

University of Kent  
School of Economics Discussion Papers

# **Trend Dominance in Macroeconomic Fluctuations**

Katsuyuki Shibayama

September 2015

KDPE 1518



# Trend Dominance in Macroeconomic Fluctuations <sup>\*</sup>

Katsuyuki Shibayama <sup>†</sup>

School of Economics

University of Kent

Sep 2015 (First Draft: Dec 2014)

## Abstract

This paper investigates multivariate Beveridge-Nelson decomposition of key macro aggregate data. We find (a) inflation seems to be dominated by its trend component, and, perhaps as a result of this, the short-term interest rate is also trend dominated; and (b) consumption also seems to be dominated by its trend component perhaps as the permanent income hypothesis suggests. What is new here is that, although the difficulty of rejecting a unit root for these variables has been long recognized, we show that these unit root processes account for a large share of the variable fluctuations. This result raises a concern about the convention that the non-stationary data is detrended in standard DSGE-type structural estimation, in the sense that a significant portion of data variation actually may come from the trend components.

**KEYWORDS:** Beveridge-Nelson Decomposition; DSGE; VECM; Detrending

**JEL CLASSIFICATION:** C32; E21; E31; E32; E52

---

<sup>\*</sup>**Acknowledgements:** We thank all seminar participants for their constructive comments. I especially thank for the comments by Jagjit S. Chadha. This version is in file "CVAR-DSGE\_15Sep03.tex".

<sup>†</sup>**Corresponding author:** School of Economics, University of Kent, Canterbury, Kent, CT2 7NP, U.K. Phone: +44(0)122782-4714. E-mail: k.shibayama@kent.ac.uk

## Non-Technical Summary

Most macroeconomic variables are trended. For example, GDP in level grows over time and hence plotting it we see upward increasing trend. Partly because, as the size of an economy increases, everything tends to increase in parallel, this is rather common phenomenon. Statistically, dealing with trended variables are not as easy as stationary variables. For example, the mean of a trended variables are not well defined. In addition, most macroeconomic models are designed to capture the economic fluctuations at business cycle frequencies (8-32 quarters, for example). Because of these reasons, it is quite common to detrend the macro variables when some statistical inferences are made.

However, detrending is not a simple job. Practically, often first differencing (in log) is used. That is, people look at the growth rate of a variable, which is often stationary (difference stationary). However, first differencing tends to magnify the noise included in data, because the first differencing accentuates the high frequency fluctuations, and presumably noises take place at very high frequencies. Other detrending methods include HP-filter (Hodrick-Prescott filter), band-pass filter and linear detrending, to name a few. But, none of them are flawless.

To consider trends in macro variables, there are a couple issues that we have to consider. First, some variables grow along deterministic trend (like linear trend), while others follow stochastic trend (unit root processes). Often both generate similar upward trending, but their statistical nature are very different. Second, it is well known that two variables or more can share the same trend components. For example, in many empirical tests, not surprisingly, people have found a lot of evidence that GDP and consumption grow along the common trend. This sharing of trends is called cointegration.

In this paper, we implement multivariate Beveridge-Nelson (MVBN) decomposition to the US key macro economic variables, based on the standard vector error correction model (VECM). MVBN decomposition decomposes a set of macroeconomic variables into the trend and cycle components; the sum of these two are the original series, by construction. The former is defined as a random walk (a special case of unit root process) and the latter is remaining stationary component. The most important strength of MVBN decomposition lies on the fact that it explicitly takes into account the cointegration relationship among macro variables. Because cointegration is so common and important, if it is ignored any inference is subject to the criticism of misspecification.

The main finding of this paper is that some variables such as inflation rate, short-term interest rate, and consumption are quantitatively dominated by their respective trend component. Certainly, in the literature, it has been long recognized that many macro variables contain unit roots and it is very hard to reject it statistically, which is not new. But, what we discuss here is not about existence or non-existence. Our discussion is rather quantitative; we argue that the major portion of some macro variable fluctuations are due to their trend component.

In terms of interest rate, it seems that the Federal Reserve Bank reacts to both output and inflation.

However, while the cycle component of output is quite important, that of inflation is dominated by its trend component. As a result, interest rate negatively correlates with output in cycle component, while it comoves with inflation in the trend component. If this story is true, detrending ahead of statistical inference does not make sense. Eliminating trend means eliminating Fed's reaction to inflation in trend.

In terms of consumption, the literature rather has focussed on the failure of the permanent income hypothesis (PIH), which states that consumption should respond to the unexpected permanent (trend) income shock but not to anticipated shock. Also, it says that consumption should react to the transitory (cycle) component of income only little. We also confirm that PIH does not hold *perfectly*, in the sense that we cannot reject the hypothesis that consumption is positively related to past income increase, which is again not new. However, based on MVBN decomposition, again, we find that the consumption fluctuation is dominated by its trend component, which is highly correlated with the trend component of income. That is, PIH does not hold perfectly, but its mechanism is still strongly working and hence the consumption is trend dominated.

We also have done additional exercises and offered some relevant discussions, but among them most importantly if, as we discussed above, there are interactions between cycle and trend components, it casts a serious doubts on the convention of detrending ahead of statistical inferences.

# 1 Introduction

While many macro time series data are stochastically or deterministically trended, the structural economic models such as dynamic stochastic general equilibrium (DSGE) models are designed to capture the business cycle fluctuations. On top, often an estimation method requires the data to be stationary. The problem caused by detrending has long been recognized. For the detrending by HP-filter, see Harvey and Jaeger (1993) and Cogley and Nason (1995). Also, linear (or higher order polynomial) detrending is quite sensitive to the change in the initial and terminal dates, while the first differencing tends to magnify noise component. Perhaps, these facts are best summarized by Canova (2009, 2012). As a part of the project, we also confirm the sensitivity of estimated parameter values to detrending method. However, more importantly, what we emphasize in this paper is the dominance of trend components in US post war macroeconomic variables. This is especially true in inflation, interest rate and consumption. Certainly, it has long been recognized that, for these variables, it is very difficult to reject the unit root hypothesis. But, what is new here is we quantify how much of their fluctuations are explained by the unit root process (trend) and the rest (cycle). This finding is similar, though in a different context, to Aguiar and Gopinath (2007); they find that, in emerging countries, shocks to trend growth are the primary source of fluctuations. Our main tool is multivariate Beveridge-Nelson (MVBN) decomposition, which, unlike its univariate counterpart, there is only a few paper that actually apply it to the real data, which is quite disproportional given significant amount of progress of theoretical developments such as Morley (2011). At our best knowledge, the only exception is Garratt, Robertson and Wright (2006) for UK macro data with exogenously assumed cointegration relationships, but, though this paper owes their work a lot, their primary motivation seems to demonstrate their theoretical results by using the data.

For inflation and interest rate, it is well-known that it is hard to reject the unit root hypothesis. But, our main interest is not about existence and non-existence of a unit root in these variables. Instead, we are interested in the quantitative importance of the trend. Actually, we find almost all their fluctuations are explained by their trend components. It seems that the interest rate is trend dominated, perhaps because Fed reacts to inflation, which is dominated by its trend component. On the contrary, Fed seems to respond to the cycle component of output. If so, the idea of detrending is questionable in the first place; removing the trend means removing the Fed's counter-inflation activity from the data.

For consumption, our results are consistent with the findings by the literature about the consumption smoothing, in the sense that there is some evidence against permanent income hypothesis (PIH). Certainly, PIH does not hold *perfectly*. But, again our interest is not statistical testing. Rather, we want to emphasize that the mechanism of PIH is strongly working, and indeed the trend component is quantitatively dominating in consumption growth rate. That is, a large, though not *all*, portion of transitory (cyclical) component of income is smoothed out in saving/consumption decision; and hence the fluctuation of consumption is dominated by the trend component of income. Note that, in the literature, some testing of PHI checks if *all* transitory (cycle) component of income is smoothed out; or checks if consumption does *not at all* respond to anticipated income shocks. This

means that, even if, say, 99% of consumers follow PIH and only 1% are non-PIH, as long as a test can detect the minority's behaviour, it is treated as a counter-evidence against PIH. We however do not do this sort of statistical tests; instead, we describe the importance of the trend quantitatively in this paper. Anyway, there is a wide consensus that consumption smoothing is an important interaction between trend and cycle, and we quantitatively confirm it. If so, again, the trend itself is an important ingredient in what we recognize as business cycles. Note that, in the literature, consumption smoothing is not the only channel to relate trend and cycle; see Gomme (1993) and Ho (1996), in which monetary policy could affect long-run economic growth.

All in all, our most important message is: regardless of the detrending method used, the idea of detrending itself may not be proper in the first place. This is not only because the trend components are dominating, but also there are some interesting interaction between trend and cycle components. This naturally suggests that we should treat trend and cycle at the same time in a consistent manner. In statistical models, it is not very difficult to do this since the emergence of cointegration estimations. However, it is not an easy task for more structured estimation such as DSGE. In this sense, it is preferable to estimate detrending parameters (under flexible trend formulation) and structural parameters at the same time as proposed by Ferroni (2011).

In this connect, we also argue that the true problem here is not the two-stage nature of the current detrending convention. Rather, it is the problem of inconsistency between the data and the model. More specifically, with our seven macro variables, which are rather standard data set used in DSGE estimation, our statistical model (VECM) indicates that the dimension of the trend components is five. But, the standard microfounded DSGE models are only in accordance with two to three trend components at maximum. Unfortunately, it seems that it is technically quite challenging to extend the dimension of the trend components in the balanced growth path.

In this paper, we first documents the details of MVBN decomposition in Section 2. Then, we show inflation, interest rate and consumption are dominated by their respective trend components in Section 3. In section 4, we also show how DSGE estimation is affected if we use MVBN decomposition as a detrending tool. We reserve Section 5 for some key discussions. The last section concludes.

## 2 Methodology and Implementation

This section describes the implementation of the multivariate Beveridge-Nelson (MVBN) decomposition and compares it with other detrending methods in details.

### 2.1 Base Data Set

We first run a Vector Error Correction Model (VECM) for seven key macro variables, which are essentially the same as those used in the benchmark DSGE estimation by Smets and Wouters (2007); it includes output (GDP), consumption, investment, hours worked, real wage rate, goods inflation and nominal policy interest rate of U.S. from 1946Q1 to 2008Q2. All but interest rate and inflation rate are in natural logarithm. There are slight differences from the data set used in Smets and

Wouters (2007). First, we extended data period, though we avoid the period of the zero lower bound (ZLB) of the policy interest rate. While VECM estimation in general requires long data series, having the ZLB period seems to distort the behaviour of nominal interest rate and consumption during the great recession.<sup>1</sup> Second, we reclassified durable goods consumption by Fisher-of-Fisher method; see Landefeld and Parker (1997) and Seskin and Parker (1998).<sup>2</sup> In this paper, we apply other detrending methods such as band-pass filter and HP-filter to this same base data set. Note that we use the level of wage, rather than wage inflation, while we do not use the level of GDP deflator, which itself seems to be  $I(2)$ . Hence, all variables in the base data set cannot reject the unit-root hypothesis, including nominal interest rate.

## 2.2 VECM Specification

As usual, we first select the lag order and then determine the cointegration rank. First, most statistics indicate that the lag order is either 2 or 3. Regardless of the lag order, however, the most statistics point to 2 for the cointegration rank.<sup>3</sup> Our choice is lag order 2 and cointegration rank 2. We eliminate the possibility of the linear trend in the first difference; otherwise, BN decomposition is not well-defined by construction. That is, we have  $\beta$  is a  $2 \times 7$  matrix,  $p = 3$  and  $\tau = 0$  in the following general specification.

$$\Delta y_t = \alpha (\beta y_{t-1} + \mu + \rho t) + \sum_{i=1}^{p-1} \Gamma_i \Delta y_{t-i} + \gamma + \tau t + \epsilon_t$$

Changing lag order to 3 alters the results quantitatively but only a little, and almost all implications still hold,<sup>4</sup> while the VECM performance is significantly deteriorated if we change the cointegration order. All in all, where it comes to our U.S. base data set, many indicators agree on the VECM specification. Here, notice that, since the cointegration rank is 2 and the number of variables is 7, the dimension spanned by the  $I(1)$  processes is  $5 = 7 - 2$ . Later, we discuss the implication of this in Section.

---

<sup>1</sup>More specifically, if the ZLB period is included, the cyclical components of interest rate and consumption increase for that period, for almost all reasonable VECM specifications. See the bottom two panels of Figure 3. Perhaps, the following story goes behind this. First, because our VECM cannot recognize any non-linearity such as ZLB, it regards as if Fed did not cut FF rate enough despite that the model predicts FF rate should be negative. Given an increase in the cyclical component of nominal interest rate, it seems that consumption did not decrease enough, which leads to the increase in the cyclical component of consumption for 2007 to 2008. However, in other respects, extending the data set to the end of 2014, it does not change qualitative nature of the MVBN decomposition even for the great recession period.

<sup>2</sup>In Smets and Wouters (2007), they deflated all nominal GDP components by the GDP deflator and subtracted durable goods consumption from total consumption and added it to investment. Their method itself is a consistent data treatment, but, since the deflator of durable goods is available, we can also construct category-wise Fisher index, which is presumably better than their reclassification method.

<sup>3</sup>More specifically, for the lag order, final prediction error and AIC both suggest 3, while SBIC and HQIC indicate 2 and 1, respectively. For the cointegration rank, for both lag orders, Johansen's trace test suggests 2 with 1% significance, and most other indicators such as max statistics, HQIC and SBIC also imply the same rank.

<sup>4</sup>This means that, on the right hand of the estimation equation, for example,  $d \ln Y_{t-1}$  is included, but not  $d \ln Y_{t-2}$ . Here, the lag order is 2 in the sense that lagged variable  $\ln Y_{t-2}$  is included in  $d \ln Y_{t-1}$ .

## 2.3 Algorithms

In this project, we wrote two sets of STATA programmes;<sup>5</sup> one is based on the long-run forecast as in the original paper by Beveridge and Nelson (1981, pp.158-9); the other is based on the impact MA matrix, which is essentially the same as the method by Garratt, Robertson and Wright (2006).<sup>6</sup> The gap is negligible, though they do not produce the exactly the same results, perhaps due to some numerical errors.<sup>7</sup> All numerical results presented in this paper are based on the algorithm with the long-run forecast mainly for the comparison reasons with univariate BN (UVBN) decomposition, which is also computed by the long-run forecast based method.

## 2.4 Reviewing Detrending Methods

One of the key motivations in this paper is to investigate the cyclical component of MVBN decomposition in comparison with other detrending methods. To be self-contained, first we summarize the properties of the MVBN decomposition and discuss briefly other detrending methods. Detrending is essential for the standard DSGE estimation. Certainly, it is sometimes, though difficult, possible to construct the (posterior) likelihood (or other optimization criteria such as the quadratic distance measure between some data moments and model correspondences) with trended data. But, most DSGE models are designed to capture the business cycle fluctuations, and we often need a fixed point to have state space representation (without expectation operators) of the equilibrium equations (with expectations).

### 2.4.1 Multivariate and Univariate Beveridge Nelson Decomposition

Univariate or multivariate whichever it is, the BN decomposition decomposes a time series into trend and cycle components, where the former is a random walk with drift, and the latter is a mean

---

<sup>5</sup>The VECM estimation is implemented on STATA command (`vec`), and the codes for the MVBN decomposition are written in STATA ado files with MATA. These files are available upon request.

<sup>6</sup>There have been proposed a lot of efficient computation methods of MVBN decomposition. To name a few, de Silva (2008) and Morley (2002) with a Kalman filter type estimation, Arino and Newbold (1998) for VARMA, and Beyaert and Quesada Medina (2001) for cointegration with  $I(0)$  system as well as the early developments by Newbold (1990), Millar (1988) and Cuddington and Winters (1987). However, most of these are more or less model specific. The algorithm based on the original Beveridge and Nelson (1981) is perhaps the most flexible in the sense that as long as a model can generate prediction we can easily decompose the time series into trend and cycle. Also, the method along the line of Garratt, Robertson and Wright (2006) is, though to the lesser extent, also quite general and flexible, in the sense that as long as the  $MA(\infty)$  representation is available it is straightforward to compute a MVBN decomposition. Third, while there are plenty of theoretical literature such as Morley (2011), there are only few empirical application of the MVBN decomposition, except for Garratt, Robertson and Wright (2006).

<sup>7</sup>Here we would like to add some technical notes. First, there are some reasons to consider that the impact MA matrix based approach is less subject to numerical errors. Most importantly, making a long-run forecast (say, 200 periods later from a certain point of time) and drawing back the trend line to today involve the summation of a lot of small numbers, which could cause the accumulation of numerical errors. Although, fortunately enough, STATA provides quad-precision running sum command – `quadrrunningsum()` –, still it is better to avoid such a computation. In the impact MA matrix based computation, there are similar running sum, but required length of the summation is limited by the data length. Second, reviewing the theoretical literature, for the impact MA matrix based computation, the key reference is Garratt, Robertson and Wright (2006). Also, the discussion of common trends by Stock and Watson (1988) provides further insights. The actual computation in this paper is hinted a lot by Johansen’s monograph (1995, Ch 4&6).

zero stationary process. In the multivariate context, trend component is essentially the same as the common trend studied by Stock and Watson (1988).

The most important feature of the BN decomposition is that it is model dependent; in our case, it depends on a VECM. Note also that the BN decomposition itself is one-sided in the sense that it only relies on the current and past data, given data generating process, which we assume to be a VECM in our case. In our implementation, however, we use the data for the entire period to estimate the VECM.<sup>8</sup> In this sense, our BN decomposition has lost its one-sided nature, if we take into account VECM estimation.

Comparing univariate and multivariate BN decompositions, in our opinion, the difference amounts to their model performance. If VECM is better than univariate ARIMA, then the BN decomposition based on the former is better than that on the latter. Again, this is a simple consequence of the model dependency nature of the BN decomposition.<sup>9</sup> We compare their performances by using data in the later sections.

### 3 Investigating MVBN Decomposition

This section discusses the multivariate Beveridge-Nelson (MVBN) decomposition, comparing with some key detrending methods. There are several observations but, among them, most importantly we find that the fluctuation of some variables are dominated by their trend components, which casts a doubt on the detrending idea for DSGE estimations to capture the business cycle behaviour of macro variables.

#### 3.1 Eye-Balling Investigation

Figure 1 shows the cyclical component of the MVBN decomposition. This section summarizes the main findings.

##### 3.1.1 Output, Consumption and Investment

For output, consumption and investment, it is obvious that the recessions that followed the two oil crises are particularly sharp. Since we excluded the period when the zero-lower bound of the nominal interest hits, we have no result for that period in our baseline estimation; however, if we include that period, the great recession between 2007 to 2010 has a slightly less deep bottom point than these two oil crises but its duration is much longer; see the bottom two panels of Figure 3.<sup>10</sup>

---

<sup>8</sup>We could do rolling estimations for example but if do so the estimation results of the early days become very unstable. Also, it implies that the presumed data generating process is changing over time, which could negate the validity of the model prediction (if the data generating process is changing in the past, it should be also changing in the future).

<sup>9</sup>In this respect, for example, given importance of low frequency fluctuations in macro data, it may be a good idea to extend our VEMC based MVBN decomposition to that based on a fractionally integrated vector autoregressive-moving average (VARFIMA) model; see, among others, Diebold and Rudebusch (1989) for ARFIMA. Or, we could add some structural breaks or the like. What matters for the BN decomposition is the performance of the underlying model.

<sup>10</sup>See also footnote 1 for the reason of eliminating the ZLB period.

Also, according to our MVBN decomposition, the recessions in the NBER business dating seem to capture the periods from peak to bottom, rather than the periods in which the cyclical component is negative.

### 3.1.2 Wage and Hours

Working hours is clearly procyclical, and indeed it closely follows the cyclical component of output with the almost same magnitude. As a result, labour productivity is acyclical. On the contrary, real wage (level) is countercyclical. Actually, its correlation with output is -0.61; see Table 1. For the standard medium scale models, this implies that wage mark-up shock is important to explain the difference in the cyclical component of hours and wages, because, if labour supply curve does not shift, the observed wage/hours relationship is along the labour supply curve, which is upward sloping. Hence, a model need to shift labour supply schedule to have countercyclical wage by, say, wage mark-up shock.

In terms of data, the countercyclical of real wage is perhaps due to the composition effect, which means that in recessions firms tend to lay-off low waged workers (e.g., part-time workers) first, but without labour quality adjustment in data, it results in a seemingly increase in wage per man-hour. In terms of microeconomic evidence<sup>11</sup>, procyclical wage at individual worker level seems to be a consensus; see Devereux (2001) for example. But, Shin and Solon (2007) find a strong composition effect which leads to acyclical aggregate wage, though the composition effect is not strong enough to overturn the wage cyclical. Presumably, in our aggregate data, the labour quality is not taken into account. This implies the limitation in matching the aggregate wage data for a representative agent model that does not explicitly consider the endogenous changes in labour quality. As shown later, in our DSGE estimation with MVBN decomposition as a detrending device, we (perhaps erroneously) find that labour supply is quite insensitive to wage change.

### 3.1.3 Inflation and Interest Rate

For goods price inflation and short-term nominal interest rate, most of their fluctuation is explained by the trend component; see the upper two panels of Figure 3. One possible explanation is that since the fluctuations in inflation occurs mostly in the trend component, the monetary policy reacts to it in the trend component as well. The cyclical component of interest rate is strongly correlated to that of output, consumption, investment and hours. For example, its correlation with GDP is 0.79 and that with consumption is 0.76; see Table 1. Hence, it seems that the monetary policy responds to both inflation and output, but in different components.

In terms of inflation non-stationarity, there are a lot of empirical supports. Some authors report that, in the univariate inflation process, unless structural break or the like is added, it is often very hard to reject the unit root hypothesis; see Rose (1988), Romero-Avila and Usabiaga (2009 for OECD countries) and Henry and Shields (2004 for UK, US and Japan), for example.<sup>12</sup> A similar results

---

<sup>11</sup>Note that most of microeconomic results are based on the first differencing.

<sup>12</sup>In this relation, see also Cogley, Primiceri and Sargent (2010, AEJ macro), who state that the inflation gap persistence is changing over time.

are also reported for interest rate; see Bierens (1997), MacKinnon (1997) and Mishkin (1992) for example. These papers only *indirectly* support our finding, in the sense that the existence of the unit root does not necessarily imply the dominance of the trend component. Certainly, as some of empirical literature suggests, our finding may be an artefact because, say, the true data generating process may be non-linear trend stationary or may have several structural breaks, both of which we do not take into account. Nonetheless, there is non-trivial issue in the dichotomy of trend and cycle in inflation and monetary policy, which is shortly discussed further.

In this relation, the bottom two panels in Figure 3 shows the results using real, instead of nominal, interest in VECM estimation.<sup>13</sup> This exercise is motivated by the criticism that the high trend correlation between inflation and nominal interest rate is simply the result of Fisher equation. In this experiment, the most observations that we find above are still valid. More specifically, (a) still the fluctuation of real interest rate is dominated by its trend component, (b) the trend components of inflation and real interest rate comove closely, though to the lesser extent, and (c) the correlation between the cyclical components of output and real interest rate is still positive (0.19). Interestingly, all cyclical and trend components of other variables than interest rate are unchanged (up to some negligible numerical errors).

### 3.1.4 Persistence and Lag/Lead Relationship

Finally, all cyclical components are persistent; see Table 2 for autocorrelations. Also, there is no lead/lag relationship among output, consumption, investment and hours, while interest rate seems to lag one period (one quarter) behind output; see Table 3 for cross-correlations.

## 3.2 Consumption Smoothing

In this subsection, we investigate our finding in light of consumption smoothing. As already repeatedly reported in the literature, we also find some evidence of excess sensitivity and excess smoothness; that is, while consumption responds to the lagged income (excess sensitivity), consumption does not fully respond to the trend component of income (excess smoothness). As is emphasized in the literature, these two are counter-evidence against PIH, which itself is not new. However, what we want to do here is not testing PIH, but quantifying the importance of the trend component in consumption. We would like to emphasize that a large share of the consumption fluctuation is accounted by the trend component of income, which is something that the existing papers have paid little attention to.

### 3.2.1 High Correlation between Consumption and Trend Income

The upper four panels of Figure 2 investigate some prominent feature of the trend component of output and consumption, and Table 7 shows relevant correlations. Under PIH/LCH, consumption responds to the trend (permanent) component of output (or income) but not (or little) to its cycle

---

<sup>13</sup>Here, real interest rate is deflated by the actual inflation (GDP deflator) rather than inflation expectation to avoid some ambiguity in defining inflation expectation.

(transitory) component. The top two panels compares consumption (actual = trend + cycle) and the trend component of output. They comove quite tightly; even comparing the growth rates of actual consumption and trend output (top right panel), the correlation (+0.63) is quite high.<sup>14</sup> In terms of growth rate, while output exhibit a similar magnitude for both trend and cycle components (their standard deviations are +0.82 and +0.87, respectively), the standard deviation of the cyclical components of consumption (+0.36) is much smaller than that of its trend component (+0.65), as is shown in the middle two panels. Our finding is rather robust, and indeed we also find much clearer results with bi-variate MVBN decomposition, in which the growth rate of the actual consumption is almost identical to that of trend output (their correlation is +0.94); see the bottom two panels of Figure 2. The trend output and consumption are almost identical (their correlation is almost +1). We will shortly discuss the other aspects of the bivariate estimation in light of excess smoothness.

### 3.2.2 Trend Dominance in Consumption

Motivated by Cochrane (1988), Table 7 shows the variance and covariance of actual, trend and cycle components for income (GDP) and consumption, in which all variables are the first difference of the log of the original data. The ratio of trend volatility to cycle volatility  $\sigma_{d(trend)}/\sigma_{d(cycle)}$  is 0.94 for output, meaning that the trend and cycle are almost equally important in the income growth, while it is 1.76 for consumption implying that consumption is much dominated by its trend component. This is again clearer for bi-variate case; the ratios are 0.87 for output and 2.51 for consumption. Notice however that, unlike Cochrane (1988) and most of the following studies, we do not impose the orthogonality between trend and cycle innovations. Indeed, our estimation results show that the correlation between actual and cycle is negative for consumption, and hence one should be cautious to compare our results with the most existing ones. Because of this, the cycle output and actual consumption is negative ( $-0.03$ ), though not significantly different from zero. To keep the discussion streamlined, we relegate further detailed discussion in Section 5.

All in all, qualitatively, our results rather confirm "excess smoothness", in the sense that actual consumption and trend income is not perfectly correlated, and that the volatility of actual consumption is smaller than that of trend component. More importantly, however, our results find that quantitatively the trend component is dominating in consumption growth.

## 3.3 Comparison with Other Detrending Methods

Below we are going to compare MVBN decomposition with other detrending methods. The first and the most obvious observation is that, as Figure 4 shows, the HP-filter and the MVBN decomposition generate relatively similar detrended series for quantities, while the correlations between MVBN and HP-filter are lower for prices; especially the correlation of wage ( $-0.09$ ) is negative; see also Table 5.<sup>15</sup> The first differencing shows very different detrended series from MVBN;

---

<sup>14</sup>To see how large it is, compare it with the following correlations. The correlation of the growth rates of output (actual) and consumption (actual) is 0.50, that of output (cycle) and consumption (cycle) is 0.44.

<sup>15</sup>As is well known, the high-pass filter (band pass filter) is quite similar to HP-filter, and hence the results with it are almost identical to those with HP-filter.

actually, it is much closer to the univariate Beveridge-Nelson (UVBN) decomposition.<sup>16</sup>

The most popular detrending method is perhaps first differencing. It is one-sided and not model dependent. However, it tends to magnify the high frequency components, which leads to the amplification of the noise component in data. Linear (and higher order polynomial) detrending is not really proper if the true data generating process includes stochastic trends. It is not one-sided, and it is quite sensitive to the start and end dates. Hodoric-Prescott filter has the problem of spurious cycles, which is documented by Harvey and Jaeger (1993) and Cogley and Nason (1995); see also Canova (2009, 2012) for the problems of HP-filter. Band-pass filter (more specifically, high-pass filter) by Baxter and King (1999) produces a quite similar detrended series to HP-filter. Both HP and band-pass filters are two-sided in their most basic version.<sup>17</sup> Finally, it is conceivable to use great ratios such as consumption/GDP, etc. However, in reality, even these ratios are trended in data; see Whelan (2006). Note that we have found that the dimension of unit root processes is 5 in our base data set; if all the ratios were stationary, it should be 1. All in all, including BN decomposition, it seems to be safe to say that all of them are not satisfactory.

## 4 DSGE Estimation with MVBN Decomposition

One of the motivation of this paper is to investigate the MVBN decomposition as a detrending tool for DSGE estimation. Here, our DSGE model is essentially the same as Smets and Wouters' (2007) medium scale model; the only differences from theirs are (a) some elasticities are estimated without any transformation; (b) priors are modified (use Beta distribution for a parameter with range  $(0, 1)$ , for example) and they are loosened for most parameters; (c) higher capital utilization increases the speed of capital depreciation (Baxter and Farr, 2005); (d) the trend growth rate is not constant and instead it follows AR(1) process; and (e) data extended from 1968Q1 to 2008Q2 and modified slightly as discussed in Section 3. Since all of these are rather minor modifications, we omit the model details.

All in all, partly because we use the same priors, most parameter estimates are more or less similar to one another among different detrending methods, which is a good news for DSGE supporters. Despite this, impulse response functions (IRFs) and variance decompositions are quite different among different detrending methods, which casts a doubt on the policy analysis based on a specific detrending methods, as previously pointed out by Canova (1998, 2009) and Canova and Ferroni (2011) for example. There are also three key findings as below.

### 4.1 Monetary Policy

First of all, before discussing tables and figures, we report that it is sometimes difficult to estimate the DSGE model with the cyclical component of MVBN decomposition, mainly because the monetary

---

<sup>16</sup>Here, UVBN results are based on ARIMA( $p, 1, q$ ) model, in which different lag order  $p$  and moving average order  $q$  are chosen for different variables based on BSIC, while the integration order is fixed at 1.

<sup>17</sup>See however Stock and Watson (1999) and Mehra (2004) for example for one-sided filtering.

policy rule does not satisfy the Taylor principle.<sup>18</sup> This finding itself is rather consistent with the discussion above, in the sense that the monetary policy reacts to inflation mainly in the trend component but not in the cycle component. In the Table 9, the estimated parameter values for the monetary policy rule seem to be more or less similar to those under the band-pass filter (BPF) and the first differencing, but this is simply because of the tight priors. If we use looser prior, especially for the response to inflation  $\psi_\pi$ , then  $\psi_\pi$  tends to move toward one (monetary policy is less responsive to inflation) under MVBN decomposition. Contrarily, the economy responds to monetary policy quite sharply especially for the very short-run. Figure 11 shows that while the initial increase in nominal interest rate is lower under MVBN decomposition (the standard deviation of the monetary policy shock is 0.192 as in Table 10), for the first couple of periods, most of variables responds more sharply to the monetary policy shock. The effects are less persistence under MVBN decomposition, perhaps because of the smaller investment adjustment cost, which is another feature of MVBN detrending. In this relation, Table 11 shows that monetary policy shock accounts for a greater share of the variation of key variables than the first differencing and the band-pass filter. In sum, under MVBN decomposition, while monetary policy is less responsive, the economy is more responsive to the monetary policy.

## 4.2 Mark-up Shocks on Goods Price and Wage

In terms of wage, the most notable feature of MVBN decomposition is that labour supply is very inelastic (elasticity is  $1/\sigma_L$  in Table 9), although other wage related parameters, such as the wage reset probability  $\phi_W$ , wage indexation  $\iota_W$ , and wage mark-up shock process  $\rho_W$  and  $\zeta_W$  are not very different from the first differencing and the band-pass filter. In terms of the variance decomposition, Table 11 shows that a large portion of the variable fluctuations is explained by wage mark-up shock; indeed, for most variables, wage mark-up shock is the most important source of variation under MVBN decomposition.

These observations seem to be because of the countercyclical wage, as found in Figure 1. Note first that, if labour supply schedule is unchanged and only labour demand is shifting over time, the observed wage-hours combination is along the labour supply, meaning that countercyclical wage implies that the labour supply is decreasing in wage. Since such a labour supply curve is excluded by the theory, the best possible labour supply curve should be very inelastic, as obtained in Table 9. It also implies that, in the labour market, the observed wage-hours relation should be along the labour demand, which implies that the shift in the labour supply must be dominating and the labour demand curve should be relatively stable. To make labour supply more variable, the model needs a large wage mark-up shock. Hence, to increase labour when wage is low, the labour supply must be inelastic. Recalling the previous discussion, however, the counter-cyclical wage is strongly denied at the individual level data, though it could be true at the aggregate level because of the composition

---

<sup>18</sup>That is, if we use a loose prior for  $\psi_\pi$ , in many cases, the posterior estimate of  $\psi_\pi$  ends up with being one, which is the minimum value that is supported by the Taylor principle. In many cases, our Dynare code estimation seems to work, but occasionally the mode point of the posterior fails to be positive definite; see Adjemian et al. (2011) for Dynare. This is perhaps because there is a cliff in the posterior density function at  $\psi_\pi = 1$ , below which the posterior is not well defined because the model does not satisfy Blanchard-Kahn condition with  $\psi_\pi < 1$ .

effect, which is missing in our model. Similarly, Figure 10 shows output, consumption and investment respond to wage mark-up shock very much.

On the contrary, the goods price mark-up shock plays only a very minor role under MVBN decomposition. Table 9 shows that goods price persistence  $\phi_F$  and indexation  $\iota_F$  are much smaller (subscript  $F$  stands for final goods). While the standard deviation of the goods price mark-up shock itself is smaller only a bit, it is much less persistence, which leads to the total variation of wage mark-up smaller; see Table 10. Table 11 and Figure 10 also confirms this finding in the sense that goods price mark-up shock little explains the variance of the model variables.

### 4.3 Investment Adjustment Cost and Other Miscellaneous Issues

Under MVBN decomposition, the elasticity of investment adjustment cost  $\psi_I$  is almost zero; it appears non zero simply because of its prior. Certainly, under our model formulation,  $\psi_I$  is smaller than in Smets and Wouters (2007) even under the first differencing, mainly because the longer data period and different specification of the cost of higher capital utilization rate. Still, the gap in  $\psi_I$  shows the non-robustness of DSGE estimation against different detrending methods. Indeed, we find that in general IRFs tend to be less persistent under MVBN decomposition, which is mainly due to low  $\psi_I$ . According to the counterfactual exercise for MVBN decomposition, in which we set  $\psi_I$  to be 1.5, we find IRFs is more persistent not only for investment but for the most quantity variables (results not shown). In our opinion, large  $\psi_I$  seems to be required perhaps to offset the noise magnified by the first differencing.

In terms of the estimation performance, the marginal density (modified harmonic mean) is  $-296.1$  for MVBN,  $-751.1$  for high-pass filter and  $-918.2$  for first differencing. Although it is hard to draw implication from these numbers since we use different detrended data, we suspect this low marginal density for the first differencing is because of its noise magnification.

## 5 Further Discussions

This section discusses some relevant issues, which we think are important but would disturb the flow of the exposition in the above sections.

### 5.1 Further Discussion on Consumption Smoothing

Consumption smoothing is not only interesting area but is a challenging issue because of the data limitation. On the one hand, since the unit of the decision making is a household, the best data to test the theory is the household level data. Also, with aggregate data, we can only study self-insurance (via risk-free bonds), while the risk sharing (under incomplete financial market) becomes increasingly important issue in the literature, which explicitly studies the heterogeneity among households; see Attanasio and Pavoni (2011). On the other hand, since the property of the income process is the key in this area, the long data along time dimension is required. In sum, ideally, we need a data set long in both time and cross section dimensions.

Because of these, in the recent literature, the focus has moved on to more disaggregated data, such as U.S. state or Canadian province level panel data and cohort base pseudo-panel data; for pseudo-panel, see Attanasio and Weber (1995), Blundell, Brwoning and Meghir (1994) and Blundell, Pistaferri and Preston (2008), to name a few; for U.S. states level panel, see Luengo-Pado and Sørensen (2008), Asdrubali, Sørensen and Yosha (1996) and Ostergaard, Sørensen and Yosha (2002) among others. Also, some papers employ founded model and compare model behaviour and microdata; see Kaplan and Violante (2010). In this sense, methodologically, using only aggregate data in reduced form statistical model may be not satisfactory.

However, one important strength of our approach is to take into account the interaction, most importantly cointegration relationship, among several variables. Actually, its importance has been long emphasized by, for example, Campbell (1987) and Morley, Nelson and Zivot (2003). This is quite important, because any of the studies in this area are model-based, whichever structural or statistical model is, meaning that, to the extent that the cointegration relationships are important, VECM type multivariate estimation is valuable. Because of this, though the followings are rather bi-product, we believe that it is worth documenting our findings. Nonetheless, unlike the most existing papers, our objective is not testing PIH. What we want to do is, acknowledging that PIH does not perfectly describe consumption, quantifying the cyclical and trend components.

### 5.1.1 Quah Critique

Note first that, as Quah (1992) shows, the decomposition of a time series into (a) a unit root process and (b) a mean-reverting process is not unique. He also discusses that we can always find a decomposition into (a) and (b) above, so that the innovations on both are orthogonal. In this respect, the Beveridge-Nelson (BN) decomposition can be viewed as a special case in which the unit root process is a random walk (with drift), but the orthogonality between trend and cycle innovations is lost. To pin down the decomposition, Quah (1990) suggests that the exact form of the unit root process can be shaped by the information set that is available to the (representative) household. In addition, he demonstrates that, whatever the true data generating process is, we can always find a decomposition in which the variance ratio of consumption growth to the innovation of the unit root component of income is matched to the theoretical prediction of the permanent income hypothesis (PIH). Note that this variance ratio is essentially the same as that in Cochrane (1988).

There are three issues here. First, given Quah's (1988, 1992) criticism, we need to justify why we have particularly chosen the BN decomposition among other (infinitely) many possible representations. Note again that, under our MVBN decomposition, by construction, the trend components of both income (GDP) and consumption are random walks. In our opinion<sup>19</sup>, these two random walks are rather preferable under the context of the consumption smoothing. For the trend income to be a random walk, we can assume that the household is rational and, because of this, the arrival of new information is orthogonal to the past information (i.e., the news is unpredictable). That is, regarding the trend components of income as the information that the household should respond

---

<sup>19</sup>See Campbell (1987) for further supporting discussions, and see also Quah (1992, footnote 12) for the opposite view.

to, the random walk type information arrival is rather consistent with the rational expectation hypothesis. For the consumption, it simply captures the idea of the martingale consumption process under PIH. On top, the theory suggests there must be a cointegration relationship between the trend components of income and consumption, which is strongly supported by data as well.

Second, under the BN decomposition, to the extent that the growth rate is positively serially correlated, the innovation to the trend and cycle tend to be negatively correlated. To see this, consider a univariate process  $X_t$ . To make the discussion concrete, suppose that its first difference follows AR(1) with positive autocorrelation; i.e.,  $X_t = X_{t-1} + \xi_t$  and  $\xi_t = \rho\xi_{t-1} + \zeta_t$ , where  $\zeta_t$  is an i.i.d innovation process and  $\rho > 0$ . In this case, the trend component is  $X_t^{trend} = X_{t-1}^{trend} + \frac{1}{1-\rho}\xi_t$ , where  $\frac{1}{1-\rho}\xi_t = E_t[\sum_{n=0}^{\infty} \xi_{t+n}]$  is the entire effect of today's innovation over the future periods. If persistence parameter  $\rho$  is positive, the initial impulse response of the trend is greater than that of the actual;  $\frac{1}{1-\rho}\zeta_t > \zeta_t$  (having  $\xi_{t-1} = 0$ , it is obvious that  $\xi_t = \zeta_t$ ). Intuitively, this is because, while the actual  $X_t$  responds only to today's shock (i.e., only  $\xi_t = \zeta_t$ ), the trend component  $X_t^{trend}$  takes into account the growth rates in the all subsequent periods (i.e.,  $E_t[\sum_{n=0}^{\infty} \xi_{t+n}]$ ), which are decaying but nonetheless are above average. This means, because  $X_t = X_t^{trend} + X_t^{cycle}$ , the initial impulse response of the cyclical component is negative;  $-\frac{\rho}{1-\rho}\zeta_t$ . Although the discussion here is nothing more than heuristics, this is the main reason why the first differences of trend and cycle components are negatively correlated in our estimation, and perhaps the same mechanism works under many cases, as long as the growth rate is positively serially correlated. See Morley, Nelson and Zivot (2003) for more formal and detailed discussion about the negative correlation of trend and cycle innovations.

Third, related to this, we would like to look at the convention in the literature in light of Quah's criticism. In the literature, say, since Cochrane (1988) to Blundell, Pistaferri and Preston (2008) among others, many of the papers "adopt" a random walk formulation for the trend component of income, perhaps due to the same reason that we prefer it. At the same time, however, they also "assume" that the innovations to the cycle component are orthogonal to those to the trend component. We are not saying that such a combination is internally inconsistent in the sense that there may exist a data generating process that satisfies both a random walk trend component and orthogonal innovations between trend and cycle.

What matters here is that whether such an orthogonality assumption between innovations is indeed empirically supported or not. In this respect, like our findings, in their univariate estimation, Morley, Nelson and Zivot (2003) find rather clear evidence to reject zero correlation between the innovations to trend and cycle. Also, Hryshko (2013) reports that allowing a negative correlation between trend and cycle components, which we believe rather plausible, improves the model performance in explaining the excess smoothness. Morley, Nelson and Zivot (2003) also demonstrate that imposing such an orthogonality assumption alters the trend cycle components significantly. In spite of these findings, in many microfounded models for consumption smoothing, the risk of orthogonality assumption is little, if not any, discussed, which we think quite unfortunate.

Finally, as is found in Quah (1992), presumably, among many possible decomposition, BN decomposition tends to give the largest variance of the innovation to the trend component. Although we believe the random walk formulation of the trend components is rather preferable, if we instead

stand on an agnostic viewpoint, our formulation could "overestimate" the importance of the trend components.

### 5.1.2 Bivariate Results

In this subsection, we would like to discuss the result of our bivariate VECM as it is. This exercise is essentially in parallel with that by Campbell (1987). VECM itself is, though an important step, not our primary interest. Nonetheless, since Campbell (1987) has offered a couple of interesting theoretical predictions, we would like to briefly document our findings in light of his predictions. We mainly discuss bivariate case because the results with seven variables are too dense to show all, hard to interpret given high dimension and less clear, while the general tendency is not very different from the bivariate case. Our bivariate MVBN decomposition is based on the bivariate VECM. The lag order is 3 chosen by BSIC, and the cointegration rank is 1 by Johansen's test.

First, the estimated cointegration relationship is

$$\ln Y_t = -44.59096 - 0.0372647t + 1.114697 \ln C_t \quad (1)$$

where constant term ( $-44.59$ ) adjusts the level gap between output  $Y_t$  and consumption  $C_t$  as well as the base year of time trend  $t$  (and hence it has no meaning). The coefficient on  $t$  is not significantly different from zero, and the coefficient on consumption is not significantly different from 1, which means that, not surprisingly, there is the one-to-one cointegration relationship between output (income) and consumption. Since this cointegration is quite clearly estimated, if it is ignored, it could fall into a pitfall of the misspecification. The cointegration relationships are also fairly clearly identified for seven variable benchmark case, but it is not straightforward to interpret them given multiple cointegration relationship.

Second, as Campbell (1987, p.1256) suggests, under PIH, all coefficients in the equation for the first difference of consumption  $d \ln C_t$  should be zero. As the left panel of Table 8 suggests, this is true except for  $d \ln Y_{t-1}$ . The cointegration error (CE on the table), which is defined by the gap between the left and right hand sides of (1), is significant only for the output equation. Our estimation result denies the martingale consumption, because of the significant coefficient on  $d \ln Y_{t-1}$  (excess smoothness). There may be hand-to-mouth households, for example.

Third, under PIH, past consumption should have some prediction power for income. That is, even though a change in permanent income has not yet fully materialized, if households know their permanent income will increase in the future, rather than waiting for the realization of the increase in permanent income, they immediately increase their consumption. This is also discussed by Campbell (1987, p.1256), though his discussion is based on savings rather than consumption. Looking at the right panel of Table 8, we indeed find that past consumption growth rates are significant with the predicted sign. That is, an increase in the consumption today implies an increase in income in the future.

The results of the benchmark MVBN with seven variables are, though less clear, in the same line with those of the bivariate case. We can summarize the findings related to the consumption smoothing

as follows; (i) past output growth ( $d \ln Y_{t-1}$ ) is significant, though the estimated coefficient is smaller (+0.13); (ii) past consumption growth ( $d \ln C_{t-1}$ ) is again a good predictor of the output growth (coefficient +0.49); (iii) other past variables have some prediction power for consumption growth, such as interest rate that has strong negative impact on the consumption growth rate ( $-0.61$ ) consistently with the theory. All in all, the seven variable case is not very different from the bivariate estimation.

## 5.2 Model Dependency

As discussed above, multivariate or univariate whichever it is, the BN decomposition is model dependent. In our case it is based on a VECM, and in univariate cases it is typically on ARIMA models. This is quite distinct from other detrending methods, such as first differencing and band-pass filter; their implementation is rather mechanical and the users' choice is limited to the selection of some computational parameters, which usually have little economic meanings. On the contrary, for BN decomposition, some economic intuition could help the model specification (e.g., the restrictions on cointegration relationships). Also, the performance of VECM affects BN decomposition. Indeed, we failed to obtain a reasonable BN decomposition for Japanese data (and hence the results not shown in this paper), perhaps because the bad VECM performance. Perhaps the main reasons are; (a) the data length is shorter, (b) some first differenced data still seem to have trends and (c) there seems to be some structural breaks; see Ohara (1999) for the importance of the structural break focussing on the Japanese economy. The key point here is, though for U.S. data VECM performs fairly well and hence the MVBN decomposition works reasonably, we cannot generalize this and indeed often it is non trivial to obtain a good VECM result.

Also, model dependency invokes a more fundamental problem. That is, one of our objectives is to detrend the data for the DSGE estimation, but, while a DSGE model is structural, a VECM is also a model that embeds some (reduced form) structure in itself. This means that, if we use BN decomposition as a detrending tool, there are two models – one is a structural model, and the other is a statistical model – in one entire estimation project, but they are not necessarily consistent with each other. We will not go further on this issue here; instead, we just would like to confine ourselves to raise the consistency issue here.

## 5.3 Balanced Growth Path and Data Trends

Given ambiguity of the validity of the detrending data prior to DSGE estimation, it seems to be a natural step to move toward estimating the trend and cycle at the same time; see Ferroni (2011) for this idea. In the standard convention, as in Smets and Wouters (2007), often first differenced model variables and first differenced data are matched in measurement equations. As long as the dimension of the trend component in the data is 1, this way of data treatment itself is consistent between data and model variables, and estimates the trend growth rate as a constant term in the measurement equations at the same time as other parameters.

However, in our opinion, the true problem does not lie on the detrending method per se, but is on the nature of a model. That is, the problem is the consistency between the dimension of the unit

root processes in the data and that of the balanced growth path in a model. More specifically, at our best knowledge, a typical DSGE model can accommodate three trend components at maximum; that is, (i) trend technology growth rate, (ii) investment specific shock á la Greenwood, Hercowitz and Krusel (1997)<sup>20</sup> and (iii) nominal trend, typically in the target inflation rate in Taylor type monetary policy rule. However, for the U.S. post world war II data, as discussed above, we find the dimension of the cointegration relationship is only two among seven variables, meaning that the dimension of the unit root processes is five. In other words, if the dimensionality of trend in the data were one (i.e., cointegration rank were six), the first differencing would be consistent because our model has a balanced growth path. But, the problem is that the data suggests that the dimension of trend is much higher than one. The point is, the problem is neither of estimation methods or detrending tools; to treat the trend seriously, we need to have a model that can accommodate a higher trend dimension.

## 6 Conclusion

In this paper, we investigate the relative importance of trend and cycle components in key U.S. macroeconomic variables by using multivariate Beveridge-Nelson (MVBN) decomposition. The main finding in this paper is that some variables are dominated by their trend component. First, inflation fluctuation seems to be dominated by its trend component, and, perhaps as a result of this, the short-term nominal interest rate is also dominated its trend component. It seems that the Fed reacts to output in the cyclical component, while it reacts to inflation in the trend component. If we take this result seriously, evaluating monetary policy after detrending data almost means that we ignore the central bank's effort to curb the inflation. Indeed, if we feed the cyclical component of the MVBN decomposition as a detrended data to a standard medium scale DSGE model, it seems that as if the Fed does not react to the inflation very much, which is perhaps true if (and only if) we focus only on the cyclical component of the data. But, it is not really what people have in mind. Second, consumption also seems to be dominated by its trend component. While the trend and cyclical components are equally important in output (income) growth, the consumption growth is governed by the trend component of the output. This is we think because of the consumption smoothing. The preceding papers have emphasized counter-evidence against the permanent income hypothesis (PIH). Certainly, we also find similar counter-evidence, in the sense that PIH does not hold *perfectly*. However, what we want to emphasize is that nonetheless the mechanism of PIH is still working, and indeed the large portion of consumption growth is determined by the trend component of income (output). This means that, if we detrend consumption, the most important part of consumption dynamics may be lost.

These findings show that the dichotomy between the trend and cycle has a fundamental pitfall. And, so does the idea of detrending ahead of a DSGE estimation. In our opinion, an ideal treatment of the trend is to estimate the trend process consistently within a model as Ferroni (2011) suggested. However, the point here does not lie on estimation techniques or detrending methods. Indeed, under

---

<sup>20</sup>See also Justiniano, Primiceri and Tambalotti (2010) for DSGE estimation.

the hypothesis that the dimension of the trend is one, for example, Smets and Wouters' (2007) data treatment (the first differenced model variables and data are matched in the measurement equations) is internally consistent. Rather, the true problem lies on the economic model per se. That is, in the case of our data set, the dimension of the trend components is five, while the model can accommodate only uni-dimensional trend. We could extend it by adding investment specific technology shock a la Greenwood, Hercowitz and Krusel (1997) and, say, non-stationary target inflation rate of the monetary policy rule (for nominal trend). But, even if do so, the dimension of a model is still short of the dimensionality that the data suggests.

## References

- Adjemian, S., Bastani, H., Juillard, M., Mihoubi, F., Perendia, G., Ratto, M., & Villemot, S. (2011). Dynare: Reference manual, version 4. Dynare Working Papers 1, CEPREMAP.
- Aguiar, M., & Gopinath, G. (2007). Emerging market business cycles: The cycle is the trend. *Journal of Political Economy* 115(1), 69-102.
- Arino, M. A., & Newbold, P. (1998). Computation of the Beveridge–Nelson decomposition for multivariate economic time series. *Economics Letters*, 61(1), 37-42.
- Asdrubali, P., Sorensen, B. E., & Yosha, O. (1996). Channels of interstate risk sharing: United States 1963-1990. *The Quarterly Journal of Economics*, 1081-1110.
- Attanasio, O. P., & Pavoni, N. (2011). Risk sharing in private information models with asset accumulation: Explaining the excess smoothness of consumption. *Econometrica*, 79(4), 1027-1068.
- Attanasio, O. P., & Weber, G. (1994). Is consumption growth consistent with intertemporal optimization? Evidence from the consumer expenditure survey. *Journal of Political Economy*, 103(6), 1121-1157.
- Baxter, M., & Farr, D. D. (2005). Variable capital utilization and international business cycles. *Journal of International Economics*, 65(2), 335-347.
- Baxter, M., & King, R. G. (1999). Measuring business cycles: approximate band-pass filters for economic time series. *Review of economics and statistics*, 81(4), 575-593.
- Bierens, H. J. (1997). Testing the unit root with drift hypothesis against nonlinear trend stationarity, with an application to the US price level and interest rate. *Journal of Econometrics*, 81(1), 29-64.
- Beveridge, S., & Nelson, C. R. (1981). A new approach to decomposition of economic time series into permanent and transitory components with particular attention to measurement of the ‘business cycle’. *Journal of Monetary economics*, 7(2), 151-174.
- Beyaert, A., & Quesada Medina, A. J. (2001). Computation of the Beveridge–Nelson decomposition in the case of cointegrated systems with  $I(0)$  variables. *Economics Letters*, 72(3), 283-289.
- Blundell, R., Browning, M., & Meghir, C. (1994). Consumer demand and the life-cycle allocation of household expenditures. *The Review of Economic Studies*, 61(1), 57-80.
- Blundell, R., Pistaferri, L., & Preston, I. (2008). Consumption inequality and partial insurance. *The American Economic Review*, 1887-1921.

- Campbell, J. (1987). Does Saving Anticipate Declining Labor Income? An alternative test of the permanent income hypothesis. *Econometrica*, 55(6), 107-118.
- Campbell, J., & Deaton, A. (1989). Why is consumption so smooth?. *The Review of Economic Studies*, 56(3), 357-373.
- Canova, F. (1998). Detrending and business cycle facts. *Journal of monetary economics*, 41(3), 475-512.
- Canova, F. (2009). Bridging cyclical DSGE models and the raw data. Manuscript, Centre de Recerca en Economia Internacional.
- Canova, F., & Ferroni, F. (2011). Multiple filtering devices for the estimation of cyclical DSGE models. *Quantitative Economics*, 2(1), 73-98.
- Cochrane, J. H. (1988). How big is the random walk in GNP?. *The Journal of Political Economy*, 893-920.
- Cogley, T., & Nason, J. M. (1995). Effects of the Hodrick-Prescott filter on trend and difference stationary time series Implications for business cycle research. *Journal of Economic Dynamics and control*, 19(1), 253-278.
- Cogley, T., Primiceri, G. E. & Sargent, T. (2010). Inflation-gap persistence in the U.S. *American Economic Journal: Macroeconomics*, 2(1) 43–69.
- Cuddington, J. T., & Winters, L. A. (1987). The Beveridge-Nelson decomposition of economic time series: A quick computational method. *Journal of Monetary Economics*, 19(1), 125-127.
- de Silva, A. (2008). Forecasting macroeconomic variables using a structural state space model.
- Devereux, P. J. (2001). The cyclicity of real wages within employer-employee matches. *Industrial & Labor Relations Review*, 54(4), 835-850.
- Diebold, F. X., & Rudebusch, G. D. (1989). Long memory and persistence in aggregate output. *Journal of monetary economics*, 24(2), 189-209.
- Ferroni, F. (2011). Trend agnostic one-step estimation of DSGE models. *The BE Journal of Macroeconomics*, 11(1).
- Garratt, A., Robertson, D., & Wright, S. (2006). Permanent vs transitory components and economic fundamentals. *Journal of Applied Econometrics*, 21(4), 521-542.
- Gomme, P. (1993). Money and growth revisited: Measuring the costs of inflation in an endogenous growth model. *Journal of Monetary economics*, 32(1), 51-77.

- Greenwood, J., Hercowitz, Z., & Krusell, P. (1997). Long-run implications of investment-specific technological change. *The American Economic Review*, 342-362.
- Harvey, A. C., & Jaeger, A. (1993). Detrending, stylized facts and the business cycle. *Journal of applied econometrics*, 8(3), 231-247.
- Henry, O. T., & Shields, K. (2004). Is there a unit root in inflation?. *Journal of Macroeconomics*, 26(3), 481-500.
- Ho, W. M. (1996). Imperfect information, money, and economic growth. *Journal of Money, Credit and Banking*, 578-603.
- Hryshko, D. (2011). Excess smoothness of consumption in an estimated life cycle. Mimeo, University of Alberta.
- Johansen, S. (1995). Likelihood-based inference in cointegrated vector autoregressive models. OUP Catalogue.
- Justiniano, A., Primiceri, G. E., & Tambalotti, A. (2010). Investment shocks and business cycles. *Journal of Monetary Economics*, 57(2), 132-145.
- Kaplan, G., & Violante, G. L. (2010). How much consumption insurance beyond self-insurance. *American Economic Journal: Macroeconomics*, 2(4), 53-87.
- Landefeld, J. S., & Parker, R. P. (1997). BEA's chain indexes, time series, and measures of long-term economic growth. *Survey of Current Business*, 77(5), 58-68.
- Luengo-Prado, M. J., & Sørensen, B. E. (2008). What can explain excess smoothness and sensitivity of state-level consumption?. *The Review of Economics and Statistics*, 90(1), 65-80.
- MacKinnon, J., G. (1996). Numerical distribution functions for unit root and cointegration tests. *Journal of applied econometrics*, 11, 601-618.
- Mehra, Y. P. (2004). The output gap, expected future inflation and inflation dynamics: another look. *Topics in Macroeconomics*, 4(1).
- Miller, S. M. (1988). The Beveridge-Nelson decomposition of economic time series: Another economical computational method. *Journal of Monetary Economics*, 21(1), 141-142.
- Mishkin, F. S. (1992). Is the Fisher effect for real?: A reexamination of the relationship between inflation and interest rates. *Journal of Monetary economics*, 30(2), 195-215.
- Morley, J. C. (2011). The two interpretations of the Beveridge–Nelson decomposition. *Macroeconomic Dynamics*, 15(03), 419-439.

- Morley, J. C. (2002). A state–space approach to calculating the Beveridge–Nelson decomposition. *Economics Letters*, 75(1), 123-127.
- Morley, J. C., Nelson, C. R., & Zivot, E. (2003). Why are the Beveridge-Nelson and unobserved-components decompositions of GDP so different?. *Review of Economics and Statistics*, 85(2), 235-243.
- Newbold, P. (1990). Precise and efficient computation of the Beveridge-Nelson decomposition of economic time series. *Journal of Monetary Economics*, 26(3), 453-457.
- Ohara, H. I. (1999). A Unit Root Test With Multiple Trend Breaks: A Theory and an Application to US and Japanese Macroeconomic Time-Series. *Japanese Economic Review*, 50(3), 266-290.
- Ostergaard, C. Bent, E. Sørensen, & Oved Yosha (2002), Consumption and Aggregate Constraints: Evidence from US States and Canadian Provinces. *Journal of Political Economy*, 110, 634-645.
- Quah, D. (1990). Permanent and transitory movements in labor income: an explanation for” excess smoothness” in consumption. *Journal of Political Economy*, 449-475.
- Quah, D. (1992). The relative importance of permanent and transitory components: identification and some theoretical bounds. *Econometrica: Journal of the Econometric Society*, 107-118.
- Romero-Ávila, D., & Usabiaga, C. (2007). Unit root tests, persistence, and the unemployment rate of the US states. *Southern Economic Journal*, 698-716.
- Rose, A. K. (1988). Is the real interest rate stable?. *The Journal of Finance*, 43(5), 1095-1112.
- Seskin, E. P., & Parker, R. P. (1998). A Guide to the NIPA’s. *Survey of Current Business*, 78(3), 26-68.
- Shin, D., & Solon, G. (2007). New evidence on real wage cyclicalities within employer–employee matches. *Scottish Journal of Political Economy*, 54(5), 648-660.
- Smets, Frank, and Rafael Wouters (2007). Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach. *American Economic Review*, 97(3), 586-606.
- Stock, J. H., & Watson, M. W. (1988). Testing for common trends. *Journal of the American statistical Association*, 83(404), 1097-1107.
- Stock, J. H., & Watson, M. W. (1999). Forecasting inflation. *Journal of Monetary Economics*, 44(2), 293-335.
- Whelan, K. (2006). New evidence on balanced growth, stochastic trends, and economic fluctuations. Mimeo.

# 7 Figures and Tables

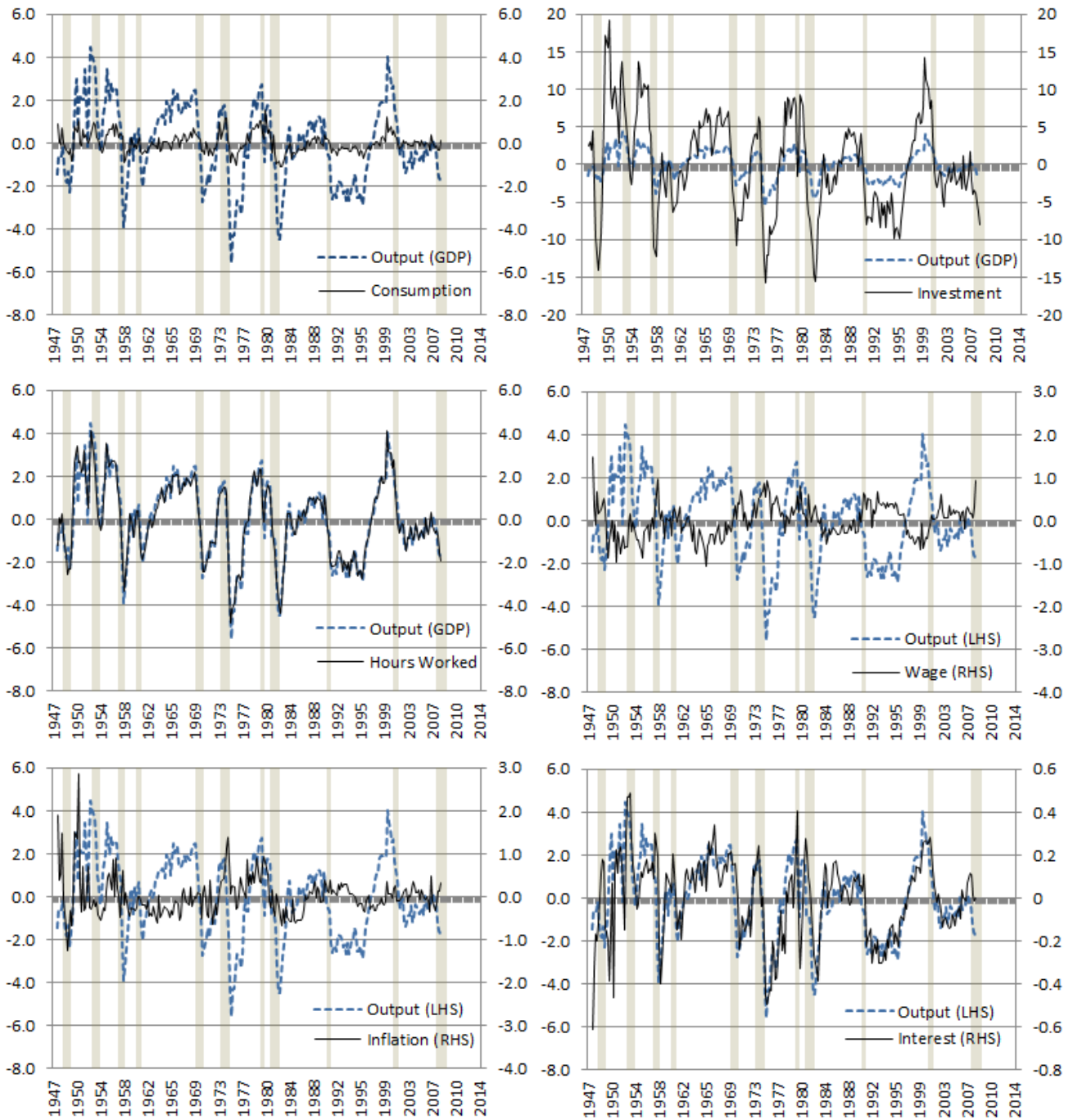


Figure 1: Cyclical Components of Beveridge-Nelson Decomposition based on VECM with lag order 2 and cointegration rank 2. The shaded area show the peak to bottom of NBER business cycle dating.

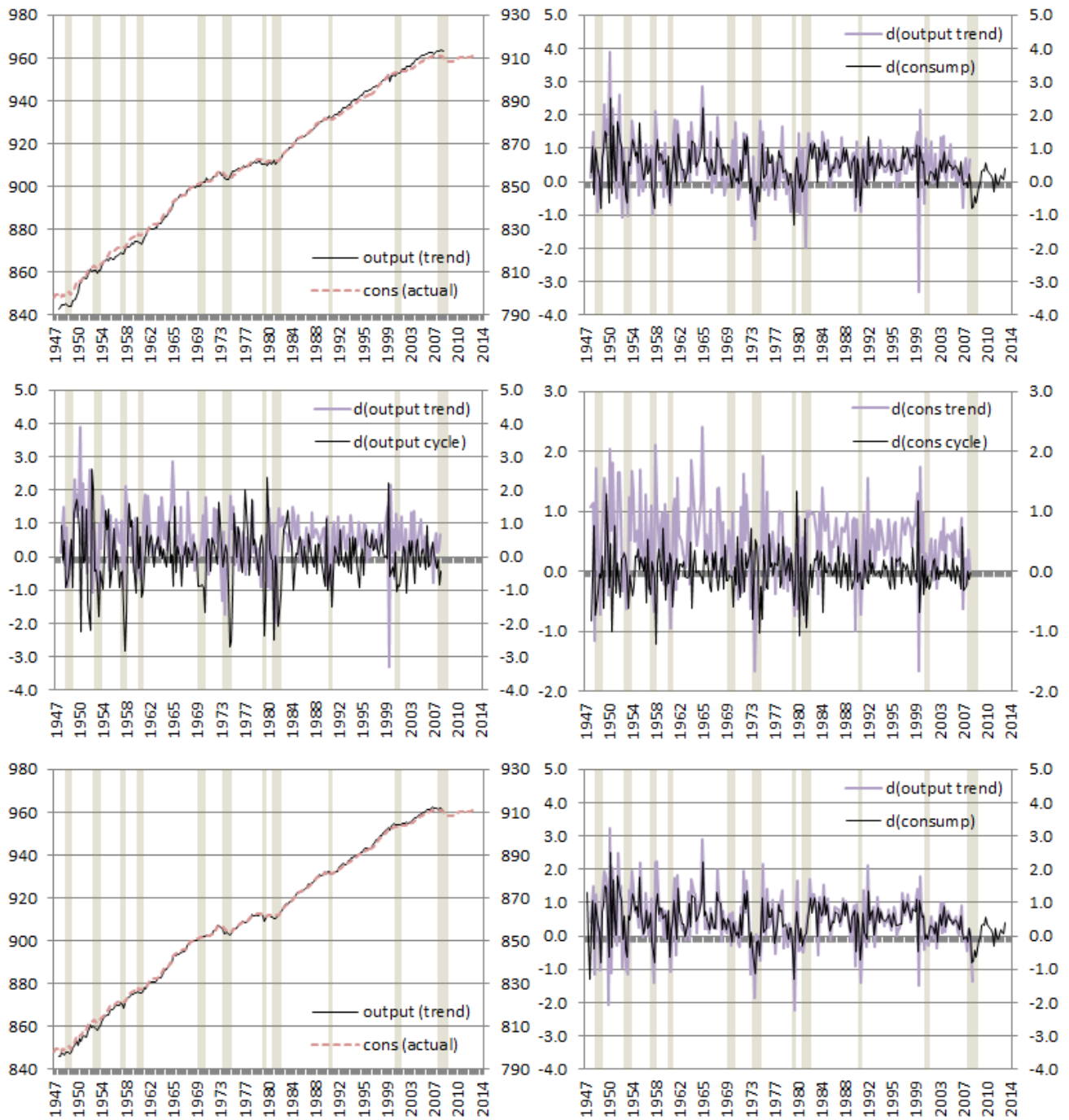


Figure 2: Multivariate Beveridge-Nelson Decomposition and Consumption Smoothing. For the top left and bottom left panels, consumption is on right axis and output is on left axis. The bottom two panels are based on bivariate MVBN decomposition with output and consumption only. For the top and middle four panels, see the note on Figure 1.

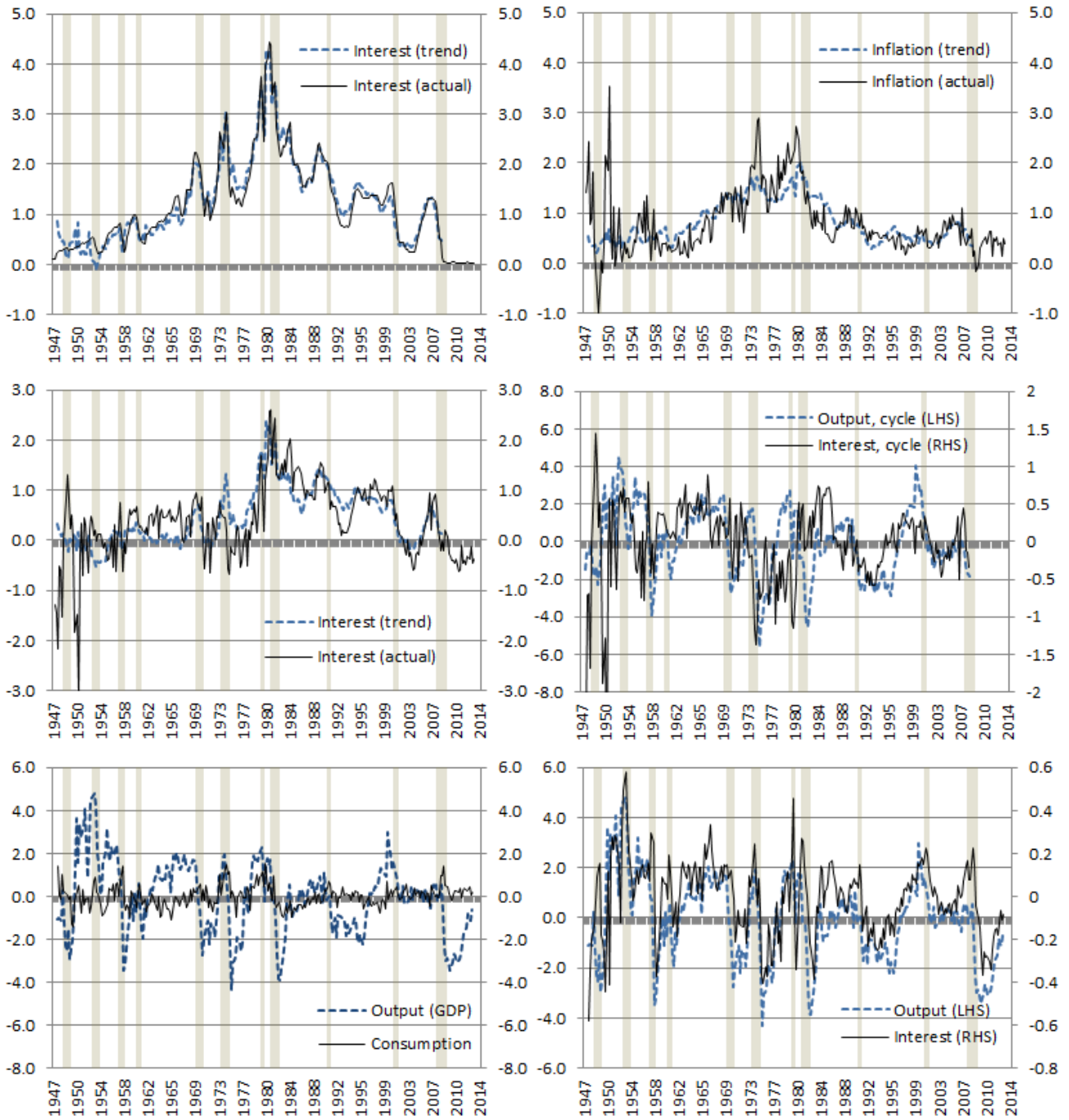


Figure 3: Miscellaneous Figures. The top two panels are the results of MVBN decomposition with nominal interest rate. The middle two panels are those with real interest rate. For all variables other than interest rate, regardless of the real or nominal interest rate, the results are identical to those in Figure 1. The bottom two panels show the consumption and interest rate with ZLB periods.

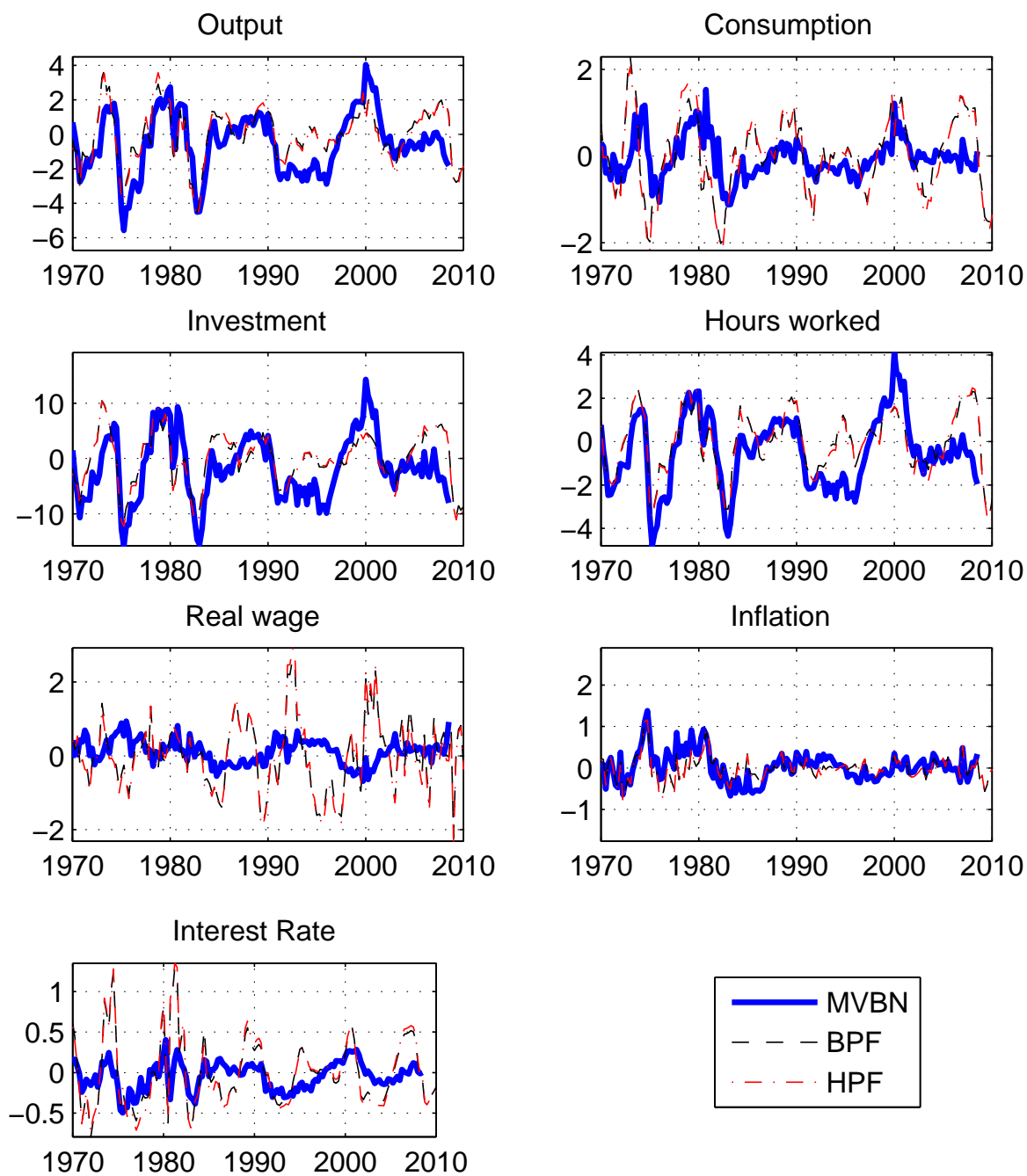


Figure 4: The cycle component of multivariate BN (MVBN) decomposition and HP-filter (penalty parameter = 1600) and High-Pass Filter (Baxter-King Band Pass filter with higher frequencies than 32 quarters). To make the results visible, the period prior to 1970 is truncated.

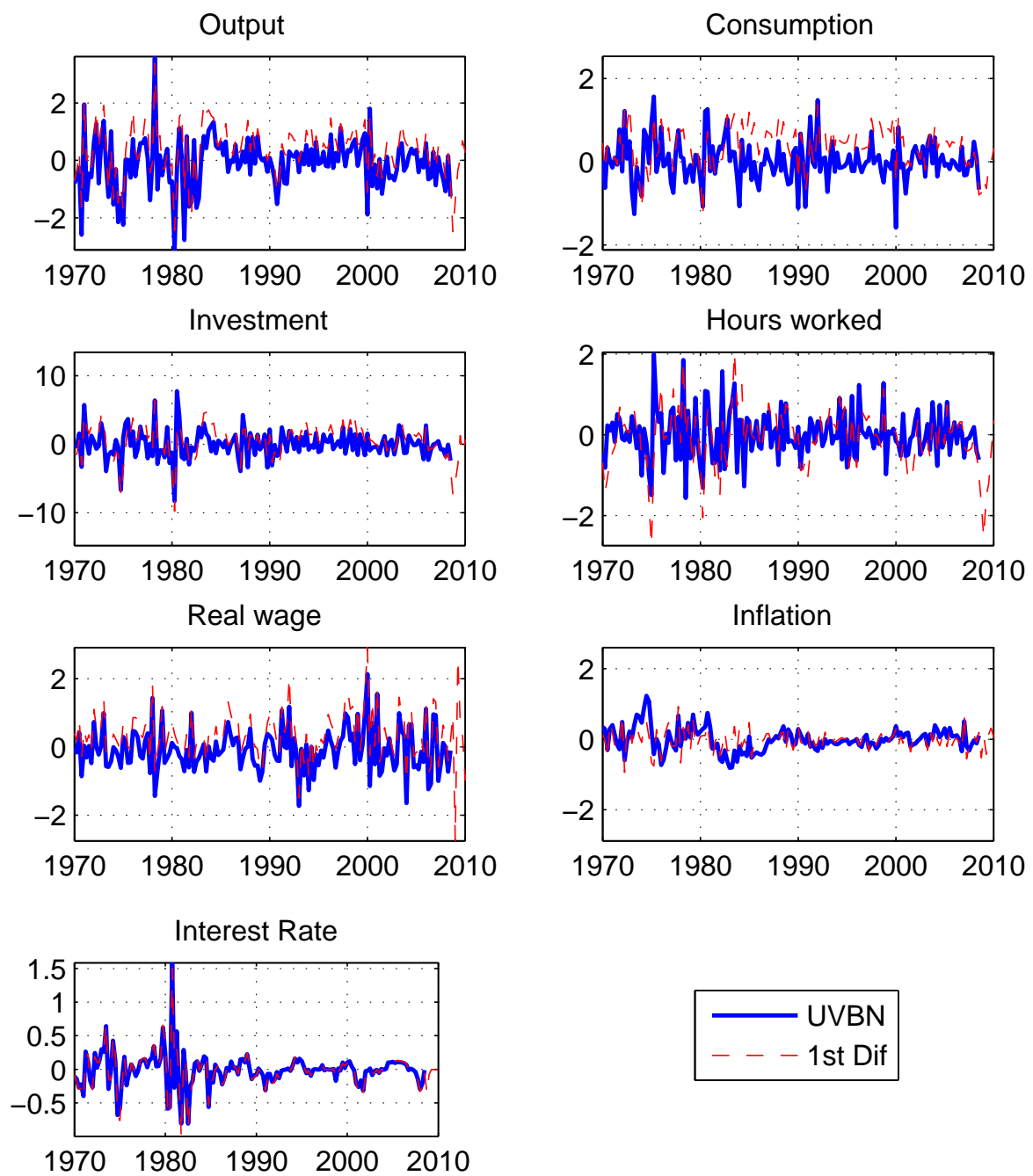


Figure 5: The cyclical component of univariate BN decomposition (UVBN) and the first differencing (1st Dif). To make the results visible, the period prior to 1970 is truncated.

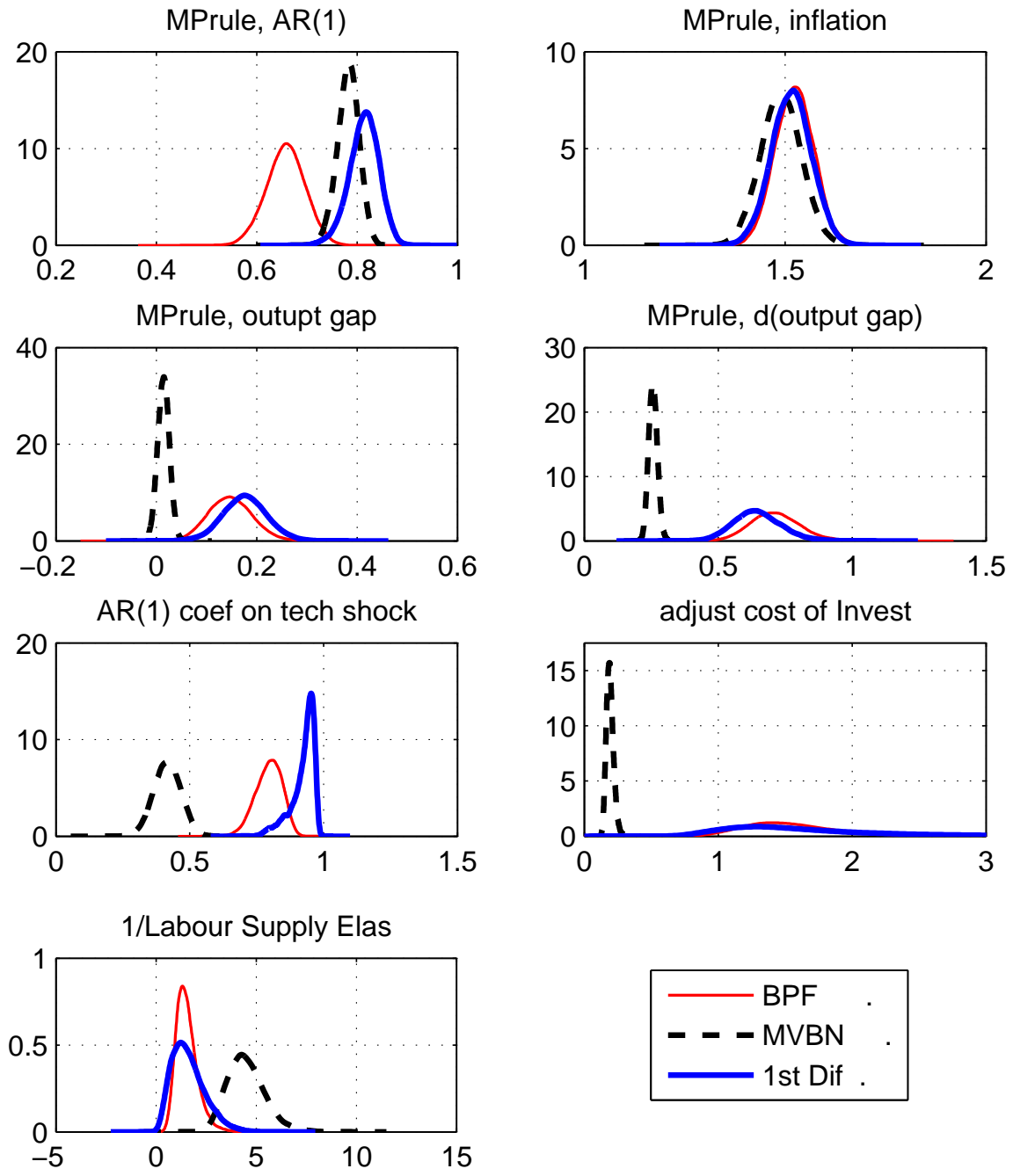


Figure 6: Posterior Density of Selected Parameters.

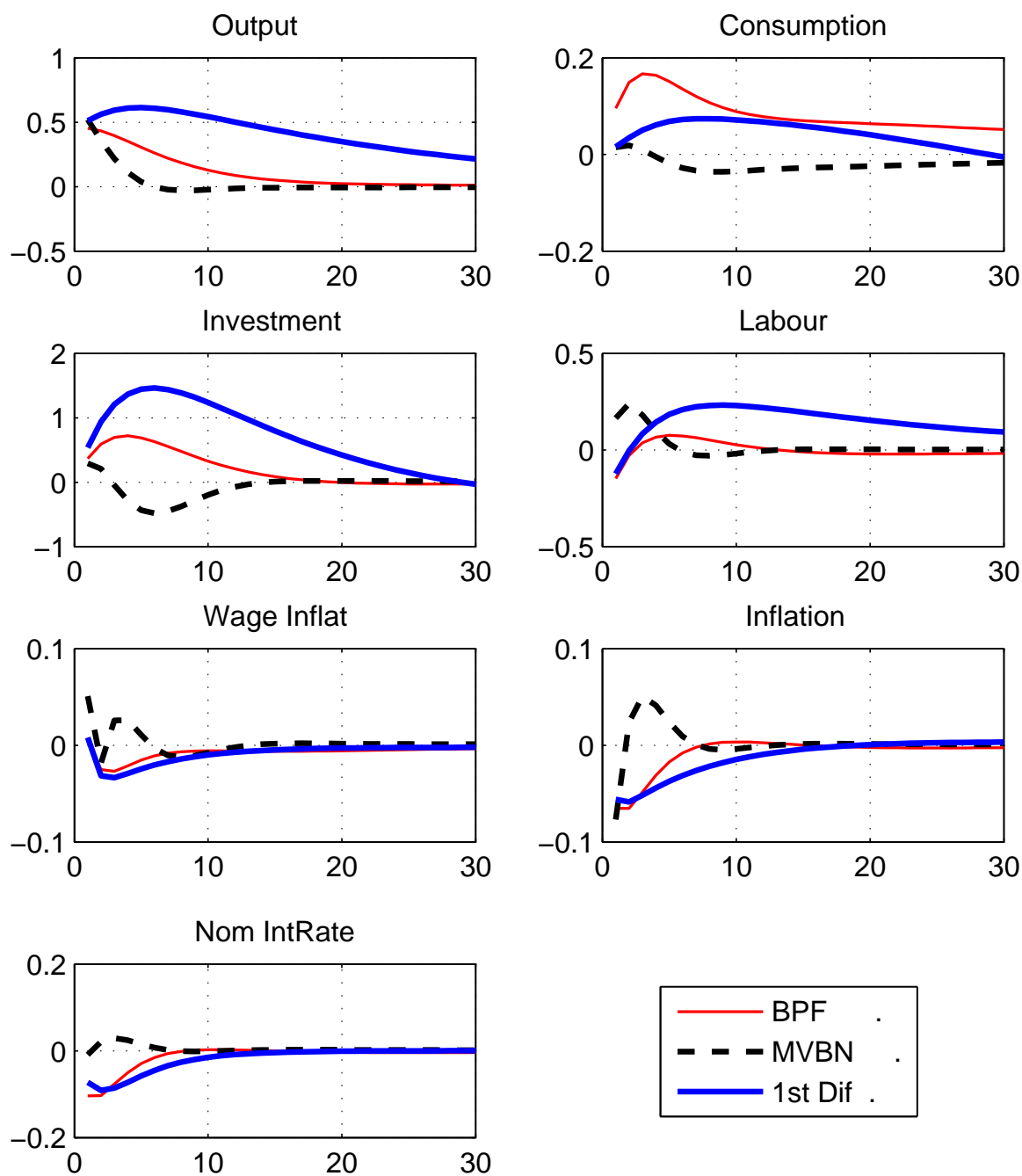


Figure 7: Selected IRFs to stationary production technology shock. BPF, MVBN and 1st Dif mean Baxter-King band-pass filter for 2-32 quarters, multivariate Beveridge-Nelson decomposition and first differencing, respectively.

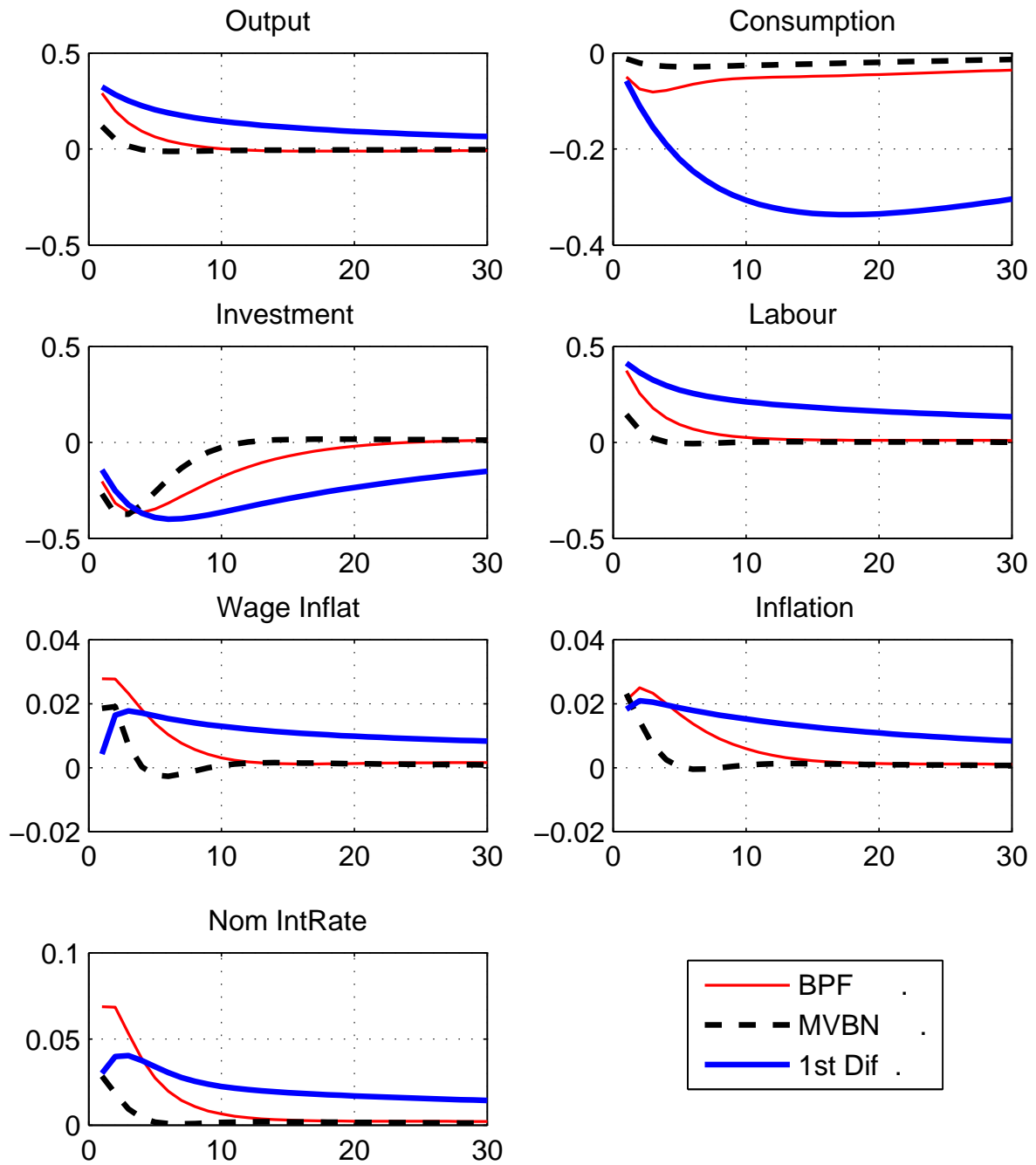


Figure 8: Selected IRFs to government expenditure shock. BPF, MVBN and 1st Dif mean Baxter-King band-pass filter for 2-32 quarters, multivariate Beveridge-Nelson decomposition and first differencing, respectively.

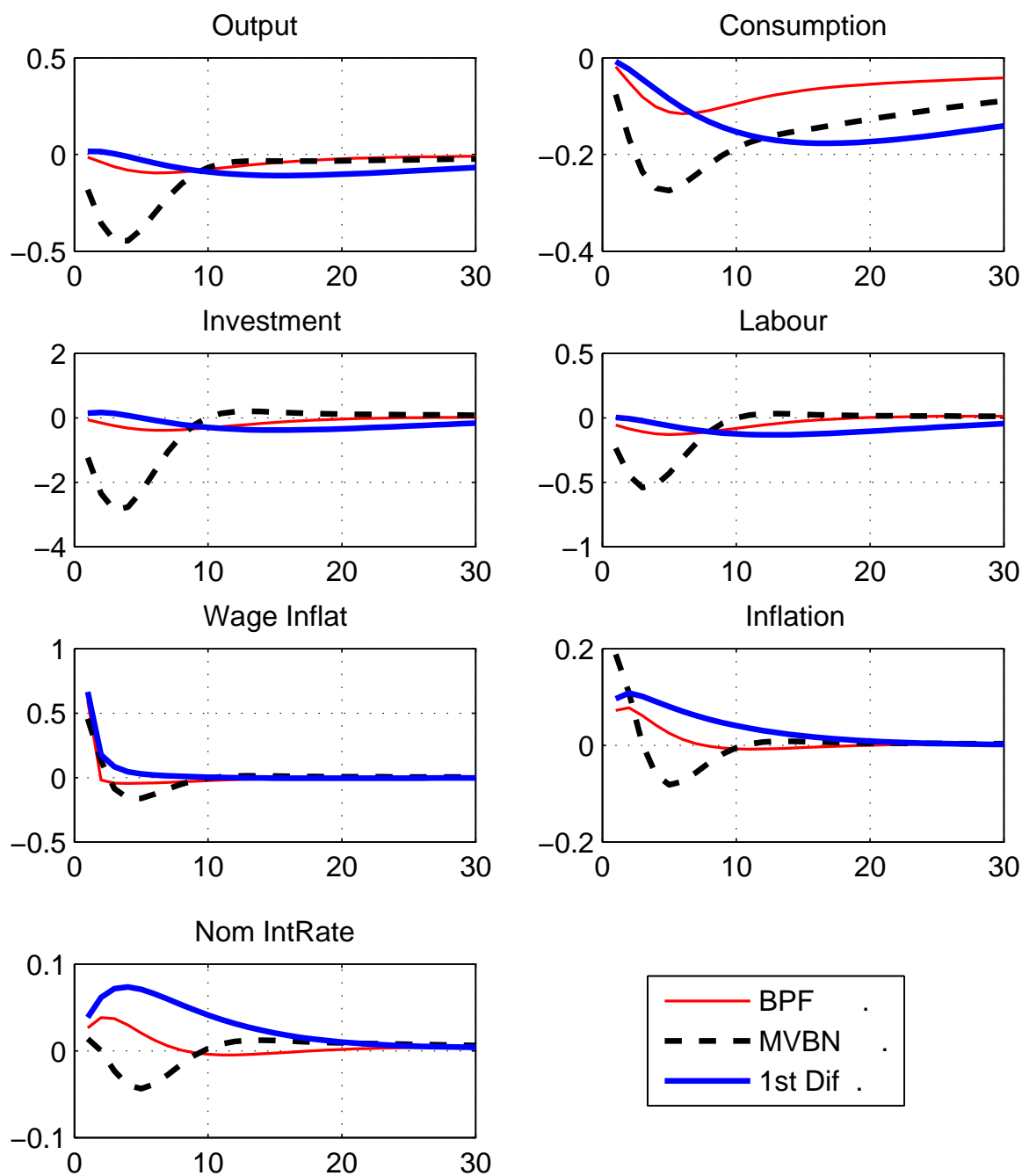


Figure 9: Selected IRFs to wage mark-up shock. BPF, MVBN and 1st Dif mean Baxter-King band-pass filter for 2-32 quarters, multivariate Beveridge-Nelson decomposition and first differencing, respectively.

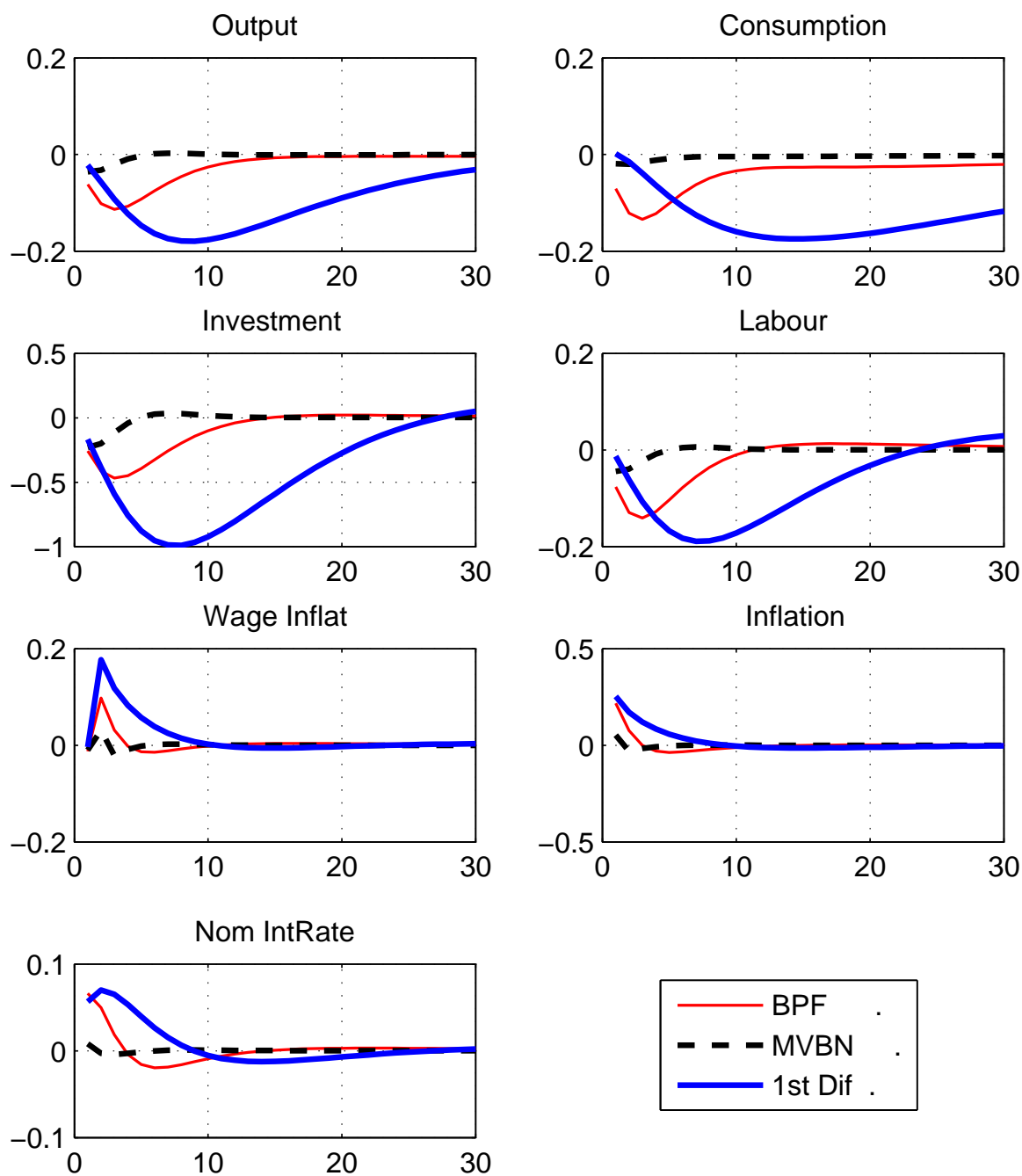


Figure 10: Selected IRFs to the shock on the marginal cost (cost push shock). BPF, MVBN and 1st Dif mean Baxter-King band-pass filter for 2-32 quarters, multivariate Beveridge-Nelson decomposition and first differencing, respectively.

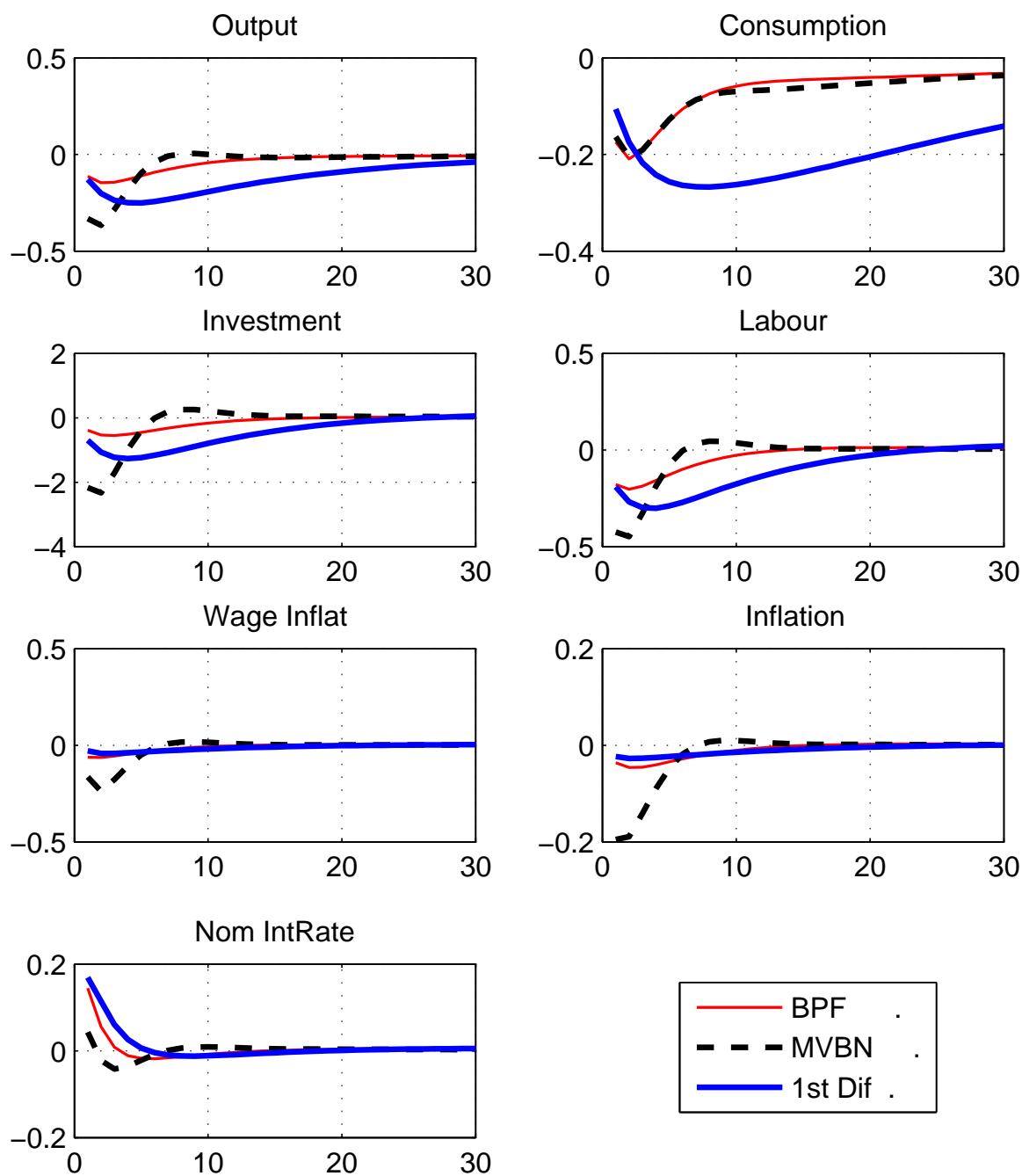


Figure 11: Selected IRFs to monetary policy shock. BPF, MVBN and 1st Dif mean Baxter-King band-pass filter for 2-32 quarters, multivariate Beveridge-Nelson decomposition and first differencing, respectively.

Table 1: Correlation Matrix for MVBN Decomposition

	<b>Output</b>	<b>Consump</b>	<b>Invest</b>	<b>Hours</b>	<b>Wage</b>	<b>Inflation</b>	<b>Int Rate</b>
<b>Output</b>	1.86	0.64	10.84	3.21	-0.42	0.01	0.27
<b>Consumption</b>	0.76	0.45	2.06	0.59	-0.00	0.08	0.05
<b>Investment</b>	0.95	0.75	6.10	10.32	-1.40	0.39	0.67
<b>Hours worked</b>	0.99	0.76	0.97	1.74	-0.41	0.05	0.24
<b>Real wage</b>	-0.61	-0.03	-0.62	-0.63	0.37	0.06	-0.03
<b>Inflation</b>	0.01	0.49	0.18	0.09	0.45	0.36	-0.01
<b>Interest Rate</b>	0.79	0.59	0.61	0.77	-0.45	-0.12	0.18

Note: (a) The lower right part shows the correlations, (b) the diagonal elements shows the standard deviations, and (c) the upper left part shows the covariances.

Table 2: Auto-Correlations for MVBN Decomposition

lead/lag:	<b>0</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>	<b>6</b>	<b>7</b>	<b>8</b>
<b>Output</b>	1.00	0.90	0.75	0.61	0.48	0.36	0.26	0.17	0.11
<b>Consumption</b>	1.00	0.71	0.60	0.49	0.33	0.20	0.10	-0.03	-0.04
<b>Investment</b>	1.00	0.89	0.74	0.59	0.46	0.32	0.21	0.11	0.03
<b>Hours worked</b>	1.00	0.92	0.78	0.64	0.50	0.37	0.26	0.17	0.10
<b>Real wage</b>	1.00	0.72	0.65	0.62	0.52	0.44	0.44	0.38	0.34
<b>Inflation</b>	1.00	0.60	0.53	0.46	0.47	0.30	0.28	0.14	0.15
<b>Interest Rate</b>	1.00	0.81	0.61	0.48	0.38	0.29	0.24	0.19	0.14

Table 3: Cross-Correlations with Output for BMVN Decomposition

lead/lag:	<b>-8</b>	<b>-4</b>	<b>-3</b>	<b>-2</b>	<b>-1</b>	<b>0</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>8</b>
<b>Output</b>	0.11	0.48	0.61	0.75	0.90	1.00	0.90	0.75	0.61	0.48	0.11
<b>Consumption</b>	-0.17	0.15	0.29	0.44	0.60	0.76	0.74	0.68	0.58	0.47	0.08
<b>Investment</b>	0.06	0.50	0.64	0.79	0.91	0.95	0.82	0.68	0.54	0.42	0.07
<b>Hours worked</b>	0.10	0.49	0.62	0.76	0.90	0.99	0.91	0.77	0.63	0.49	0.11
<b>Real wage</b>	-0.40	-0.64	-0.67	-0.69	-0.68	-0.61	-0.54	-0.42	-0.30	-0.21	-0.09
<b>Inflation</b>	-0.40	-0.24	-0.15	-0.06	-0.03	0.01	0.13	0.17	0.17	0.16	-0.03
<b>Interest Rate</b>	0.13	0.28	0.36	0.46	0.59	0.79	0.90	0.82	0.70	0.57	0.18

Note: The minus sign in the header shows it leads relative to output.

Table 4: Comparison of Standard Deviations among Several Filters

	Output		Consump		Invest		Hours		Wage		Inflation		Int Rate	
<b>MVBN</b>	1.86	1.00	0.45	0.24	6.10	3.27	1.74	0.93	0.37	0.20	0.36	0.19	0.18	0.10
<b>UVBN</b>	0.92	1.00	0.56	0.61	2.35	2.57	0.66	0.72	0.60	0.66	0.33	0.36	0.22	0.24
<b>1st Dif</b>	0.92	1.00	0.50	0.55	2.34	2.54	0.69	0.75	0.61	0.66	0.31	0.34	0.23	0.25
<b>BPF</b>	1.52	1.00	0.82	0.54	4.12	2.71	1.32	0.86	0.79	0.52	0.28	0.18	0.36	0.24
<b>HPF</b>	1.59	1.00	0.86	0.54	4.41	2.78	1.38	0.87	0.84	0.53	0.28	0.18	0.38	0.24
<b>Linear</b>	3.60	1.00	2.70	0.75	8.75	2.43	2.92	0.81	3.34	0.93	0.58	0.16	0.83	0.23

Notes: MVBN, UVBN, 1st Dif, BPF, HPF and Linear stand for multivariate Beveridge-Nelson decomposition, univariate Beveridge-Nelson decomposition, first differencing, band pass filter (for less than 32 quarters), Hodrick-Presscot filter and linear detrending, respectively. Each column item shows (a) the standard deviations, and (b) their relative values to output volatility.

Table 5: Correlation between Several Filters and MVBN Decomposition

	Output			Consumption			Invest			Hours			Wage			Inflation			Int Rate		
<b>MVBN</b>	1.00	1.00	0	1.00	1.00	0	1.00	1.00	0	1.00	1.00	0	1.00	1.00	0	1.00	1.00	0	1.00	1.00	0
<b>UVBN</b>	0.07	0.31	-3	-0.30	-0.30	0	-0.05	-0.33	1	-0.16	-0.30	1	-0.26	-0.26	0	0.74	0.74	0	-0.10	0.30	-2
<b>1st Dif</b>	0.01	-0.35	2	-0.35	-0.35	0	0.08	-0.41	3	0.04	-0.44	3	-0.31	-0.31	0	0.45	0.45	0	-0.07	0.32	-2
<b>BPF</b>	0.69	0.70	-1	0.32	0.59	-3	0.62	0.62	0	0.67	0.67	0	-0.09	0.26	-12	0.77	0.77	0	0.48	0.53	-1
<b>HPF</b>	0.69	0.71	-1	0.33	0.61	-3	0.62	0.62	0	0.67	0.67	0	-0.09	0.26	-12	0.76	0.76	0	0.47	0.52	-1
<b>Linear</b>	0.53	0.53	-1	0.19	0.30	-3	0.54	0.54	0	0.53	0.53	0	0.09	0.36	-12	0.70	0.70	0	0.10	-0.31	19

Notes: Each column item shows (a) the contemporaneous correlation with MVBN decomposition, (b) the maximum absolute value of the cross-correlation, and (c) the lead/lag period of (b), where minus means it leads relative to MVBN decomposition. See also the note on Table 4.

Table 6: Correlation between Several Filters and First Difference

	Output			Consumption			Invest			Hours			Wage			Inflation			Int Rate		
<b>MVBN</b>	0.01	-0.37	-2	-0.37	-0.37	0	0.08	-0.42	-3	0.04	-0.45	-3	-0.31	-0.31	0	0.45	0.45	0	-0.07	0.33	2
<b>UVBN</b>	0.86	0.86	0	0.62	0.62	0	0.77	0.77	0	0.61	0.61	0	0.96	0.96	0	0.65	0.65	0	0.97	0.97	0
<b>1st Dif</b>	1.00	1.00	0	1.00	1.00	0	1.00	1.00	0	1.00	1.00	0	1.00	1.00	0	1.00	1.00	0	1.00	1.00	0
<b>BPF</b>	0.26	-0.51	-3	0.22	-0.49	-4	0.22	-0.51	-3	0.22	-0.54	-4	0.35	0.35	0	0.55	0.55	0	0.30	-0.43	-2
<b>HPF</b>	0.25	-0.50	-3	0.21	-0.47	-4	0.20	-0.49	-3	0.21	-0.52	-4	0.33	0.33	0	0.54	0.54	0	0.28	-0.41	-2
<b>Linear</b>	0.13	0.26	2	0.08	-0.27	-8	0.13	-0.31	-4	0.11	-0.30	-4	0.05	-0.18	-11	0.26	-0.26	-1	0.14	-0.24	-6

Note: See the notes on Tables 4 and 5.

Table 7: Variance-Covariance Matrices for Consumption Smoothing

<b>7 variables</b>	<b>Output, actual</b>	<b>Consum, actual</b>	<b>Output, trend</b>	<b>Consum, trend</b>	<b>Output, cycle</b>	<b>Consum, cycle</b>
<b>Output, actual</b>	0.9791	0.2592	0.4353	0.2712	0.5234	-0.0120
<b>Consum, actual</b>	0.4961	0.5336	0.2733	0.2853	-0.0141	-0.0005
<b>Output, trend</b>	0.5435	0.6261	0.8179	0.4217	-0.2337	-0.1484
<b>Consum, trend</b>	0.4268	0.8238	0.7944	0.6490	-0.1505	-0.1359
<b>Output, cycle</b>	0.6144	-0.0303	-0.3283	-0.2665	0.8701	0.1364
<b>Consum, cycle</b>	-0.0332	-0.0028	-0.4932	-0.5691	0.4262	0.3679
<b>2 variables</b>	<b>Output, actual</b>	<b>Consum, actual</b>	<b>Output, trend</b>	<b>Consum, trend</b>	<b>Output, cycle</b>	<b>Consum, cycle</b>
<b>Output, actual</b>	0.9791	0.2592	0.3752	0.3366	0.5836	-0.0773
<b>Consum, actual</b>	0.4961	0.5336	0.4155	0.3728	-0.1563	-0.0880
<b>Output, trend</b>	0.4645	0.9440	0.8249	0.6104	-0.3052	-0.1949
<b>Consum, trend</b>	0.4645	0.9440	1.0000	0.7400	-0.2738	-0.1748
<b>Output, cycle</b>	0.6322	-0.3107	-0.3925	-0.3925	0.9428	0.1175
<b>Consum, cycle</b>	-0.2680	-0.5596	-0.8018	-0.8018	0.4231	0.2947

Notes: All variables are first difference of log, and "actual" is the sum of trend and cycle. The upper panel shows the results based on the benchmark VECM with seven variables, while the lower panel is based on the bivariate VECM. In each panel, (a) the lower right part shows the correlations, (b) the diagonal elements shows the standard deviations, and (c) the upper left part shows the covariances.

Table 8: Bivariate VECM

$d \ln Y_t$	coef.	s.d.	P-val	95% interval		$d \ln C_t$	coef.	s.d.	P-val	95% interval	
CE	-0.1689	0.0316	0.0000	-0.2309	-0.1069	CE	0.0120	0.0201	0.5520	-0.0274	0.0513
$d \ln Y_{t-1}$	0.1903	0.0674	0.0050	0.0583	0.3223	$d \ln Y_{t-1}$	0.1161	0.0428	0.0070	0.0323	0.1999
$d \ln Y_{t-2}$	0.1154	0.0661	0.0810	-0.0142	0.2449	$d \ln Y_{t-2}$	0.0440	0.0420	0.2950	-0.0383	0.1262
$d \ln C_{t-1}$	0.4721	0.1136	0.0000	0.2494	0.6948	$d \ln C_{t-1}$	0.0494	0.0721	0.4940	-0.0920	0.1907
$d \ln C_{t-2}$	0.2727	0.1150	0.0180	0.0473	0.4981	$d \ln C_{t-2}$	0.0457	0.0730	0.5320	-0.0975	0.1888
const.	0.0233	0.0799	0.7710	-0.1334	0.1800	const.	0.3291	0.0508	0.0000	0.2296	0.4286

Notes: This table has the estimation results of bivariate VECM with cointegration rank 1 and lag order 3. The left panel shows the estimation results of output (income) equation, while the right panel has those of consumption equation. CE stands for cointegration error, and const means constant term.

Table 9: DSGE Parameter Estimation with Different Detrending Methods: Structural Parameters

parameter	Prior			1st Diff			MVBN			BPF		
	dist	mean	s.d.	mean	5%	95%	mean	5%	95%	mean	5%	95%
$l_{ave}$ average labour	$N$	0.000	1.000	-0.659	-2.259	0.980	-0.018	-1.652	1.613	0.020	-1.604	1.682
$\pi_{ss}$ inflation at steady state	$\Gamma$	0.625	0.100	0.708	0.549	0.861	0.627	0.463	0.784	0.621	0.456	0.778
$\delta$ discount rate ( $1/\beta - 1$ )	$\Gamma$	0.250	0.100	0.187	0.084	0.288	0.167	0.059	0.270	0.183	0.064	0.299
$\gamma$ trend growth rate	$\Gamma$	0.400	0.500	0.268	0.184	0.361	0.000	0.000	0.000	0.000	0.000	0.000
$h$ external habit	$B$	0.500	0.200	0.860	0.761	0.956	0.677	0.605	0.750	0.585	0.480	0.690
$\sigma_C$ elas of intertemporal subs of C	$N$	1.500	1.500	1.070	0.946	1.198	1.071	1.032	1.110	1.134	1.013	1.256
$\sigma_L$ elas of labour supply; 1/Frisch	$N$	2.000	1.500	1.607	0.325	2.875	4.484	2.952	5.931	1.528	0.695	2.325
$\varphi_I$ adj cost on investment	$N$	4.000	1.500	1.652	0.764	2.568	0.192	0.149	0.234	1.552	0.982	2.100
$\varphi_Z$ elas of capital utilization	$\Gamma$	0.200	0.100	0.162	0.016	0.314	0.113	0.025	0.198	0.237	0.075	0.397
$\alpha$ capital share in production	$N$	0.300	0.050	0.258	0.220	0.297	0.219	0.205	0.233	0.305	0.263	0.347
$\Xi_F$ elas of subs among goods type	$N$	2.000	0.125	2.283	2.124	2.449	2.548	2.400	2.698	2.250	2.089	2.415
$\phi_W$ 1 - prob[reset wage]	$B$	0.500	0.100	0.872	0.826	0.920	0.682	0.607	0.761	0.686	0.598	0.775
$\phi_F$ 1 - prob[reset goods price]	$B$	0.500	0.100	0.692	0.632	0.749	0.185	0.168	0.202	0.524	0.444	0.600
$\iota_W$ indexation, wage	$B$	0.500	0.150	0.716	0.556	0.876	0.689	0.520	0.859	0.501	0.287	0.715
$\iota_F$ indexation, goods price	$B$	0.500	0.150	0.209	0.083	0.334	0.122	0.066	0.178	0.392	0.160	0.616
$\psi_\rho$ Taylor Rule, AR(1)	$B$	0.500	0.250	0.813	0.765	0.862	0.782	0.747	0.818	0.657	0.594	0.721
$\psi_\pi$ Taylor Rule, inflation	$N$	1.500	0.050	1.515	1.437	1.600	1.489	1.405	1.572	1.523	1.445	1.601
$\psi_Y$ Taylor Rule, output gap	$N$	0.125	0.050	0.176	0.106	0.248	0.014	-0.005	0.034	0.147	0.076	0.219
$\psi_{dY}$ Taylor Rule, d(output gap)	$N$	0.125	0.250	0.646	0.499	0.787	0.257	0.230	0.284	0.713	0.563	0.861

Note: BPF means band-pass filter for 2-32 quaters. For the prior distribution (dist),  $N, B, \Gamma, IG$  meand normal, beta, gamma and inverse gamma distribution, respectively. Means and 90% intervals are obtanied by 250,000 times MCMC resampling.

Table 10: DSGE Parameter Estimation with Different Detrending Methods: Parameters of Exogenous Processes

parameter	dist	Prior		1st Diff			MVBN			BPF		
		mean	s.d.	mean	5%	95%	mean	5%	95%	mean	5%	95%
$\zeta_\gamma$ Std.Dev, shock to trend growth rate	<i>IG</i>	0.250	2.000	0.403	0.336	0.475	0.373	0.334	0.411	0.129	0.053	0.216
$\zeta_a$ Std.Dev, shock to productivity	<i>IG</i>	0.250	2.000	0.614	0.553	0.677	0.387	0.340	0.434	0.573	0.520	0.627
$\zeta_b$ Std.Dev, shock to time preference	<i>IG</i>	0.250	2.000	0.059	0.043	0.076	0.049	0.039	0.059	0.143	0.113	0.176
$\zeta_{gv}$ Std.Dev, shock to govt expenditure	<i>IG</i>	0.250	2.000	0.643	0.557	0.729	0.256	0.225	0.289	0.607	0.533	0.678
$\zeta_i$ Std.Dev, shock to investment	<i>IG</i>	0.250	2.000	0.761	0.312	1.153	0.181	0.068	0.298	0.850	0.738	0.960
$\zeta_R$ Std.Dev, shock to monetary policy	<i>IG</i>	0.250	2.000	0.263	0.231	0.293	0.192	0.169	0.216	0.250	0.222	0.278
$\zeta_f$ Std.Dev, shock to goods price mark-up	<i>IG</i>	0.250	2.000	0.116	0.089	0.141	0.111	0.098	0.124	0.171	0.146	0.197
$\zeta_w$ Std.Dev, shock to wage mark-up	<i>IG</i>	0.250	2.000	0.280	0.234	0.325	0.247	0.213	0.280	0.334	0.285	0.382
$\rho_\gamma$ AR(1), shock to trend growth rate	<i>B</i>	0.500	0.250	0.428	0.100	0.786	0.023	0.000	0.049	0.205	0.005	0.403
$\rho_A$ AR(1), shock to productivity	<i>B</i>	0.500	0.250	0.922	0.852	0.979	0.415	0.333	0.499	0.794	0.713	0.873
$\rho_B$ AR(1), shock to time preference	<i>B</i>	0.500	0.250	0.622	0.388	0.836	0.195	0.054	0.333	0.364	0.190	0.540
$\rho_{gv}$ AR(1), shock to govt expenditure	<i>B</i>	0.500	0.250	0.977	0.948	0.999	0.675	0.610	0.742	0.801	0.732	0.870
$\rho_I$ AR(1), shock to investment	<i>B</i>	0.500	0.250	0.989	0.977	1.000	0.637	0.235	0.960	0.174	0.041	0.298
$\rho_R$ AR(1), shock to monetary policy	<i>B</i>	0.500	0.250	0.146	0.026	0.254	0.083	0.022	0.139	0.062	0.001	0.120
$\rho_F$ AR(1), shock to goods price mark-up	<i>B</i>	0.500	0.250	0.806	0.704	0.911	0.129	0.002	0.255	0.321	0.025	0.568
$\rho_W$ AR(1), shock to wage mark-up	<i>B</i>	0.500	0.250	0.346	0.080	0.595	0.340	0.180	0.503	0.215	0.010	0.391
$m_F$ MA(1), shock to goods price mark-up	<i>N</i>	0.000	0.200	0.542	0.292	0.798	0.304	0.147	0.458	0.188	-0.072	0.432
$m_W$ MA(1), shock to wage mark-up	<i>N</i>	0.000	0.200	0.163	-0.126	0.442	-0.115	-0.306	0.076	0.154	-0.081	0.380
$m_{a,g}$ covariance btw $A_t$ and $G_t$	<i>N</i>	0.500	0.200	1.259	1.096	1.417	2.034	1.893	2.176	1.154	0.985	1.327

Note: See the note on Table 9.

Table 11: DSGE Parameter Estimation with Different Detrending Methods: Variance Decomposition

	<b>Output</b>			<b>Consumption.</b>			<b>Investment</b>			<b>Hours Worked</b>			
	1st Dif	BND	BPF	1st Dif	BND	BPF	1st Dif	BND	BPF	1st Dif	BND	BPF	
<b>Trend Growth</b>	0.033	0.038	0.010	0.553	0.398	0.132	0.005	0.009	0.002	0.007	0.005	0.011	
<b>Production Tech</b>	0.505	0.245	0.580	0.006	0.011	0.184	0.106	0.022	0.212	0.109	0.068	0.053	
<b>Time Preference</b>	0.108	0.019	0.057	0.181	0.011	0.161	0.249	0.025	0.071	0.086	0.029	0.157	
<b>Govt Expenditure</b>	0.201	0.010	0.093	0.043	0.008	0.064	0.007	0.012	0.058	0.193	0.014	0.249	
<b>Investment</b>	0.091	0.001	0.108	0.128	0.001	0.137	0.469	0.004	0.405	0.461	0.002	0.184	
<b>Monetary Policy</b>	0.047	0.199	0.066	0.066	0.123	0.146	0.119	0.283	0.105	0.076	0.278	0.156	
<b>Cost Push, price</b>	0.011	0.002	0.036	0.013	0.001	0.060	0.036	0.002	0.068	0.038	0.002	0.074	
<b>Cost Push, Wage</b>	0.004	0.486	0.049	0.010	0.447	0.117	0.009	0.643	0.080	0.029	0.603	0.115	
	<b>Wage</b>			<b>Inflation</b>			<b>Interest Rate</b>						
	1st Dif	BND	BPF	1st Dif	BND	BPF	1st Dif	BND	BPF				
<b>Trend Growth</b>	0.015	0.127	0.006	0.017	0.011	0.004	0.002	0.195	0.034				
<b>Production Tech</b>	0.078	0.085	0.212	0.065	0.059	0.113	0.015	0.071	0.173				
<b>Time Preference</b>	0.312	0.171	0.290	0.010	0.015	0.044	0.002	0.003	0.020				
<b>Govt Expenditure</b>	0.048	0.048	0.104	0.030	0.004	0.026	0.001	0.004	0.011				
<b>Investment</b>	0.309	0.012	0.115	0.097	0.001	0.021	0.041	0.001	0.065				
<b>Monetary Policy</b>	0.105	0.235	0.175	0.022	0.537	0.095	0.003	0.079	0.035				
<b>Cost Push, price</b>	0.043	0.004	0.060	0.492	0.021	0.530	0.152	0.009	0.156				
<b>Cost Push, Wage</b>	0.090	0.319	0.038	0.266	0.351	0.166	0.784	0.638	0.507				

Note: See the note on Table 9.

## Recent Kent Discussion Papers in Economics

15/17: 'Efficient estimation with many weak instruments using regularization techniques', Marine Carrasco and Guy Tchuente

15/16: 'High school human capital portfolio and college outcomes', Guy Tchuente

15/15: 'Regularized LIML for many instruments', Marine Carrasco and Guy Tchuente

15/14: 'Agglomeration Economies and Productivity Growth: U.S. Cities, 1880-1930', Alexander Klein and Nicholas Crafts

15/13: 'Microcredit with Voluntary Contributions and Zero Interest Rate - Evidence from Pakistan', Mahreen Mahmud

15/12: 'Act Now: The Effects of the 2008 Spanish Disability Reform', Matthew J. Hill, Jose Silva and Judit Vall

15/11: 'Testing for Level Shifts in Fractionally Integrated Processes: a State Space Approach', Davide Delle Monache, Stefano Grassi and Paolo Santucci de Magistris

15/10: 'German Wage Moderation and European Imbalances: Feeding the Global VAR with Theory', Timo Bettendorf and Miguel A. León-Ledesma

15/09: 'Repaying Microcredit Loans: A Natural Experiment on Liability Structure', Mahreen Mahmud

15/08: 'Fundamental shock selection in DSGE models', Filippo Ferroni, Stefano Grassi and Miguel A. León-Ledesma

15/07: 'Direct calibration and comparison of agent-based herding models of financial markets', Sylvain Barde

15/06: 'Efficiency in a forced contribution threshold public good game', Edward Cartwright and Anna Stepanova

15/05: 'Military Aid, Direct Intervention and Counterterrorism', María D.C. García-Alonso, Paul Levine and Ron Smith

15/04: 'A Practical, Universal, Information Criterion over Nth Order Markov Processes', Sylvain Barde

15/03: 'Public Good Provision in Indian Rural Areas: the Returns to Collective Action by Microfinance Groups', Paolo Casini, Lore Vandewalle and Zaki Wahhaj

15/02: 'Fiscal multipliers in a two-sector search and matching model', Konstantinos Angelopoulos, Wei Jiang and James Malley

15/01: 'Military Aid, Direct Intervention and Counterterrorism', María D.C. García-Alonso, Paul Levine and Ron Smith