University of Kent

School of Economics Discussion Papers

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December 2014

KDPE 1417

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February 2014

Abstract

We estimate a macro-finance yield curve model for both the nominal and real forward curve for the UK from 1993 to 2008. Our model is able to accommodate a number of key macroeconomic variables and allows us to estimate the instantaneous response of the yield curve and so gauge the impact of Quantitative Easing on forward rates. We find that 10 year nominal interest rates on average are lower by 46 basis points which can largely be explained by three main channels: portfolio balance; liquidity premium and signalling but there is no sizeable impact on real interest rates.

JEL Classifications: E43; E44; E47; E58; E65.

Keywords: Term Structure of Interest Rates; Monetary Policy; Quantitative Easing.

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1 Introduction

Over the past couple of decades, researchers have taken a particular interest in trying to determine the macroeconomic factors driving the dynamics of the term structure of interest rates (see, for example, Knez, Litterman and Scheinkman, 1994). The workhorse model has been the affine term structure model that relies on a no-arbitrage condition and allows all bond maturities to be priced. The literature here includes Ang and Piazzesi (2003), Rudebusch and Wu (2008), Dewachter and Lyrio (2006), Hördahl *et al* (2006) and Ang *et al* (2011) as examples. Another direction in which the macro-finance literature moved was to use the parsimonious Nelson-Siegel (1987) set-up in which dynamic yield curve factors are estimated.[1] The flexible Nelson-Siegel curve can approximate the cross-sectional shape of the yield curve and imposing further no-arbitrage assumptions may depress the model's ability to forecast the yield curve and condition on many macroeconomic variables (Diebold and Li, 2006 and Diebold *et al,* 2006). The literature often focuses on the three macroeconomic variables that are associated with monetary policy: inflation, real output and the policy interest rate. In recent years research has started to extend beyond these variables, such as Afonso and Martins (2012) who study the effects of fiscal variables on the term structure or Dewachter and Iania (2011) on the effects of financial variables. Whilst the number of macroeconomic variables used to explain yield curve dynamics in affine models remain quite limited. This is because affine models have to be solved under both historical and risk neutral measures, for which parameter estimation is computationally burdensome (Borgy *et al*, 2011).

In this paper we extend a new methodology that allows us to explore the macroeconomic underpinnings of the UK's nominal and real term structure of interest rates that can accommodate a much larger number of macroeconomic variables.[2] In total we examine 31 different macroeconomic variables over five key groups of data: inflation, real activity, monetary and fiscal policy, financial prices and international factors. The estimation is performed in two stages; firstly adopting the state space methodology similar to that of Diebold *et al* (2006) and Afonso and Martins (2012). We estimate a variant of Nelson-Siegel, which is Svensson's four factor model (1994, henceforth referred to as Svensson), for the forward curve for both the UK nominal and real term structures using a Kalman filter and maximum likelihood estimation. This methodology does not impose no-arbitrage, which reduces the number of parameters that need to be estimated but allows for more flexible forecasting of the macroeconomic variables. Then, by using seemingly unrelated

[1] Some have argued that it is not completely clear whether the no-arbitrage condition is a necessary assumption in a joint macro-finance experiment because bond markets are so actively traded, at least in developed countries, that any arbitrage opportunities would be traded away instantly (Diebold *et al*, 2005).

[2] See Breedon *et al.* (2012) for a version of this method applied to the nominal term structure.

regression (henceforth, SUR) we test down from the 31 different macroeconomic and financial variables to determine which macroeconomic factors can explain the nominal and real forward curve. We do not allow for bidirectional interaction between the macroeconomy and the term structure. We limit our study to the effects that the macroeconomic and financial variables have on the term structure because in the majority of cases there will be lags from the changes in the yield curve and their impact on macroeconomic variables and, in the case, of overseas factors, the impact of the UK yield curve on overseas may be rather limited.

In this paper we make two main contributions. The first is a detailed analysis of the macroeconomic and financial factors that affect either or both of the nominal and the real term structure of interest rates. This literature has previously focused on the nominal term structure but we have also isolated macroeconomic and financial variables that impact on the real term structure. From the 31 macroeconomic and financial variables we identify 10 different variables that have an effect on the nominal term structure in the UK. Amongst these are the variables that concern monetary policy makers, debt-to-GDP and international variables such as the effective exchange rate, measures of German real activity and the Federal Funds Rate. For the real dynamic factors the macroeconomic variables do less well and there are four variables that drive the real curve: debt-to-GDP, inflation expectations, the Libor spread and notes and coins. We identify a net supply effect on government bonds across our included maturities, inflation expectations are more important than actual inflation, and that the exchange rate and international macroeconomic announcements from Germany and the US have an effect on the nominal curve.

This paper builds on the approach of Breedon *et al* (2012), who analyse the effects that the first round of QE had on the nominal UK term structure. We develop a more detailed methodology and extend our analysis to include the real term structure of interest rates, as well as offering a decomposition of the impact of QE on the term structure based on the portfolio balance, liquidity and signalling effects. Our model allows for a more detailed and richer conditioning of movements in the term structure than can be undertaken by an events study. Our work is in a similar vein to Bernanke *et al* (2004), who use their model to assess the impact of Japanese Quantitative Easing. Also Christensen, Lopez and Rudebusch (2009), who analyse the effectiveness of the central bank liquidity facility that was provided to financial institutions to improve and ease liquidity constraints in the interbank lending market. To analyse the impact of such policy they use a multi-factor affine term structure model of the US government yields and bank credit risk. Both sets of authors perform counterfactual analysis to determine if the path of interest rates had changed drastically in light of the policy action.

Given that central banks typically use the short-term interest rate as their main policy tool, the term structure of interest rates forms a key element of the transmission mechanism of monetary policy. And so the second contribution that our paper makes to the existing literature is a methodology that is well suited for analysing specific monetary policy episodes. We develop this particular contribution further by analysing the impact that Quantitative Easing (QE) in the UK had on the term structure of interest rates. In the UK, the Bank of England's first episode of QE operated from March 2009 to January 2010. We define QE as large scale purchases of government bonds funded by borrowed Central Bank reserves that are implemented when the policy rate is at its effective zero lower bound.[3] To examine the impact of QE we conduct an out-of-sample forecast of the term structure for both the nominal and the real term structure of interest rates across the QE period. The forecasts in each period are conditioned on the estimated coefficients of the statistically significant macroeconomic and financial variables over our estimation sample from March 1993 to December 2008. We assume that the forecasted path is the counterfactual path of interest rates that would have occurred if QE had not been employed by the Bank of England.[4]

Our out-of-sample forecast are similar in size to other recent literature on the first round of QE in the UK such as Caglar *et al* (2012) and Meaning and Zhu (2012) with the five year forward and the ten year forward on average overestimating the actual curve by 60-70 and 40-50 basis points respectively. We find the overestimate of nominal forward rates is plausible both in terms of timing and maturities targeted: the overestimate occurs from March 2009 and maturities greater than 24 months show an over-prediction relative to the actual curve.

Furthermore, we analyse the forecast error by decomposing it into the three channels: portfolio balance, liquidity premium and signalling. The portfolio balance channel represents the supply effect within the bond market in which imperfect substitutability of different assets means that the relative supply of bonds can determine their price. The liquidity premium can be alleviated when a Central Bank intervenes and becomes a large scale purchaser of bonds and improve the functioning of the bond market. Both of these channels should exert downward pressure on longer-term yields and foster an easing in financial conditions and stimulate growth. The signalling channel refers to the market's expectations of the future path of interest rates based on the signals the market receives from both the monetary

[3]At the time of writing the Bank of England was still using QE as the tool for conducting monetary policy and we focus our analysis solely on what was called the first round of QE which occurred from the March of 2009 to January 2010.

[4]QE intended to stimulate nominal spending and so it could be surmised that the conditioning macroeconomic variables may include the positive impact of QE directly on macroeconomic variables. Implying the impact on the term structure within our forecasts was not independent from QE. But given the policy lag between the implementation of QE and any impact on the macroeconomy we assume that this impact is at most minimal. Also, with regards to the efficacy of QE, we only concern ourselves with the first part of the monetary transmission mechanism, which is the immediate impact on asset prices, in this particular case, bonds. We leave the greater impact of QE on the macroeconomy to others and for future research.

authorities and the wider macroeconomy leaving the impact on the yield curve ambiguous. One way to uncover the signalling effect is with the use of risk-adjusted market interest rates used to gauge the expected path of interest rates.

Overall we find that all three channels exerted downward pressure on the term structure and we show that it is the signalling channel that plays the most prominent role when QE was first implemented but this effect dissipates as QE purchases were extended. The portfolio balance channel had the largest effect at the end of the sample as the amount of purchases increased. This channel alone is found to reduce yields by as much as 136 basis points at 10 years. The liquidity premium channel does play a small role in reducing yields for the 5 year forward but not for the 10 year forward. On average, the forecast error at 5 and 10 years is 67 and 46 basis points respectively and the average estimated impact that the three channels have on the term structure are 88 and 86 basis points respectively.[5]

The forecast of the real curve does not demonstrate any persistent deviation from the realised path of interest rates. This is an appealing result as the Bank of England did not undertake any QE operations using real bonds. The three channels of QE would have had a limited or no impact on the real curve, particularly the portfolio balance channel. It appears that any net supply effects of debt were offset by the sensitivity of the real curve to changes in the Libor spread.

This paper is further divided into five sections. Section Two outlines the methodology used to fit the term structure, the econometric specification and estimation techniques as well as the instantaneous responses. Section Three outlines the forward curve data, the estimated yield curve factors and the macroeconomic variables data and the overall fit of the estimated term structure across the sample period. Section Four presents the empirical results from the macro-finance model and post-estimation analysis. Section Five outlines the forecast exercise performed over the QE period as well as the decomposition of the forecast error and finally, Section Six concludes.

2 Methodology

We outline our two-stage methodology. The first stage begins with outlining the Svensson (1994) methodology used to fit the term structure and employing a state-space approach similar to that of Diebold *et al* (2006) and Afonso and Martins (2012). We estimate four latent forward curve factors which we call the level, slope and two curvatures by means of the Kalman filter. The second stage is then to take these estimated latent factors, and using

[5]The impact of the three channels sum to more than the forecast error which suggests a further factor such as credit risk may exist. Such a channel may have exerted some upward pressure on the term structure over this time period. We provide some anecdotal evidence in Section 5 for such a factor.

Seemingly Unrelated Regression with macroeconomic and financial variables, we produce instantaneous responses that illustrate how each macroeconomic variable affects the shape of the forward curve.

2.1 *Term Structure Model*

The functional form that has been used to fit the term structure of interest rates in this analysis is that of Svensson (1994). This parametric model is simple to implement and provides a parsimonious description of the term structure of interest rates. Svensson, as a function of different factor weights and parameters β_i, produces a smooth fit of the term structure. The functional form of Svensson is:

$$y(\tau) = \beta_1 + \beta_2 \left(\frac{1 - e^{-\tau\lambda_1}}{\tau\lambda_1} \right) + \beta_3 \left(\frac{1 - e^{-\tau\lambda_1}}{\tau\lambda_1} - e^{-\tau\lambda_1} \right) + \beta_4 \left(\frac{1 - e^{-\tau\lambda_2}}{\tau\lambda_2} - e^{-\tau\lambda_2} \right). \quad (1)$$

The Svensson model is a four factor extension of the widely used Nelson and Siegel model (1987), which uses three different factors to fit the term structure of interest rates. Both models are parametric models which specify a single-piece function that is defined across the span of the maturity domain. The Nelson and Siegel methodology only has the one curvature factor which implies that β_4 and λ_2 are equal to zero. The choice of the Svensson methodology over that of Nelson and Siegel allows for a better fit of the yield curve and can accommodate a more flexible term structure than Nelson-Siegel. Like most recent macro-finance literature, we include maturities up to 120 months because the inclusion of longer maturities may hinder the estimation of the latent factors. The focus of this paper is not solely on the fitting ability of the yield curve model but to be able to give a very good approximation of the term structure so that the macroeconomic determinants of the changes in the yield curve can be better understood. For the cost of estimating an additional factor we can determine if an additional latent factor is important to understanding the yield curve.

The Svensson representation can then be interpreted as a dynamic latent factor model where β_1, β_2, β_3 and β_4 become time varying parameters that capture the level L_t, slope S_t, the first curvature $C_{1,t}$ and the second curvature factor $C_{2,t}$ at time t. The functional form of Svensson can then be expressed as:

$$y(\tau) = L_t + S_t \left(\frac{1 - e^{-\tau\lambda_1}}{\tau\lambda_1} \right) + C_{1,t} \left(\frac{1 - e^{-\tau\lambda_1}}{\tau\lambda_1} - e^{-\tau\lambda_1} \right) + C_{2,t} \left(\frac{1 - e^{-\tau\lambda_2}}{\tau\lambda_2} - e^{-\tau\lambda_2} \right). \quad (2)$$

The zero-coupon forward is denoted as $y(\tau)$ and τ is the maturity. The parameters L_t, S_t, $C_{1,t}$ and $C_{2,t}$ are the time-varying parameters that correspond to their appropriate factor loadings. The level, L_t is a long-term factor that has a loading equal to 1 for every maturity across the term structure. The factor S_t is the slope of the term structure and is a short-term factor that has a maximum loading of 1 at the shortest maturity and decays monotonically to 0 as the maturities increase. The final two factors are the curvatures that represent the 'humps' of the yield curve: $C_{1,t}$ and $C_{2,t}$ have loadings that start with values of 0 at the shortest maturity which then increase over medium-term maturities before decaying back to 0 as the maturities increase. The parameters λ_1 and λ_2 are held constant throughout estimation. They determine the maximum loadings of $C_{1,t}$ and $C_{2,t}$ as well as the rate of decay of S_t.[6] These curvature factors are expected to capture variations in the curvature across the term structure, which may be visually represented as a peak and a trough.

The term structure factors have often been given a macroeconomic interpretation. The level term is found to be very closely related to inflation and inflation expectations and the slope is referred to as a business cycle factor as the short-term interest rate changes over the business cycle. Notable examples include Dewachter and Lyrio (2006), Hördahl, Tristani and Vestin (2006), Diebold, Rudebusch and Aruoba (2006), Rudebusch and Wu (2008) and Bekaert, Cho and Moreno (2010). However an economic interpretation of the curvature factor has proved more intractable. Dewachter and Lyrio (2006) believe that the curvature measures the stance of the monetary authorities whereas Dewachter and Iania (2011) find that the curvature can be explained by a measure of market liquidity and credit risk. A similar result is shared with Christensen, Lopez and Rudebusch (2009), they find that the Libor spread as a measure of credit quality has an effect on the curvature. This may suggest that the level of risk in the financial system and the stance of monetary policy may provide an economic interpretation for the curvature.

We assume that L_t, S_t, $C_{1,t}$ and $C_{2,t}$ follow a first order vector autoregressive process which allows the model to form a state-space system and, by means of the Kalman filter, we are able to obtain the maximum-likelihood estimates of the parameters and the implied estimates of L_t, S_t, $C_{1,t}$ and $C_{2,t}$ (for an overview of applying the Kalman filter to estimate term structure models, see, Bolder, 2001). The exact relationship between the yields and the state variables are defined within the measurement equation. The relationship between the yields and the state variables in the case of the Svensson representation are related by the factor loadings at each included maturity.[7]

[6] Diebold, Rudebusch and Aruoba (2006) fit the zero-coupon spot rates derived from bid-ask average price quotes using the unsmoothed Fama-Bliss approach. They estimate a value of 0.077 for their lambda which implies a maximum-loading on the curvature at 23.3 months. Afonso and Martins (2012), using zero-coupon rates at the same maturities as Diebold *et al* (2006), find that for the US the value of lambda is approximately equal to 0.037 and has a maximum loading at 48 months. For Germany lambda is approximately equal to 0.041 and this corresponds to a maximum loading at 43 months.

[7] The Kalman filter routine was programmed in Matlab 2010a. Both Hamilton (1994) and Kim and Nelson (1999) provide a good overview and an application of the Kalman filter algorithm. Our chosen optimiser is

The unobserved system is captured by the transition equation, which determines the dynamics of the state vector is:

$$\begin{pmatrix} L_t \\ S_t \\ C_{1,t} \\ C_{2,t} \end{pmatrix} = \begin{pmatrix} \mu_L \\ \mu_S \\ \mu_{C_1} \\ \mu_{C_2} \end{pmatrix} + \begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{pmatrix} \begin{pmatrix} L_{t-1} \\ S_{t-1} \\ C_{1,t-1} \\ C_{2,t-1} \end{pmatrix} + \begin{pmatrix} \eta_{L,t} \\ \eta_{S,t} \\ \eta_{C_1,t} \\ \eta_{C_2,t} \end{pmatrix}. \tag{3}$$

Where $t = 1, ..., T$, in this case μ_L, μ_S, μ_{C_1} and μ_{C_2} are constants and $\eta_{L,t}$, $\eta_{S,t}$, $\eta_{C_1,t}$ and $\eta_{C_2,t}$ are the disturbances of the autoregressive process of the latent factors. The measurement equation relates a set of N observed zero-coupon yields of different maturities to the four latent factors of the yield curve and is given by:

$$\begin{pmatrix} y_t(\tau_1) \\ y_t(\tau_2) \\ \vdots \\ y_t(\tau_N) \end{pmatrix} = \begin{pmatrix} 1 & \left(\frac{1-e^{-\tau_1\lambda_1}}{\tau_1\lambda_1}\right) & \left(\frac{1-e^{-\tau_1\lambda_1}}{\tau_1\lambda_1} - e^{-\tau_1\lambda_1}\right) & \left(\frac{1-e^{-\tau_1\lambda_2}}{\tau_1\lambda_2} - e^{-\tau_1\lambda_2}\right) \\ 1 & \left(\frac{1-e^{-\tau_2\lambda_1}}{\tau_2\lambda_1}\right) & \left(\frac{1-e^{-\tau_2\lambda_1}}{\tau_2\lambda_1} - e^{-\tau_2\lambda_1}\right) & \left(\frac{1-e^{-\tau_2\lambda_2}}{\tau_2\lambda_2} - e^{-\tau_2\lambda_2}\right) \\ \vdots & \vdots & \vdots & \vdots \\ 1 & \left(\frac{1-e^{-\tau_N\lambda_1}}{\tau_N\lambda_1}\right) & \left(\frac{1-e^{-\tau_N\lambda_1}}{\tau_N\lambda_1} - e^{-\tau_N\lambda_1}\right) & \left(\frac{1-e^{-\tau_N\lambda_2}}{\tau_N\lambda_2} - e^{-\tau_N\lambda_2}\right) \end{pmatrix}$$

$$\times \begin{pmatrix} L_t \\ S_t \\ C_{1,t} \\ C_{2,t} \end{pmatrix} + \begin{pmatrix} \varepsilon_{\tau_1,t} \\ \varepsilon_{\tau_2,t} \\ \vdots \\ \varepsilon_{\tau N,t} \end{pmatrix}. \tag{4}$$

Where $t = 1, ..., T$ and $\varepsilon_{\tau_1,t}$, $\varepsilon_{\tau_2,t}$,..., $\varepsilon_{\tau N,t}$ are the measurement errors of the observed forwards at every maturity at time t and the implied yields determined by the shape of the fitted Svensson curve. In matrix notation, the state-space form of the model can be written as:

$$\mathbf{F}_t = \boldsymbol{\mu} + \mathbf{A}\mathbf{F}_{t-1} + \boldsymbol{\eta}_t \qquad \eta_t \sim i.i.d.(0, \Sigma), \tag{5}$$

$$\mathbf{Y}_t = \mathbf{B}\mathbf{F}_t + \boldsymbol{\varepsilon}_t \qquad \varepsilon_t \sim i.i.d.(0, \sigma^2 \cdot I). \tag{6}$$

Where **A** and **B** are the transition and measurement matrices respectively. For the Kalman filter to be the optimal linear filter it is assumed that the initial conditions of the state vector are uncorrelated with the innovations in both systems. The disturbances

fminsearch which is a non-linear minimisation technique which uses the Nedler-Mead (1965) technique. This is a heuristic search method that allows convergence to a non-stationary point. A particularly useful feature when one or more of the unobserved state variables may be non-stationary.

of both the measurement and the transition equations are i.i.d. and uncorrelated. The variance-covariance matrix of the disturbances for the transition equation Σ is non-diagonal and the variance-covariance matrix of the measurement equation $\sigma^2 \cdot I$ is diagonal. This implies that the residual between the fitted and the zero-coupon yield at any maturity is not correlated with the residual at any other maturity.

Given some set of initialisations for the four latent factors and the parameters (the coefficients of the **A** and **B** matrices, the variance-covariance matrices and the choice of lambdas) the one step-ahead prediction errors and the variance of these prediction errors are used to compute the log-likelihood function. The Kalman filter produces the maximum likelihood estimates of the parameters as well as the optimal filtered and smoothed estimates of the underlying latent factors L_t, S_t, $C_{1,t}$ and $C_{2,t}$.

2.2 *Macro-Finance Specification and Estimation*

The joint macro-finance model is specified as a system of regression equations with a lagged dependent variable. The dependent variables are the estimated forward curve factors for the nominal and real curve and only the lagged dependent variable of the respective forward curve factor will appear on the right hand side. The system of equations has the following form:

$$\mathbf{F}_t = \boldsymbol{\alpha} + \boldsymbol{\gamma}\mathbf{t} + \boldsymbol{\rho}\mathbf{F}_{t-1} + \mathbf{X}_t\boldsymbol{\theta} + \mathbf{d}_t\boldsymbol{\delta} + \boldsymbol{\varepsilon}_t. \tag{7}$$

F is the vector of dependent variables, α is a vector of constants and γ is a time trend, all of which are 4×1 vectors. **X** is a $x \times 1$ vector of the independent variable; the macroeconomic variables and θ is the $4 \times x$ matrix of their coefficients where x can be as large as 31. We use dummy variables to explain any large residuals, so d is an $m \times 1$ vector of dummy variables and δ is a $4 \times n$ matrix of coefficients for the dummy variables.

From equation (7) it is clear that our model only explores the impact of macroeconomic variables on the yield curve factors. This may appear to be a shortfall of the modelling technique but other researchers have found relatively little impact from the yield curve factors on the macroeconomy. For example, Diebold *et al* (2006) find that there is a relationship from the slope on inflation and capacity utilisation but they suggest that it is most likely that market participants are anticipating the Federal Reserve's actions. Also, a variance decomposition finds very little of the variance of macroeconomic variables explained by the yield curve factors. The variance decomposition of Afonso and Martin (2012) also shows that the yield curve factors explain very little of the forecast error variance of the fiscal variables within a forecast horizon of a year. The affine term structure literature also finds little response of yield curve factors on the macroeconomic variables. Rather, the level is treated as a measure of medium or long-term inflation expectations (see, for example, Rudebusch and Wu, 2008). These results suggest that the contemporaneous direction of causality is from the macroeconomy to the term structure. It would also appear, that the exclusion of any bidirectional relationship may not be a source of estimation bias, instead, by focusing only on the macroeconomic variables that influence the term structure factors we can explore a considerably larger number of macroeconomic variables than previously seen. We exclude lags of the macroeconomic variables so that we can explore the contemporaneous effects macroeconomic variables have on the yield curve but we include the lag of the dependent variables so that the shape of the yield curve changes gradually through time.

However, there may be a remaining issue with the extent to which financial and exchange rate data (outlined in Section 3) may impact on the yield curve contemporaneously and vice versa. Without discussing too many results prematurely, we find that most of the financial variables do not explain well the UK term structure. Of all the financial variables that we examine (see Table 1 for a list of all variables analysed) only the Libor spread has a statistically significant effect on the term structure. We also examine the UK effective exchange rate and three bilateral exchange rates, and of these, only the effective exchange rate is statistically significant.

The transition equation of the Kalman filter assumes that the error terms of the four factors are correlated and so it is appropriate to take account of this in the econometric analysis and so we use seemingly unrelated regression. We estimate the system of equations with feasible generalised least squares (henceforth, FGLS) and the standard errors are bootstrapped. FGLS is preferable to OLS for two reasons, the greater the correlation between the residuals in each equation, the greater the efficiency gain obtained by FGLS. Second, the less correlation there is between the $\mathbf{X}$ matrices, the greater the gain to FGLS. Thus, estimating SUR with all of the information within the system of equations makes it more efficient than estimating each individual equation.

Within each term structure factor equation we adopt a general-to-specific selection criterion for each macroeconomic variable. Only those macroeconomic variables that persist to be significant will remain in each equation. Post-estimation we perform exclusion restrictions on the coefficients within each equation as well as testing each macroeconomic variable across each equation to determine if they are jointly significant.[8]

[8] After initial estimations we found that the residuals were non-stationary for both the nominal and real equations (not shown). So to deal with these non-stationary residuals and any trending independent variables we include a deterministic time trend, rendering the residuals stationary.

2.3 *Instantaneous Response of the Yield Curve*

We calculate and show how the instantaneous response from each significant macroeconomic variable alters the shape of the yield curve. The effect of the macroeconomic variables on the latent variables is determined by the coefficients of the SUR estimations. Using equation (2) we substitute the coefficients from the SUR estimations in for the parameter values that fit the curve to produce the yield curve response as shown below:

$$YR(\tau)_j = \hat{\theta}_{j,L} + \hat{\theta}_{j,S}\left(\frac{1-e^{-\tau\lambda_1}}{\tau\lambda_1}\right) + \hat{\theta}_{j,C_1}\left(\frac{1-e^{-\tau\lambda_1}}{\tau\lambda_1} - e^{-\tau\lambda_1}\right) + \hat{\theta}_{j,C_2}\left(\frac{1-e^{-\tau\lambda_2}}{\tau\lambda_2} - e^{-\tau\lambda_2}\right). \quad (8)$$

Where YR is the yield curve response at a maturity (τ) and $\hat{\theta}_j$ are the coefficients of any one of the macroeconomic variables j, where $j = 1, 2, .., 31$. The lambda values used for the instantaneous response are the same as those used to estimate the term structure factors. One set back of these responses is the weighting placed on the level. As it has the largest factor weighting, any possible level effect may dominate the instantaneous response (albeit, this may be appropriate for some variables) but given this, the overall dynamics of the instantaneous responses do not change. The instantaneous responses are presented along with joint 95% confidence intervals. The instantaneous responses derived from equation (8) are dependent on multiple coefficients, such that, multivariate standard errors need to be calculated. This multivariate standard error and the confidence intervals are calculated as:[9]

$$YR(\tau) \pm 1.96 \times S \quad \text{where } S = \sqrt{\left(\hat{\mathbf{X}}'\left(\mathbf{X}'\hat{\mathbf{\Omega}}^{-1}\mathbf{X}\right)^{-1}\hat{\mathbf{X}}\right)}.$$

The confidence interval of the yield curve response is calculated from the estimated coefficients and standard deviation of each independent variable, $\hat{\mathbf{\Omega}}$. The components of the vector $\mathbf{X}$ are the four estimated coefficients from each one of the forward curve factors with respect to each separate macroeconomic variable.

3 Data

The choice of UK government liability forwards we use to define the nominal curve are 9, 12, 15, 18, 21, 24, 30, 36, 48, 60, 72, 84, 96, 108 and 120 maturities expressed in months giving a

[9] The degrees of freedom are the number of data observations minus the number of variables on the right-hand-side of equation 8, which gives a t-value of 1.96 at 95% confidence. See Draper and Smith. (1998), pages 158-160.

total of 15 different nominal forwards. To fit the real curve we use 10 different index-linked forwards with maturities of 48, 54, 60, 66, 72, 78, 84, 96, 108 and 120 months. We start at 9 months for the nominal and 48 months for the real as the shorter maturities have too many missing observations. All forward rates are zero-coupon forwards and all observations are end-of-month as the majority of macroeconomic announcements are made towards the latter end of the month.[10]

The estimation spans from March 1993 to December 2008 giving a total of 190 observations. This sample period covers the Bank of England's use of inflation targeting, the financial crisis, and the recession but ends two months before the use of Quantitative Easing. Recall that the purpose is to forecast across the QE period to determine whether or not macroeconomic variables can provide a counterfactual nominal and real term structure that may highlight the effects of QE on the term structure.

The macroeconomic variables are divided into 5 groups, inflation, real activity, policy, financial prices and international. We test down from 31 different macroeconomic time series but present only the macroeconomic variables that were significant. The full list of variables is shown in Table 1. The initial set of macroeconomic and financial variables are guided by Clarke and Mortimer-Lee (2008) who analyse the relationship between UK interest rates and key announcements of macroeconomic variables. For inflation we use the one-year ahead median inflation forecast from HM Treasury's survey.[11] For real activity we use a real activity index. For policy we include the Bank rate and for a measure of fiscal policy we use debt-to-GDP expressed as a percentage. To measure narrow money supply we use a series of notes and coins.[12] For foreign variables we include the effective exchange rate, German retail sales, IFO index of business climate and the Federal Funds Rate.[13] The final group of macroeconomic variables are the financial group and for this we use an index of annual returns for three different equity series and we use a measure of the Libor spread, which is calculated as the difference between the three month Libor rate and the monetary policy

[10] Anderson and Sleath (1999) present the methodology that constructs the zero-coupon forward and spot rates. The methodology makes adjustments for coupon payments and in the case of the index-linked bonds, the indexation lag. The estimates for the forwards are derived from a cubic spline based technique.

[11] We refer the reader to "Forecasts for the UK Economy: a comparison of independent forecasts" for more information. This publication is available monthly from HM Treasury at the following website:

http://www.hm-treasury.gov.uk/data_forecasts_index.htm

The data was accessed on the 1^{st} of July 2010.

[12] This is a measure of real money and we use the Bank of England's notes and coins in circulation (series code LPMAVAB) deflated by RPI. Notes and coins are used to represent narrow money where M0 was the Bank's main narrow money measure. But when the Bank introduced its Money Market Reform in May 2006, the Bank ceased publication of M0 and instead began publishing a series for reserve balances to accompany Notes and Coin in circulation. Notes and coins are the longest available measure of narrow money.

[13] The IFO Index of the business climate is a leading indicator of economic growth in Germany. The measure is used to capture the turning points in economic growth in the near future. This variable captures the business managers' sentiment on the current business environment and whether or not the same manager expects the business situation to improve or deteriorate in 6 months time.

rate of the Bank of England. Normally the Libor spread is the difference between the 3 month Libor and Overnight Interest Rate Swaps but, as the OIS data does not go back to 1993, we use the policy interest rate as a proxy for the OIS.

3.1 *Principal Components of Output and Financial variables*

At a monthly frequency a measure of real GDP is not available and so other measures of real activity need to be used. Such measures include an index of production, retail sales (the volume of sales measure is used which removes the effects of price changes) and the claimant count. All three variables were statistically significant and each had a similar affect on the term structure but they were also highly correlated with one another. So to reduce the number of variables in the estimations we took the first principal component of the correlated series. The first principal component explains 69% of the variance of all three variables. The NIESR construct a monthly estimate of real GDP from monthly indicators such as industrial production and retail sales. The NIESR measure was included amongst the initial set of variables but was found to be statistically insignificant in each equations but our constructed index was found to be significant. Both share a correlation of 93% and the relative standard deviation of the index to the NIESR measure is 0.68.[14]

We also construct a measure of equity returns using principal components. From 1996 to 2010 an average of 22.5% of the UK government debt portfolio was held by overseas investors.[15] Some developments in foreign financial markets may have an impact on the demand for UK bonds such as episodes of 'flight to quality' where investors sell risky assets and purchase safer government bonds in times of economic turmoil. We include the UK's FTSE 100 but to capture an additional international factor and possible 'flight to quality' episodes we include two further equity indexes that represent larger international markets. The Standard and Poor's index of the largest 500 US companies and the Dax 30 of Germany as a measure of European markets. Just like the real activity index we take the first principal component of the three series particularly because of the very high degree of correlation between these three equity indexes. This first principal component explains 90% of the variance of the constituent variables. The correlations between the principal component and the individual series are 97% with the FTSE 100, 95% for the S & P 500 and 93% with the DAX 30.

[14] The correlations between this variable and the three measures of real activity are 88% with production, -95% with the claimant count and 48% with that of retail sales.

[15] This data is publicly available from the UK Debt Management Office and is available at a quarterly frequency from the first quarter of 1996.

3.2 *Fitting the Forward Curve*

For the nominal curve there are 15 maturities, vectors $\mathbf{Y}_t$ and $\boldsymbol{\varepsilon}_t$ are 15×1. $\mathbf{B}$ is a 15×4 matrix and $\sigma^2 \cdot I$ is a 15×15 matrix. For the transition equation there are 30 parameters that are estimated. The 16 elements of the 4×4 transition matrix $\mathbf{A}$, four elements of $\boldsymbol{\mu}$ and 16 elements of the 4×4 variance-covariance matrix Σ. The 15 diagonal elements of $\sigma^2 \cdot I$ also need to be estimated for the nominal forwards and 10 for the real forwards. So in total 45 parameters must be estimated for the nominal curve and 40 for the real. The values of lambda were fixed prior to estimation.[16] For the nominal curve we use the values 0.07456 for λ_1 and 0.03 for λ_2 and for the real curve 0.0557 λ_1 and 0.0238 for λ_2. For the nominal curve, the maximum loading of lambdas λ_1 and λ_2 are at 24 and 60 months respectively and for the real curve, the 32 and 75 months.[17] Table 2 presents the estimates of the $\mathbf{A}$ and Σ matrices and the vector of constants $\boldsymbol{\mu}$. Knowing that the transition equation follows an AR(1) process the estimates of $\mathbf{A}$ for the nominal curve show that the level and slope are non-stationary but the two curvatures are not. As with the Nelson-Siegel literature, the diagonal elements of $\mathbf{A}$ show declining persistence as we move from the level through to the second curvature but the shock volatility in $\boldsymbol{\Sigma}$ increases. The estimates of the real factors do not exhibit the same relationship as their nominal counterparts in $\mathbf{A}$. The level and first curvature are non-stationary unlike the slope and second curvature but there is increasing volatility through the $\boldsymbol{\Sigma}$ matrix.

The Svensson methodology fits the data well for the nominal forwards; Table 3 shows the mean and standard deviation of the fitting error in basis points at each maturity. The average fitting error for all maturities except 9 months is less than a basis point.[18] The estimated real factors fit the real curve very well at the short-end but do less well from 108 months onwards with the 120 month forward being over fitted by an average of 3 basis points. The nominal level is positive throughout the entire sample period and remains very persistent.[19] The nominal term structure is upward sloping across our sample period 54% of the time. Over the QE period the high level factor and the negative slope suggests that

[16] We solve for the optimal values of β_1, β_2, β_3, β_4, λ_1 and λ_2 using constrained optimisation by minimising the residual sum of squares between the average term structure across the entire sample period and the fitted curve using the Svensson methodology. The imposed constraints are λ_1 and λ_2 have to be greater than zero and $\beta_1 + \beta_2$ are greater than zero, so that all forward rates will be greater than zero.

[17] For completeness and comparison, the estimated values of λ_1 and λ_2 for the nominal curve were 0.100 and 0.033 which maximise the curvature at 18 and 54 months respectively. For the real curve the estimated values were 0.038 and 0.020 and maximise the curvature at 46 and 90 months respectively.

[18] We found the Svensson methodology has an improved fit over the Nelson and Siegel methodology, particularly at longer-term maturities but the differences were not statistically different at 5% or 1% for the nominal curve. For the real curve the difference in the fitting errors between the Nelson-Siegel function form and Svensson are statistically significant so we continue the analysis for both curves with the functional form of Svensson.

[19] The estimated nominal and real term structure factors are presented in Figures that are available on request.

there is a considerable spread between the short-term interest rate and the long-term interest rate. Figure 1 shows the average nominal yield curve across three different sub-samples, first, March 1993 to June 2007 which could be characterised as 'normal' economic conditions with monetary policy using the short-term rate as the policy tool. The next sub-sample is from July 2007 to February 2009 which encompasses the start of the financial crisis and includes the recession but where monetary policy makers are still using the short-term interest rate as their policy tool and the final sub-sample is across the period of active QE from March 2009 to January 2010. During 'normal' economic conditions the yield curve is almost flat with only, on average, 32 basis points difference between the 9 month nominal forward and the 10 year nominal forward, with some positive curvature but the ten year forward is lower than that of the 2 year forward. This has been widely attributed to inflation expectations in the UK being anchored, for example see Gürkaynak *et al* (2003). When the financial turmoil of 2007 began and the policy rate was being cut, the yield curve becomes more upward sloping. It is over the QE period, when the policy rate is at its zero lower bound, that spreads between the short and long rates are at their widest.

The real level displays less volatility than that of its nominal counterpart and follows a downward trend. The values of the real slope factor are mostly positive, with the real curve being on average downward sloping 53% of the time. Figure 2 shows the average real forward curve across the same three sub-samples as its nominal counterpart. Across the longest sub-sample the real yield curve is very flat with the long-term (120 months) rates being 9 basis points lower than the shortest maturity of 48 months. With the onset of the financial crisis long-term real rates fall considerably to a low of approximately 83 basis points and the spread between the shortest and longest rates is approximately 90 basis points. Over the QE period the real curve begins to flatten, in fact it is upward sloping from 48 to 96 months but longer rates are slightly lower. Across the data sample it appears the first curvature contains a structural break. There is a sharp fall in this series when the government introduced the Pensions Act in 1995 which became effective in April 1997, and seems responsible for an abrupt reduction in long-term yields.[20]

From the macro-finance literature one of the most common results is that the level factor is closely attributed to inflation or inflation expectations.[21] As we estimate both the nominal and the real curve, we can see if there is a high degree of contemporaneous correlation between the actual measure of inflation expectations that we use and breakeven inflation, in

[20]This is the Minimum Funding Requirement of the Pension Act. This requirement was introduced by the government to ensure the solvency of UK pension funds which led to an increase in demand for long-dated nominal and index-linked bonds by institutional investors. A simple Chow test confirms that there is no structural break in the data at either one or five percent significance so we feel that we are able to continue our analysis with these estimated factors.

[21]For a UK perspective, see Bianchi *et al* (2009) and Lildholdt *et al* (2007).

this case we measure the breakeven inflation rate as the difference between the nominal and real level factors. Breakeven rates are not a perfect measure of inflation expectations due to the presence of various time-varying premia. Over normal economic conditions the degree of correlation between the breakeven measure and inflation expectations is quite high at 55% and inflation expectations are seen to be relatively constant given the average nominal and real curve in Figures 1 and 2. But in the two later sub-samples the correlation becomes negative. From July 2007 to February 2009 the correlation is -58% and over the QE period the correlation is as low as -87%.

4 Results of the Macro-Finance Specification

Following estimation we perform two types of exclusion restriction tests on the independent variables and present the results in Table 4. The first test is shown in the top portion of the table and tests whether or not all macroeconomic variables in each equation are statistically significant. As the results show, the macroeconomic variables in each equation are jointly significant. The second exclusion restriction test is shown in the lower portion of the table and this tests whether or not each macroeconomic variable is significant across the four equations. The results show that each macroeconomic variable is jointly significant across the different equations. Collectively these results imply that macroeconomic variables can explain changes in forward rates and some of the variation in each of the factors. The descriptive statistics of the estimations are shown in Table 5. The R^2 is high across all four of the nominal equations, particularly the level and slope equations. We find it difficult to explain the variability of the curvature factors but, as already stated in Section 2, this is a result shared with other macro-finance literature. For the real factor equations the R^2 are not as high as those of the nominal except for the first curvature which has an R^2 value of 0.95. The second curvature is poorly explained by the macroeconomic variables. The RMSE of the real curve is larger than that of the nominal which highlights that the real curve is harder to predict with our macroeconomic and financial variables.

The tests for autocorrelation, heteroscedasticity and normality show that the residuals do not suffer with autocorrelation. Heteroscedasticity is a concern for the first curvature factor of the real curve. The only problem that is apparent is normality, particularly for the nominal level and the second curvature but also for the first curvature factor of the real curve. The estimated residuals have been shown to be stationary around a trend, rendering the forward curve factors stationary.[22] Dummy variables were used to take account of some of the larger residuals. For the nominal curve only one dummy variable was used and this was for

[22] Table available upon request.

the November of 2008. For the real curve there were three dummy variables used: October, November and December of 2008. The remaining results present the instantaneous responses outlined in Section 2.3. Rather than directly using the coefficients of the macroeconomic variables from the SUR estimations we use the total effects, $\hat{\theta}_i/1-\hat{\rho}$. All instantaneous responses can be read as being increasing or decreasing in hundreds of basis points and all responses are shown with 95% confidence intervals. Figure 3a presents the instantaneous responses for both the nominal and the real term structure where both curves respond to the same macroeconomic variables and Figure 3b present the response of the nominal term structure to additional macroeconomic variables.

4.1 *Inflation Expectations*

A 1% shock to inflation expectations increases the nominal curve by about 70 basis points at the short-end and 140 basis points at the long-end. The increasing yields stand in contrast to those of Gürkaynak *et al* (2003) who suggest that post-independence of the Bank of England (which was announced in the May of 1997) that announcements of various inflation measures had no impact on the curve which would suggest that the Bank of England's monetary policy is seen to be credible. Also Chadha *et al* (2007) find that post independence the medium and long-term interest rates fell by 50 basis points which is consistent with the sudden increase in the monetary authority's aversion to inflation. We find, with our explicit market measure of inflation expectations that an increase in inflation expectations causes long-term forwards to increase more than short-term rates following this shock. This suggests that markets expect a future monetary tightening but given that the 10 year forward rates increase quite considerably suggest that our results do not necessarily subscribe to the notion of completely credible monetary policy. For the real curve, a shock to inflation increases the yield curve over the medium term by approximately 100 basis points but there is very limited movement through to the longer term yields. The response of the two curves suggest that an increase in inflation expectations today leads to widening of breakeven rates in the future, capturing some possible inflation premium. Although there is a very large response in the real term structure the confidence bands[23] suggest that the results are insignificant and so the greatest impact is on the nominal forward curve which suggests markets may expect the short-term rate to rise in the future.

Both the post-estimation exclusion restriction test results presented in Table 4 and the SUR results suggest the measure of inflation expectations are significant and should be included within the real term structure system of equations. However, as demonstrated by the instantaneous responses and the joint standard error, inflation expectations are in fact

[23]Figure 3a presents both nominal and real term structure responses to variables that impact on both term structures. For Figure 3a we do not present the confidence intervals but rather, we discuss the results.

insignificant. The probable cause for this result is that the estimated coefficients are scaled by the Svensson factor weights to give the instantaneous response as shown in equation (8). The standard errors however are not scaled by the factor weights, thus making the standard errors for some coefficients proportionately larger in the instantaneous response than in the SUR estimations rendering inflation expectations and subsequently some of the variable insignificant.

4.2 *Debt-to-GDP*

The real and nominal curves both share a similar result for an increase in the supply of debt relative to GDP. The nominal curve shows very little response at the short-end of the curve but at the ten year forward, a one percent increase in debt-to-GDP increases forward rates by 12 basis points. For the real curve, following a similar increase in debt-to-GDP all forward rates increase by 9 basis points.[24] The confidence bands of both the nominal and the real curve suggest that the estimated responses are statistically significant which suggests that fiscal variables are important for determining the shape of the yield curve.

4.3 *Notes and Coins*

A £1 billion increase in notes and coins causes nominal yields to fall, with 24 month forwards falling by as much as 10 basis points. This may be a slight overestimate in the elasticity of the effect in notes and coins but we would expect these dynamics to be correct as private agents substitute their excess money balances for nominal bonds. But the increase in real yields suggests that real money balances and real bonds are not substitutes. Although the rate of return on real bonds are inflation protected it is possible the opportunity cost is not large enough relative to nominal bonds to encourage agents to switch out of money and into real bonds, explaining why the fall in nominal bonds is larger in absolute term compared to the negligible rise in real yields. The confidence intervals for both the nominal and the real term structure are quite narrow suggesting the impact on both yield curves are jointly significant.

4.4 *Libor Spread*

The effects of an increase in the Libor spread by 100 basis points only impacts the slope of both the nominal and the real curve which implies a very dominant effect on the short-end of the term structure. The Libor spread is often used as a measure of health for the banking

[24] Using the total net debt for the UK over the sample period this would suggest an average issuance of approximately £10bn of bonds at a time.

sector because an increase in the Libor rate is destabilising for the economy as lending to other banks and business is rendered more expensive. Dewachter and Iania (2011) find a similar result; they call a shock to their Libor spread a 'credit crunch' shock. They find that the level and slope increases as a result of this shock, leading to a flattening of the yield curve, and an increase in the short-end of the curve reduces the curvature. They report that there is a stagflationary response from a 'credit crunch' shock but in fact, we find a deflationary responses with real yields increasing more than the nominal curve at identical maturities which may suggest that the short-rate is not an expected monetary policy response to stagflationary pressures but possible premia effects originating from an increase in perceived credit risk. The confidence intervals are very wide for both the nominal and the real term structure, implying that jointly they are insignificant.

4.5 *Real Activity*

A 1% shock to real activity causes the long-end of the curve to increase over the short end, this stems from an increase in the second curvature factor. Although significant across within the equation, the instantaneous response does however suggest that the result is insignificant. As with the other literature we examine the correlation between the estimated level and inflation expectations as well as the slope and real activity. We find correlation between inflation expectations and the nominal level is 0.49 suggesting that there is a link between inflation expectations and the level. The real activity index and the slope have a correlation of 0.65 which provides evidence that the slope factor is indeed a business cycle factor with short-term forwards rising when real activity increases.

4.6 *Bank of England Policy Rate*

A 100 basis point increase in the policy rate leads to a fall in forward rates. The effect from a rise in the policy rate reduces the level but the first curvature and the slope increases. The negative level effect dominates the movement of the curve which gives the impression that the actual term structure falls in response to an increase in the policy rate. Gürkaynak *et al* (2003) find a similar result, ten year forward rates fall when there is an increase in the policy rate. Diebold *et al* (2006) suggest that an increase in the policy rate from a credible policy maker will cause long-run inflation expectations to fall, thus lowering the level of the term structure. Coupling such an interpretation with our inflation expectations result would suggest that if a surprise monetary tightening were to lead to a decline in inflation expectations then long-term rates would also fall.

4.7 *Effective Exchange Rate*

The effect of an appreciating effective exchange rate reduces the level of the term structure causing long-term rates to fall relative to short-term rates. This is consistent with a slowdown in real activity and an easing of inflationary pressures. It appears that a one point increase in the effective exchange rate has very little impact on short-term rates but reduces the nominal forward by approximately 20 basis points suggesting that future interest rates are slow to adjust to an exchange rate appreciation.

4.8 *IFO and German Retail Sales*

For the IFO business climate index for Germany, we estimate a dominant slope effect as short-term interest rates rise relative to long-term interest rates. A one percent increase in German Retail Sales has a similar result to that of the IFO index, with the slope effect being the only significant result. At the 120 month forward the results for both variables are similar but at the short-end the impact varies with the forward curve being influenced more by the impact of retail sales than the business manager's sentiment on the prospects of the German economy. The instantaneous response shows that the short-end of the curve increases which may capture an expected increase in the policy rate. This result captures possible demand shocks that arise from highly correlated business cycles both at home and abroad as the correlation between the UK real activity index and German retail sales is 79%. A similar result is found by Diebold, Li and Yue (2008), they find that the slope factors across the UK, Germany, US and Japan are highly correlated, as is real activity across countries.

4.9 *Federal Funds Rate*

The Federal Funds Rate only has an effect on the slope of the nominal term structure which generates an increase in short-term maturities of approximately 80 basis points for a one percent increase in the Fed's Funds Rate and there is some pass through to long-term rates with the 120 months forward increasing by approximately 12 basis points. Following the interpretation that an increase in the policy rate follows the business cycle (see Dewachter and Lyrio, 2006 and Hördahl *et al*, 2006 for example) then the impact of the Federal Funds Rate on the UK term structure is similar to that of the IFO Index and German retail sales above, that business cycles, both home and abroad are highly correlated.

5 Assessment of Quantitative Easing by Out-of-Sampling Forecasting

In Spring 2008 there was a large contraction in output and a rise in the level of unemployment, which later led to the short-term policy rate being cut from 5% in November to 0.5% on the 5^{th} of March 2009. On that very same day, the Bank of England's Monetary Policy Committee (MPC) announced that it would undertake large scale asset purchases, involving the borrowing of central bank reserves and the subsequent purchase of financial assets such as gilts and corporate bonds. This operation would change the magnitude and composition of the central bank balance sheet and swap reserves for, mostly, long duration government paper. Bank rate was maintained at 0.5% (the effective zero lower bound) whilst some £200bn of nominal government liabilities were bought.[25] On March 5^{th} the Bank of England announced that it would initially purchase £75bn pounds worth of UK gilts. On May 7^{th} and August 6^{th} the Bank announced it would extend purchases by another £50bn on both dates and then, finally on 5^{th} of November the last £25bn was purchased bringing the total to £200bn. The aim of these operations was to lower long-term interest rates in an attempt to provide further monetary and financial easing. The Bank of England (Dale, 2012) considered that there were three important channels by which the transmission mechanism of QE would work: the portfolio balance effect, the role of interest rate expectations and the role of liquidity in the market. The importance of each of these channels is contested from both the UK and US experiences of large scale asset purchases.

Only nominal paper was purchased because the nominal market is much larger than the inflation index-linked market,[26] as there were fears that large bond purchases might distort segments of the real yield curve where issuances were often less than £4 billion.[27] The actions undertaken by the MPC were initially intended to lower the yields on gilts with a remaining maturity that exceeded 5 years yet lower than 25 years but in the August, the Bank of England announced that the maturities eligible for purchase were to be extended to any nominal bonds with a remaining maturity greater than 3 years. To test our models abilities we adopt an approach similar to Bernanke *et al* (2004)[28] and D'Amico *et al* (2012), where we use our model to provide a counterfactual path of interest rates given the model

[25] By the end of QE1 the Bank of England had also purchased £1.956 billion of sterling corporate bonds.

[26] In 2009 the amount of nominal gilts in issue was approximately 4 times larger than the amount of inflation-indexed gilts in issue. See the UK Debt Management Office website for more details: http://www.dmo.gov.uk.

[27] On the 5^{th} March 2009 the Asset Purchase Facility issued a notice to the market stating which gilts were eligible for purchase. This note can be found at: http://www.bankofengland.co.uk/markets/Documents/marketnotice090305.pdf

[28] The authors examine the effectiveness of monetary policy at the zero lower bound. By using an affine term structure model the authors analyse the experience the Japanese had with Quantitative Easing over the sample period of December 1998 (prior to the introduction of QE) up to June 2001.

structure and conditional results from Section 4. We then decompose our forecast error into the three different channels in an attempt to identify which channels were responsible for the fall in yields between March 2009 and January 2010.

Using a dynamic forecast we project the estimated model of the previous section over the period of QE to produce a counterfactual forward curve that can aid in the identification of the total impact of QE on both the nominal and real curves.[29] The out of sample forecasting uses a dynamic system where the initial conditions commence from T, the system is then:

$$\hat{\mathbf{F}}_t = \sum_{i=1}^{m} \hat{\mathbf{\Pi}}_i \hat{\mathbf{F}}_{t-1} + \sum_{j=0}^{r} \hat{\mathbf{\Pi}}_{j+m+1} \hat{\mathbf{X}}_{t-j} \text{ for } t = T+1, ..., T+H \quad \text{where } \hat{\mathbf{F}}_{t-i} = \hat{\mathbf{F}}_{t-i} \text{ for } t-i \leq T. \tag{9}$$

The forecasts require data on the independent variables $\hat{\mathbf{X}}_{tT+1}, ..., \hat{\mathbf{X}}_{T+h}$ for H periods ahead. For the best forecasts, the independent variables must be exogenous and that the coefficients within $\hat{\Pi}$ must remain fixed at all points in time. The forecast is performed explicitly over the period of Quantitative Easing in the UK using the estimated coefficients from Section 4 with the forecasts beginning in January 2009 and ending in January 2010, so that $H = 13$. The forecasts then become a function of the lag, trend, constant and the independent variables with their respective coefficients in each equation. The purpose of this exercise is to see where the nominal and real forward curves are placed as a function of macroeconomic variables to examine the extent to which QE purchases had on the term structure.

Table 6 gives the point estimates of the forecast error (in basis points) for the nominal macro-finance forward curve when it is projected out-of-sample from January 2009 to January 2010. The first column in Table 6 reports the one-step ahead standard error. Figure 4 shows the out of sample forecast for 10 year forwards. From March 2009 to January 2010 the forecast error averages approximately 67 basis points at the 60 month forward and 46 basis points for the 120 month forward. This result is somewhat consistent with the implementation of QE with regards to its timing; maturities greater than 21 months appear to be affected by QE and the over prediction begins from March with the start of purchases.

Prior to QE the model under-predicts the curve at the 60 month forward by 62 and 35 basis points in January and February respectively. At 120 month, the model under-predicts the curve by 62 and 104 basis points in January and February. At this time there may have existed some concerns over the solvency of the UK government which possibly lead to higher yields but our fiscal variable is unable to capture the markets forward looking concerns.

[29] Our model is able to forecast the path of interest rates at each maturity quite accurately over other sample periods (not shown). We feel this allows us to perform the forecasting exercise over QE. Any deviation from the estimated path of interest rates can then be attributed to QE. With this methodology we are unable to test whether or not QE had an effect on macroeconomic variables. If QE were to lead to higher output, that would raise both the actual and our fitted curve without exerting any influence on the residual.

Given this, it is reasonable to ascribe the total change in forwards from the pre-QE period to the QE period to the events of QE, thus the overall change in the model residuals around the QE event from March 2009 to January 2010 relative to the January and February of 2009 is a reduction in the 60 month and 120 month nominal forwards by some 120-150bp.

Also in Figure 4 we present the forecast of 120 month real forward rates along with the actual path of the real rate and their significance bands. The forecasts of all maturities included within the sample, along with the standard errors are presented in Table 7. We use the forecasts of the real curve as a way to test the credibility of our nominal forecasts. We would not expect to observe any continuous over-or-under-estimate of the real rates as no index-linked debt was purchased as a part of QE. Figure 4 shows that the actual real forwards move in a very narrow range whereas the forecasted curve is somewhat more volatile. At 60 months, the average forecast error is an over-prediction of 6 basis points and given the average forecast error from March 2009 to January 2010 is only 3 basis points, the impact on the real curve is negligible at best. For the 120 month forward the average forecast error is smaller than that of the 60 months and the average error over the QE purchases is an over-estimate of 19 basis points. At every maturity there is no persistent deviation away from the actual path of real bonds, such a result is consistent with the Bank of England not purchasing any index-linked bonds as part of their initial QE program.

5.1 *Forecast Error Decomposition*

The forecasts of the nominal and real forward curves offer appealing results because the deviation between the actual and the forecasted nominal curve, as well as the timing of the overestimate suggests that QE had reduced actual yields. Second, the forecast of the real curve shows no persistent deviation from actual rates, consistent with there being no purchases of index-linked debt. However it may not be appropriate to attribute all of the difference between the actual and forecasted curve to QE. The primary reason for this being that both the nominal and the real forecasted paths are within one standard deviation of the actual path of interest rates (see Figure 4) and over the forecast period there were many news factors that would have altered the shape and dynamics of the yields curve. To explain how QE may have impacted on the forward curve we provide a decomposition of the 60 and 120 month forecast errors over the QE period based on some rather strong assumptions about QE and the channels that were expected to have worked their way through the transmission mechanism. For the UK's experience, Dale (2012) outlines three channels in which QE was intended to have worked through the transmission mechanism and directly impact the term structure: portfolio balance, liquidity and signalling.

First the modelling assumptions may be unrealistic as we tend to assume that financial markets are frictionless and there are no liquidity constraints that prevent assets from being perfectly priced and hedged. In reality, these assumptions may not hold and financial assets may have a downward sloping demand curve. A downward sloping demand curve then means that a change in the relative supply of assets will lead to a change in the assets price. It is this mechanism that gives the portfolio balance channel its traction. By reducing the available supply of bonds and thus increasing the price of bonds leading to lower interest rates.

Second, economic agents will hold a quantity of money up until the marginal benefit of the liquidity service it provides equals its marginal cost, the nominal interest rate. However, at the zero lower bound the assets become perfect substitutes and as such a change in the relative holdings will not impact on inflation, or real activity, leaving the economy in a liquidity trap. Eggerston and Woodford (2003) show that the zero bound represents a constraint to the monetary policy maker's stabilisation policy in an environment of low inflation, or deflation. such that the only way for the monetary policy maker to influence expectations is to commit to low interest rates for a period longer than what is normally expected, allowing inflation to overshoot its target over the medium term but return back to conventional policy when it is feasible to do so. So by signalling to the market its commitment, the monetary policy maker will allow inflation to overshoot its target and altering the market's expectations, generating an increase in nominal spending as agents bring their consumption forward.

The final channel, and quite possibly the smallest, is the liquidity channel. If market participants are accosted by greater uncertainty about the timing and the ability to trade, they are unable to hedge liquidity risk because of borrowing constraints. Such that, less liquid assets are imperfect substitutes for more liquid ones. This liquidity risk causes asset prices to diverge from the fundamental price determined by the underlying cash flow. Brunnermeir and Pedersen (2009) show that the availability of funding affects trades and this funding is affected by market liquidity. When funding is tight (the amount that the market participant can borrow against their collateral, haircut, margin etc) market participants will reduce the size of the positions they take in illiquid assets.

In light of the QE experience, both in the UK and the US, there has been a growing amount of empirical research into what channels exactly caused the observed fall in yields. We present a brief discussion of the most important literature. The earliest literature was in the form of event studies which determined the impact on yields immediately following asset purchase announcements. Such studies include Bauer and Rudebusch (2011), Gagnon *et al* (2011) and Swanson (2011). For the UK experience, the majority of the literature adopt an event study approach, which measures the change of various interest rates across the term structure (or other assets) over a one or two day window. The window in which the announcement is to be analysed needs to be very small so to limit the impact that other announcements may have on asset prices. At the same time, the window needs to be large enough so that market participants can correctly price the news following the surprise announcement. Examples of the event study include Meier (2009), Glick and Leduc (2010), Caglar *et al* (2012) and Meaning and Zhu (2012). Meaning and Zhu (2012) also adopt the approach of D'Amico and King (2010) to estimate the portfolio balance effect of Gilt purchases on the yield curve. A related analysis to ours is undertaken by Breedon *et al* (2012) who examine the impact of debt purchases on both monthly and daily bond yields over the same period as well as duration effects of the changing nominal bond portfolio brought about by large scale asset purchases. They found that the portfolio effect was likely to reduce yields somewhere between 30 to 90 basis points. The duration effect initially offset the declining yields from the portfolio balance effect because of increased government issuance at long-term maturities when the primary focus of QE purchases were focused on a residual maturity range from 5 to 15 years. Once the purchase of long-term bonds increased (and the maturity extension of eligible bonds) the duration effects were reversed.

Typically the different studies found that QE lowered the term structure of interest rates somewhere between 40 to 110 basis points. Caglar *et al* (2012) point out the normal event study methodology may have overestimated the effects because the first announcement of QE was truly a 'surprise' to the market rather than the subsequent six announcements (furthermore, the announcement coincides with the cut in the policy rate by 0.5%). For the remaining events the markets were expecting further rounds of purchases and as such, muted the effect of subsequent announcements. Table 6 shows the largest divergence between the forecast and the actual path of interest rates occurs between the February and the March but it should be noted that actual bond purchases started in the March of 2009 and Breedon *et al* (2012) show that asset prices increased around the times of auctions. We are unable to separate the surprise announcement and reverse auctions effects but given the change between the end of February and the end of March, the average decline of the nominal forwards rates over the same period was 76 basis points, which again falls in line with the findings of others. We already account for the cut in the policy rate of 50bps, so it can be expected that a large proportion of the decline in yields would be closely attributed to the surprise announcement rather than the asset purchases themselves.

For the US, D'Amico and King (2010) and D'Amico *et al* (2011) analyse the impact of QE using cross-sectional regressions on bond transactions data. D'Amico and King (2010) analyse the flow effects-the response of prices as a result of the ongoing purchases over the QE period and the stock effects- the change from the withdrawal of supply. Using a counterfactual path of interest rates the authors show that a withdrawal of $300bn worth of assets leads to a flattening of the yield curve between the 10 and 15 year maturity segment and lowering yields by approximately 45 basis points. D'Amico et al (2011) provide a description of the term premium effects for which they believe lowered long-term yields, a 'scarcity' channel which caused yields with substitute maturities to fall, a 'duration' channel whereby market participants can swap duration risk and a signalling channel. They find statistically significant results for both the scarcity and duration channels, with the duration channel accounting for approximately a quarter of the total impact. There were no statistically significant results for the signalling channel.

Thornton (2012), using the methodology of Gagnon *et al* (2011), presents evidence as to why the portfolio balance effect may be over-stated within their methodology; their measure of the term premium has a similar trend to that of their measure of public debt. By removing the trend, Thornton (2012) shows that there is no longer any statistically significant impact. We find from our results, there is a supply effect from the issuance of more debt (relative to GDP) on the term structure. This can be interpreted in the same fashion as D'Amico and King's (2010) stock effect. We have also included a linear trend in our initial estimations. Our statistically significant result (see Figure 3a) suggests that when there is an increase in the relative supply of assets, long-term yields increase. When we reverse the impact, the withdrawal of supply will lead to a flattening of the curve as there is very little impact on the short-end of the yield curve but there is an impact at the long-end. The three channels that we consider for the decomposition are the signalling, the portfolio balance and the liquidity channel. All of which, should have an immediate effect on the forward curve thus allowing us to show the impact that QE had on the nominal forward curve. Below we outline the different ways in which we intend to measure these different channels.

5.1.1 *Debt-to-GDP Effect*

The debt-to-GDP variable captures a pure supply effect on the forward curve and not any other factors such as the perceived credit risk of the UK government. The purchase of gilts made by the APF for the purpose of QE is the same as a reduction in the private sector's holdings of gilts, as the purchases reduced the amount of available supply to the private sector whilst at the same time reducing the total supply of available bonds but in particular, long-duration bonds.

We calculate the total number of bonds purchased each month at the reverse auctions and determine the equivalent size of debt-to-GDP, for example in March the Bank of England purchased 1.1% of the total supply of bonds relative to GDP and by the January 2010 they had purchased approximately 14% of the total supply of bonds relative to GDP.[30] Like D'Amico and King (2010) we treat the purchases as a stock effect, we take the cumulative

[30] Breedon *et al* (2012) calculate the debt effect using the amounts announced by the Bank of England to be purchased in the March, May, August and November. Our more granular analysis allows for a more detailed breakdown of the impact of the three different channels across time.

amount of purchases made from one month to the next. Given this we can estimate the impact of QE on bond yields at five and 10 years using the elasticity for a change in forward rates given a change in debt-to-GDP as seen in Figure 3a. This effect measures the change in the relative supply of the bonds, and assuming that bonds do have a downward sloping demand curve, the change in the supply of bonds will change the price of the bond. So by purchasing bonds, the monetary authorities are reducing the available supply, which increases the price and lowers yields. So, we expect to observe the portfolio balance channel to reduce yields, and the extent of the decline in yields to increase with the amount of purchases.

5.1.2 *Liquidity Channel*

The second channel in which QE may have worked through is the liquidity channel. The asset purchases made by the central bank should ease any liquidity issues that would have arisen from the financial crisis and the recession. The Bank of England's behaviour as a large purchaser of gilts should improve the functioning of the gilt market and reduce the liquidity premium that is embedded within the term premia. The size of the liquidity premium will vary from one maturity to another depending on how deep and liquid the market is at that particular maturity, in this case bonds that have a remaining maturity of five and ten years. The Bank of England paid a liquidity premium to ensure that bond holders would wish to give up their bond holdings. As we only examine the effects of this channel on the most liquid part of the curve, we may not observe a large liquidity premium, or even a premium that should depress yields. This premium is calculated and the liquidity effect is subtracted from forecast error.[31] The liquidity premium is there as a guide only to suggest which direction the liquidity premium (if any exists) impacts on the curve because the premium is determined by spot rates as opposed to the forward rates that we use but the affects would pass through to forwards rates.

5.1.3 *Signalling Channel*

To measure the signalling channel we use a risk adjusted measure of a market interest rate. Statistically the chosen interest rate should be able to predict the policy rate but the presence of term premia and differences in the maturity, liquidity and credit quality may

[31] One way to measure the liquidity premium is to look at the spread between an off-the-run bond (a bond in the secondary market that is at 10 years to maturity) and the on-the-run bond (a bond that is issued into the primary market). Any difference between the two would represent a liquidity premium. In this case we subtract the average weighted accepted yield at each auction for 5 and 10 year nominal bond purchases and subtract this from the closing price of the same day. This premium is not too dissimilar to the market liquidity measure of Brunnermeir and Pedersen (2009); the difference between the fundamental price (the off-the-run price) and the transaction price (the reverse auction price). We sum the results across each month and as this is a 'flow' variable we do not take the cumulative sum. We would like to thank the UK DMO for providing closing prices for five and ten year nominal bonds.

blur the signal for the future path of the policy rate. We forecast the policy rate using a simple linear regression model similar to that of Gürkaynak *et al* (2007) and Piazzesi and Swanson (2008) who use a similar type of regression to analyse market-based measures of the expected path of the Fed rate in the US:

$$\Delta i_{t,t+1} = \alpha + \beta(f_{t,t+1} - i_t) + \varepsilon_{t,t+1}. \tag{10}$$

Where $i_{t,t+1}$ is the realised Bank rate one year ahead, i_t is Bank rate observed today, $f_{t,t+1}$ is the forward rate observed today for one year ahead and $\varepsilon_{t,t+1}$ is an error term. The parameters α and β should be equal to 0 and 1 respectively if the pure expectations hypothesis is to hold. A more relaxed version of the expectations hypothesis[32] would expect $\alpha \neq 0$, implying there is a constant term premium and if β is statistically different to 1 this implies that there is some time-varying term premium. The data we use is the Bank of England's own commercial bank liability curve which is based on Libor interest rates and other market rates of various derivative instruments linked to the Libor such as forward rate agreements, short-term interest rate futures and Libor swaps.[33] This measure is based on unsecured lending and so the interest rate at one year should include default risk and as the Libor is a money market rate, some liquidity risk will also be present. We look at the expected path of interest rates up to one year only and we use end-of-month data over the same sample period as that of Sections 3 and 4. We correct the standard errors for heteroscedasticity and autocorrelation using the Newey-West procedure with 5 lags. The R^2 over this sample period is 0.37 suggesting that our chosen measure, the difference in the one-year forward and the policy rate can explain some of the variation in the policy rate in one-years time. The coefficient α has a value of -0.85 which suggests that the forward rates overestimate the actual outcome by 85 basis points, a result consistent with a positive forward premium. The coefficient on β of 0.96 is very close to one and with a general restriction test with a null hypothesis that $\beta = 1$, we do not reject the null hypothesis implying there is no time-varying term premium at this maturity for this particular interest rate.

What we call the signalling effect here is the difference between the policy rate and the markets expected path of interest rates. Using the one year commercial liability forward we subtract the estimated constant term premium (which is alpha in equation 10). This gives the risk adjusted forecast of the policy rate. What we call the signalling channel is then the difference between the risk adjusted bank liability forward and the Bank rate. The signalling effect is expected to occur within the month and should be different from one month to the

[32] This still requires well behaved residuals. We remove a unit root by taking the policy rate away from both sides.

[33] For more information we direct the reader to the following note by the Bank of England: http://www.bankofengland.co.uk/publications/Documents/quarterlybulletin/qb000404.pdf

next and we assume it has the same impact on both the 5 and 10 year forwards. The effect that the signalling channel may have had on the yield curve is somewhat ambiguous. When the Bank of England first undertook QE, market participants may have believed that the Bank will hold interest rates at the current level or maybe reduce Bank rate further. But if inflationary pressures were to have increased as a result of this monetary easing, the market may expected the Bank to pursue lower and stable inflation as outlined in its remit and change the policy rate.

5.1.4 *A Trial Decomposition*

Now that we have outlined our proposal to measure the three different channels we now decompose the forecast error into these three channels. Tables 8 and 9 show the decomposition of the residual at both five and ten years respectively from when QE was implemented in March 2009. The debt-to-GDP effect becomes larger as the amount of gilt purchased increased with this channel explaining as much as 98 to 126 basis points at 5 and 10 years respectively. For almost all of the gilts purchased at a maturity of 5 years there is a fall in the liquidity premium. Suggesting that non-bank financial institutions were willing to substitute their 5 year bonds for central bank reserves and that the Bank of England were willing to pay a premium to ensure this was so. For ten year government paper the liquidity premium is positive (except for March and April) although the reverse auctions were well covered.[34] Although market participants were willing to sell ten year bonds at a premium, the Bank of England's Asset Purchase Facility was not willing to pay a premium, which may simply be because the medium-term segment of the curve was the focus of the purchase strategy.[35]

Whereas the debt-to-GDP effect becomes larger and exerts more downward pressure on the yield curve as time progresses and the amount of purchases increased, it is the signalling channel that has the largest impact at the start of QE but becomes less prominent as bond purchases increased. In March when QE was announced the MPC made the decision to cut the policy rate to 0.5%, our model already takes account of this change in the out-of-sample forecast. Initially, the signalling channel has a large impact on the curve with the market expecting interest rates to fall a further 48 basis points below the new policy rate of 0.5% suggesting that market participants were pricing the policy rate to be closer to zero. As

[34] Individual auction data is available for each bond at each reverse auction from the QE data set compiled by the Bank of England for the conference 'Learning the lessons from QE and other unconventional monetary policies'. The data set is available at http://www.bankofengland.co.uk/publications/Pages/events/qeconference/qedataset.aspx

[35] The short-term is defined as any maturity between 3 to 7 years and medium-term maturities are from 7 to 15 years. Data regarding the breakdown of the UK government liability portfolio is available from the UK Debt Management Office and their website page http://www.dmo.gov.uk/index.aspx?page=Gilts/Portfolio_Statistics

was observed with the announcement effects literature, the largest impact of the signalling channel occurs in the March when QE was first announced but in the following months when the Bank announced further bond purchases the market expected interest rates to be lower than the current level of the base rate (except for August). The fact that the difference between the forecasted Bank rate and actual Bank rate becomes smaller suggests that market participants had already been expecting further rounds of purchases.

Over this period inflation expectations had begun to rise from 1.9% in the March to 2.4% in December. The market may then feel that if inflation follows its trend it would be close to 3% in a years' time, prompting a response from the Bank of England to contain the rise in inflation. But with the announcement of further asset purchases the market can expect the Bank to hold interest rates steady for the foreseeable future. Clearly the signalling channel had a large initial impact on the curve which explains some of the downward pressure on forward rates but in the latter stages of QE it was the debt effect that became dominant.

Now that the different channels have been outlined it is possible to explain why QE was unlikely to have had an impact on the real term structure, beyond the fact that no index-linked bonds were purchased by the Asset Purchase Facility. Figures 3a show that a change in debt-to-GDP is likely to had an effect on the real term structure through the change in relative supply of bonds but unlike their nominal counterparts, the holdings of index-linked bonds by insurance companies and pension funds is enforced by the Pensions Act of 1995, thus making the demand for index-linked bonds more elastic than conventional bonds. It is also very likely that escalating credit risk within the financial system was likely to have offset any possible debt-to-GDP effects. The instantaneous response for the effect of the Libor rate in Figure 3a shows that at identical maturities, the real curve is more sensitive to credit risk. So any perception that the UK government has become less credit worthy may be offsetting any possible debt-to-GDP effects. Furthermore, as no index-linked bonds were purchased it is unlikely that there would be a significant pass through from the liquidity channel. The Bank of England hoped that by buying nominal bonds, the new liquidity in the market would be spent on other asset classes, reducing their liquidity premium, thus providing one mechanism for the liquidity channel to have worked. The results of Tables 8 and 9 show the direct liquidity premium for nominal bonds is quite small, the impact on index-linked bonds, if they were to be purchased would most likely be very small also.

The policy rate and the money market curve are nominal rates and so any signalling effect would most likely be restricted to the nominal government curve, which is often used as a measure of forward guidance for the policy rate. The implication being very little or no impact from the signalling channel on the real term structure.

5.1.5 *Credit Effects*

The trial decomposition suggests that QE had a large impact on the forward curve but there remains a persistent residual to be explained. These three channels account for more than the forecast residual, suggesting that another factor may have been driving up nominal yields. The remaining residual might well reflect some change in the perceived credit worthiness of the UK government or some time variation in the estimated impact of the other channels, in particular, the debt-to-GDP variable which only captures supply effects. Any default risk premium would be a source of upward pressure on the yield curve and so we experimented with the spread of commercial rate forwards over nominal government forwards at the same maturity. We find that the spread has a correlation with the remaining residuals at 5 and 10 years of -0.51 and -0.81 respectively. The remaining forecast residual and the commercial-government liability spread are presented in the last two columns of Tables 8 and 9. The ten year commercial-government liability forward spread is also displayed in Figure 5. We observe that this spread at ten years is negative, implying that the perceived likelihood of default by the government has increased over this period. Furthermore, it appears that the impact is greatest on long-term rates but due to data limitations we cannot estimate the impact the spread has on the term structure so we are unable to make specific assertions about this channel but we want to develop a fully specified model of default risk to assess this in future work.

6 Conclusion

In this paper we study how a variety of macroeconomic and financial variables affect the nominal and real term structure of UK government liabilities from March 1993 to December 2008. The results are consistent with recent work undertaken by others in a joint macroeconomic and financial economic approach to understand the dynamics of the forward curve but our methodology is able to accommodate a larger number of explanatory variables than much of the available literature. However, unlike some of the macro-finance literature, we only explore the impact that macroeconomic variables have on the term structure of interest rates rather than the impact the term structure has on macroeconomic variables.

Our instantaneous responses show how the shape of the term structure is altered following a macroeconomic shock. From 31 different macroeconomic variables we identify ten key macroeconomic variables that impact on the nominal yield curve, all of which provide intuitively appealing results such as inflation expectations leading to an increase in yields and a positive supply effect in the gilt market. Further to this, we find that key economic indicators from Germany and the US impact on the UK curve. For the real curve we find that there are only four variables that shift the real curve but the confidence bands of the instantaneous responses suggest that these results are not significant. Overall we find that the factors of the nominal curve are well explained by the macroeconomic variables but these same variables to less well at explaining the real curve. We leave further work on the real curve to future work.

Then we forecast out-of-sample across the first round of QE in the UK from March 2009 to January to 2010. Our expectation for the future path of forwards rates is conditioned on the macroeconomic variables and the estimated macro-finance model parameters. The nominal curve was over-estimated by 60-70 and 40-50 basis points for the five and ten year forwards and for the real curve there is no persistent *over* or *under* prediction of the curve. Both the nominal and the real curve forecasts are consistent with the timing and the implementation of QE in the UK. Our results are similar to those found by Caglar *et al* (2012) and Meaning and Zhu (2012) who examine the immediate financial market impact of QE in the UK. We find that the forecast error can be readily explained by the portfolio balance, liquidity premium and signalling channels. When the Bank of England purchased gilts they were willing to pay a premium to purchase 5 year bonds from the market but not for 10 year bonds suggesting that the Bank was willing to pay a higher price for bonds in relatively less liquid segments of the curve. The liquidity channel alone can explain up to as much as 30 basis points in any one month but it was the portfolio balance channel that had the greatest impact on the nominal yield curve which can account for as much as 125 basis points in January 2010. From the initial implementation of QE it was the signalling channel that exerted most of the downward pressure on the forward curve as market participants expected the policy rate to be cut further or at least to be held at the zero lower bound for the foreseeable future. This channel can explain up to as much as 50 basis points. The forecasts of the real term structure across the first round of QE show no significant deviation away from the actual path of real forward rates which is consistent with the UK experience of QE, that no real bonds were purchased as part of the QE programme but it is also likely, that the three transmission channels of QE were unlikely to have an effect on the real curve. The focus of QE purchases was on nominal government bonds but also it is likely that credit conditions in the economy were offsetting any possible debt-to-GDP effects at that time.

Our work has to be extended to consider credit risk effects on nominal and real bonds and also the possibility of time variation of other potential channels but the identification of a supply effect on bond yields at this frequency allows us to have some confidence that the first round of QE did have some impact on lowering bond yields.

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Tables

Table 1. Initial Macroeconomic and Financial Variables.

Inflation	Real Activity	Policy	Foreign	Financial
Inf. Expectations	Real Activity*	BoE	UK Eff. ER	Equity Returns*
RPI	Unemployment	$\left(\frac{M}{P}\right)$	IFO Index	Libor
CPI	NIESR GDP	PSNCR	German R.S	Nationwide House Prices
RPIX	Terms of Trade	Debt-to-GDP	US NFP	Oil
RPIY	Trade Balance		FFR	Gold
PPI			ECB Policy	Vix Volatility Index
			Spot E.R:	
			$/£	
			€/£	
			¥/£	

Note: Those variables marked with an asterisk are those constructed using principal component analysis. For further details please see Subsection 3.1. The abbreviations are as follows: PSNCR is the public sector net cash requirement, BoE is the Bank of England policy rate, US NFP is US Non Farm Payrolls, FFR is the Federal Funds Rate, UK Eff. ER is the UK Effective Exchange Rate. The various inflation rates are as follows: RPI is the retail price index, RPIX is the retail price index minus mortgage payments, RPIY is the retail price index minus mortgage payments and indirect taxes, CPI is the consumer price index and PPI is the producer price index.

Table 2. Coefficients of the A , Σ and μ Matrices.

Nominal Parameter Estimates					Real Parameter Estimates				
	Estimated **A** Matrix					Estimated **A** Matrix			
	L_{t-1}	S_{t-1}	$C_{1,t-1}$	$C_{2,t-1}$		L_{t-1}	S_{t-1}	$C_{1,t-1}$	$C_{2,t-1}$
L_{t-1}	0.980	-0.034	-0.018	0.038	L_{t-1}	0.948	0.408	-0.007	0.260
S_{t-1}	0.030	1.038	0.052	-0.034	S_{t-1}	-0.374	0.095	-0.224	-0.390
$C_{1,t-1}$	0.079	-0.090	0.861	0.053	$C_{1,t-1}$	0.648	1.047	1.221	0.449
$C_{2,t-1}$	-0.146	0.096	0.161	0.824	$C_{2,t-1}$	0.295	-0.086	0.062	0.211
	Estimated Σ Matrix					Estimated Σ Matrix			
	L_t	S_t	$C_{1,t}$	$C_{2,t}$		L_t	S_t	$C_{1,t}$	$C_{2,t}$
L_t	0.019				L_t	0.046			
S_t	-0.026	0.042			S_t	-0.052	0.132		
$C_{1,t}$	0.034	-0.061	0.107		$C_{1,t}$	0.048	-0.178	0.263	
$C_{2,t}$	-0.073	0.106	-0.151	0.293	$C_{2,t}$	-0.147	0.201	-0.214	0.491
	Estimated μ Matrix					Estimated μ Matrix			
L_t	0.027				L_t	0.038			
S_t	-0.163				S_t	1.00			
$C_{1,t}$	-0.410				$C_{1,t}$	-1.66			
$C_{2,t}$	0.856				$C_{2,t}$	-0.646			

Note: These estimates correspond to the transition equation that is shown in Equation 5, which explains the dynamics of the estimated latent factors. The estimated standard deviations of Equation 6 are not shown. **A** are the autoregressive coefficients of the lagged latent factors. The Σ matrix is the variance-covariance matrix of the transition equation and the μ matrix are the constants of the transition equations.

Table 3. Average Fitting Error and Standard Deviations at Each Maturity.

	Nominal			Real	
Maturity	Mean	Std. Dev.	Maturity	Mean	Std. Dev.
9	1.00	4.159	48	0.018	0.14
12	-0.85	2.109	54	-0.018	0.27
15	-0.72	3.437	60	-0.026	0.23
18	-0.22	2.622	66	0.019	0.24
21	0.28	1.449	72	0.019	0.26
24	0.54	1.267	78	-0.013	0.16
30	0.48	2.682	84	-0.181	0.71
36	0.17	3.305	96	-0.741	2.17
48	-0.34	2.578	108	-1.649	4.52
60	-0.33	1.768	120	-3.077	7.70
72	-0.04	2.403			
84	0.07	2.700			
96	0.06	2.178			
108	0.08	1.353			
120	0.05	3.319			

Note: The results in this table are the average Svensson fitting errors as given by the arithmetic mean and are presented in basis points. Maturity is measured in months and the fitting error is given as the actual curve minus the fitted curve at each maturity and as an example: the 9 month forward is under-fitted by an average of 1 basis point and the 120 month nominal forward, the average fitting error is five hundredths of a basis point.

Table 4. Exclusion Restrictions Tests.

	Nominal	Real
Exclusion restrictions within equations		
Level	46.66 [0.0000]**	34.38 [0.0000]**
Slope	113.45 [0.0000]**	29.33 [0.0000]**
Curv. 1	56.47 [0.0000]**	27.55 [0.0000]**
Curv. 2	12.06 [0.0072]**	44.53 [0.0000]**
Exclusion restrictions across equations		
Inf. Exp.	8.004 [0.0046]**	59.60 [0.0000]**
RA	9.86 [0.0017]**	
BoE Policy Rate	58.07 [0.0000]**	
Debt to GDP	13.92 [0.0010]**	22.37 [0.0000]**
UK Eff. ER	38.01 [0.0000]**	
IFO	12.17 [0.0005]**	
GRS	27.85 [0.0000]**	
FFR	21.13 [0.0000]**	
$\left(\frac{M}{P}\right)$	22.70 [0.0000]**	60.54 [0.0000]**
Libor	19.02 [0.0000]**	5.28 [0.0216]*

Note: Curv.1 is the first curvature factor and Curve. 2 is the second curvature factor. All exclusion restriction tests have a null hypothesis that all coefficients are jointly equal to zero. Each test has a Chi^2 distribution with four degrees of freedom. The top part of Table 4 presents the within equation exclusion restriction tests. This test determines whether or not all included macroeconomic variables used to explain each factor are jointly equal to zero. The lower portion of the table tests whether or not an individual macroeconomic variable can be excluded from each equation in which it is present.

Table 5. Estimation Results for the Nominal and Real Curves from March 1993 to December 2008.

	Nominal				Real			
	Level	Slope	Curv.1	Curv.2	Level	Slope	Curv.1	Curv. 2
R^2	0.95	0.95	0.86	0.73	0.88	0.90	0.94	0.59
RMSE	0.49	0.53	1.14	1.87	0.46	0.46	1.18	1.57
Durbin-Alt	0.02 [0.8865]	0.31 [0.5798]	0.76 [0.3834]	4.84 [0.0279]*	14.49 [0.0001]**	0.16 [0.6881]	7.34 [0.0067]**	2.16 [0.1413]
Breusch-Pagan	2.98 [0.0844]	1.63 [0.2018]	1.48 [0.2238]	6.56 [0.0104]*	2.93 [0.0869]	1.05 [0.3046]	10.19 [0.0014]**	0.20 [0.6548]
Normality	19.09 [0.0000]**	6.29 [0.0162]*	0.62 [0.7319]	39.28 [0.0000]**	2.37 [0.3053]	3.52 [0.1720]	11.04 [0.0040]**	3.51 [0.1728]

Note: The Durbin-Alternative test (with one lag), has a null hypothesis that the residuals are homoscedastic. This tests has one degree of freedom. The Breusch-Pagan test has 1 degree of freedom and the null hypothesis is that the residuals are not autocorrelated. The normality test has two degrees of freedom and the null hypothesis is that the residuals are normally distributed. All tests have a Chi^2 distribution and ** represents rejection of null at the 1% level and * represents rejection of the null at the 5% level.

Table 6. Forecast Errors and Standard Errors in Basis Points for Nominal Forwards from January 2009 to January 2010.

Maturity	Std Err.	Jan '09	Feb '09	Mar '09	Apr '09	May '09	Jun '09	Jul '09	Aug '09	Sep '09	Oct '09	Nov '09	Dec '09	Jan '10
9	118	5.96	73.42	35.12	60.49	7.23	96.71	98.82	37.57	52.05	79.55	30.59	13.30	27.96
12	118	-28.07	35.98	15.82	34.18	-22.82	70.03	71.83	-12.18	-9.72	31.14	-17.57	-30.01	-13.88
15	117	-41.15	13.56	6.14	24.02	-32.14	59.38	58.39	-40.17	-51.86	-1.73	-47.53	-58.53	-40.39
18	116	-41.45	0.34	1.93	21.17	-31.12	54.20	49.59	-58.66	-81.44	-24.14	-67.48	-76.57	-59.45
21	115	-36.69	-8.91	-2.77	19.14	-28.41	48.62	42.31	-70.80	-102.49	-41.29	-80.00	-89.80	-73.40
24	113	-29.29	-13.99	-8.75	16.72	-25.15	42.39	34.07	-79.86	-117.19	-53.77	-89.13	-98.33	-83.14
30	110	-12.76	-16.13	-25.18	6.96	-21.30	26.17	18.27	-91.47	-134.07	-69.45	-98.48	-106.83	-95.74
36	106	4.60	-11.42	-43.59	-7.02	-21.15	8.20	2.92	-95.10	-138.50	-75.95	-102.19	-106.07	-99.29
48	99	35.59	9.01	-78.70	-33.10	-22.31	-28.25	-21.77	-88.41	-128.07	-74.26	-98.27	-89.17	-89.83
60	92	62.19	34.83	-99.74	-46.93	-18.14	-54.09	-38.34	-74.69	-109.85	-65.29	-90.86	-67.16	-72.88
72	87	79.86	58.54	-105.18	-48.91	-4.12	-66.13	-47.26	-60.49	-91.74	-55.90	-85.34	-47.86	-57.46
84	82	85.56	75.38	-98.77	-41.03	14.32	-64.54	-50.81	-50.47	-81.60	-50.29	-82.40	-37.18	-48.05
96	78	82.07	87.86	-85.29	-29.25	32.84	-56.05	-49.67	-48.43	-79.09	-47.71	-81.00	-34.16	-45.28
108	74	71.90	96.38	-67.42	-17.63	45.69	-44.67	-47.73	-55.45	-84.91	-50.42	-82.99	-39.71	-47.47
120	71	61.82	103.73	-50.22	-7.77	50.21	-35.20	-46.72	-69.69	-98.60	-57.39	-86.76	-50.21	-54.33

Note: For Tables 6 and 7 the forecast errors are measured as actual forward minus the forecasted forward and the results are presented in basis points. A negative forecast error implies that the forecast over estimates the actual curve.

Table 7. Forecast Errors and Standard Errors in Basis Points for Real Forwards from January 2009 to January 2010.

Maturity	Std Err.	Jan '09	Feb '09	Mar '09	Apr '09	May '09	Jun '09	Jul '09	Aug '09	Sep '09	Oct '09	Nov '09	Dec '09	Jan '10
48	52	-56.23	26.16	59.26	-34.82	23.13	-30.22	-16.91	-19.69	-40.55	-16.22	-12.62	24.01	32.36
54	51	-43.14	23.22	54.26	-28.38	18.54	-44.14	-17.91	-16.44	-38.44	-16.10	-13.10	28.87	36.38
60	50	-34.55	22.39	50.42	-23.94	14.87	-55.15	-19.55	-14.85	-36.86	-16.60	-11.91	31.46	39.92
66	49	-28.77	23.84	46.62	-20.85	12.68	-63.87	-23.46	-12.76	-33.98	-17.78	-12.17	32.64	43.66
72	48	-26.31	25.67	43.48	-18.61	10.32	-71.04	-27.33	-12.06	-31.91	-19.65	-11.96	33.27	46.36
78	47	-25.78	29.90	40.45	-17.91	9.02	-76.46	-31.95	-11.66	-29.75	-21.24	-11.38	32.26	47.84
84	46	-27.93	33.55	37.86	-17.51	7.91	-81.01	-37.13	-12.50	-27.53	-23.60	-11.48	30.55	49.94
96	44	-35.46	43.01	30.81	-20.05	4.53	-87.18	-45.72	-15.72	-25.06	-26.69	-11.95	25.79	50.52
108	43	-48.83	51.79	22.54	-23.48	1.38	-91.12	-55.33	-20.46	-25.64	-31.11	-13.71	18.69	49.39
120	42	-64.42	59.60	13.62	-29.51	-3.62	-94.64	-65.49	-28.60	-28.33	-36.99	-16.00	9.03	45.06

Table 8. Decomposition of the Five Year Forward Forecast Error.

	Forecast Error	Debt Effect	Liquidity	Signalling	Total	Forecast Error -Total	Libor-Forward
	A	B	C	D	B+C+D	A-(B+C+D)	
Mar '09	-100	-8	0	-48	-56	-44	49
Apr '09	-47	-22	-17	-35	-74	27	24
May '09	-18	-34	-32	-41	-107	89	54
Jun '09	-54	-50	0	10	-41	-14	86
Jul '09	-38	-62	0	15	-47	9	65
Aug '09	-75	-67	-2	5	-64	-11	58
Sept '09	-110	-78	-5	5	-78	-32	38
Oct '09	-65	-87	-33	2	-118	53	22
Nov '09	-91	-89	-29	-23	-142	51	1
Dec '09	-67	-93	-17	15	-95	28	-15
Jan '10	-73	-98	-32	-19	-150	77	-13

Note: For Tables 8 and 9 the forecast error is calculated as actual forward rates minus the forecasted forward rates. All forecast errors and individual effects are presented in basis points and have been rounded to the nearest whole basis point. An entry marked with zeros in the liquidity premium channel implies that there were no auctions that month for 5 or 10 year bonds. The last column is the difference between forward Libor rates and the forward rates on nominal government liabilities of the same maturity. The final column is not an important part of the decomposition as we cannot formally test whether or not these credit effects influence the yield curve.

Table 9. Decomposition of the Ten Year Forward Forecast Error.

	Forecast Error	Debt Effect	Liquidity	Signalling	Total	Forecast Error -Total	Libor-Forward
	A	B	C	D	B+C+D	A-(B+C+D)	
Mar '09	-50	-10	9	-48	-49	-1	-23
Apr '09	-8	-29	-17	-35	-80	72	-70
May '09	50	-48	-9	-41	-98	149	-66
Jun '09	-35	-64	0	10	-55	19	-28
Jul '09	-47	-80	31	15	-35	-12	-19
Aug '09	-70	-87	14	5	-67	-3	-9
Sept '09	-99	-100	10	5	-84	-15	-28
Oct '09	-57	-112	10	2	-99	42	-26
Nov '09	-87	-115	9	-23	-129	43	-25
Dec '09	-50	-119	0	15	-104	54	-47
Jan '10	-54	-126	0	-19	-145	91	-39

Note: See footnote for Table 8.

Figures

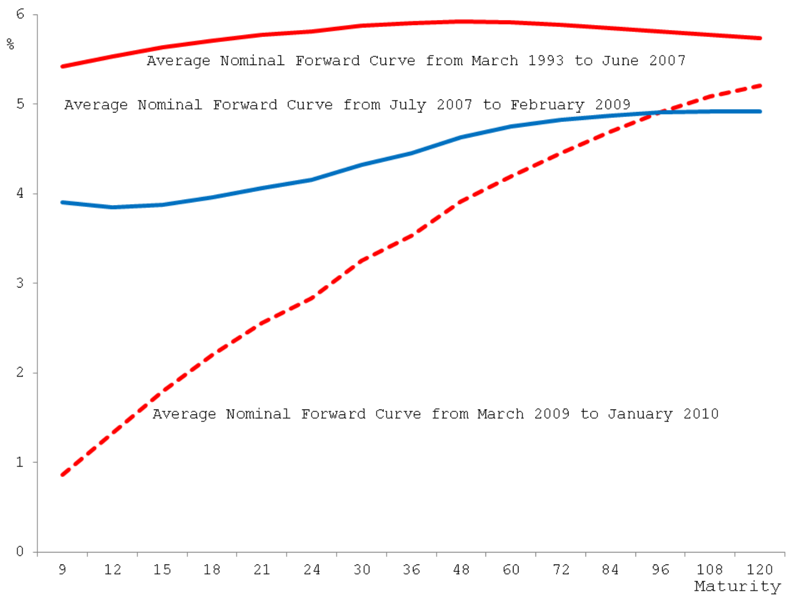


Figure 1. Average Nominal Forward Curves

Note: Figures 1 and 2 present the average nominal and real forward curves across three different subsamples.

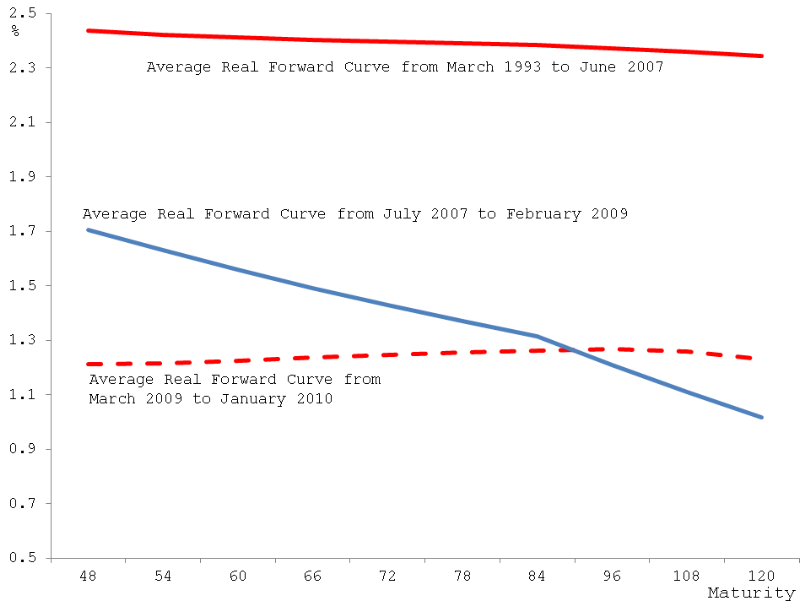


Figure 2. Average Real Forward Curves

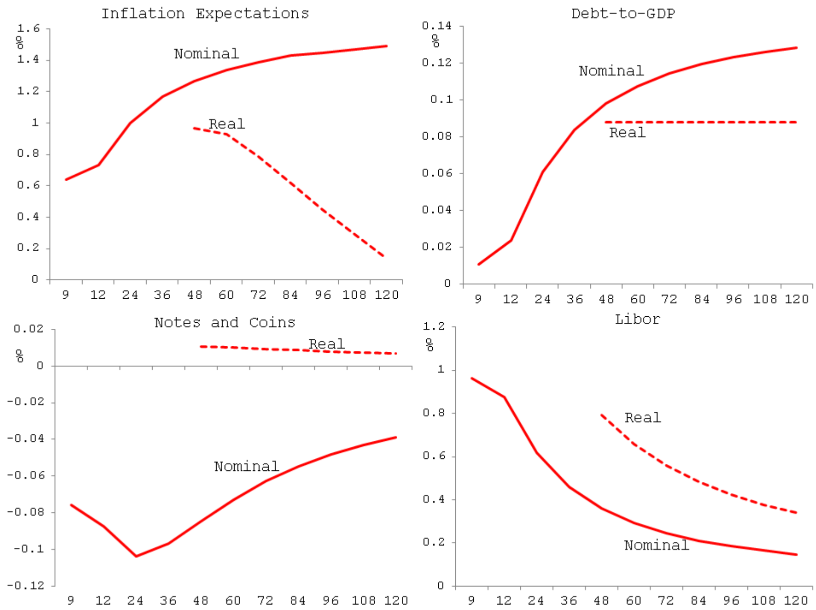


Figure 3a. Nominal and Real Instantaneous Responses to a 1% Shock in Macroeconomic and Financial Variables

Note: Included in Figure 3a are the nominal and the real instantaneous responses for the same macroeconomic and financial variables. The shortest maturity available for the real curve is the 48 month forward. For Figure 3a we exclude the standard errors for ease of representation. The size of one standard error for the nominal curve responses are as follows: Inflation expectations 57bp, debt-to-GDP 9bp, notes and coins 5bp and the Libor spread 46 basis points. The size of one standard deviation for the real curve response: inflation expectation 216bp, debt-to-GDP 7bp, notes and coins 0.1 basis points and the Libor spread 46 basis points. Please note that in Figure 3b the confidence interval is the 95th percentile which is 1.96 times the standard error.

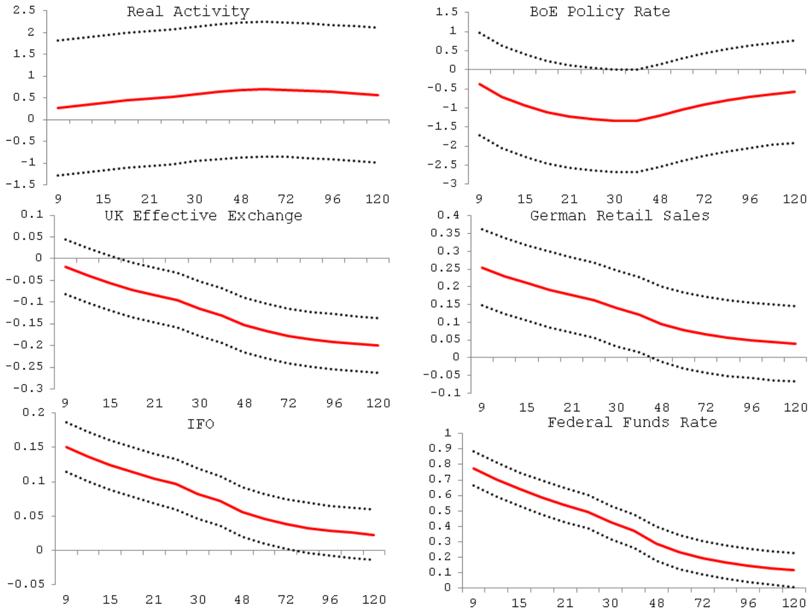
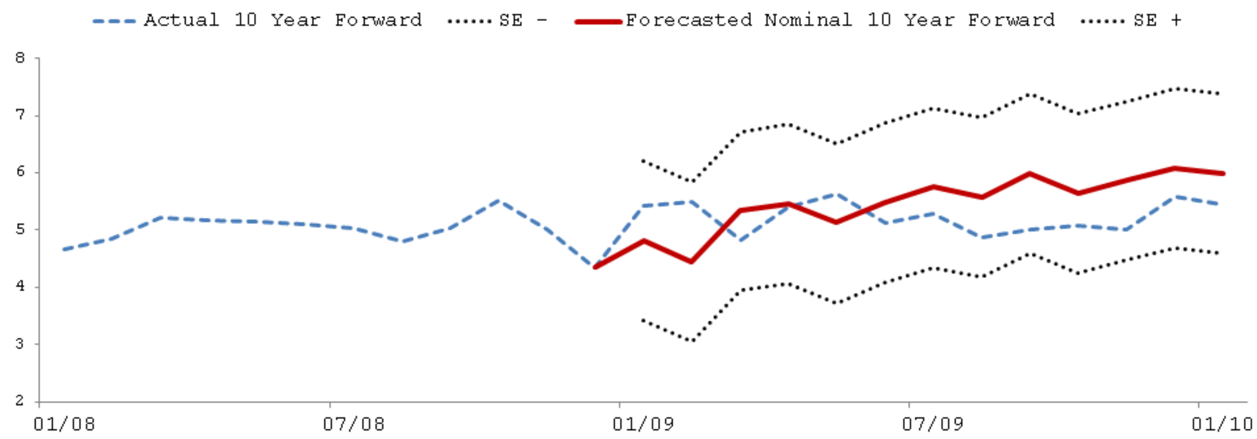


Figure 3b. Additional Nominal Instantaneous Responses to a 1% Shock in Macroeconomic and Financial Variables

Note: Figure 3b includes the remaining macroeconomic and financial variables that have an effect on the nominal term structure. The red solid line is the term structure response to the macroeconomic variables and the black dotted lines are the 95 percent confidence intervals. See page 11 for an overview of the standard errors and the confidence bands.



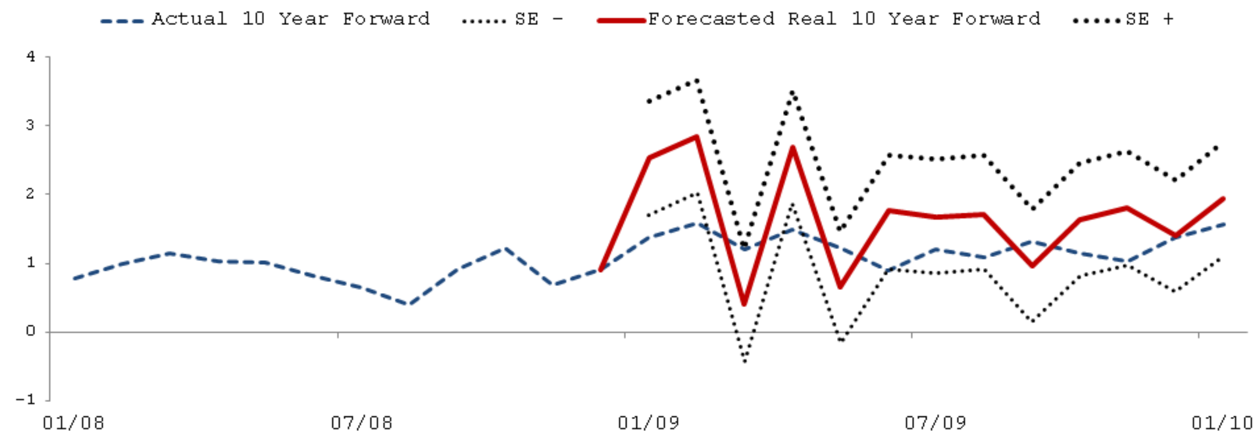


Figure 4. Nominal and Real 10 Year Forward Forecasts with Actual 10 Year Forwards and Standard Errors

Note: The forecasted interest rates are calculated by taking the forecasted term structure factors and multiplying by the factor weights from the Svensson methodology for the 10 year rate, which gives the ten year forward rate. The joint forecast error from each of the different factors for each forecast observation is calculated by multiplying the forecast error by the ten year factor weight so it is scaled accordingly.

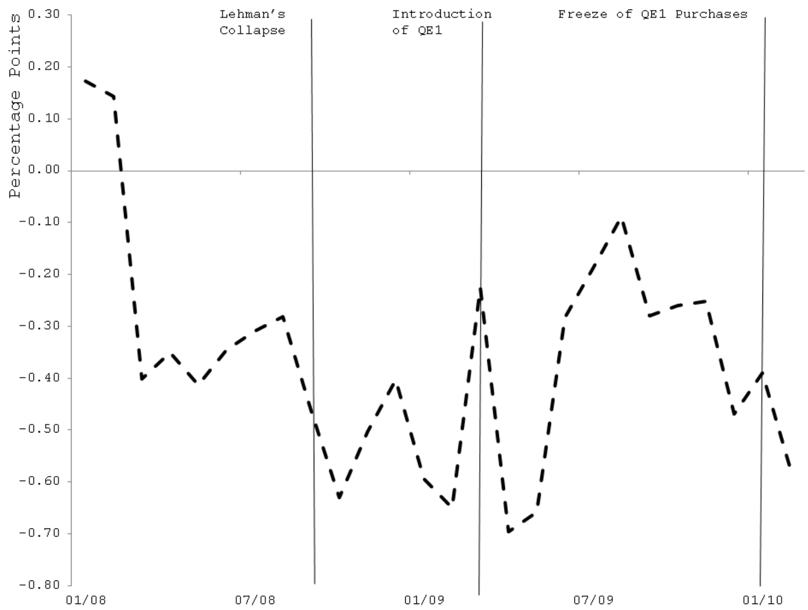


Figure 5. The Ten Year Commercial Rate - Government Liability Forward Spread

Note: The spread is calculated as the ten year commercial liability forward minus the ten year government liability forward. A negative spread implies that the government forward is higher than the Libor forward. This series is used as a demonstration of possible fall in the credit worthiness of the UK government in Tables 8 and 9.