run moments of the labor share, the CES model generates an excessive counter-cyclical behaviour, although it matches well its volatility. The TC model does a better job at generating a reasonable counter-cyclical behavior, especially when compared to the moments for other countries in Table 2. Overall, in the short run, the RBC and TC models perform better in terms of matching data moments, with the TC model performing best in terms of the behavior of the labor share.

It is for the medium-run where the performance of the model with technology choice (TC) is best relative to the data and other models. The model outperforms the basic RBC and CES in most counts. Importantly, the short run counter-cyclical behavior of the labor share now becomes slightly pro-cyclical, but not significant, in line with the data. There is also a substantial fall in its volatility relative to output. However, this volatility still appears lower than in the data. As the long-run elasticity of substitution approaches unity, both its volatility and counter-cyclicality fall when we look at medium run frequencies.

In conclusion, the TC model does the best job at fitting labor share moments, and only slightly worse at fitting some of the labor market moments only in the short run. The lack of success at reproducing labor market moments is, however, common to most standard macroeconomic models without frictions. The introduction of other mechanisms such as indivisible labor or search and matching frictions (see di Pace and Villa, 2016) should improve all models' moments, but is beyond the scope of our illustration. Our conclusions are also supported by the fuller comparison of models provided in supplementary appendix D.

It is also worth mentioning that a model comparison based on posterior odds ratios between the models with choice of techniques and a standard RBC with Cobb-Douglas or CES only, favor dramatically the former. In most cases, if we assign equal prior model probabilities, the posterior probability of the models with choice of techniques is always higher than 0.99.

[28]We introduced indivisible labor in several versions of the model following Hansen (1985). Indivisible labor increases the volatility of hours and reduces the volatility of wages as expected. But, in the models with choice of technology, the short- and medium-run behavior of the labor share remains consistent with the data. This is a promising avenue for future research as it would help matching not only the labor share, but the *joint* cyclical behavior of its components.

3.5 Dynamic transmission in the data and in the model

The performance of the model to reproduce the behavior of the labor share can also be assessed in terms of its ability to match the dynamic transmission of shocks observed in the data. In order to do so, we first identify the effect of technology shocks on the labor share using a structural VAR (SVAR) on the data. We then compare this response with that of model-generated data. To do so, we take the following steps:

1. Using a SVAR, identify investment-specific and "neutral" technology shocks in the data, and analyze the impulse responses of the labor share to these shocks.

2. Generate simulated data from the theory model driven by investment-specific and "neutral" stochastic shocks.

3. Apply the same SVAR to the artificial data and obtain impulse responses.

4. Compare impulse responses from data and simulated data.

By "neutral" in this context we mean shocks that do not affect the relative price of investment goods in the long run (i.e. X_t in our model), following naming conventions in the literature. The reason we compare model and data this way follows Chari et al. (2008) who express concerns about the ability of SVARs with long-run restrictions to identify model shocks. Comparing data and model with the *same* (potentially mispecified) metric, we ensure we are carrying out an appropriate model evaluation.

For the identification of the two shocks, we follow Fisher (2006). Intuitively, the identification strategy is that only investment-specific technology shocks can have permanent effects on the price of investment relative to consumption and that both investment-specific and neutral shocks can have a long-run impact on labor productivity. This intuition can then be used to construct a SVAR with long run restrictions. The basic information set in Fisher (2006) consists of $[\Delta Prod \quad \Delta Prel \quad \ln h]'$. $\Delta Prod$ is the rate of growth of labor productivity, $\Delta Prel$ is the inverse rate of growth of the relative price of investment, and $\ln h$ is the log of hours worked. We use this information set and add the (log) labor share ordered last.[29] The SVAR can be represented as a structural Vector Moving Average (VMA) if it satisfies invertibility and stability conditions. Thus, the structural VMA in our case is:

$$\begin{pmatrix} \Delta Prod_t \\ \Delta Prel_t \\ \ln h_t \\ LSH_t \end{pmatrix} = \begin{pmatrix} \mathbb{C}_{1,1}(L) & \mathbb{C}_{1,2}(L) & \mathbb{C}_{1,3}(L) & \mathbb{C}_{1,4}(L) \\ \mathbb{C}_{2,1}(L) & \mathbb{C}_{2,2}(L) & \mathbb{C}_{2,3}(L) & \mathbb{C}_{2,4}(L) \\ \mathbb{C}_{3,1}(L) & \mathbb{C}_{3,2}(L) & \mathbb{C}_{3,3}(L) & \mathbb{C}_{3,4}(L) \\ \mathbb{C}_{4,1}(L) & \mathbb{C}_{4,2}(L) & \mathbb{C}_{4,3}(L) & \mathbb{C}_{4,4}(L) \end{pmatrix} \begin{pmatrix} \epsilon_{\Delta prod,t} \\ \epsilon_{\Delta prel,t} \\ \epsilon_{h,t} \\ \epsilon_{lsh,t} \end{pmatrix}, \tag{38}$$

[29] Ordering the labor share last or before (log) hours does not affect the results.

Table 4: Theoretical and data moments: short run

	RBC		CES		TC		Data	
	Std(X)/Std(Y)	Cor(X,Y)	Std(X)/Std(Y)	Cor(X,Y)	Std(X)/Std(Y)	Cor(X,Y)	Std(X)/Std(Y)	Cor(X,Y)
C	0.641	0.775	0.789	0.003	0.650	0.967	0.370	0.799
Inv	2.790	0.900	4.270	0.866	2.173	0.976	2.101	0.884
W	0.692	0.898	0.632	0.184	0.651	0.968	0.441	0.166
L	0.486	0.779	0.631	0.547	0.313	0.499	0.853	0.866
Prod	0.692	0.898	0.841	0.779	0.886	0.952	0.490	0.452
LSH	na	na	0.371	-0.933	0.207	-0.669	0.510	-0.362

Table 5: Theoretical and data moments: medium run

	RBC		CES		TC		Data	
	Std(X)/Std(Y)	Cor(X,Y)	Std(X)/Std(Y)	Cor(X,Y)	Std(X)/Std(Y)	Cor(X,Y)	Std(X)/Std(Y)	Cor(X,Y)
C	0.932	0.953	0.867	0.937	0.811	0.967	0.803	0.837
Inv	1.583	0.844	1.707	0.881	1.732	0.940	2.070	0.918
W	0.940	0.974	0.876	0.950	0.812	0.969	0.750	0.656
L	0.229	0.368	0.114	0.480	0.323	0.668	0.902	0.574
Prod	0.940	0.974	0.951	0.995	0.820	0.956	0.722	0.664
LSH	na	na	0.154	-0.473	0.108	*0.056*	0.390	*0.248*

Italics indicate not significantly different from zero using GMM 95% confidence intervals.

where $\epsilon_{\Delta prod,t}$, $\epsilon_{\Delta prel,t}$, $\epsilon_{h,t}$, $\epsilon_{lsh,t}$ are contemporaneously correlated shocks with variance covariance matrix $\mathcal{V}$. Since these shocks are correlated, they cannot be interpreted as structural innovations. The problem is to identify structural shocks $[\varepsilon_{N,t}, \varepsilon_{IST,t}, \varepsilon_{h,t}, \varepsilon_{lsh,t}]'$. Here, $\varepsilon_{N,t}$ and $\varepsilon_{IST,t}$ are the neutral and investment specific structural innovations of interest to us. The variance of ε_t is normalised to 1 so that $\mathrm{E}(\varepsilon_t \varepsilon_t') = I$. To transform ϵ_t into orthogonal innovations, we pre-multiply times matrix $\mathbb{D}^{-1}$ such that $\varepsilon_t = \mathbb{D}^{-1}\epsilon_t$. $\mathbb{D}$ is an invertible 4x4 matrix such that $\mathbb{D}\mathbb{D}' = \mathcal{V}$.

Defining the vector of observables as $\mathcal{Y}_t$, then the VMA model (38) can be written in terms of structural shocks as $\mathcal{Y}_t = \mathbb{G}\varepsilon_t$, where $\mathbb{G} = \mathbb{C}\mathbb{D}$ and the elements of $\mathbb{C}$ are $\mathbb{C}_{i,j}(L)$ in (38). The restriction that $\mathbb{D}\mathbb{D}' = \mathcal{V}$ gives us 10 equations, but $\mathcal{V}$ has 16 elements. Thus, to identify the orthogonal shocks we need 6 restrictions. We use restrictions on the long-run impact matrix derived from theory. Define $\mathbb{G}(1)$ as the long-run cumulative impact matrix such that $\mathbb{G}(1)_{i,j} = \sum_{s=1}^{\infty} \mathbb{G}_{i,j}^{(s)}$. $\mathbb{G}(1)_{i,j}$ is the cumulative impact on variable i of shock j. By imposing restrictions on these elements, we can identify the orthogonal shocks ε_t.

The first set of restrictions we use come from Fisher (2006). It implies that only IST shocks can have a cumulative impact on $\Delta Prel_t$ (i.e. a permanent *level* effect on the relative price of investment). This implies that $\mathbb{G}(1)_{1,2} = \mathbb{G}(1)_{1,3} = \mathbb{G}(1)_{1,4} = 0$. The second set of restrictions come from the well known Galí (1999) identification assumption that only productivity shocks can have long-run effects on labor productivity. In our case this implies that both IST and neutral shocks can have effects on labor productivity, but not the other two shocks: $\mathbb{G}(1)_{2,3} = \mathbb{G}(1)_{2,4} = 0$. This gives us 5 restrictions. The final restriction comes from the chosen VAR ordering, but it is not important in our context as we are only interested in the effect of neutral and IST shocks on the labor share since these are the shocks present in our theory model. Since we ordered the labor share last, we impose the restriction that $\mathbb{G}(1)_{3,4} = 0$, that is, that labor share shocks cannot have an effect on the cumulative impulse response of hours.[30] Note that, since both hours and the labor share enter in levels, none of the shocks can have an impact on their level in the long run.

The data used for the vector of observables are the same used for the analysis of the data moments and estimation of the DSGE model. The results reported below use the $LSH2$ measure of the labor share but, as explained below, we also used the other measures. The VAR is estimated using Bayesian methods with a standard Minnesota prior. We used a lag length of 2 for the VAR. We then draw 1,000 times from the posterior distribution of the parameters and obtain impulse responses to the neutral and IST technology innovations and plot the median and the 68% credible sets. We repeat this with the simulated data for the model as explained above. We used the TC model with permanent X_t and Q_t shocks reported above. We simulate 2,000 time series for the variables of interest and drop the first 1,738 so we are left with the same

[30] If we ordered hours last, the restriction implies that hours shocks cannot have a long-run effect on the cumulative labor share.

number of observations as for the actual data. The parameters used for the simulation come from the estimation/calibration in the previous sub-section.

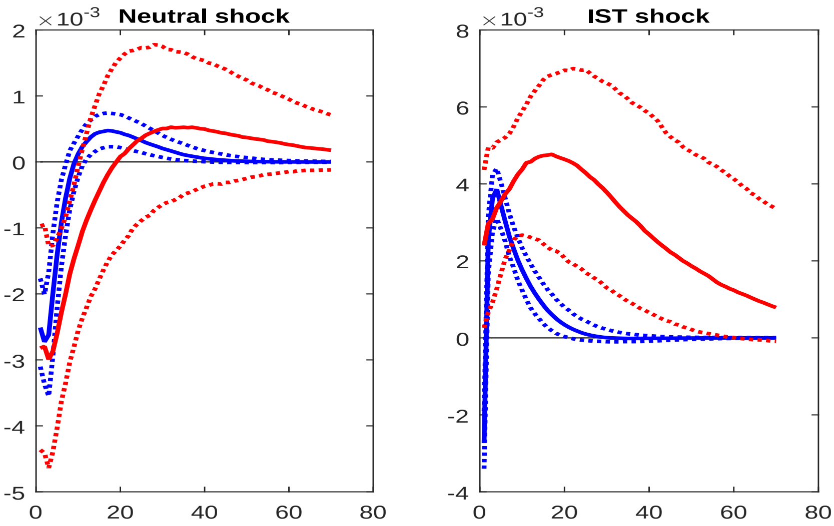


Figure 6: Impulse responses of the labor share to a 1% Neutral (left), and IST (right), shock from the structural VAR. In red, the IRF obtained using actual data, in blue the IRF using model-simulated data. The solid lines are the median IRF and the dotted lines the 68% credible sets.

Figure 6 displays the resulting impulse responses for the data (in red) and the model-generated data (in blue). For the neutral shock, the pattern and the shape of the response of the labor share in both datasets look remarkably similar, save for two aspects. First, the IRF for the data seems to peak later than in the model generated data. Second, the model generated data appears to display less persistence. Nevertheless, the credible sets overlap for most of the periods. For the IST shock, the result is not as satisfactory. However, save for the impact response, both data and model generate IFRs displaying an initial increase of the labor share. The model, however, cannot generate the persistence present in the data IRF. The simple structure of the theory model is not well suited to generate the amplification and persistence we observe in the data. However, this is a well known common problem of DSGE models, particularly those that do not include standard short-run rigidities. With that in mind, the IRF comparison shows that the model does a reasonably good job at reproducing the dynamic transmission of technology shocks for the labor share. More complex structural models including technology choice are left for future research.

We finally carried out some robustness exercises that we only comment briefly for reasons of space. We first experimented with the three measures of the labor share. With the exception of the first measure ($LSH1$), for which the response to IST shocks leads to a fall with large uncertainty bands, the other measures give similar patterns, especially regarding neutral shocks. We also used the Fernald (2010) measure of utilization adjusted TFP instead of labor productivity, leading to results that were remarkably close to the original ones. We also carried out a sample split before and after 1983, a period around which important changes in the US macroeconomy took place. The results for the post-1983 sample are close to those for the whole sample. For the pre-1983 sample, the neutral shocks display a similar pattern, but the IST shocks show the opposite response, albeit with a large degree of uncertainty. Finally, since the identification of the orthogonal innovations does not require the use of hours as an observable, we dropped it for both the actual and simulated data. The results in this case still show a close link between theory generated and actual data IRFs, but the pattern for the neutral shocks does not display the hump-shaped response we see for the original set of observables.[31]

4 Conclusions

We argue that modelling firms' technology choice on a technology frontier presents at least two distinct advantages for macroeconomic modelling. The first is that the shape of a technology frontier determines the long-run elasticity of substitution between capital and labor. Thus, the frontier determines jointly how the capital/labor share and the long-run elasticity of substitution evolve along the long-run growth path. We provide a theoretical characterisation of this process for any generic well behaved production function.

The second advantage is that technology choice naturally leads to a situation where the elasticity of substitution between capital and labor is larger in the long run than in the short run. Our framework allows for a long-run elasticity that can be unity (Cobb-Douglas) or any other value (larger than the short-run elasticity). If there are adjustment costs to technology choice, then the short- and long-run elasticities will differ after an exogenous shock. A particular focus of our framework is to provide a tractable and easily implementable resolution to the 'balanced growth conundrum' created by the balance growth path theorem without requiring explicit models of R&D. If balanced growth is a good description of the long-run growth path, this prevents the inclusion of certain kinds of *permanent* technical progress in macroeconomic models when, in accordance with empirical evidence, the elasticity of capital-labor substitution is below one. Using the above framework, we show that,

[31]Note that the VAR identification scheme is only able to distinguish between neutral and IST shocks. However, the theory model can have two types of neutral shocks (X_t and Z_t). Identifying separately these two neutral shocks would require additional restrictions as the model would be under-identified. A combination of long-run and sign restrictions has the potential of dealing with this problem by selecting among IRFs from the under-identified model. We leave this for future research as it departs from the main focus of this paper. However, provisional estimates give promising results in support of our model with technology choice.

if the technology frontier is long-linear, the elasticity of capital-labor substitution is less than one in the short run but converges to one in the long run. This leads to a class of production functions that are consistent with balanced growth even in the presence of permanent investment specific or other kinds of biased technical progress, but where short-run dynamics are characterised by gross complementarity.

As an application, we present a stochastic general equilibrium business cycle model with the production technology and estimate it using US data for the 1948:Q1-2013:Q3 period. We show that the model does a good job at matching the behavior of the labor share of income at short and medium run frequencies: the labor share is counter-cyclical and volatile in the short run, and almost a-cyclical and smoother in the medium run. The model also performs well in terms of data moments and statistical behavior against a standard RBC model with Cobb-Douglas, and an RBC model with short and long run CES only. It is also capable of reproducing the overshooting of the labor share in reaction to a technology innovation obtained from structural VAR estimates. Extensions of this approach for further research could consider its introduction in multi-sector growth models, the estimation of technology frontiers, a more detailed specification of the labor market, and a richer set of non-technology shocks.

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Online appendix

“Appropriate technology and balanced growth”

Miguel A. León-Ledesma and Mathan Satchi

Contents

A Proofs

A.1 Proof of Proposition 2

The solution to the first order conditions (k^*, θ^*) locally maximises firm profits if and only if

$$\eta(\theta^*) < 1 - \sigma_{SR}(k^*, \theta^*). \tag{A1}$$

Proof. Given te $Y = F(e^{a(\theta)}K, e^{b(\theta)}L)$, we need to show that the Hessian

$$H = \begin{pmatrix} Y_{KK} & Y_{KL} & Y_{K\theta} \\ Y_{KL} & Y_{LL} & Y_{L\theta} \\ Y_{K\theta} & Y_{L\theta} & Y_{\theta\theta} \end{pmatrix}$$

is negative semi-definite whenever the first-order conditions are satisfied. Due to constant returns, we have

$$Y_{LL} = -kY_{KL} = k^2 Y_{KK}. \tag{A2}$$

We can also write $Y = F(e^{a(\theta)}K, e^{b(\theta)}L) = F(e^{a(\theta)}K, \theta e^{a(\theta)}L) = \theta e^{a(\theta)} L f({}^{k}/_{\theta})$. Then, taking partial derivatives of both sides, we can show that

$$Y_K = e^{a(\theta)} f'\left({}^{k}/_{\theta}\right) \quad ; \quad Y_L = \theta e^{a(\theta)} \left\{ f\left(\frac{k}{\theta}\right) - \frac{k}{\theta} f'\left(\frac{k}{\theta}\right) \right\} \quad ; \quad Y_{KK} = \frac{e^{a(\theta)}}{\theta L} f''\left(\frac{k}{\theta}\right). \tag{A3}$$

It follows that

$$Y_{K\theta} = a'(\theta) Y_K - \frac{K}{\theta} Y_{KK} \quad ; \quad Y_{L\theta} = b'(\theta) Y_L + \frac{kK}{\theta} Y_{KK}. \tag{A4}$$

We now evaluate the Hessian at the solution to the first order conditions, where we have $Y_\theta = 0$. Since

$$Y_\theta = a'(\theta) K Y_K + b'(\theta) L Y_L = 0 \tag{A5}$$

we have

$$-\frac{b'(\theta)}{a'(\theta)} = -\frac{1}{\theta a'(\theta)} - 1 = k\frac{Y_K}{Y_L} = \frac{Y}{LY_L} - 1. \tag{A6}$$

The following three equations are implied by ((A4)) and ((A6)):

$$Y_{L\theta} = -kY_{K\theta} \tag{A7}$$

$$Y_{K\theta} = a'(\theta) Y_K \left(1 - \frac{KY_{KK}}{\theta a'(\theta) Y_K}\right) = a'(\theta) Y_K \left(1 - \frac{Y_{KL} Y}{Y_L Y_K}\right) = a'(\theta) Y_K \left(1 - \frac{1}{\sigma_{SR}}\right) \tag{A8}$$

$$Y_{KK} = \frac{\theta a'(\theta)}{K \sigma_{SR}} Y_K. \tag{A9}$$

Equations (A2) and (A7) imply that $\det H = 0$, so, as in the standard case, there is one zero eigenvalue. This simplifies the form of the characteristic polynomial $-\det(H - \lambda I)$, which then is

$$\lambda^3 - \left[Y_{KK} + k^2 Y_{KK} + Y_{\theta\theta}\right] \lambda^2 + \left\{ \left(1 + k^2\right) \left(Y_{\theta\theta} Y_{KK} - Y_{K\theta}^2\right) \right\} \lambda. \tag{A10}$$

The two remaining eigenvalues will be strictly negative if and only if the term in the square brackets in (A10) (the sum of the eigenvaules) is strictly negative and the term in the curly bracket (their product) is strictly positive. Since $Y_{KK} < 0$, the latter condition

$$Y_{\theta\theta} Y_{KK} > Y_{K\theta}^2. \tag{A11}$$

implies the former and so it both necessary and sufficient.

To analyse (A11), note that from (A5) and (A2)

$$Y_{\theta\theta} = \frac{KY_K a'(\theta)}{\theta}\left\{\frac{\theta a''(\theta)}{a'(\theta)} - \frac{\theta b''(\theta)}{b'(\theta)}\right\} + KY_{K\theta}\left\{a'(\theta) - b'(\theta)\right\}. \tag{A12}$$

Noting that $\eta(\theta) = {}^{\theta b''(\theta)}\!/_{b'(\theta)} - {}^{\theta a''(\theta)}\!/_{a'(\theta)}$, we have from (A8)

$$Y_{\theta\theta} = -\frac{KY_K a'(\theta)}{\theta}\left\{\eta(\theta) + 1 - \frac{1}{\sigma_{SR}}\right\}. \tag{A13}$$

From (A8), (A9) and (A13), the condition (A11) holds if and only if the condition given in the proposition (A1) holds.

■

A.2 Additional regularity conditions

Proposition 2 gives a necessary and sufficient condition for the a solution to the first order conditions to locally maximise firm profits. Clearly, we would be interested in whether such a solution would also be a global maximum. In general this will depend on the global properties of both the frontier and the production function $F(.,.)$. Suppose we replace condition (10) in the proposition by the condition that,

$$\eta(\theta) < 1 - \max_k \sigma_{SR}(k,\theta)\ \forall\, \theta \tag{A14}$$

i.e. condition (10) holds for all k and θ. Clearly if $F(.,.)$ is given by a CES form such as in equation (1), the right-hand side of (A14) takes a very simple form. Condition (A14) implies (the derivation being almost identical to that of equation 15) that we always have ${}^{\partial \ln MRTS}\!/_{\partial \ln k} > 0$ where $MRTS$ is the marginal rate of technical substitution between labor and capital (${}^{MPL}\!/_{MPK}$). Hence any solution to the first order conditions must be unique.

Any global solution to the firm's problem must be an interior one or a corner solution. Hence if (A14) holds, since any interior solution is unique, then we only need to verify that no corner solutions maximise firm profits, that is no firm chooses $k = 0$, $k = \infty$, $\theta = 0$ or $\theta = \infty$. Note that if a firm makes a corner choice in k it will be optimal to make a corner choice in θ, so any corner choice must have the property that $\theta = 0$ or $\theta = \infty$. Sufficient conditions for this yielding zero profits are as follows.

Let us assume that $F(.,.)$ satisfies a slight modification of the property of *strict essentiality* (see e.g. Barro and Sala-i-Martín, 2003). Strict essentiality states that a strictly positive amount of each input is required for strictly positive production. We assume the slightly stronger condition that as the quantity of any input tends to zero, then $F(.,.)$ always does too even if the quantity of the other tends to infinity. If $F(.,.)$ takes the form given in (1), this is true if and only if $\rho < 0$ (it excludes Cobb-Douglas). In this case, a sufficient condition to exclude a corner solution is that the shape of the frontier satisfies a property we might refer to as *weakly bounded efficiency*: that is choosing an infinite efficiency in one input must imply choosing zero efficiency in the other. Weakly bounded efficiency can be shown to be equivalent to the conditions

$$\lim_{\theta\to\infty} a(\theta) = \lim_{\theta\to 0} b(\theta) = -\infty. \tag{A15}$$

Clearly, $\theta = 0$ or $\theta = \infty$ must then imply zero output: thus the global solution to the firm's problem must be the unique interior solution to the firm's first order conditions. Note that these conditions are sufficient but not necessary for the solution to be a global maximum. This is further discussed in the proof of proposition 3.

Finally note that if $F(.,.)$ takes the CES form (1) and the log-frontier is linear, then a necessary and sufficient condition for a solution to the first order conditions to be a global solution to the firm's problem is that $\rho \leq 0$ i.e. capital and labor are gross complements. If $\rho < 0$, then the two conditions above are satisfied. If $\rho = 0$, we just have a standard Cobb-Douglas. If $\rho > 0$ then the solution to the first order conditions cannot maximise the firm's profits from proposition 2.

A.3 Proof of proposition 3

In problem P_2, firms choose K and L to maximise $Y - (r+\delta)K - wL$ given factor prices $r+\delta$ and w and a CES production technology with elasticity of substitution $\sigma_{LR} = \frac{1}{1-\mathcal{R}}$ with $\mathcal{R} < 1$:

$$Y = \begin{cases} X\left(\alpha K^{\mathcal{R}} + (1-\alpha)L^{\mathcal{R}}\right)^{\frac{1}{\mathcal{R}}} & \text{when } \mathcal{R} \neq 0 \\ XK^{\alpha}L^{1-\alpha} & \text{when } \mathcal{R} = 0. \end{cases} \tag{A16}$$

In problem P_1, firms choose K, L and θ to maximise $Y - (r+\delta)K - wL$ where, writing $x = \ln X$, Y is given by

$$Y = \left[\mathrm{e}^{\rho a(\theta;x)}K^{\rho} + \theta^{\rho}\mathrm{e}^{\rho a(\theta;x)}L^{\rho}\right]^{1/\rho}, \tag{A17}$$

where we assume $\mathcal{R} > \rho$. Proposition 3 then is as follows.

Proposition 3: Consider the following function form for the shape of the frontier $a(\theta;x)$:

$$a(\theta;x,\mathcal{R}) = \begin{cases} x + \frac{1}{\mathcal{R}\zeta}\ln\left(\alpha^{\zeta} + (1-\alpha)^{\zeta}\theta^{-\mathcal{R}\zeta}\right) & \text{when } \mathcal{R} \neq 0 \\ x + \frac{1}{\rho}\left[\alpha\ln\alpha + (1-\alpha)\ln(1-\alpha)\right] - (1-\alpha)\ln\theta & \text{when } \mathcal{R} = 0 \end{cases} \tag{A18}$$

where the constant $\zeta \equiv \frac{\rho}{\rho-\mathcal{R}}$.[32] The functions for the slope and elasticity of the frontier implied by (A18) are:

$$s(\theta;\mathcal{R}) = -\left(\frac{\alpha}{1-\alpha}\right)^{\zeta}\theta^{\mathcal{R}\zeta}\ ;\ \eta(\theta;\mathcal{R}) = \mathcal{R}\zeta = (1-\sigma_{SR})\frac{\sigma_{LR}-1}{\sigma_{LR}-\sigma_{SR}} < 1-\sigma_{SR}. \quad \text{(A19)}$$

Then, if P_2 has an interior equilibrium solution, the unique interior solution to P_1 will result in identical outcomes for $y \equiv {}^{Y}/_{L}$ and $k \equiv {}^{K}/_{L}$ if and only if the function takes the form given by equation (A18).

Proof. For problem P_2, we obtain

$$\frac{1-\alpha}{\alpha}k^{1-\mathcal{R}} = \Lambda. \tag{A20}$$

If problem P_1 has an interior solution, equations (5) and (11) imply that

$$\theta^{\rho}k^{1-\rho} = \Lambda, \tag{A21}$$

$$-\frac{k^{\rho}}{\theta^{\rho}} = s(\theta). \tag{A22}$$

Equations (A20), (A21) and (A22) must have the same solution for k for any factor price ration Λ. Hence we must have

$$\theta^{\rho} = \frac{1-\alpha}{\alpha}k^{\rho-\mathcal{R}} \tag{A23}$$

and

$$s(\theta) = \frac{1}{\theta a'(\theta)}1 = -\left(\frac{\alpha}{1-\alpha}\right)^{\zeta}\theta^{\zeta\mathcal{R}}. \tag{A24}$$

where $\zeta \equiv \frac{\rho}{\rho-\mathcal{R}}$. The differential equation (A24) has the following solution for $a(\theta)$

$$a(\theta;\mathcal{R}) = \begin{cases} c_1 + \frac{1}{\mathcal{R}\zeta}\ln\left(\alpha^{\zeta} + (1-\alpha)^{\zeta}\theta^{-\mathcal{R}\zeta}\right) & \text{when } \mathcal{R} \neq 0 \\ c_2 - (1-\alpha)\ln\theta & \text{when } \mathcal{R} = 0 \end{cases} \tag{A25}$$

For problem P_2, writing $y = {}^{Y}/_{L}$, (17) implies

$$y = \begin{cases} X\left(\alpha k^{\mathcal{R}} + (1-\alpha)\right)^{\frac{1}{\mathcal{R}}} & \text{when } \mathcal{R} \neq 0 \\ Xk^{\alpha} & \text{when } \mathcal{R} = 0. \end{cases} \tag{A26}$$

[32] Note that, since $\zeta \to 1$ as $\mathcal{R} \to 0$ and that $\frac{\partial\zeta}{\partial\mathcal{R}}|_{\mathcal{R}=0} = \frac{1}{\rho}$, it follows by L'Hôpital's rule that $a(\theta; x, \mathcal{R})$ is continuous in $\mathcal{R}$.

Substituting from (A25) and (A22) into equation (A25) should give us expressions for y equivalent to (A26). This enables us to extract the constants c_1 and c_2:

$$\begin{aligned} c_1 &= \ln X = x \\ c_2 &= x + \frac{1}{\rho}\left[\alpha \ln \alpha + (1-\alpha)\ln(1-\alpha)\right]. \end{aligned} \tag{A27}$$

So assuming that P_1 has an interior solution, the shape of the frontier specified by (A25) and (A27) is then both necessary and sufficient to ensure that P_1 and P_2 have identical outcomes for y and k. The functional forms for $s(\theta;\mathcal{R})$ and$\eta(\theta;\mathcal{R})$ given in the proposition follows immediately from (A24).

It remains to justify the assumption of an interior solution for P_1. Because equations (A19) imply that (A14) if always satisfied in section A.2, there is at most one interior solution. Thus we need only exclude corner solutions. Since again if a firm makes a corner choice in k it will be optimal to make a corner choice in θ, any corner choice must have the property that $\theta = 0$ or $\theta = \infty$. When $\mathcal{R} < 0$, the property of weakly bounded efficiency given in section (A.2) of the appendix also applies, and hence there must be an interior solution. Suppose $\mathcal{R} > 0$. Using (A18), when $\rho < 0$ and $\theta = \infty$, or when $\rho > 0$ and $\theta = 0$, the production function takes the form

$$Y = X\alpha^{\frac{1}{\mathcal{R}}}K. \tag{A28}$$

Similarly when $\rho > 0$ and $\theta = \infty$, or when $\rho < 0$ and $\theta = 0$, the production function takes the form

$$Y = X(1-\alpha)^{\frac{1}{\mathcal{R}}}L. \tag{A29}$$

Thus if P_2 has an interior solution, i.e. conditions (18) and (19) hold, then so must P_1 since a corner solution must yield negative profits.

■

B The user cost of capital and the capital share

Since the production technology developed in the main paper is Cobb-Douglas in the long run, there is no long-run relationship between the capital share and the real user cost of capital. Thus a trend in the real user cost of capital – due, for example, to changes in depreciation rates – would cause no trend in the capital income share. Because the production function is CES in the short run, from the marginal product of capital condition, the short-run elasticity between these two quantities will be $1-\sigma$. Suppose instead we had a standard CES production function in both the short and long run, with purely labor augmenting progress for compatibility with balanced growth. In that case, in addition to the short-run correlation, we would expect a long-run co-integrating relationship between the log of the capital share and the log of the real user cost with an elasticity $1-\sigma$.

Figure A1 plots the joint evolution of the log of the capital income share of the non-farm private business sector and the log of the real user cost for the US for the 1952:Q1-2004:Q4 period.[33] The capital income share is calculated as a residual after deducting wages and imputed self-employed income from the private sector GDP. The real user cost is simply the ratio between this imputed capital income and the capital stock.

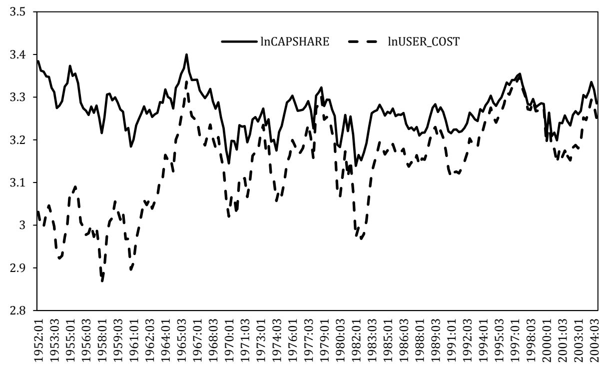

Figure A1: Log of the capital income share (solid) and log of the real user cost (dashed)

A close correlation between the two series can be observed in the short run. Using a Band Pass filter to separate trend and cyclical components, the correlation coefficient between the cyclical components is 0.91. Regressing the cyclical components yields an estimated σ of 0.42. In the long-run, however, there is a sizeable departure between the two series, and the two series are clearly not co-integrated. Though the short run correlation is consistent with a CES production function, the long-run pattern is more akin to the one arising from a Cobb-Douglas. Clearly, this is far short of making this empirical distinction a rigorous one, but it provides evidence that the short/long-run properties of the production technology may at least be consistent with the data.

C Data sources and construction

C.1 The labor share

Measuring the share of labor in total income is complicated by problems associated with how to impute certain categories of income to labor and capital owners. The existence of self-employment income, the treatment of the government sector, the role of indirect taxes and subsidies, household income accruing from owner occupied housing, and the treatment of capital depreciation, are common problems highlighted

[33] We used data from Klump et al (2007).

in the literature. These have been discussed at length in Gollin (2002) and Gomme and Rupert (2004). In constructing the labor share data for the US, where data on income sources is richest, we use three different measures. The first is directly taken from the BLS, and the other two are based on Gomme and Rupert (2004). For other countries, where available, we will use similar measures. However, data availability limits the extent to which we can obtain corrected labor share measures and, in many cases, we work with rough estimates of labor shares.

C.1.1 US labor share

The three measures used for the US are constructed using data from the BLS and the BEA NIPA Tables and are as follows:

1. *Labor share 1*: Labor share in the non-farm business sector. This is taken directly from BLS.[34] The series considers only the non-farm business sector. It calculates the labor share as compensation of employees of the non-farm business sector plus imputed self-employment income over gross value added of the non-farm business sector. Self-employment imputed income is calculated as follows: an implicit wage is calculated as compensation over hours worked and then the imputed labor income is the implicit wage times the number of hours worked by the self-employed.

2. *Labor share 2*: Labor share in the domestic corporate non-financial business sector. This follows Gomme and Rupert's (2004) first alternative measure of the labor share. The use of data for the non-financial corporate sector only has the advantage of not having to apportion proprietors income and rental income, two ambiguous components of factor income. It also considers the wedge introduced between the labor share and one minus the capital share by indirect taxes (net of subsidies), and only makes use of unambiguous components of capital income. The labor share is thus calculated as:

$$\text{LSH2} = 1 - \frac{\text{CORPORATE PROFITS + NET INTEREST - NET IND. TAXES}}{\text{NET VALUE ADDED}}.$$

3. *Labor share 3*: imputing ambiguous income for the macroeconomy. This corresponds to the second alternative measure of the labor share proposed in Gomme and Rupert (2004). The measure excludes the household and government sectors. They define unambiguous labor income (Y^{UL}) as compensation of employees, and unambiguous capital income (Y^{UK}) as corporate profits, rental income, net interest income, and depreciation. The remaining (ambiguous) components are then proprietors' income plus indirect taxes net of subsidies. These are

[34] FRED series PRS85006173 provided as an index number.

apportioned to capital and labor in the same proportion as the unambiguous components. The resulting labor share measure is:

$$\text{LSH3} = \frac{Y^{UL}}{Y^{UK} + Y^{UL}}.$$

C.1.2 Other countries

We constructed measures of the labor share on a quarterly basis for some countries for which data were available for a sufficiently long period of time. In the descriptive analysis, we used data for Australia, Canada, The Netherlands, Spain, and the UK. For most of these countries, the level of detail of the income accounts did not allow us to construct measures consistent with those of the US. In what follows, we describe the different measures used for these countries:

1. *Australia*. Quarterly data for the 1959:Q1-2013:Q3 from the Australian Bureau of Statistics. One minus gross operating surplus of private non-financial corporations plus all financial corporations as a percentage of total factor income.[35]

2. *Canada*. Quarterly data for the 1981:Q1-2013:Q3 period from Statistics Canada. Compensation of employees over total factor income. [36]

3. *The Netherlands*. Quarterly data for the 1988:Q1-2013:Q3 from the Central Bureau of Statistics. The longest time series available allowed us to construct the series for the labor share as one minus gross operating surplus over GDP net of indirect taxes less subsidies.[37]

4. *Spain*. Quarterly data for the 1995:Q1-2011:Q2 period from the National Institute for Statistics. We used compensation of employees over GDP.[38]

5. *UK*. Quarterly data for the 1955:Q1-2013:Q3 period from the Office for National Statistics. We used compensation of employees over gross value added at factor costs.[39]

[35] Web link: `http://www.abs.gov.au/AUSSTATS/abs@.nsf/DetailsPage/5206.0Sep%202013?OpenDocument`, Table 7.

[36] Web link: `http://www.statcan.gc.ca/nea-cen/hr2012-rh2012/data-donnees/cansim/tables-tableaux/iea-crd/c380-0063-eng.htm`.

[37] Web link: `http://statline.cbs.nl/StatWeb/selection/?DM=SLEN&PA=81117ENG&LA=EN&VW=T`.

[38] Web link: `http://www.ine.es/jaxiBD/tabla.do?per=03&type=db&divi=CNTR&idtab=10`.

[39] Web link: `http://www.ons.gov.uk/ons/datasets-and-tables/data-selector.html?table-id=D&dataset=qna`.

C.2 Other US macroeconomic aggregates

Other macroeconomic aggregates were constructed following standard convention in the calibrated business cycle literature. All data were obtained from either the FRED database or directly at the Bureau of Economic Analysis (indicated in parentheses). A brief description follows:

- Output: output in the non-farm business sector over civilian non-institutionalized population (FRED).
- Consumption: real non-durable and services consumption over civilian non-institutionalized population (BEA).
- Investment: real private fixed investment plus durable consumption over civilian non-institutionalized population (BEA).
- Wages: compensation per hour in the non-farm business sector (FRED).
- Hours: all hours in the non-farm business sector over civilian non-institutionalized population (FRED).
- Productivity: output per hour in the non-farm business sector (FRED).
- Relative price of investment: price deflator for durables and investment relative to the deflator for non-durables and services (BEA).
- Civilian non-institutionalized population over 16 from the FRED database.

D Model comparison: complete set of models

We provide here a fuller comparison of models other than those in the main body of the paper. The procedure followed is the same as in the comparison provided there. However, estimation here uses the posterior mode of the parameters rather than the posterior mean of the whole MCMC. Nevertheless, for the models where we used both methods, the results were always very close and did not change any conclusion. We discuss here the following set of representative models:

1. RBC1: a standard RBC model with Cobb-Douglas and only labor-augmenting shocks.
2. CES1: a model with CES technology but no technology choice (short and long run CES) with only labor-augmenting shocks.
3. CES2: a model with short and long run CES with a permanent labor-augmenting shock and a temporary Hicks-neutral shock.

4. CESCD1: a model with technology choice and only permanent Hicks-neutral shocks.

5. CESCD2: a model with technology choice and permanent labor-augmenting and investment-specific shocks.

6. CESCD3: a model with technology choice and permanent labor-augmenting and Hicks-neutral shocks.

Tables A1 to A4 present the moments calculated with the simulated data for the different models. Italics denote statistically insignificant correlations using 95% GMM confidence intervals. Model RBC1 reflects standard results in the literature. Consumption volatility and its correlation with output are higher than in the data, whereas the medium-run fall in the volatility of investment is higher than observed in the data. Labor market moments reflect the standard labor market puzzle.

A natural benchmark is to compare the performance of models featuring only one permanent shock and then models with two shocks. Comparing models RBC1, CES1, and CESCD1, we can see that CESCD1 does better at matching investment and consumption in the medium run. It also does a better job at matching wages and hours, although the standard labor market puzzle remains. None of the models is good at matching productivity at both frequencies. Regarding the labor share, CES1 matches better the short-run volatility but generates too strong a negative correlation with output. CESCD1 also generates a strong negative short-run correlation but does slightly better in the medium run where it generates a positive but not significant correlation with output. Overall, CES1 and CESCD1 outperform the basic RBC with Cobb-Douglas and only permanent labor-augmenting shocks. CESCD1 does slightly better at matching the labor share.

We turn now to the models with two shocks. CES2 generates some counterfactual results such as excess short-run volatility of investment. In the medium run, the produces a three-fold reduction in investment volatility. The volatility of hours increases relative to the RBC models in the short run, but in the medium run it goes to almost zero. As for the labor share, CES2 generates a low short-run volatility and too high a negative correlation. In the medium run, the model does better as the negative correlation falls and becomes insignificant, whereas the standard deviation relative to output goes to almost zero.

Overall, thus, for models featuring either one or two shocks, the models with short- and long-run CES do not do a good job at matching the behaviour of the labor share. Those that do better such as CES2 do it at a substantial cost in terms of matching the behaviour of expenditure components and the labor market. Models with short- and long-run Cobb-Douglas, by definition, cannot generate cyclical fluctuations in the labor share.

The models that do a better job at matching the behavior of the labor share are the CESCD models. Specifically, model CESCD2 does a very good job at matching the short-run volatility and cyclicality of the labor share. At medium-run frequencies, the correlation falls and becomes insignificant in all cases, and the volatility also falls substantially. Importantly, this is done at a lower cost than the CES models in terms of other data moments, such as consumption, investment, and the labor market. The exception is CESCD2, which generates a high volatility of productivity and a negative correlation of hours and output in the short run. The reason is that, as shown by Cantore et al. (2014), an RBC model with a sufficiently low elasticity of substitution can generate negative responses of hours to a labor-augmenting technology shock. These shocks have a substitution and an output effect on labor demand such that, for a sufficiently low elasticity, the negative substitution effect can outweigh the positive output effect. If this shock dominates, the unconditional correlation can also fall and become negative.[40] In general, CESCD models do the best job at fitting labor share moments, and only slightly worse at fitting some of the labor market moments in the short run.

Table A1: Theoretical moments using posterior modes: RBC1

	RBC1 (Z shock)			
	Short-run		Medium run	
	Std(X)/Sdt(Y)	Cor(X,Y)	Std(X)/Sdt(Y)	Cor(X,Y)
C	0.604	0.977	0.983	0.997
Inv	2.269	0.985	1.085	0.977
W	0.699	0.990	0.987	0.998
L	0.323	0.954	0.060	0.247
Prod	0.699	0.990	0.987	0.998
LSH	0	0	0	0

[40] Adding demand shocks such as preference shocks to the coefficient of hours in the utility function would eliminate this negative correlation. However, we have deliberately kept the models simple to highlight the role of the supply side.

Table A2: Theoretical moments using posterior modes: CES1 and CES2

	CES1 (permanent Z shocks)				CES2 (permanent Z temporary X shocks)			
	Short-run		Medium run		Short-run		Medium run	
	Std(X)/Sdt(Y)	Cor(X,Y)	Std(X)/Sdt(Y)	Cor(X,Y)	Std(X)/Sdt(Y)	Cor(X,Y)	Std(X)/Sdt(Y)	Cor(X,Y)
C	0.363	0.820	0.984	0.994	0.232	0.645	0.991	0.987
Inv	2.953	0.982	1.094	0.966	3.336	0.990	1.148	0.924
W	0.392	0.864	0.985	0.995	0.314	0.888	0.990	0.988
L	0.130	0.961	0.020	0.204	0.441	0.892	0.040	*0.073*
Prod	0.876	0.999	0.996	1.000	0.639	0.950	0.998	0.999
LSH	0.361	-0.955	0.052	-0.195	0.248	-0.848	0.075	*-0.159*

Table A3: Theoretical moments using posterior modes: CESCD1 and CESCD2 (technology choice models)

	CESCD1 (permanent X shocks)				CESCD2 (permanent Z and Q shocks)			
	Short-run		Medium run		Short-run		Medium run	
	Std(X)/Sdt(Y)	Cor(X,Y)	Std(X)/Sdt(Y)	Cor(X,Y)	Std(X)/Sdt(Y)	Cor(X,Y)	Std(X)/Sdt(Y)	Cor(X,Y)
C	0.640	0.991	0.787	0.974	0.865	0.814	0.870	0.986
Inv	2.119	0.992	1.778	0.953	2.409	0.788	1.486	0.957
W	0.687	0.992	0.836	0.982	0.764	0.812	0.895	0.989
L	0.165	0.963	0.203	0.897	0.722	-0.385	0.217	0.423
Prod	0.842	0.999	0.823	0.994	1.441	0.887	0.929	0.977
LSH	0.131	-0.805	0.070	*0.066*	0.684	-0.648	0.164	*-0.091*

Table A4: Theoretical moments using posterior modes: CESCD3 (technology choice models)

	CESCD3 (permanent Z and X shocks)			
	Short-run		Medium run	
	Std(X)/Sdt(Y)	Cor(X,Y)	Std(X)/Sdt(Y)	Cor(X,Y)
C	0.730	0.897	0.864	0.986
Inv	2.222	0.912	1.489	0.960
W	0.730	0.938	0.893	0.992
L	0.386	0.262	0.171	0.652
Prod	0.973	0.924	0.898	0.990
LSH	0.268	-0.535	0.070	*-0.009*

Italics indicate not significantly different from zero (GMM 95% CIs).

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