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**Abstract**

We contrast and compare three ways of predicting efficiency in a forced contribution threshold public good game. The three alternatives are based on ordinal potential, quantal response and impulse balance theory. We report an experiment designed to test the respective predictions and find that impulse balance gives the best predictions. A simple expression detailing when enforced contributions result in high or low efficiency is provided.

**Keywords**: Public good, threshold, impulse balance theory, quantal response, forced contribution, ordinal potential.

**JEL codes**: C72, H41, C92

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**Non-technical summary**

There are many important contexts where 'success' requires a critical amount of activity. Consider, for example, a political party deciding whether to adopt a policy which is socially efficient but, for some reason, unpopular with voters. The policy will be enacted if and only if enough party members are willing to back it. Individual members face a difficult choice: the policy is socially efficient and so they would want to back it, but to do so would risk the wrath of voters. Potentially we could end up with an inefficient outcome where members shy away from backing the policy.

The scenario we have just described above is, in the terminology of game theory, a binary threshold public good game. In the current paper we study a particular type of threshold public good game called a forced contribution game. In this game all group members are 'forced' to contribute if sufficiently many members 'voluntarily' contribute. In the context of our earlier example, this would be to say that all party members pay the cost of an unpopular policy if the party enacts the policy.

Our interest in the forced contribution game stems from prior evidence that enforcing contributions increases inefficiency. In particular, we know that in general, groups are relatively inefficient at providing threshold public goods. Some prior experimental studies, however, have shown that high efficiency is obtained in forced contribution games. Our objective in this paper is to explore in detail, both theoretically and experimentally, the conditions under which enforcing contributions works.

In our theoretical analysis we contrast and compare three alternative models of behaviour: ordinal potential, quantal response and impulse balance. Ordinal potential says that group members will act to maximize aggregate payoff; this results in the 'optimistic' prediction of maximum efficiency. Quantal response is a model that allows for decision making 'with mistakes'; it gives a 'pessimistic' prediction of low efficiency unless the return from the public good is very high. Finally, impulse balance theory assumes that people make decisions based on expected ex-post regret; here we get an intermediate prediction where efficiency depends on the return to the public good and size of the threshold.

We complement the theory with an experimental study where the number of players and return to the public good are systematically varied in order to test the respective predictions of the three models detailed above. We find that impulse balance provides the best fit with the experimental data. This allows us to derive a simple expression predicting when enforced contributions result in high or low efficiency. Our predictions are consistent with the uniformly high efficiency observed in previous studies. We also find, however, that enforced contributions are not a guarantee of high efficiency.

# 1 Introduction

A threshold public good is provided if and only if sufficiently many people contribute towards its provision. The classic example would be a capital project such as a new community school (Andreoni 1998). The notion of threshold public good is, however, far more general than this classic example. Consider, for example, a charity that requires sufficient funds to cover large fixed costs. Or, consider a political party deciding whether to adopt a policy which is socially efficient but, for some reason, unpopular with voters; the policy will be enacted if and only if enough party members are willing to back the policy (Goeree and Holt 2005).

In a threshold public good game the provision of the public good is consistent with Nash equilibrium. There are, however, typically multiple equilibria (Palfrey and Rosenthal 1984). This leads to a coordination problem that creates a natural uncertainty about total contributions. The literature has decomposed this uncertainty into a fear and greed motive for non contribution (Dawes et al. 1986, Rapoport 1987, see also Coombs 1973). The fear motive recognizes that a person may decide not to contribute because he is pessimistic that sufficiently many others will contribute. The greed motive recognizes that a person may decide not to contribute in the hope that others will fund the public good.

Dawes et al. (1986) noted that the fear motive can be alleviated by providing a refund (or money back guarantee) if contributions are short of the threshold. Similarly, the greed motive can be alleviated by *forcing* everyone to contribute if sufficiently many people volunteer to contribute. In three independent experimental studies Dawes et al. (1986) observed significantly higher efficiency in a forced contribution game. On this basis they concluded that inefficiency was primarily caused by the greed motive. Rapoport and Eshed-Levy (1989) challenged this conclusion by showing that the fear motive can cause inefficiency (see also Rapoport 1987). They still, however, observed highest efficiency in a forced contribution game.[1]

These experimental results suggest that enforcing contributions is an effective way to obtain high efficiency. This is a potentially important finding in designing mechanisms for the provision of public goods. Existing evidence, however, is limited to the two experimental studies mentioned above. Our objective in this paper is to explore in detail, both theoretically and experimentally, the conditions under which forced contributions leads to high efficiency in binary threshold public good games. We do so by applying three alternative models that are, respectively, based on ordinal potential (Monderer and Shapley 1996), quantal response (McKelvey and

[1] For a general overview of the experimental literature on threshold public goods see Croson and Marks (2000), Schram, Offerman, and Sonnemans (2008), and Cadsby et al. (2008).

Palfrey 1995), and impulse balance (Selten 2004). We show that the three models give very different predictions on the efficiency of enforcing contributions.

We complement the theory with an experimental study where the number of players and return to the public good are systematically varied in order to test the respective predictions of the three models. We find that impulse balance provides the best fit with the experimental data. This allows us to derive a simple expression predicting when enforced contributions result in high or low efficiency. Our predictions are consistent with the uniformly high efficiency observed in previous studies. We also find, however, that enforced contributions are not a guarantee of high efficiency. The interpretation of this finding will be discussed more in the conclusion.

At this stage a brief discussion on the interpretation of enforced contributions may be useful. In particular, we want to clarify why forced contribution is not inconsistent with the notion of *voluntary* provision of a public good. To illustrate, we provide three examples of a forced contribution game.[2] First, consider a firm trying to acquire an apartment block for redevelopment; rules stipulate that the firm can forcibly purchase the entire block if sufficiently many residents volunteer to leave. Second, consider a firm attempting a takeover of a publicly listed competitor; rules stipulate that the firm can forcibly purchase the competitor if sufficiently many shareholders sell their shares. Finally, consider a political party deciding whether to endorse a particular policy; if sufficiently many members back the policy then others are forced to pay the cost of introducing the policy even if they initially opposed it.

In all of the three examples above it is endogenously determined whether the public good is provided. If not enough people voluntarily 'contribute' then the public good is not provided and no one is forced to contribute. In this sense public good provision is voluntary: it is voluntary at the level of the group. Another thing these examples illustrate is that enforcing contributions is a practical possibility in numerous situations. Our analysis will provide insight on when this possibility is worth pursuing. In particular, enforcing contributions is likely to be costly to implement and so it is crucial to know whether enforcement will lead to high efficiency.

As a final preliminary we highlight that an important contribution of the current paper is to apply impulse balance theory in a novel context. Impulse balance theory, which builds on learning direction theory, says that players will tend to change their behavior in a way that is consistent with ex-post rationality (Selten and Stoeker 1986, Selten 1998, Selten 2004, Ockenfels and Selten 2005, Selten and Chmura 2008, see also Cason and Friedman 1997, 1999). In Alberti, Cartwright and Stepanova (2013) we apply impulse balance to look at continuous threshold public good games. Here we focus on the binary forced contribution game. As already previewed, we

[2] The first example is taken from Dawes et al. (1986).

find that impulse balance successfully predicts observed efficiency. This is clearly a positive finding in evaluating the merit of impulse balance theory.[3]

We proceed as follows: In section 2 we describe the forced contribution game. In section 3 we provide some theoretical preliminaries, in section 4 we describe three models to predict efficiency and in section 5 we compare the three models predictions. In section 6 we report our experimental results and in section 7 we conclude.

# 2 Forced contribution game

In this section we describe the forced contribution threshold public good game. There is a set of players $N = \{1, ..., n\}$. Each player is endowed with $E$ units of private good. Simultaneously, and independently of each other, every player $i \in N$ chooses whether to contribute 0 or to contribute $E$ towards the provision of a public good. Note that this is a binary, all or nothing, decision. For any $i \in N$, let $a_i \in \{0, 1\}$ denote the action of player $i$, where $a_i = 0$ indicates his choice to contribute 0 and $a_i = 1$ indicates his choice to contribute $E$. Action profile $a = (a_1, ..., a_n)$ details the action of each player. Let $A$ denote the set of action profiles. Given action profile $a \in A$, let

$$c(a) = \sum_{i=1}^{n} a_i$$

denote the number of players who contribute $E$.

There is an exogenously given threshold level $1 < t < n$.[4] The payoff of player $i$ given action profile $a$ is

$$u_i(a) = \begin{cases} V & \text{if } c(a) \geq t \\ E(1 - a_i) & \text{otherwise} \end{cases},$$

where $V > E$ is the value of the public good. So, if $t$ or more players contribute then the public good is provided and every player gets a return of $V$. In interpretation, every player is *forced* to contribute $E$ irrespective of whether they *chose* to contribute 0 or $E$. If less than $t$ players contribute then the public good is not provided and there is no refund for a player who chose to contribute $E$.

[3] Impulse balance theory and quantal response are compared by Selten and Chmura (2008) (see also Chmura, Georg and Selten 2012), and Berninghaus, Neumann and Vogt (2014). No strong difference in predictive power is found between the two.

[4] If $t = n$ then we have the weak link game. If $t = 1$ then we have a form of best shot game. For simplicity we exclude these 'special cases' from the analysis.

For any player $i \in N$ the strategy of player $i$ is given by $\sigma_i \in [0,1]$ where $\sigma_i$ is the probability with which he chooses to contribute $E$ (and $1-\sigma_i$ is the probability with which he chooses to contribute 0). Let $\sigma = (\sigma_1, ...., \sigma_n)$ be a strategy profile. With a slight abuse of notation we use $u_i(\sigma_i, \sigma_{-i})$ to denote the expected payoff of player $i$ given strategy profile $\sigma$, where $\sigma_{-i}$ lists the strategies of every player except $i$.

# 3 Theoretical preliminaries

We say that a strategy profile $\sigma = (\sigma_1, ...., \sigma_n)$ is symmetric if $\sigma_i = \sigma_j$ for all $i, j \in N$. Given that choices are made simultaneously and independently, it is natural to impose a homogeneity assumption on beliefs (Rapoport 1987, Rapoport and Eshed-Levy 1989).[5] This justifies a focus on symmetric strategy profiles. Symmetric strategy profiles $\sigma^0 = (0, ..., 0)$ and $\sigma^1 = (1, ..., 1)$ will prove particularly important in the following. We shall refer to $\sigma^0$ as the zero contribution strategy profile and $\sigma^1$ as the full contribution strategy profile.

Any symmetric strategy profile $\sigma = (\sigma_1, ..., \sigma_n)$ can be summarized by real number $p(\sigma) \in [0,1]$ where $p(\sigma) = \sigma_i$ for all $i \in N$. Where it shall cause no confusion we simplify notation by writing $p$ instead of $p(\sigma)$. Given symmetric strategy profile $\sigma$, the expected payoff of player $i$ if he chooses, ceteris paribus, to contribute $E$ is

$$\begin{aligned} u_i(1, \sigma_{-i}) &= V \Pr(t-1 \text{ or more other players contribute } E) \\ &= V \sum_{y=t-1}^{n-1} \binom{n-1}{y} p^y (1-p)^{n-1-y}. \end{aligned}$$

If he chooses to contribute 0 his expected payoff is

$$\begin{aligned} u_i(0, \sigma_{-i}) &= E + (V-E) \Pr(t \text{ or more contribute } E) \\ &= E + (V-E) \sum_{y=t}^{n-1} \binom{n-1}{y} p^y (1-p)^{n-1-y}. \end{aligned}$$

Note that player $i$'s expected payoff from strategy profile $\sigma$ is

$$u_i(\sigma_i, \sigma_{-i}) = p(\sigma) u_i(1, \sigma_{-i}) + (1 - p(\sigma)) u_i(0, \sigma_{-i}).$$

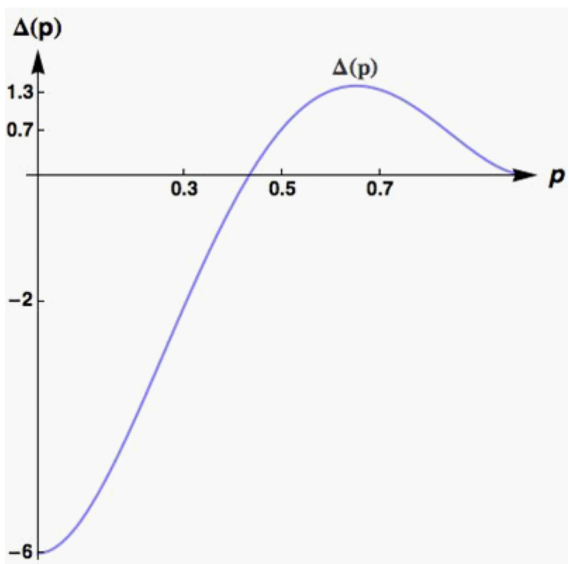
[5] See Offerman, Sonnemans and Schram (1996) for an alternative perspective.

The following function will prove useful in the subsequent analysis,

$$\begin{aligned}\Delta\left(p\left(\sigma\right)\right) &= u_i\left(1,\sigma_{-i}\right)-u_i\left(0,\sigma_{-i}\right) \\ &= V\binom{n-1}{t-1}p^{t-1}\left(1-p\right)^{n-t}-E\sum_{y=0}^{t-1}\binom{n-1}{y}p^{y}\left(1-p\right)^{n-1-y}.\end{aligned} \tag{1}$$

To illustrate, Figure 1 plots $\Delta(p)$ for $p\in[0,1]$ when $n=5, t=3, E=6$ and $V=13$. If $\Delta\left(p\right)<0$ then player $i$'s expected payoff is highest if he chooses to contribute 0. If $\Delta\left(p\right)=0$ then player $i$ is indifferent between choosing to contribute 0 and $E$. Finally, if $\Delta\left(p\right)>0$ then player $i$'s expected payoff is highest if he chooses to contribute $E$.

Figure 1: Function $\Delta\left(p\right)$ for $n=5, t=3, E=6$ and $V=13$.




### 3.1 Nash equilibrium

Previous theoretical analysis of binary threshold public good games has largely focussed on Nash equilibria (see, in particular, Palfrey and Rosenthal 1984). The forced contribution game has not, however, been explicitly studied and so we begin the analysis by solving for the set of Nash equilibria. Strategy profile $\sigma^*=(\sigma_1^*,...,\sigma_n^*)$ is a Nash equilibrium if and only if $u_i\left(\sigma_i^*,\sigma_{-i}^*\right)\geq u_i\left(s,\sigma_{-i}^*\right)$ for any $s\in[0,1]$ and all $i\in N$. In the following we focus on symmetric Nash equilibria.[6]

[6]There are many assymetric Nash equilibria. For example, it is a Nash equlibrium for $t$ players to contribute $E$ (with probability 1) and $n-t$ players to contribute 0 (with probability 1). If

The set of symmetric Nash equilibria is easily discernible from the function $\Delta(p)$. To illustrate, consider again Figure 1. In this example there are three Nash equilibria. The zero contribution strategy profile $\sigma^0$ is a Nash equilibrium because $\Delta(0) < 0$. The 'mixed' strategy profile $\sigma^m$ where $p(\sigma^m) = 0.43$, and the full contribution strategy profile $\sigma^1$ are also Nash equilibria because $\Delta(0.43) = \Delta(1) = 0$.

Our first result shows that Figure 1 is representative of the general case (see also Rapoport 1987).

**Proposition 1**: For any value of $V > E$ and $n > t > 1$ there are three symmetric Nash equilibria: (i) the zero contribution strategy profile $\sigma^0$, (ii) a mixed strategy profile $\sigma^m$ where $p(\sigma_i^m) \in (0,1)$, (iii) the full contribution strategy profile $\sigma^1$.

**Proof**: For $p = 0$ it is simple to show that $\Delta(p) = -E$. This proves part (i) of the proposition. For $p = 1$ it is simple to show that $\Delta(p) = 0$. This proves part (iii) of the proposition. In order to prove part (ii) consider separately the two terms in $\Delta(p)$ by writing $\Delta(p) = V\alpha(p) - E\beta(p)$. Term $\alpha(p)$ is the probability that exactly $t-1$ out of $n-1$ players contribute $E$ and so it takes a bell shape. Formally, $\alpha(0) = 0, \alpha(1) = 0$ and

$$\frac{d}{dp}\alpha(p) = \binom{n-1}{t-1} p^{t-2} (1-p)^{n-t-1} (t-1-p(n-1))$$

implying $\frac{d}{dp}\alpha(p) \gtreqless 0$ for $p \lesseqgtr \frac{t-1}{n-1}$. Term $\beta(p)$ is the probability $t-1$ or less of $n-1$ players contribute $E$ and so is a decreasing function of $p$. Formally, $\beta(0) = 1, \beta(1) = 0$ and

$$\frac{d}{dp}\beta(p) = -(n-1)(1-p)^{n-2} - \sum_{y=1}^{t-1} \binom{n-1}{y} \left(p^{y-1}(1-p)^{n-2-y}\right)(n-y-1) < 0.$$

For $p < 1$ it is clear that $\alpha(p) < \beta(p)$. As $p \to 1$ we know $\beta(p) - \alpha(p) \to 0$. Given that $V > E$ this means there exists some $\overline{p} \in (0,1)$ such that $\Delta(\overline{p}) > 0$. This proves part (ii) of the theorem. Note that we have also done enough to show that there exists a unique value $p^* \in (0,1)$ where $\Delta(p^*) = 0$.■

Proposition 1 shows that there are multiple symmetric Nash equilibria. We shall now consider and contrast possible approaches to select one of these equilibria.

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players have some form of pre-play communication such equilibria have been seen to arise in related games (van de Kragt, Orbell and Dawes 1983). If, however, players choose simultanesouly and independently it is difficult to see how players could coordinate on such equilibria.

Before doing that let us briefly comment on the experimental evidence concerning $\Delta(p)$. Rapoport (1987) and Rapoport and Eshed-Levy (1989) proposed the relatively weak hypothesis (their monotonicity hypothesis) that a player is more likely to contribute the higher is $\Delta(p)$. Rapoport and Eshed-Levy (1989) experimentally elicit subjects beliefs in order to test this hypothesis and find only weak support for it. Offerman, Sonnemans and Schram (2001) obtain similar results. The challenge, therefore, is to develop a model that can not only select an equilibrium but also capture the forces behind individual choice.

# 4 Three models of equilibrium selection

In this section we introduce three different ways of selecting a Nash equilibrium. We shall then suggest that these three alternatives allow us to make predictions on whether the zero contribution or full contribution strategy profile is more likely to be observed.

## 4.1 Ordinal potential

One method to refine the set of Nash equilibria, if a game is an ordinal potential game, is to find the Nash equilibria that maximize potential (Monderer and Shapley 1996). Function $W : A \rightarrow \mathbb{R}$ is an ordinal potential of the forced contribution game if for every $i \in N$ and $a \in A$[7]

$$u_i(a_i, a_{-i}) > u_i(1 - a_i, a_{-i}) \text{ if and only if } W(a_i, a_{-i}) > W(1 - a_i, a_{-i}).$$

A game is an ordinal potential game if it admits an ordinal potential. Our next result shows that the forced contribution game admits an ordinal potential. Moreover, the full contribution strategy profile maximizes potential. In this sense the full contribution Nash equilibrium is 'selected'.

**Proposition 2**: The forced contribution game is an ordinal potential game and the ordinal potential is maximized at the full contribution strategy profile $\sigma^1$.

**Proof**: The aggregate payoff, given action profile $a = (a_1, ..., a_n)$, is

$$W(a) = \begin{cases} nV & \text{if } c(a) \geq t \\ E(n - c(a)) & \text{otherwise} \end{cases}.$$

[7] See equation (2.1) of Monderer and Shapley (1996).

If $W$ is an ordinal potential then the potential is maximized for $c(a) \geq t$. In order to verify that $W$ is an ordinal potential there are five cases to consider:

(i) If $c(a) > t$ or $c(a) = t$ and $a_i = 0$ then $c(1 - a_i, a_{-i}) \geq t$ implying $u_i(a_i, a_{-i}) = u_i(1 - a_i, a_{-i}) = V$ and $W(a_i, a_{-i}) = W(1 - a_i, a_{-i}) = nV$.

(ii) If $c(a) = t$ and $a_i = 1$ then $c(1 - a_i, a_{-i}) = t - 1$ implying $u_i(a_i, a_{-i}) = V > u_i(1 - a_i, a_{-i}) = E$ and $W(a_i, a_{-i}) = nV > W(1 - a_i, a_{-i}) = E(n - t + 1)$.

(iii) If $c(a) = t - 1$ and $a_i = 0$ then $c(1 - a_i, a_i) = t$ implying $u_i(a_i, a_{-i}) = E < u_i(1 - a_i, a_{-i}) = V$ and $W(a_i, a_{-i}) = E(n - t + 1) < W(1 - a_i, a_{-i}) = nV$.

(iv) If $c(a) \leq t - 1$ and $a_i = 1$ then $u_i(a_i, a_{-i}) = 0 < u_i(1 - a_i, a_{-i}) = E$ and $W(a_i, a_{-i}) = E(n - c(a)) < W(1 - a_i, a_{-i}) = E(n - c(a) + 1)$.

(v) If $c(a) < t - 1$ and $a_i = 0$ then $u_i(a_i, a_{-i}) = E > u_i(1 - a_i, a_{-i}) = 0$ and $W(a_i, a_{-i}) = E(n - c(a)) > W(1 - a_i, a_{-i}) = E(n - c(a) - 1)$.■

With a slight abuse of terminology we shall interpret Proposition 2 as saying *ordinal potential predicts perfect efficiency* in the forced contribution game. Interestingly, this prediction is consistent with the prior experimental evidence (Dawes et al. 1986, Rapoport and Eshed-Levy 1989). However, while Monderer and Shapley (1996) show that ordinal potential can be used to refine the set of Nash equilibria they also openly admit that they have no explanation for why ordinal potential would be maximized. So, to paraphrase Monderer and Shapley (p. 136), 'it may be just a coincidence' that ordinal potential is consistent with the prior evidence. The conjecture, therefore, that ordinal potential can predict behavior in the forced contribution game needs a more rigorous empirical test.

## 4.2 Logit equilibrium

Quantal response provides a way to model behavior that allows for 'noisy' decision making (McKelvey and Palfrey 1995). Offerman, Schram and Sonnemans (1998) apply a quantal response model to a no refund threshold public good game (see also Goeree and Holt 2005).[8] Here we apply the approach to a forced contribution game. Specifically, we consider the logit equilibrium (McKelvey and Palfrey 1995).

Symmetric contribution profile $\sigma$ is a logit equilibrium if

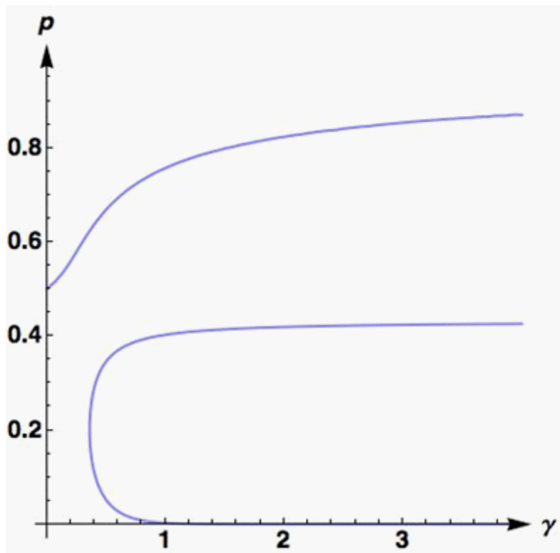
$$p(\sigma) = \frac{e^{\gamma u_i(1, \sigma_{-i})}}{e^{\gamma u_i(1, \sigma_{-i})} + e^{\gamma u_i(0, \sigma_{-i})}} = \frac{1}{1 + e^{-\gamma \Delta(p(\sigma))}}$$

for any player $i \in N$ where $\gamma \geq 0$ is a parameter. In interpretation, $\gamma$ is inversely related to the level of error, where error can be thought of as resulting from random

[8] They also consider a naive Bayesian quantal response model.

mistakes in calculating expected payoff.[9] Figure 2 plots the logit equilibria for the example $n = 5, t = 3, E = 6$ and $V = 13$. We see that there is a unique equilibrium for small $\gamma$ (i.e. a high level of error) and three equilibria for large $\gamma$. Clearly the set of logit equilibria may differ from the set of Nash equilibria.

Figure 2: Logit equilibria when $n = 5, t = 3, E = 6$ and $V = 13$.

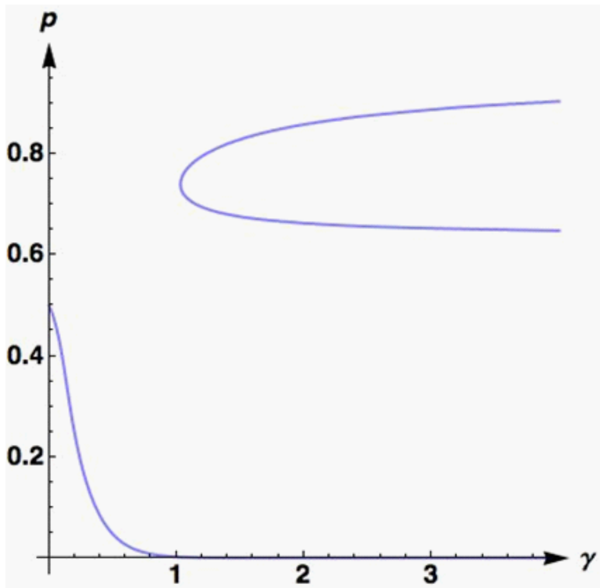



McKelvey and Palfrey (1995) demonstrate that a graph of the logit equilibrium can be used to select a Nash equilibrium. Specifically, the graph of logit equilibria contains a unique branch starting at 0.5 and converging to a Nash equilibrium as $\gamma \to \infty$. This is called the limiting logit equilibrium. In the example of Figure 2 the limiting logit equilibrium is the full contribution Nash equilibrium $\sigma^1$. For different parameter values the limiting logit equilibrium can be the zero contribution Nash equilibrium $\sigma^0$. To illustrate, Figure 3 plots the logit equilibria when $n = 7, t = 5, E = 6$ and $V = 13$.

The proceeding examples demonstrate that the limiting logit equilibrium can be $\sigma^0$ or $\sigma^1$ depending on the parameters of the game. We shall pick up on this point further in Section 5. For now we note that there exists a critical value $\widetilde{V}$ such that $\sigma^0$ is the limiting logit equilibrium for $V < \widetilde{V}$ and $\sigma^1$ is the limiting logit equilibrium for $V > \widetilde{V}$. With a further abuse of terminology we shall say that *quantal response predicts zero efficiency if $V < \widetilde{V}$ and perfect efficiency if $V > \widetilde{V}$.*

[9] Conventionally $\lambda$ is used rather than $\gamma$. We use $\gamma$ to avoid confusion with a $\lambda$ term used in impulse balance theory.

Figure 3: Logit equilibria when $n = 7, t = 5, E = 6$ and $V = 13$.




## 4.3 Impulse Balance Theory

A key contribution of the current paper is to apply impulse balance theory. Impulse balance theory provides a quantitative prediction on outcomes based on ex-post rationality (Ockenfels and Selten 2005, Selten and Chmura 2008, Chmura, Georg and Selten 2012). To apply the theory we need to determine the direction and strength of impulse of each player for any action profile (Selten 1998). In order to do this we distinguish the four *experience conditions* defined below.

Take as given an action profile $(a_1, ..., a_n)$ and a player $i \in N$. Let $\overline{u}_i = u_i (a_i, a_{-i})$ denote the realized payoff of player $i$ and let $\overline{gu}_i = u_i (1 - a_i, a_{-i})$ denote the payoff player $i$ would have got from choosing the opposite action.

*Zero no*: Player $i$ faces the zero no experience condition if $c(a) < t - 1$ and $a_i = 0$. In this case $\overline{u}_i = \overline{gu}_i = E$ and so we say player $i$ has no impulse. Equivalently, the strength of impulse is 0.

*Wasted contribution*: Player $i$ faces the wasted contribution experience condition if $c(a) \leq t - 1$ and $a_i = 1$. In this case $\overline{u}_i = 0$ while $eu_i = E > 0$. We say that player $i$ has a downward impulse of strength $\overline{gu}_i - \overline{u}_i = E$.

*Lost opportunity*: Player $i$ faces the lost opportunity experience condition if $c(a) = t-1$ and $a_i = 0$. In this case $\overline{u}_i = E$ while $\overline{gu}_i = V > E$. We say that player $i$ has an upward impulse of strength $\overline{gu}_i - \overline{u}_i = V - E$.

*Spot on*: Player $i$ faces the spot on experience condition if $c(a) \geq t$. In this case $\overline{u}_i = \overline{gu}_i = V$ and so we say player $i$ has no impulse.

The direction and size of impulse for each of the experience conditions are summarized in Table 1.

Table 1: The conditions on $a_i$ and $c(a)$, the direction and size of impulse for each experience condition

| Experience condition | Properties of $a$ | | Impulse | |
|---|---|---|---|---|
| | $a_i$ | $c(a)$ | Direction | Size |
| Zero no | 0 | $< t-1$ | – | 0 |
| Wasted contribution | 1 | $\leq t-1$ | ↓ | $E$ |
| Lost Opportunity | 0 | $t-1$ | ↑ | $V-E$ |
| Spot on | 0 or 1 | $\geq t$ | – | 0 |

We can now define expected upward and downward impulse. In doing this we retain a focus on symmetric strategy profiles. The upward impulse of player $i \in N$ comes from the lost opportunity experience condition. So, given a symmetric strategy profile $\sigma$ the expected upward impulse is

$$\begin{aligned} I^{+}(p(\sigma)) &= (V-E)\Pr(i \text{ chooses to contribute } 0)\Pr(t-1 \text{ others contribute } E) \\ &= (V-E)\binom{n-1}{t-1}p^{t-1}(1-p)^{n-t+1}. \end{aligned}$$

We note at this point that

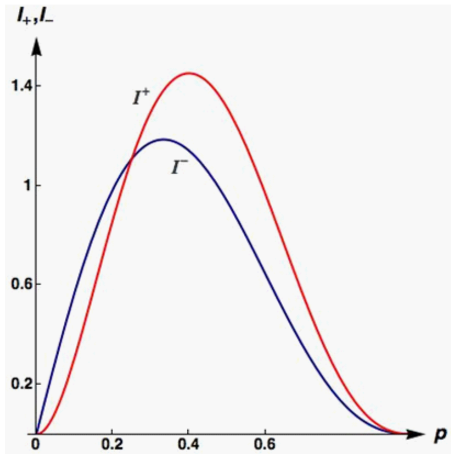
$$\frac{dI^{+}(p)}{dp} = (V-E)\binom{n-1}{t-1}(1-p)^{n-t}p^{t-2}(t-1-pn) \tag{2}$$

implying that

$$\frac{dI^{+}(p)}{dp} \gtrless 0 \text{ as } p \lessgtr \frac{t-1}{n}.$$

Thus, the upward impulse is an inverse U shaped function of $p$ (on interval $[0,1]$). To illustrate, Figure 4 plots $I^{+}(p)$ (and $I^{-}(p)$ to be defined shortly) for the example $n = 5, t = 3, E = 6$ and $V = 13$.

Figure 4: Upward and downward impulse as a function of $p$ when $n = 5, t = 3, E = 6$ and $V = 13$.




The expected downward impulse of player $i$ comes from the wasted contribution experience condition. It is given by

$$\begin{aligned} I^{-}(p(\sigma)) &= E \Pr(i \text{ contributes } E) \Pr(t-2 \text{ or less others contribute } E) \\ &= E \sum_{y=0}^{t-2} \binom{n-1}{y} p^{y+1} (1-p)^{n-1-y}. \end{aligned}$$

Note that

$$\frac{dI^{-}(p)}{dp} = E \sum_{y=0}^{t-2} \binom{n-1}{y} p^{y} (1-p)^{n-2-y} (y+1-np) \tag{3}$$

and so the downward impulse is also an inverse U shaped function of $p$. Moreover,

$$\frac{dI^{-}(p)}{dp} < 0 \text{ if } p > \frac{t-1}{n}$$

implying that the maximum downward impulse occurs for a lower value of $p$ than the maximum upward impulse. This is readily apparent in Figure 4.

Symmetric strategy profile $\sigma^*$ is a weighted impulse balance equilibrium if $I^{+}(p(\sigma^*)) = \lambda I^{-}(p(\sigma^*))$, where $\lambda$ is an exogenously given weight on the downward impulse. We shall say that an impulse balance equilibrium $\sigma^*$ is stable if $I^{+}(p) > \lambda I^{-}(p)$ for $p \in (p(\sigma^*) - \varepsilon, p(\sigma^*))$ and $I^{+}(p) < \lambda I^{-}(p)$ for $p \in (p(\sigma^*), p(\sigma^*) + \varepsilon)$ for some $\varepsilon > 0$.[10] Otherwise we say the equilibrium is unstable. Intuitively, an equilibrium

[10] If $p^* = 0$ or $p^* = 1$ the definition is amended as appropriate.

is stable if a small deviation from the equilibrium does not result in impulses that drive strategies further away from the equilibrium.

In Figure 4, where $\lambda = 1$, there are two stable impulse balance equilibria: (i) the zero strategy profile $\sigma^0$, and (ii) full contribution strategy profile $\sigma^1$. There is also (iii) an unstable mixed strategy equilibrium $\sigma^m$ where $p(\sigma^m) = 0.25$. Note that this mixed strategy impulse balance equilibrium is different to the mixed strategy Nash equilibrium.

We are now in a position to state our main theoretical result.

**Proposition 3**: (a) If $V \leq \overline{V}(\lambda)$ where

$$\overline{V}(\lambda) = \frac{E(n - (t-1)(1-\lambda))}{n - t + 1}$$

then there are two impulse balance equilibria: the zero strategy profile $\sigma^0$ is a stable equilibrium, and the full contribution strategy profile $\sigma^1$ is an unstable equilibrium.
(b) If $V > \overline{V}(\lambda)$ and $t \geq 3$ there are three impulse balance equilibria: the zero strategy profile $\sigma^0$ is a stable equilibrium, the full contribution strategy profile $\sigma^1$ is a stable equilibrium, and there is an unstable mixed strategy equilibrium $\sigma^m$ where $p(\sigma^m) \in (0,1)$.
(c) If $V > \overline{V}(\lambda)$ and $t = 2$ there are two impulse balance equilibria: the zero strategy profile $\sigma^0$ is an unstable equilibrium, and the full contribution strategy profile $\sigma^1$ is a stable equilibrium.

**Proof**: Let $C_k^\nu = \binom{\nu}{k}$ and let

$$DI(p) = I^+(p) - \lambda I^-(p)$$

denote the difference between upward and downward impulse. We have

$$DI(p) = C_{t-1}^{n-1} p^{t-1} (1-p)^{n-t+1} (V - E) - \lambda E \sum_{y=0}^{t-2} C_y^{n-1} p^{y+1} (1-p)^{n-1-y}.$$

Symmetric strategy profile $\sigma$ is an impulse balance equilibrium if and only if $DI(p(\sigma)) = 0$. If $p = 0$ then $DI(p) = 0$ implying the zero strategy profile is an impulse balance equilibrium. If $p = 1$ then $DI(p) = 0$ implying the full contribution strategy profile is also an impulse balance equilibrium.

Suppose for now that $t \geq 3$. Then

$$\begin{aligned} DI(p) &= p^{t-1}(1-p)^{n-t+1}\left[C_{t-1}^{n-1}(V-E) - \lambda E C_{t-2}^{n-1}\right] \\ &\quad - \lambda E \sum_{y=0}^{t-3} C_y^{n-1} p^{y+1} (1-p)^{n-1-y} \\ &= p^{t-1}(1-p)^{n-t+1} C_{t-1}^{n-1}\left(V - \overline{V}\right) - \lambda E \sum_{y=0}^{t-3} C_y^{n-1} p^{y+1}(1-p)^{n-1-y} \\ &= p(1-p)^{n-t+1}\left(p^{t-2} C_{t-1}^{n-1}\left(V-\overline{V}\right) - \lambda E \sum_{y=0}^{t-3} C_y^{n-1} p^y (1-p)^{t-2-y}\right). \end{aligned}$$

If $V \leq \overline{V}$ then $DI(p) < 0$ for all $p \in (0,1)$. This implies that there is no mixed strategy impulse balance equilibrium. It also implies that the zero strategy profile is stable and the full contribution strategy profile is unstable.

If $V > \overline{V}$ we need look in more detail at

$$G(p) = p^{t-2} C_{t-1}^{n-1}\left(V - \overline{V}\right) - \lambda E \sum_{y=0}^{t-3} C_y^{n-1} p^y (1-p)^{t-2-y}.$$

It is simple to see that $G(0) < 0$ and $G(1) > 0$. Continuity of $G(p)$ implies at least one value $p^* \in (0,1)$ such that $G(p^*) = 0$. At $p^*$ we obtain an impulse balance equilibrium. Moreover, we obtain stable equilibria corresponding to $p = 0$ and $p = 1$.

It remains to consider the case $t = 2$. Now

$$\begin{aligned} DI(p) &= (n-1)p(1-p)^{n-1}(V-E) - \lambda E p (1-p)^{n-1} \\ &= p(1-p)^{n-1}(n-1)\left(V - \overline{V}\right). \end{aligned}$$

As before, if $V \leq \overline{V}$ then $DI(p) < 0$ for all $p \in (0,1)$. In this case there are two equilibria, the zero strategy profile is stable and the full contribution strategy profile is unstable. If $V > \overline{V}$ then $DI(p) > 0$ for all $p \in (0,1)$. In this case there are still only two equilibria but the zero strategy profile is unstable and the full contribution strategy profile is stable.■

Proposition 3 shows that if $V \leq \overline{V}(\lambda)$ impulse balance theory gives a sharp prediction - the zero strategy profile is the unique stable impulse balance equilibrium. If $V > \overline{V}(\lambda)$ then, with the exception of the extreme case $t = 2$, we obtain a less sharp prediction - both the zero and full contribution equilibria are stable. In this case we shall hypothesize that play converges to the Pareto optimal, full contribution equilibrium. Thus, we say that *impulse balance predicts zero efficiency if* $V \leq \overline{V}(\lambda)$ *and perfect efficiency if* $V > \overline{V}(\lambda)$.

# 5 Comparing model predictions

Having introduced three alternative ways for predicting outcomes in the forced contribution game we will now demonstrate that they can give very different predictions. To do so we begin by analyzing the four games detailed in Table 2. This will serve to illustrate the stark differences between model predictions. Our focus is on games with $n = 5$ or 7 players where we vary $V$ keeping $E = 6$ and $n - t = 2$ fixed. When comparing models we shall analyze predicted efficiency measured by the probability of the public good being provided.[11]

Table 2: Parameters in the four games

| Name | $n$ | $t$ | $V$ | $E$ |
|---|---|---|---|---|
| Few-small | 5 | 3 | 7 | 6 |
| Few-large | 5 | 3 | 13 | 6 |
| Many-small | 7 | 5 | 7 | 6 |
| Many-large | 7 | 5 | 13 | 6 |

Ordinal potential (see Proposition 1) predicts perfect efficiency for all four games. Consider next quantal response. For $n = 5$ and $t = 3$ one can show numerically that the full contribution strategy profile is the limiting logit equilibrium if and only if $V > \widetilde{V}$ where $\widetilde{V} \approx 11$. Otherwise, the zero strategy profile is the limiting logit equilibrium. For $n = 7$ and $t = 5$ the analogous cut-off point is $\widetilde{V} \approx 22.8$. Only in the few-large game, therefore, efficiency is predicted to be high. This prediction does not change significantly if we consider (non-limiting) logit equilibria. To illustrate, Table 3 details predicted efficiency for a range of values of $\gamma$. There is clearly a stark contrast between the predictions obtained using ordinal potential and quantal response.

Table 3: Predicted efficiency with logit equilibrium.

| Game | $\gamma = 0.2$ | $\gamma = 0.5$ | $\gamma = 1$ | $\gamma = 4$ | $\gamma = \infty$ |
|---|---|---|---|---|---|
| Few-small | 0.24 | 0.001 | 0 | 0 | 0 |
| Few-large | 0.61 | 0.79 | 0.90 | 0.98 | 1 |
| Many-small | 0.01 | 0 | 0 | 0 | 0 |
| Many-large | 0.01 | 0 | 0 | 0 | 0 |

[11] The logit equilibrium and impulse balance equilibrium give a value for $p$, the probability of a player chooing to contribute $E$. From this one can obtain the probability of the public good being provided.

Consider next impulse balance and the case $n = 5$ and $t = 3$. From Proposition 3 we know that the full contribution strategy profile $\sigma^1$ is a stable impulse balance equilibrium if and only if

$$\lambda < \frac{3}{2}\left(\frac{V}{E} - 1\right).$$

So, if $V = 7$ (recalling $E = 6$) the equilibrium $\sigma^1$ is stable if and only if $\lambda < 0.25$. If $V = 13$ the equilibrium $\sigma^1$ is stable if and only if $\lambda < \frac{7}{4}$. When $n = 7$ and $t = 5$ we obtain analogous condition

$$\lambda < \frac{3}{4}\left(\frac{V}{E} - 1\right).$$

So, if $V = 7$ the equilibrium $\sigma^1$ is stable if and only if $\lambda < \frac{1}{8}$ and if $V = 13$ it is stable if and only if $\lambda < \frac{7}{8}$.

Prior estimates of $\lambda$ are in the range of 0.2 to 1 (Ockenfels and Selten 2005, Alberti, Cartwright and Stepanova 2014). Recall, that we predict play will converge to the full contribution equilibrium if and only if it is stable. Table 4 summarizes predicted efficiency for five different values of $\lambda$. Efficiency is predicted to be high in the few-large game and low in the many-small game. In the few-small and many-large game predictions depend on $\lambda$. Comparing Tables 3 and 4 we see that predicted efficiency with impulse balance lies somewhere in-between the extremes obtained with ordinal potential and quantal response.

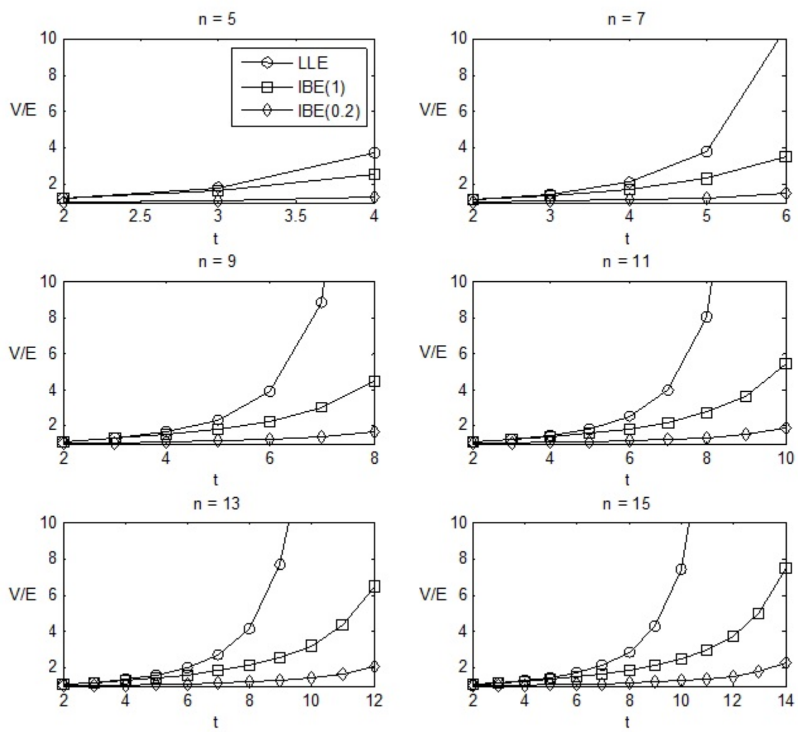
Table 4: Predicted efficiency with impulse balance.

| Game | $\lambda = 0.2$ | $\lambda = 0.4$ | $\lambda = 0.6$ | $\lambda = 0.8$ | $\lambda = 1.0$ |
|---|---|---|---|---|---|
| Few-small | 1 | 0 | 0 | 0 | 0 |
| Few-large | 1 | 1 | 1 | 1 | 1 |
| Many-small | 0 | 0 | 0 | 0 | 0 |
| Many-large | 1 | 1 | 1 | 1 | 0 |

Having looked at the four games above as illustrative examples let us now turn to the general setting. We have already shown (Proposition 2) that ordinal potential gives the 'optimistic' prediction of perfect efficiency. We have also shown (Proposition 3) that impulse balance gives a less optimistic prediction of zero efficiency if $V < \overline{V}(\lambda)$. While a general prediction for quantal response is not possible, one can show numerically that it gives the least optimistic prediction. In particular, the critical value above which $\sigma^1$ is the limiting logit equilibrium is greater than the critical value above which $\sigma^1$ is a stable impulse balance equilibrium, $\widetilde{V} > \overline{V}(\lambda)$ for $\lambda \leq 1$. This is clear in the examples, and illustrated more generally in Figure 5.

Figure 5 plots the critical values $\overline{V}(\lambda)/E$ and $\widetilde{V}/E$ above which the full contribution equilibrium $\sigma^1$ is a stable impulse balance equilibrium (for $\lambda = 0.2$ and 1) or a limiting logit equilibrium. We consider 6 possible values of $n$ and all relevant values of $t$. As one would expect, the higher is the threshold $t$ the higher has to be the return on the public good $V$ in order to predict efficiency. The main thing we wish to highlight is that $\widetilde{V} > \overline{V}(1)$ across the entire range of $n$ and $t$. In other words, there are always values of $V$ where impulse balance predicts high efficiency and quantal response predicts inefficiency. This gap widens the higher is $t$.

Figure 5: The critical value $\widetilde{V}/E$ for the limiting logit equilibrium (LLE) and of $\overline{V}(1)/E$ and $\overline{V}(0.2)/E$ for the impulse balance equilibrium IBE(1) and IBE(0.2) for different combinations of $n$ and $t$.




Recall, see the introduction, that forced contributions have been suggested as a means to promote efficiency in public good games. This conjecture is consistent with the predictions of ordinal potential but not of impulse balance or quantal response. It is natural, therefore, to want to test which model is more powerful at predicting efficiency, and to explore whether efficiency can be low despite forced contributions.

That motivates the experiments that we shall discuss shortly. Before doing that we briefly comment on experimental results from the previous literature.

Table 5 summarizes the forced contribution experiments reported by Dawes et al. (1986) and Rapoport and Eshed-Levy (1989).[12] For the game in experiment 1 of Dawes et al. and that of Rapoport and Eshed-Levy all three models predict high efficiency and this is essentially what was observed. Experiment 2 of Dawes et al. is more interesting in that quantal response predicts inefficiency while impulse balance predicts inefficiency ($\lambda = 1$) or high efficiency ($\lambda = 0.2$) depending on the value of $\lambda$. The observed high efficiency appears inconsistent with the former prediction. It is difficult, however, to infer much from this one experiment. We shall now introduce our experiments, which provide a more detailed test of the three models.

Table 5. Parameters, observed efficiency and critical values of $V$ for games considered in the literature.

| | $n$ | $t$ | $E$ | $V$ | $\overline{V}(0.2)$ | $\overline{V}(1)$ | $\widetilde{V}$ | Observed efficiency |
|---|---|---|---|---|---|---|---|---|
| Dawes et al. experiment 1 | 7 | 3 | 5 | 10 | 5.4 | 7.0 | 7.3 | 1.00 |
| Dawes et al. experiment 2 | 7 | 5 | 5 | 10 | 6.4 | 11.7 | 19.0 | 0.93 |
| Rapoport & Eshed-Levy | 5 | 3 | 2 | 5 | 2.3 | 3.3 | 3.7 | 0.72 |

# 6 Experiment design and results

Our experiment was designed to test the predictions of the three models discussed above. In order to do this we used a between subject design in which the four games introduced in Table 2 were compared. This gives four treatments corresponding to the four games.

Subjects were randomly assigned to a group and interacted anonymously via computer. We used z-Tree (Fischbacher 2007). The instructions given to subjects were game specific, in detailing $n, t$ and $V$, and so subjects could not have known that these differed across groups. In order to observe dynamic effects subjects played the game for 30 periods in fixed groups. The instructions given to subjects are available in the appendix. As detailed in Table 6, we observed a total of 27 groups and 155 subjects. A typical session lasted 30 to 40 minutes and the average payoff was £9.

[12]Dawes et al. (1986) report the results of 3 experiments. We have combined their experiments 2 and 3 because they are identical for our purposes.

Table 6: Treatments and the number of observations per treatment

| Treatment | Subjects | Groups |
|---|---|---|
| Few-small | 45 | 9 |
| Few-large | 40 | 8 |
| Many-small | 35 | 5 |
| Many-large | 35 | 5 |
| | 155 | 27 |

## 6.1 Observed efficiency

Table 7 summarizes average efficiency (measured by the proportion of periods the public good was provided) in the four treatments. In the few-large treatment efficiency was very high. This is as predicted by all three models. In the many-small treatment efficiency was very low (and significantly lower than in all other treatments, $p \leq 0.04$, Mann-Whitney). This matches the prediction of impulse balance and quantal response but not that of ordinal potential. That efficiency was so low in the many-small treatment is clear evidence that enforcing contributions does not guarantee high efficiency.

In the few-small and many-large treatments efficiency was not as high as that in the few-large treatment but the differences are statistically insignificant ($p > 0.15$, Mann-Whitney). The success rate in the many-large treatment did decline over the 30 periods ($p = 0.02$, LR test). Even if we focus on periods 11 to 30, however, the differences between the many-large, few-small and few-large treatments are insignificant ($p > 0.21$, Mann-Whitney). The relatively high level of efficiency in the few-small and many-large treatments matches the predictions of ordinal potential and impulse balance (provided the weight on the downward impulse is low, $\lambda < 0.25$), but not that of quantal response.

Table 7: Average efficiency in the four treatments where efficiency is measured as the proportion of periods the public good is provided.

| Treatment | Observed efficiency | | | |
|---|---|---|---|---|
| | Overall | Periods 1-10 | Periods 11-20 | Periods 21-30 |
| Few-small | 0.71 | 0.68 | 0.71 | 0.73 |
| Few-large | 0.90 | 0.89 | 0.93 | 0.88 |
| Many-small | 0.04 | 0.12 | 0.00 | 0.00 |
| Many-large | 0.69 | 0.84 | 0.66 | 0.58 |

We see that the only model consistent with observed efficiency in all four games is impulse balance. Ordinal potential does not capture the low efficiency in the many-small treatment and quantal response does not capture the high efficiency in the few-small and many-small treatments. The predictive power of impulse balance is, however, dependent on $\lambda$ and so we shall now look at this in more detail.

## 6.2 Impulse and behavior

Both impulse balance and quantal response have one degree of freedom, the weight on downward impulse $\lambda$ and the inverse error rate $\gamma$, respectively. Table 8 provides estimates of $\lambda$ and $\gamma$ obtained from fitting the probability with which subjects contributed to the public good. In terms of $\lambda$ we get consistent estimates of 0.2 to 0.25 across three treatments.[13] Such estimates are similar to those obtained by Ockenfels and Selten (2005) for a first price auction. By contrast, it is difficult to find any consistency in the estimates of $\gamma$. Impulse balance appears, therefore, to provide relatively robust predictions.

Table 8: Estimates of $\lambda$ for impulse balance and $\gamma$ for quantal response given observed contributions.

| Treatment | $\lambda$ | | $\gamma$ | |
|---|---|---|---|---|
| | Overall | Periods 11-30 | Overall | Periods 11-30 |
| Few-small | 0.21 | 0.21 | 0 | 0 |
| Few-large | 0.24 | 0.24 | 2.55 | 2.84 |
| Many-small | 0.09 | 0.06 | 0.31 | 0.42 |
| Many-large | 0.22 | 0.22 | 1.44 | $\infty$ |

It remains to question why the weight on the downward impulse is relatively low. To get some insight on this we shall look at how subjects changed contribution from one period to the next. Recall that impulse balance theory assumes players will change contribution based on ex-post rationality. We want to check whether subjects behaved consistent with this assumption. A relatively low weight on the downward impulse would imply that subjects are less likely to act on a downward impulse than an upward impulse. Figure 6 details the proportion of players who

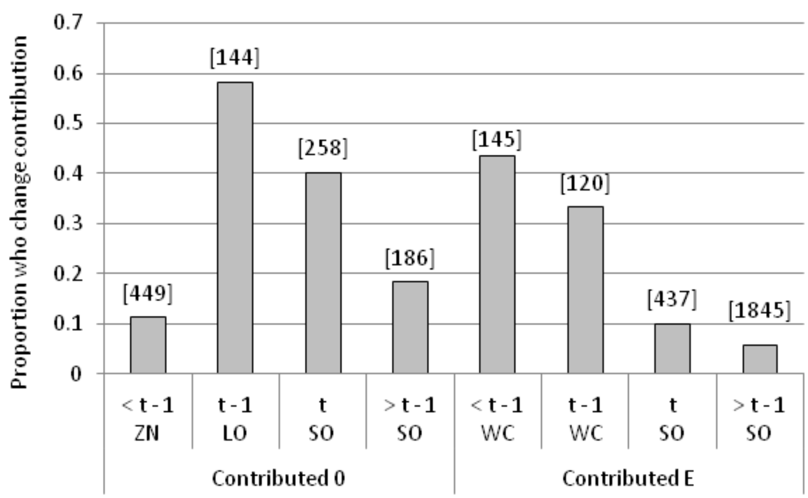
[13] The low efficiency in the many-small treatment makes it difficult to estimate $\lambda$ and so a value of $\lambda$ near 0 is to be expected. This is essentially because the equilibrium condition $I^+ = \lambda I^-$ becomes $0 = \lambda \times 0$.

changed contribution aggregating across all four treatments. We distinguish three cases. Recall that $c(a)$ denotes the number of players who contributed $E$.

(i) If $c(a) < t - 1$ then any player who contributed $E$ has a downward impulse (faces the wasted contribution experience condition) and any player who contributed 0 has no impulse (zero no). Consistent with this we see, in Figure 6, a strong tendency for those who contributed $E$ to reduce their contribution and a weak tendency for those who contributed 0 to increase their contribution.

(ii) If $c(a) = t - 1$ then any player who contributed $E$ has a downward impulse (wasted contribution) and any player who contributed 0 has an upward impulse (lost opportunity). Consistent with this we see a strong tendency for both those who contributed $E$ and those who contributed 0 to change their contribution. Importantly, those who contributed 0 are more likely to increase contribution than those who contributed $E$ are to decrease contribution. This is consistent with a low weight on the downward impulse and pushes the group towards successful provision of the public good in the next period.

Figure 6: The proportion of subjects who changed contribution from one period to the next distinguishing by initial contribution and the number of players who contributed $E$. The number of observations is given in brackets [·].




(iii) If $c(a) \geq t$ then no player has an impulse (spot on). What we observe is a relatively strong tendency for those who contributed 0 to increase their contribution, particularly when $c(a) = t$. This could be interpreted as a reaction to the 'near-miss' of the lost opportunity experience condition (Kahneman and Miller 1986, De Cremer and van Dijk 2011). The effect is to push the group towards sustained provision of the public good.

In all the three cases discussed above we observe that subjects change contribution consistent with ex-post rationality. Of particular note is that for $c(a) \geq t-1$ we see a stronger tendency to increase than decrease contributions. This explains why we find that the weight on the downward impulse is relatively low. Not only, therefore, does impulse balance theory predict aggregate success rates it is also consistent with individual behavior.

# 7 Conclusion

In this paper we contrast three methods of predicting efficiency in a forced contribution threshold public good game. The three methods are based on ordinal potential, quantal response and impulse balance theory. We also report an experiment to test the respective predictions. We found that impulse balance theory provides the best predictions.

To put our results in context we highlight that impulse balance theory allows us to derive a simple expression with which we can predict when forced contributions result in high or low efficiency. This prediction depends on the number of players $n$, threshold $t$, relative return to the public good $V/E$ and weight on the downward impulse $\lambda$. From our experiments we estimated $\lambda$ around 0.2 to 0.25. Setting $\lambda = 0.25$ we get a prediction of high or low efficiency as

$$\frac{V}{E} \gtrless \frac{n - \frac{3}{4}(t-1)}{n-(t-1)}.$$

We see that a ceteris paribus increase in the number of players lowers the critical value of the return to the public good. In other words, an increase in the number of players is predicted to enhance efficiency. Conversely, a ceteris paribus increase in the threshold is predicted to lower efficiency.

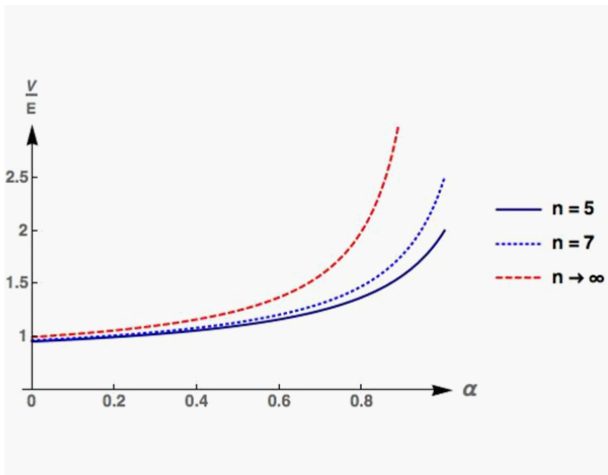
Consider next what happens if we fix the ratio between $t$ and $n$ at $t = \alpha n$. Figure 7 plots the critical value of the return to the public good as a function of $\alpha$. One can also derive that high efficiency is predicted if

$$\frac{V}{E} \geq \frac{1 - \frac{3}{4}\alpha}{1-\alpha}.$$

High efficiency is predicted, therefore, provided $t$ is not ‘too large’ a proportion of $n$. For example, if the relative return to the public good is 2 then we need

$\alpha \leq 0.8$. This prediction is consistent with the high efficiency observed in previous forced contribution experiments (Dawes et al. 1986, Rapoport and Eshed-Levy 1989). It also shows, however, that enforcing contributions does not always lead to high efficiency. This is clearly demonstrated in our many-small treatment where $\alpha = 5/7 \approx 0.71$, $V/E = 7/6 \approx 1.17$ and efficiency is near zero.

Figure 7: The value of $V/E$ above which high efficiency is predicted, where $\alpha = t/n$.




# 8 Appendix - Experiment instructions

In this experiment you will be asked to make a series of decisions. Depending on the choices that you make you will accumulate 'tokens' that will subsequently be converted into money. Each token will be converted into £0.02. You will be individually paid in cash at the end of the experiment.

At the start of the experiment you will be randomly assigned to a group of 5 people. You will remain with the same group throughout the experiment.

The experiment will last 30 rounds.

At the beginning of each round you will be allocated 6 tokens. You must decide whether to contribute these six tokens towards a group project. This is a yes or no decision, i.e. you either contribute all 6 tokens towards the group project or contribute none.

Everybody in the group faces the same choice as you do. And all group members will be asked to make their choice at the same time. Everybody, therefore, makes their choice without knowing what others in the group have chosen to do.

Your payoff will be determined by your choice whether or not to contribute towards the group project and the choices of others in the group as explained on the next page.

**If the group project goes ahead successfully**

If three or more group members contribute towards the group project then it goes ahead successfully. As a consequence, everyone in the group who initially opted (earlier in the round) not to contribute towards the project will now be required to contribute. And, everyone in the group will receive a return from the group project worth 7 tokens. Thus, everyone in the group will get a payoff of 7 tokens irrespective of whether they initially opted to contribute or not.

If three of more contribute towards the group project:

Your payoff = 7 tokens

**If the group project does not go ahead successfully**

If less than three group members contribute towards the group project then it does not go ahead successfully. Those who opted to contribute towards the project will get a payoff of 0 tokens. Those who opted not to contribute towards the project will get a payoff of 6 tokens.

If less than three contribute towards the group project:

$$\text{"Your payoff"} = \begin{cases} \text{0 tokens if you chose to contribute } E \\ \text{6 tokens if you chose not to contribute} \end{cases}.$$

At the end of the round you will be told the number of people that initially opted to contribute towards the group project, whether or not the project went ahead successfully, and your payoff for the round.

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