University of Kent

School of Economics Discussion Papers

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December 2013

KDPE 1320

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Abstract

We study the exchange rate effects of monetary policy in a balanced macroeconometric two-country model for the US and UK. In contrast to the empirical literature on the 'delayed overshooting puzzle', which consistently treats the domestic and foreign countries unequally in the modelling process, we consider the full model feedback, allowing for a thorough analysis of the system dynamics. The consequential inevitable problem of model dimensionality is tackled in this paper by invoking the approach by Aoki (1981) commonly used in economic theory. Assuming country symmetry in the long-run allows to decouple the two-country macro dynamics of country averages and country differences such that the cointegration analysis can be applied to much smaller systems. Secondly the econometric modelling is general-to-specific, a graph-theoretic approach for the contemporaneous effects combined with an automatic general-to-specific model selection. The resulting parsimonious structural vector equilibrium correction model ensures highly significant impulse responses, revealing a delayed overshooting of the exchange rate in the case of a Bank of England monetary shock but suggests an instantaneous response to a Fed shock. Altogether the response is more pronounced in the former case.

Keywords: Two-country model; Cointegration; Structural VAR; *Gets* Model Selection; Monetary Policy; Exchange Rates.

JEL classification: C22; C32; C50.

1 Introduction

When studying the effects of monetary policy on exchange rates the central theory contributions are the overshooting models of Dornbusch (1976) and Frankel (1979), predicting an instantaneous jump of the exchange rate after an interest rate shock, due to different speeds of adjustment in goods and asset

*Corresponding author, email: r.heinlein@keele.ac.uk (R. Heinlein). We are grateful to audiences at seminars at Kent, University of Konstanz, DIW Macroeconometric Workshop 2011, Berlin, SNDE Symposium, Istanbul, the ICMAIF 2012, Rethymno, the VfS Congress 2012, Göttingen and the RCEA Time Series Workshop 2013, Rimini. The usual disclaimer applies.

markets, followed by a slow convergence to the steady state exchange rate. For the post-Bretton-Woods era Eichenbaum and Evans (1995) found instead of a sudden jump of the exchange rate a steady but slow appreciation for several months, followed eventually by a depreciation. This belated peak of the exchange rate is referred to as the 'delayed overshooting puzzle'. Several papers followed after this surprising result, some confirming the delayed response others supporting the theory predictions.

The question of the effects of monetary policy, in form of an increase in short-term interest rates, on the $/£ exchange rate is revisited in this paper utilizing a fully balanced two-country setup with the possibility of full model feedback between both countries. A thorough analysis of the system dynamics requires a balanced approach with both economies being modelled equally detailed and monetary policy shocks are studied in each country. This is not standard in the VAR model based literature on this topic, as shown in Table 1, which reports the sets of variables used in the literature.[1]

Table 1 *Model specifications in the VAR model based literature on the delayed overshooting hypothesis*

Papers	US Variables	Non-US	Exchange rate	Shocks
Eichenbaum and Evans (1995)	p, y, R^X	$i - i^{us}$	$s(e)$	$R^{X,us}$
	p, y, R^X, i	y, i	$s(e)$	$R^{X,us}, i^{us}$
	p, y, R^X, i^f	y, i	s	$R^{R,us}$
Grilli and Roubini (1996)	π, y, i	π, y, i	e	i
	π, y, i	$\pi, y, r - i$	$e(s)$	$r - i$
	i	p, y, m, i, p^o	e	i
Cushman and Zha (1997)	p, y, i^f	p, y, m, i, Ex, Im, p^c	e	m
Kim and Roubini (2000)	i^f	p, y, m, i, p^o	$e(s)$	$i, i^{f,us}$
Faust and Rogers (2003)	p, y, R^X, i	y, i	e	$R^{X,us}$
	p, y, R^N, R^T, m, i, r	p, y, m, i, r, p^c	e	$R^{N,us}$
Scholl and Uhlig (2008)	p, y, R^X, i	y, i	e	$R^{X,us}$
Bjørnland (2009)	i^w	π, y, i	Δs^w	i
	i^w	π, y, i, p^o	Δs^w	i
Voss and Willard (2009)	p, y, u, m, i^f	p, y, i, p^c	$e(s)$	$i, i^{f,us}$
Bouakez and Normandin (2010)	p, y, R^N, R^T, i^f	$i - i^{us}, p^c$	e	$R^{N,us}$
	p, y, R^N, R^T, i^f, i	p, y, i, p^c	e	$R^{N,us}$
Heinlein and Krolzig (2012)	$\pi - \pi^{us}, \Delta y - \Delta y^{us}, i - i^{us}, r - r^{us}$		e	$i - i^{us}$

The variables are defined as follows: price index, p, inflation, π, oil price, p^o, commodity price, p^c, output, y, unemployment rate, u, export to the US, Ex, import from the US, Im, non-borrowed reserves, R^N, total reserves, R^T, ratio of non-borrowed to total reserves, R^X, Romer and Romer (2004) monetary policy index, R^R, monetary aggregate, m, the federal funds rate, i^f, short-term interest rate, i, long-term interest rate, r, nominal exchange rate, e, real exchange rate, s, trade weighted, ^{w}.

In the literature it is common to apply a small-country framework, see Cushman and Zha (1997), Bjørnland (2009) and Voss and Willard (2009). The predominant number of papers model only one country in detail, like in Kim and Roubini (2000), Faust and Rogers (2003) or in Bouakez and Normandin (2010). Grilli and Roubini (1996) use an identical set of variables for both countries, but treat the countries differently at the theory-driven recursive identification stage.[2] Heinlein and Krolzig (2012) build a fully symmetric two-country model but that implies perfectly symmetric responses to foreign and domestic policy shocks. A key reason for the use of unbalanced information sets for the two countries

[1] There has been some criticism about the limited information set of small-scale VAR approaches. Mumtaz and Surico (2009) applied a factor augmented VAR, Binder et al. (2010) used a Global VAR model, both find no delayed overshooting.

[2] Scholl and Uhlig (2008) state in a robustness chapter that they performed analysis with a model with a fully symmetric set of variables and it would deliver similar results than in their baseline specification.

is the problem of dimensionality arising with an increasing number of model variables. The motivation for an asymmetric treatment in restraining the number of variables, is clearly stated in Eichenbaum and Evans (1995) and Bouakez and Normandin (2010) among others. Monetary policy shocks in both countries are studied in Kim and Roubini (2000) and Voss and Willard (2009), but in both papers in a non-balanced setup. The necessity to analyse monetary policy shocks in the domestic and the foreign country is emphasised by Voss and Willard (2009). Using an unbalanced setup they found that only policy shocks by the Reserve Bank of Australia are affecting the AUD/USD exchange rate significantly. Grilli and Roubini (1995) find puzzling results in the case of the non-US monetary policy shock, but theory-conform overshooting results on impact in the case of a US monetary shock. The possibility of asymmetric responses of the US and the other countries is not generally discussed in the literature.

We develop in this paper a small econometric two-country model for the UK and the US macroeconomy for the post-Bretton-Woods period. The system consists of nine variables: inflation, output growth, 3-month interest rates and 10-year government bond yields for both countries and the nominal USD/GBP exchange rate. To adequately study the long-run and the short-run properties of the macroeconomic time series we construct a CVAR model. An econometric model with a well-defined long-run equilibrium imposes important data-coherent constraints on impulse response functions, which are critical when assessing the effects of macroeconomic stabilization policies. The results of our two-country model will show a pronounced asymmetry: a delayed overshooting response of the exchange rate in the case of a Bank of England shock, but an instantaneous jump in the case of a Fed shock.[3] Altogether the exchange rate response is larger in size after an increase of the short-term interest rate in the UK. This asymmetric result illustrates the need for a rigorous analysis of this issue and may shed light on the conflicting results in the literature.

This paper seeks to contribute to the knowledge on the delayed overshooting puzzle by improving on the existing literature in four economically and econometrically important aspects:

(i) Lack of balance: The empirical literature on the 'delayed overshooting puzzle' consistently treats the domestic and foreign countries unequally in the modelling process. To allow for a thorough analysis of the system dynamics, the possibility of full model feedback is necessary, with the need for a balanced VAR model. We apply a balanced approach with both economies being modelled with an equally detailed information set. Further we identify the long-run and the short-run empirically, with homogeneous treatment of the two countries. Finally we study shocks with an origin in each of the two countries. This allows to detect differences in responses, which are purely data driven and not predetermined by the model set-up.

(ii) Lack of a well-specified long-run: Despite the involvement of possibly integrated time series, most of the relevant literature employs VAR models in levels. By commencing from an unrestricted

[3] Some recent papers have revived the interest in finding an explanation for the delayed-overshooting phenomenon. According to Gourinchas and Tornell (2004), the puzzle is caused by systematic distortion in investors' beliefs about the interest rate process. Suppose investors overestimate the relative importance of transitory interest rate shocks. Confronted with a higher than expected interest rate in the next period, investors revise their beliefs. This 'updating effect' has been suggested as a cause of the forward premium effect and the delayed overshooting puzzle. Kim (2005) proposed that foreign exchange rate interventions of the central bank as driving factors of the delayed overshooting puzzle for the Canadian-US bilateral exchange rate. Exchange rate appreciation on impact might be counteracted by policy interventions in the foreign exchange market. According to Bacchetta and van Wincoop (2010) infrequent foreign currency portfolio decisions of agents can explain the forward discount and the delayed overshooting puzzle. They suggest that the infrequent portfolio decisions can be optimal as the welfare gain from active currency management may be smaller than the corresponding fees.

cointegrated VAR model and developing a parsimonious structural vector equilibrium correction model, which is the adequate $I(0)$ representation of the system, we will be able to carefully study the long-run and short-run properties of the macroeconomic time series.

(iii) Ad-hoc modelling/Specification of instantaneous causality: Since the seminal contribution of Eichenbaum and Evans (1995), there has been an intense discussion about the arbitrary assumptions leading to the identification of the direction of instantaneous causality. Many of the proposed alternative schemes are based on theoretical ad-hoc assumptions. In this paper, we seek to overcome these limitations by taking advantage of recent advances in graph theory and its application to the search for causality among variables. When the applied algorithm is not capable of finding a directed acyclical graph, i.e. a causal chain, we use information criteria for selecting or pooling to facilitate the policy experiment.

(iv) Curse of dimensionality: Highly parameterized unrestricted VAR or just-identified structural VAR models require the estimation of a large number of parameters with the majority being irrelevant, the degrees of freedom being exhausted and estimation uncertainty inflated. The impulse responses become inconclusive due to a growing width of confidence intervals, which will eventually include the zero line. To avoid this problem we make use of the breakthrough in automatic general-to-specific model reduction procedures in reducing the complexity of the model while preserving the characteristics of the data. The graph-theoretic approach to identify the direction of the instantaneous causality generally delivers testable overidentifying restrictions which in contrast to the ad-hoc just identified Cholesky decomposition approach, allows to ask interesting questions. Altogether we construct a overidentified parsimonious structural vector equilibrium correction model for a statistically precise and economically meaningful impulse response analysis.

Our data-driven econometric model selection combines the VAR based cointegration analysis of Johansen (1995) and Juselius (2006), see also Garratt et al. (2006), with the graph-theoretic approach of Spirtes et al. (2001) implemented in TETRAD for the search for instantaneous causal relations (see Demiralp and Hoover, 2003, for its application to econometrics) and the automatic general-to-specific model selection algorithm implemented in *PcGets* of Krolzig and Hendry (2001) and Krolzig (2003) for the selection of a congruent parsimonious structural vector equilibrium correction model.

To keep the cointegration analysis feasible for a balanced high-dimensional system, we utilize an approach suggested by Aoki (1981) for the macro dynamics of two-country models. Aoki has shown for linear dynamic two-country models that under symmetry the system of linear differential equations can be decomposed into the autonomous subsystems of country differences and country averages. In the paper, we will ensure that the symmetry of the US and UK economies holds in the long-run such that the cointegration relations of the full two-country system consist of equilibrium conditions for the aggregated 'global' economy as well as arbitrage and convergence conditions for the UK-US spreads and the exchange rate. This allows to perform the cointegration analysis separately for VAR submodels of country averages and differences.

The assumption that advanced economies behave similar in the long-run presupposes that large economies like the UK or the US do not differ in their aggregate behaviour in a systematic way. In contrast the speed of adjustments to equilibria may vary markedly, due to unequal sizes of the countries or structural differences. We allow for country-specific contemporaneous effects, short-run dynamics, and speeds of adjustment, but determine the long-run equilibria to be symmetric over the two countries, like the international parity conditions are established. The symmetry assumption in the long-run allows to apply the method, proposed by Aoki (1981) and applied by others, for example Turnovsky (1986), to determine the long-run properties of the model in two smaller subsystems. It makes the cointegration analysis in smaller subsystems feasible. Symmetry is rejected for the short-run, thus for the given cointegration vectors further modelling of the short-run is based on the full two-country system.

The idea of breaking the analysis of a system down into submodels, is related to the integrated model approach of Juselius (2006) and the GVAR approach of Pesaran et al. (2004). In the integrated model of Juselius (2006), the long-run structures of different sectors are analysed separately and then combined to a complete model. For example in Juselius (2006) inflation is modelled by combining submodels representing the money market, the external sector and the labour market. This is extended in Tuxen (2007) for the public sector. In the GVAR approach country-specific models are specified including the domestic economy and country-specific global variables. A large number of individual country models are linked together usually via a trade weighted matrix, to form the global system.

Empirically the paper follows the analysis on the international transmission mechanism of monetary policy with respect to its effects on the $\$/£$ exchange rate in Heinlein and Krolzig (2012), where symmetry is imposed by constructing a CVAR in country differences only. Consequently an interest rate shock in the UK has the same effect on the exchange rate as a US shock of opposite sign and identical magnitude. Here we only assume long-run symmetry thus allowing for assymmetries in the adjustment processes and short-run dynamics. The long-run is in the present paper not only analysed in respect to country specific features in a country difference model, but also in respect to the global features in a country average model. The role of the country average model can be compared to the global factors in dynamic factor models (see Forni et al., 2000). While we work with monthly data we have been able to replicate central results of the cointegration part derived for quarterly time series in Heinlein and Krolzig (2012).

The structure of the paper is as follows. In §2 we present the methodological aspects of the economic modelling approach and the econometric model selection procedure. In §3 we then introduce the data set and provide a brief overview of the UK-US macro history since the breakdown of Bretton Woods particular with regard to the international parity conditions. This will provide us with valuable insights for the formation of the-two-country model to be discussed in §4 for the long-run and in §5 for the full 9-dimensional system. §6 investigates the effects of monetary policy shocks with focus on the presence of a delayed overshooting puzzle. Finally §7 concludes.

2 Methodology

Our modelling approach proceeds in four major steps.

2.1 Identification of the cointegration relations among country differences and country averages

The first step is studying the long-run of our model with a likelihood based analysis of the cointegration properties. Due to the size of the dynamic system the analysis is performed in two subsets of country differences and country averages.

- *Separation of long-run dynamics in country differences and country averages.*

  We are utilising a modelling approach by Aoki (1981) frequently used in economic theory in a dynamic macroeconometric two-country model. Masanao Aoki showed that for a system of linear differential equations, the assumption of country symmetry allows to decouple the dynamics of the system into two autonomous subsystems of country averages and country differences. In Krolzig and Heinlein (2013) we generalize Aoki's approach to a stochastic setup in discrete time. In this paper we demonstrate how it can be used for the analysis of the long-run of a large dynamic system.

  Suppose we have two countries and our focus is on a balanced set of K key macroeconomic variables for both economies, respectively, and their exchange rate. Let $\boldsymbol{y}_t$ be the $K \times 1$ vector of domestic variables, $\boldsymbol{y}_t^*$ the $K \times 1$ vector of the same variables for the foreign country, and e_t denotes the exchange rate. The full system vector $\boldsymbol{x}_t = (\boldsymbol{y}_t, \boldsymbol{y}_t^*, e_t)'$ is hence $2K+1$ dimensional.

  Following Aoki (1981) the set of domestic-foreign variables is transformed into a set of country-average-difference variables:

$$\boldsymbol{y}_t^a = \frac{1}{2}(\boldsymbol{y}_t + \boldsymbol{y}_t^*) \quad \text{and} \quad \boldsymbol{y}_t^d = \boldsymbol{y}_t - \boldsymbol{y}_t^* \tag{1}$$

  The system $\tilde{\boldsymbol{x}}_t = (\boldsymbol{y}_t^a, \boldsymbol{y}_t^d, e_t)'$ is a linear isomorphic transformation of the system $\boldsymbol{x}_t = (\boldsymbol{y}_t, \boldsymbol{y}_t^*, e_t)'$. Defining $\tilde{\boldsymbol{x}}_t^a \equiv \boldsymbol{y}_t^a$ and $\tilde{\boldsymbol{x}}_t^d \equiv (\boldsymbol{y}_t^d, e_t)'$, the two-country CVAR($p$) model in its country-averages-differences VECM form without deterministic terms looks like follows:

$$\begin{bmatrix} \Delta\tilde{\boldsymbol{x}}_t^a \\ \Delta\tilde{\boldsymbol{x}}_t^d \end{bmatrix} = \begin{bmatrix} \boldsymbol{\alpha}_{aa} & \boldsymbol{\alpha}_{ad} \\ \boldsymbol{\alpha}_{da} & \boldsymbol{\alpha}_{dd} \end{bmatrix} \begin{bmatrix} \boldsymbol{\beta}'_{aa} & \boldsymbol{\beta}'_{ad} \\ \boldsymbol{\beta}'_{da} & \boldsymbol{\beta}'_{dd} \end{bmatrix} \begin{bmatrix} \tilde{\boldsymbol{x}}_{t-1}^a \\ \tilde{\boldsymbol{x}}_{t-1}^d \end{bmatrix} + \sum_{i=1}^{p-1} \begin{bmatrix} \boldsymbol{\Gamma}_{aa,i} & \boldsymbol{\Gamma}_{ad,i} \\ \boldsymbol{\Gamma}_{da,i} & \boldsymbol{\Gamma}_{dd,i} \end{bmatrix} \begin{bmatrix} \Delta\tilde{\boldsymbol{x}}_{t-i}^a \\ \Delta\tilde{\boldsymbol{x}}_{t-i}^d \end{bmatrix} + \begin{bmatrix} \boldsymbol{u}_t^a \\ \boldsymbol{u}_t^d \end{bmatrix}, \tag{2}$$
$$\textit{where} \quad \begin{bmatrix} \boldsymbol{u}_t^a \\ \boldsymbol{u}_t^d \end{bmatrix} \sim \mathsf{IID}\left(\begin{bmatrix} \mathbf{0} \\ \mathbf{0} \end{bmatrix}, \begin{bmatrix} \boldsymbol{\Sigma}_{aa} & \boldsymbol{\Sigma}_{ad} \\ \boldsymbol{\Sigma}_{da} & \boldsymbol{\Sigma}_{dd} \end{bmatrix} \right).$$

  As in Aoki (1981), symmetry of the two countries implies separapility into two autonomous subsystems $\tilde{\boldsymbol{x}}_t^a$ and $\tilde{\boldsymbol{x}}_t^d$. Symmetry requires block-diagonality of all parameter matrices in (2):

  (i) $\boldsymbol{\beta}_{ad} = \boldsymbol{\beta}_{da} = \mathbf{0}$,

  (ii) $\boldsymbol{\alpha}_{ad} = \boldsymbol{\alpha}_{da} = \mathbf{0}$,

  (iii) $\boldsymbol{\Gamma}_{ad,i} = \boldsymbol{\Gamma}_{da,i} = \mathbf{0}$ for all $i = 1, \ldots, p-1$ and

  (iv) $\boldsymbol{\Sigma}_{ad} = \boldsymbol{\Sigma}_{da} = \mathbf{0}$.

  It is worth noting that we assume symmetry for the long-run only. In the final full two-country model, only the first assumption $\boldsymbol{\beta}_{ad} = \boldsymbol{\beta}_{da} = \mathbf{0}$ will remain. After analysing the two subsystems we will run thorough tests for the validity of the symmetry assumption. We will also investigate the degree of symmetry in the adjustment process, the short-run dynamics and the contemporaneous effects.

- *Specification of the general unrestricted system of country differences and country averages.*

We commence from a p-th order reduced-form vector autoregressive (VAR) model without any equation-specific restrictions to capture the characteristics of the data:

$$\tilde{\boldsymbol{x}}_t^s = \boldsymbol{\nu}^s + \sum_{j=1}^{p} \boldsymbol{A}_j^s \tilde{\boldsymbol{x}}_{t-j}^s + \boldsymbol{u}_t^s, \tag{3}$$

where $\boldsymbol{u}_t^s \sim \mathsf{NID}(\mathbf{0}, \boldsymbol{\Sigma})$ is a Gaussian white noise process and $s = a, d$ are country average variables or country differences. This step involves the specification of the deterministic terms, selection of the lag length p and misspecification test to check the validity of the assumptions made.

- *Johansen cointegration tests* and *identification of the cointegration vectors*.

  The Johansen procedure for determining the cointegration rank, r, is then applied to the system (3) mapped into its vector equilibrium-correction mechanism (VECM) representation:

$$\Delta\tilde{\boldsymbol{x}}_t^s = \boldsymbol{\nu}^s + \boldsymbol{\Pi}^s \tilde{\boldsymbol{x}}_{t-1}^s + \sum_{j=1}^{p-1} \boldsymbol{\Gamma}_j^s \Delta\tilde{\boldsymbol{x}}_{t-j}^s + \boldsymbol{u}_t^s, \tag{4}$$

  For a cointegrated vector process, the reduced-rank matrix, $\boldsymbol{\Pi}^s$, can be decomposed into loading matrix, $\boldsymbol{\alpha}^s$, and cointegration matrix, $\boldsymbol{\beta}^s$, containing the information of the long-run structure of the model. The Johansen procedure delivers unique estimates of $\boldsymbol{\alpha}^s$ and $\boldsymbol{\beta}^s$ as a result of requiring $\boldsymbol{\beta}^s$ to be orthogonal and normalized. These estimates provide a value for the unrestricted log-likelihood function to be compared to the log-likelihood under economically meaningful overidentifying restrictions, $\boldsymbol{\beta}^{s,r}$:

$$\Delta\tilde{\boldsymbol{x}}_t^s = \boldsymbol{\nu}^s + \boldsymbol{\alpha}^s \boldsymbol{\beta}^{s,r'} \tilde{\boldsymbol{x}}_{t-1}^s + \sum_{j=1}^{p-1} \boldsymbol{\Gamma}_j^s \Delta\tilde{\boldsymbol{x}}_{t-j}^s + \boldsymbol{u}_t^s, \tag{5}$$

  with $\mathsf{E}[\boldsymbol{u}_t^s] = 0$ and $\mathsf{E}[\boldsymbol{u}_t^s \boldsymbol{u}_t^{s\prime}] = \hat{\boldsymbol{\Sigma}}^s$. The empirical modeling procedure for finding the cointegration relations follows Juselius (2006).

After having analysed the long-run in two subsets we return for the further modelling to the full 9D system.

2.2 Graph-theoretic search for instantaneous causal relations

The determination of the contemporaneous causal links between the variables has been advanced by modern graph-theoretic methods of searching for causal structure based on relations of conditional independence developed by computer scientists (Pearl, 2000) and philosophers (Spirtes et al., 2001). Following Demiralp and Hoover (2003), who introduced this approach to econometrics, we use the PC algorithm implemented in TETRAD 4 (see Spirtes et al., 2005 for details). The PC algorithm exploits the information embedded in the residual variance-covariance matrix, $\hat{\boldsymbol{\Sigma}}$, of the full 9D system. A causal structure is represented by a graph with arrows from causes to caused variables. To find the directed acyclic graph, the algorithm performs an elimination and an orientation stage.

In the elimination stage, a significance level α for Fisher's Z-statistic is the criterium to decide on correlation between variables. The algorithm starts by assuming that all variables are linked to each other through an undirected link. Firstly connections are removed between two variables which are unconditionally uncorrelated. Then connections are eliminated for variables which are uncorrelated conditional on other variables. Here the correlation of a pair of variables is conditioned on each other variable individually, then on all possible pairs of variables, hereafter on all subsets of three variables and so on up to all possible subsets of conditioning. If there is no more link to be removed the elimination stage is finished and the skeleton of the graph is identified.

This skeleton is the basis for the orientation stage of the algorithm, seeking to direct the undirected edges. Triples of linked variables $A — B — C$ are analysed. If $A \to B \to C$ is true, i.e. A causes B causes C, then A and C would be correlated, but independent when conditioned on B. It is said B screens A from C. If $A \leftarrow B \to C$ then A and C are again dependent but independent when conditioned on B. Here B is the common cause of A and C. An important case is the so called 'unshielded collider'. When $A \to B \leftarrow C$ is true, B is an unshielded collider. A and C are independent when conditioned on possible sets of variables, but dependent when conditioned on B. The algorithm systematically searches for unshielded colliders and directs accordingly. Thereafter, remaining undirected edges are observed by logical reasoning. If $A \to B — C$, where A and C are not directly connected, then the later link can be only as $B \to C$. An important final criterium is the search for possible circular relations. Cyclicality of directed graphes is excluded, hence also bidirectional links are not allowed. If there is a directed path leading from A to B through other variables and an edge between A and B, then $A \to B$ holds. These are the main steps of the orientation stage. It is possible that not all links can be directed.

A directed acyclic graph (DAG) results if all edges could be oriented. Based on the identified contemporaneous causal structure of the system, the 9D VECM can be represented as a *recursive* structural vector equilibrium correction mechanism (SVECM). By suitable ordering of the variables, the DAG can be mapped to a lower-triangular contemporaneous matrix, $\boldsymbol{B}^r$, with units on the diagonal and non-zero lower-off-triangular elements representing the causal links found by the PC algorithm. In contrast to the traditional orthogonalisation with the help of a Choleski decomposition of $\hat{\boldsymbol{\Sigma}}$, this approach results in an overidentified SVECM in the majority of cases. The zero lower-triangular elements of $\boldsymbol{B}^r$ provide testable overidentifying constraints allowing to verify the validity of the selected contemporaneous structure. Most importantly, as the contemporaneous causal structure captured by $\boldsymbol{B}^r$ is data determined, it avoids the problems associated with the ad-hoc nature of orthogonalised structural VAR models.

2.3 Computer-automated Gets single-equation reductions of the SVECM

Starting point is the structural VECM with long-run relations $\beta^{s,r}$ determined in §2.1 and contemporaneous structure $\boldsymbol{B}^r$ given by the corresponding directed acyclic graph:

$$\boldsymbol{B}^r \Delta \boldsymbol{x}_t = \boldsymbol{\delta} + \tilde{\boldsymbol{\alpha}} \begin{pmatrix} \beta^{a,r\prime} \tilde{\boldsymbol{x}}^a_{t-1} \\ \beta^{d,r\prime} \tilde{\boldsymbol{x}}^d_{t-1} \end{pmatrix} + \sum_{j=1}^{p-1} \boldsymbol{\Upsilon}_j \Delta \boldsymbol{x}_{t-j} + \boldsymbol{\omega}_t, \quad \boldsymbol{\omega}_t \sim \mathsf{NID}(\mathbf{0}, \boldsymbol{\Omega}), \tag{6}$$

where $\boldsymbol{B}^r$ is the lower-triangular matrix found by TETRAD and $\boldsymbol{\Omega}$ is a diagonal variance-covariance matrix. A single-equation based *Gets* reduction procedure such as *PcGets* can be applied to the equations in (6) straightforwardly and, as shown in Krolzig (2001), without a loss in efficiency. The parameters of interest are the coefficients collected in the intercept, $\boldsymbol{\delta}$, the adjustment matrix $\tilde{\boldsymbol{\alpha}}$ and the short-run matrices $\boldsymbol{\Upsilon}_j$ in the structural VECM. The result is a parsimonious structural vector equilibrium correction model denoted PSVECM, which is nested in (6) and defined by the selected $\boldsymbol{\delta}^*$, $\tilde{\boldsymbol{\alpha}}^*$ and $\boldsymbol{\Upsilon}^*_j$ with $j = 1, \ldots, p-1$.

2.4 Selection of the dominant PSVECM or model pooling

In case that the PC algorithm finds a link but has insufficient information to identify the direction an undirected edge emerges. In this case, there exists a set of contemporaneous causal structures, $\{\boldsymbol{B}^{(i)}\}$, that are all consistent with the data evidence. Two proceedings are suggested in this situation: a best model is chosen out of the set of possible models and an average of the models is supposed to be closest to the real data generating process.

If the graph-theoretical search in §2.2 produces an acyclic graph with at least one undirected edge, the determination of the direction of instantaneous causal relations has to rely on the information from the PSVECMs resulting from the *Gets* reduction of the SVECMs as defined by the set of contemporaneous causal structures. As the PSVECMs are mutually non-nested and the union is usually unidentified, we propose to select the PSVECM with the greatest penalized likelihood. Thus the dominant design of the contemporaneous effects matrix according to information criteria such as Akaike or Schwarz would be used as the baseline model.

As an alternative to choosing the best model out of the set of models consistent with the graph-theoretic finding in §2.2, we are invoking the idea of model pooling, called 'thick' modelling in Granger and Jeon (2004). All possible model options of the graph-theoretic approach are reduced by the automatic general-to-specific model reduction algorithm to parsimonious models. The output of interest of these models, here the impulse responses, are combined to an average impulse response that can be seen as the best possible model simulation. A trimming and weighting scheme could be applied.

3 The UK-US macroeconomic history since the end of Bretton Woods

3.1 Time series

The macroeconometric two-country model for the UK and the US we develop in this paper, consists of nine variables: inflation, output growth, 3-month interest rates, 10-year government bond yields for both countries, and the nominal exchange rate. We are using monthly data from 1972M3 to 2010M8 giving a total of 462 monthly observations. The paper is written from an UK perspective, so we will refer to UK variables as the domestic and US variables as foreign ones, marked by a star. Table 2 gives an overview over the macro time series under consideration.

Table 2 *Time series definitions and source*

Variable	Description	Source	EcoWin code
P_t	UK Retail Prices, all items excluding mortgage interest payments (RPIX), Index, (1987M1=100), spliced with RPI (before 1975)	ONS	$ew:gbr11815$
Y_t	UK Industrial Production, SA , (2005=100), USD	IFS	$ifs:s1126600czfm$
I_t	UK Treasury bills, Bid, 3 month, Yield, End of Period, GBP	Reuters	$ew:gbr14010$
R_t	UK Government Benchmarks, Bid, 10 year, Yield, End of Period GBP	Reuters	$ew:gbr14020$
P_t^*	US Consumer Prices, all items, SA, Index, (1982-1984=100)	BLS/Reuters	$ew:usa11970$
Y_t^*	US Industrial Production, SA , (2005=100), USD	IFS	$ifs:s1116600czfm$
I_t^*	US Treasury bills, 3 month, Yield, Close, USD	Reuters	$ew:usa14430$
R_t^*	US Government Benchmarks, Bid, 10 Year, Yield, End of Period, USD	Reuters	$ew:usa14021$
e_t	Spot rates, GBP/USD transformed to USD/GBP, End of period	Reuters	$ew:gbr19005$

The price index P_t is seasonally adjusted with Seats/Tramo. Both industrial production series Y_t, Y_t^* are outlier corrected with Seats/Tramo. Variables without a superindex are of the domestic country (UK), a * indicates the foreign country (US), ^{a} is a country average and ^{d} indicates a country difference. All financial variables are end-of-period series.

To guarantee the consistency of the parity conditions to be considered in §3.2, the variables have been transformed to ensure that interest, inflation and growth rates are measured as monthly log returns. Table 3 explains in detail how each variable entering the model has been created.

Table 3 *Model variables*

Variable	Description
$\pi_t = \Delta \log P_t$	rate of inflation
$\Delta y_t = \Delta \log Y_t$	output growth rate
$i_t = \log(1 + I_t/1200)$	short-term interest rate
$r_t = \log(1 + R_t/1200)$	long-term interest rate
$e_t = \log E_t$	exchange rate

3.2 Discussion

In the following, we discuss the properties of the macro time series as far as relevant for the econometric modelling to follow in §4 and §5.

3.2.1 Inflation and the output growth

Focussing first on the real economy, Figure 1 plots the rates of inflation and output growth in the UK and the US, as well as the differences and averages between the two countries. It can be seen that, except for the most recent years, the UK macro economy is characterized by a far more volatile output growth and a higher rate of inflation.

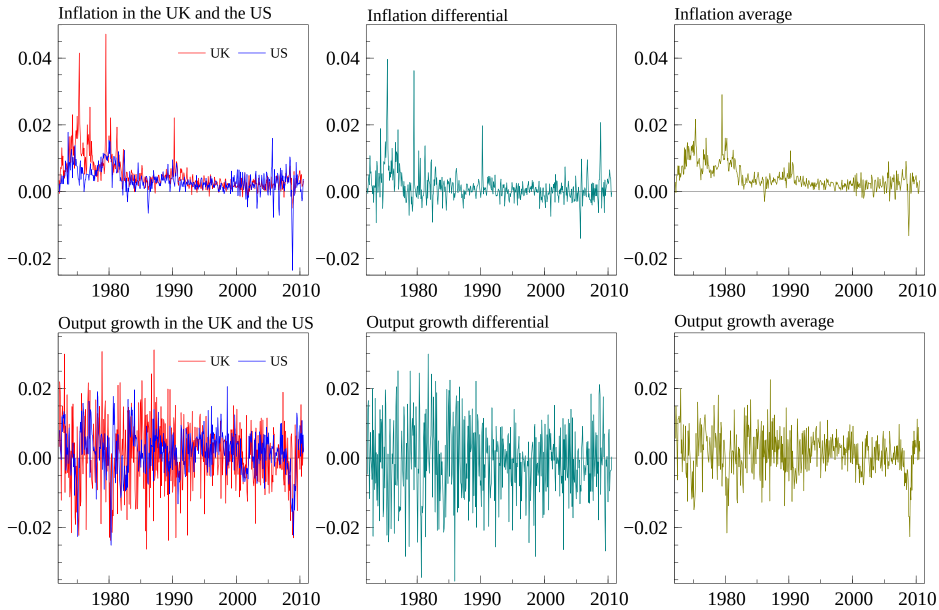

Figure 1 *Inflation rates and output growth rates*

Moving to the asset markets, the further discussion is structured along some of the central international parity conditions.

3.2.2 Purchasing power parity

It might have come as a surprise to some readers that we included in our analysis the inflation rates of the UK and US but not the price levels. In light of the purchasing power parity (PPP) theory, one would have expected that the nominal exchange rate follows the relative price level of the two countries. Thus, the real exchange rate $s_t = e_t + p_t - p_t^*$, measuring the deviation of the nominal exchange rate from the relative price level, should be mean-reverting, as that the law of one price holds at least in the long term.

However, as can be seen in Figure 2, purchasing power parity clearly does not hold for the \$/£ exchange rate over the sample period. The Pound Sterling appreciated in real terms by more than 70% from the end of 1984 to the beginning of 2008. In our judgement, the non-stationarity of the real exchange rate can not be explained within the set of macro variables considered here. We therefore leave this issue for further investigations.

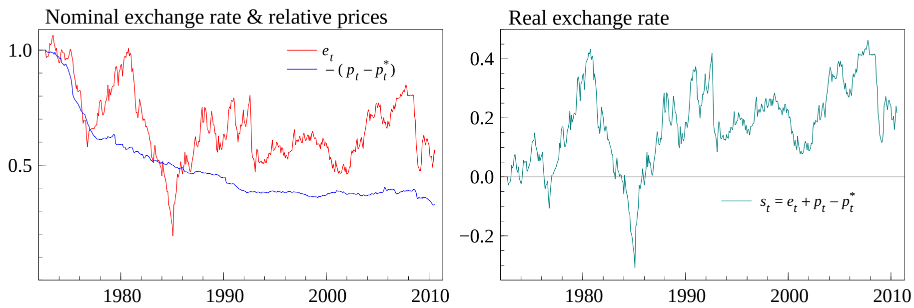

Figure 2 *Nominal exchange rate, relative prices and the real exchange rate*

3.2.3 Expectations hypothesis of the term structure

In the expectations model of the term structure, the yield of a zero bond with a maturity of T periods equals the average of the expected one-period interest rates plus a potential risk premium, ϕ_t:

$$r_t = \frac{1}{T} \sum_{j=0}^{T-1} \mathsf{E}_t i_{t+j} + \phi_t. \tag{7}$$

After substracting i_t from both sides and rearranging the right hand side we get the equation (8):

$$r_t - i_t = \frac{1}{T} \sum_{j=0}^{T-1} \sum_{h=1}^{j} \mathsf{E}_t \Delta i_{t+h} + \phi_t. \tag{8}$$

If the short-term rate and the long-term rate are $I(1)$ variables and the risk premium is a stationary process, it follows from (8) that the spread between r_t and i_t is stationary, $r_t - i_t \sim I(0)$, because Δi_t is $I(0)$ and a finite sum of stationary variables is stationary. Hence the interest rates are expected to be cointegrated.

Figure 3 plots the term spread for the UK and the US, as well as their differences and averages. While the term spread appears potentially stationary for the US, this clearly is not the case for the UK. These conjectures were confirmed by ADF tests. We are not expecting, that short and long-term interest rate differentials cointegrate, but the cointegration between short and long-term interest averages may hold.

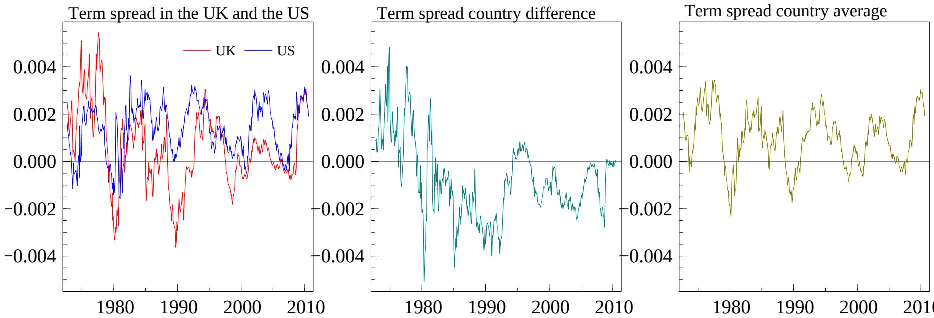

Figure 3 *Term structure in the UK and the US, and their differences and averages*

3.2.4 Nominal interest rate parity

Figure 4 looks at the potential cointegration between the nominal interest rates in the UK and the US. Due to the accommodating UK monetary policy in the 1970s, the long-term interest-rate differential shows clear signs of non-stationarity. As the UK short-term interest rates do not fully reflect the inflation problem of that time period, the short-term interest-rate differential conversely is a potential candidate for a cointegration relation. The interest rate averages are both non-stationary. An ADF test for the long-term interest rate average rejects marginally a unit root in favour of trend stationarity. This, however, is highly sample dependent due to the 30 year old bull market for long-term government bonds since 1981. since 1981.

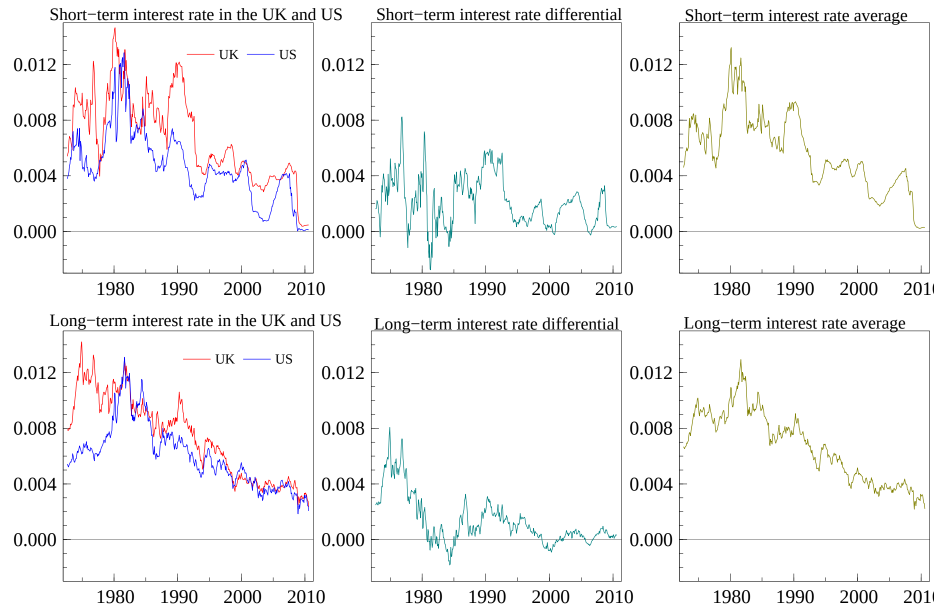

Figure 4 *Nominal short- and long-term interest rates, and country differences and averages*

3.2.5 The Fisher hypothesis and the real interest rate parity

Another important relation for our empirical analysis is the Fisher hypothesis. It states that the nominal interest rate equals the real interest rate ρ_t, invariant to monetary policy, plus inflation expectations,

$$i_t = \rho_t + \mathsf{E}_t \pi_{t+1}, \tag{9}$$

where the real interest rate is determined by the marginal product of capital and thus expected to be stationary with a low variance.

The Fisher relation motivates the real interest rate parity, according to which the ex-ante real interest rates in home and foreign country should equalize in the long run, i.e.:

$$\rho_t - \rho_t^* = (i_t - \mathsf{E}_t \pi_{t+1}) - (i_t^* - \mathsf{E}_t \pi_{t+1}^*) \sim I(0). \tag{10}$$

The calculation of ex-ante real interest rate involves future inflation expectations. As those are empirically difficult to measure, we focus here on a naive definition of the real interest rate using the current backward-looking inflation.[4] These are plotted in Figure 5. Both the short-term and the long-term real interest rates for the UK and the US show a level shift at the time of the Volcker disinflation. Since then a downward trend is present. Overall, the real long-term interest rate differential is more likely to be stationary than the real short-term differential. The level shift in 1981 is also present in the real rate averages.

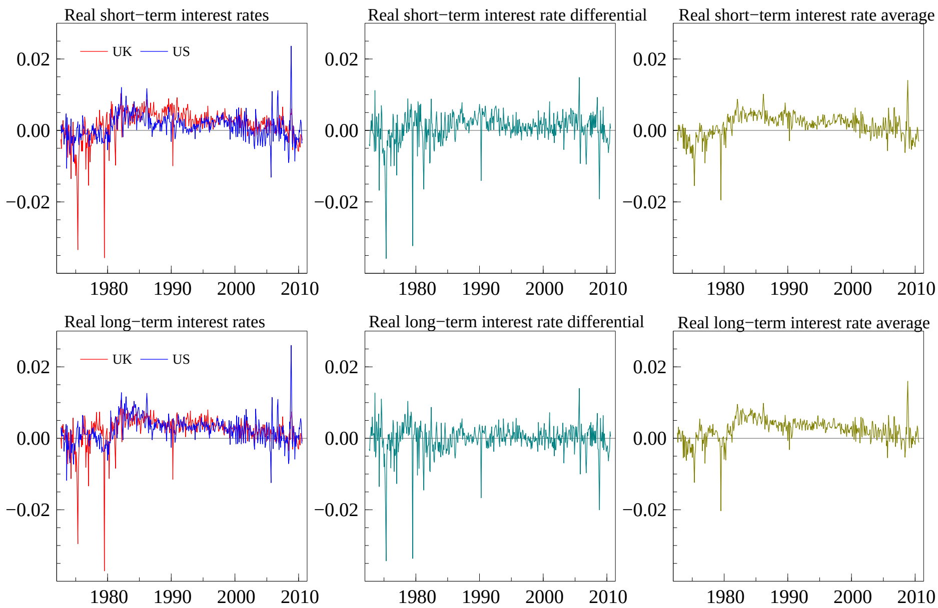

Figure 5 *Naive real short- and long-term interest rates, country differences and averages*

[4] A common alternative measurement approach would involve the use of realized future inflation rates based on the rational expectations hypothesis, which excludes systematic forecast errors of the agents. This procedure is, however, not compatible with the VAR modelling approach used in this paper.

3.2.6 Uncovered interest parity

A central parity condition is the uncovered interest rate parity (UIP), which requires that the expected return on the domestic asset, in equilibrium, is equal to expected return, measured in the home currency, on a foreign asset with otherwise identical characteristics. For a one-period bond, this implies:

$$i_t = i_t^* - \mathsf{E}_t \Delta e_{t+1}. \tag{11}$$

Under rational expectations, there are no systematic forecast errors and equation (11) can be rewritten as:

$$\xi_t = i_t^d + \Delta e_{t+1}, \tag{12}$$

where ξ_t is a martingale difference sequence and measures the excess return of the UK bond. The realized excess returns over the sample period and their cumulation can be seen in Figure 6.

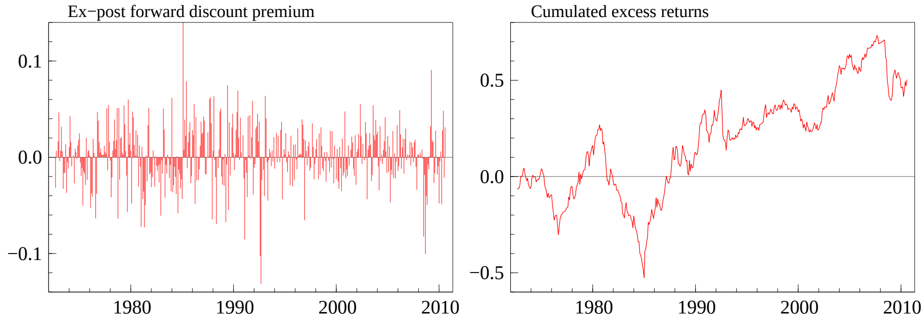

Figure 6 *Deviations from UIP: Ex-post excess returns and their cumulation*

The UIP condition in (11) has been formulated for a one-period bond. We now consider its generalization to bonds with multi-period maturities. According to the expectations hypothesis of the term structure, we have that the long-term interest rate, or more precisely the yield of a zero bond of maturity of T periods, equalizes the expected average return of one-period bonds over T periods:

$$r_t^d = \frac{1}{T} \sum_{j=0}^{T-1} \mathsf{E}_t i_{t+j}^d. \tag{13}$$

Combining (13) with the forward solution of the UIP relation in (11) for e_t,

$$e_t = \mathsf{E}_t e_{t+T} + \sum_{j=0}^{T-1} \mathsf{E}_t i_{t+j}^d, \tag{14}$$

we get the multi-period form of UIP,

$$e_t = \mathsf{E}_t e_{t+T} + T(r - r^*)_t, \tag{15}$$

which states that the exchange rate is determined by the long-term exchange rate expectation, $\mathsf{E}_t e_{t+T}$, and T times the long-term interest rate differential. Note that this relation will not hold exactly in our data set due to the different type of bonds under consideration, in which case the impact of the bond yield differential is expected to be systematically smaller.

4 The long-run of the two-country model

4.1 Application of the Aoki method

In the following, we will assume symmetry in the long-run but allow for country-specific contemporaneous effects, short-run dynamics and speeds of adjustment. Long-run symmetry invokes the approach of Aoki (1981) for the analysis of the cointegration properties of the system. Separating the 9D dynamics into two autonomous subsystems makes the likelihood based cointegration analysis of the two-country model feasible.

For the cointegration analysis, the variables of the model are transformed from domestic-foreign into country-average-difference measures. The exchange rate is included in the country-difference system. Thus, the cointegration analysis is performed in a country difference model of dimension 5 and the country average model of dimension 4, see chapters 4.2 and 4.3 respectively. Subsequently the variables will be transformed back into the original system with the error correction terms preserved and the short-run being analysed in the full 9-dimensional system.

4.2 Cointegrated vector autoregression I - Country difference model

In the following we seek to develop a congruent statistical model for the macro dynamics of the country differences involving the inflation differential, $\pi_t^d = \pi_t - \pi_t^*$, the output growth differential, $\Delta y_t^d = \Delta y_t - \Delta y_t^*$, the short-term interest rate differential, $i_t^d = i_t - i_t^*$, the long-term interest rate differential, $r_t^d = r_t - r_t^*$, and the exchange rate e_t. The results of Augmented Dickey Fuller tests indicate that the output growth differential Δy_t^d is stationary and the short-term interest rate differential i_t^d is marginally stationary. The other time series were found to be $I(1)$. Thus, the vector process, $\tilde{x}_t^d = (\pi_t^d, \Delta y_t^d, i_t^d, r_t^d, e_t)'$ is integrated of order one: $\tilde{x}_t^d \sim I(1)$.

As discussed in §2.1, the first step involves the specification of the deterministic terms, selection of the lag length and misspecification tests to check the validity of the assumptions made. The lag structure analysis of the unrestricted VAR, commencing from a maximum lag length of thirteen with consecutive F-tests for excluded individual and joint lags, indicated a lag order of four. An unrestricted constant is included. It is the only deterministic term as a linear time trend was found to be statistically insignificant.

The results of tests for misspecification are displayed in Table 4. Most importantly, non of the equations suffers from autocorrelation. As one would expect in financial time series data the normality test shows serious non-normality mainly due to excess kurtosis in all but the exchange rate equation. Also, heteroscedasticity and ARCH effects in the interest rate equations are detected. The non-normality will not negatively influence the point estimates. However the distributions of the F, t, and χ^2 tests are less exact under non-normality, but they are still good approximations with increasing sample size. Some of the non-normalities can be traced back to the reduction in volatility during the Great Moderation as well as outliers, for which dummy variables will be included in the PSVECM in §5.3. The presents of heteroscedasticity is not causing a bias on the estimations, but the variance of the coefficients may be over or under estimated. Therefore the significance levels have to be treated with some caution. The general-to specific model selection algorithm is reducing the non-normality by including dummy variables, but is not fully robust to heteroscedasticity. Another important question is the impact of the problems of misspecification on the rank test. Rahbek et al. (2002) demonstrate that the Johansen trace test, which has originally been designed for i.i.d. Gaussian errors, is robust to general martingale difference sequences under some regularity conditions. Hence the validity of the performed rank test is not in danger due to the misspecification problems. Some problems of misspecification are detected, but overall the residuals are sufficiently well behaved to proceed with the system.

Table 4 *Misspecification tests for the unrestricted VAR(4) of the country difference model*

Test		π_t^d	Δy_t^d	i_t^d	r_t^d	e_t
AR 1-13	$F(13,428)$	0.947	1.473	1.438	1.167	1.182
		[0.530]	[0.124]	[0.138]	[0.302]	[0.289]
Normality	$\chi^2(2)$	149.11**	0.27	125.83**	30.20**	39.70**
		[0.000]	[0.875]	[0.000]	[0.000]	[0.000]
ARCH 1-13	$F(13,415)$	1.591	1.660	7.918**	5.371**	1.816*
		[0.085]	[0.067]	[0.000]	[0.000]	[0.039]
Hetero	$F(40,400)$	2.300**	0.908	2.742**	3.394**	1.386
		[0.000]	[0.634]	[0.000]	[0.000]	[0.065]

** significant at 1% level, * significant at 5% level.

We continue by analyzing the long-run properties of the system. The number of stable long-run relations $\beta'\tilde{x}_t^d$, which is equal to the rank of the matrix $\mathbf{\Pi}$ of the vector equilibrium-correction mechanism in (4), is determined by the Johansen (1995) test for $I(1)$ cointegration. The eigenvalues and trace test results are shown in Table 5. The long-run properties of the system are characterized by four cointegration relations, $\text{rank}(\mathbf{\Pi}) = 4$, and one stochastic trend.

Table 5 *Johansen likelihood ratio trace test of* H_0 : rank $\leq r$

r	eigenvalue	trace test	prob
0	0.263	274.60 **	[0.000]
1	0.189	133.48 **	[0.000]
2	0.042	36.73 **	[0.006]
3	0.030	16.69 *	[0.031]
4	0.005	2.48	[0.115]

** significant at 1% level, * significant at 5% level.

In Table 6 we test for long-run weak exogeneity of the variables of the system. Under the null hypothesis of a particular zero row in $\boldsymbol{\alpha}$, the corresponding variable is not adjusting towards the long-run equilibrium. The LR test results of the restrictions on $\boldsymbol{\alpha}$ show that, with a p-value of 0.72, the bond yield differential is the only weakly exogenous variable. Thus, we identified the long-term interest rate differential r_t^d as the unique common stochastic trend in the system.[5]

Table 6 *Testing for weak exogeneity*

	π_t^d	Δy_t^d	i_t^d	r_t^d	e_t
$\chi^2(4)$	–	136.14	15.97	2.07	14.84
	[0.00]	[0.00]	[0.00]	[0.72]	[0.00]

Following the modelling approach suggested by Juselius (2006), the following cointegration vectors were identified by paying attention not only to statistical acceptability but also to consistency with

[5] In the following, we will see that the long-term interest rate differential appears to be driven by long-term inflation expectations as predicted in Fisher hypothesis.

economic theory. Three linearly independent cointegration vectors related to parity conditions (i) to (iii) were found by means of simple hypothesis tests. With $r^d = 4$, one further composite cointegration vector had to be identified as (iv).

(i) *Stationary output growth differential.*

$$\Delta y_t^d = \Delta y_t - \Delta y_t^* \sim I(0). \tag{16}$$

The first cointegration vector is the difference between the UK and US output growth rates, which follows from the stationarity of the output growth rates of both countries.

(ii) *Stationary nominal short-term interest rate differential.*

$$i^d = i_t - i_t^* \sim I(0). \tag{17}$$

This is somewhat surprising given that the *long-term* interest rate differential constitutes the stochastic trend of the system. In other words, while the nominal interest rate parity holds for the money markets, it is violated for the bond markets.

The opposite holds for the real interest rate parity:

(iii) *Stationary real long-term interest rate differential.*

$$\rho_t^d = r_t^d - \pi_t^d = (r - \pi)_t - (r^* - \pi^*)_t \sim I(0). \tag{18}$$

The third cointegrating vector reflects the real interest rate parity and is closely related to the Fisher hypothesis, where the real long-term interest rates are calculated naively with the current rather than the expected future inflation. Since r_t^d is nonstationary this must also hold for the inflation differential, which is driven by the same stochastic trend. It is also worth noting that due to (17) and (18) the UK and US term structures do not cointegrate.

(iv) *Nominal long-term interest-rate differential based exchange rate determination.* The last cointegration vector is a UIP inspired exchange rate determination relation:

$$e_t - 87.2(r - r^*)_t \sim I(0). \tag{19}$$

This cointegration vector should be interpreted in light of the multi-period form of UIP. For zero bonds with a maturity of 10 years, respectively $T = 120$ months, equation (15) results in:

$$e_t = \mathsf{E}_t e_{t+120} + 120 r_t^d. \tag{20}$$

While, for the type of government bonds analyzed here, the relation above only holds approximately, the estimated multiplier of 87.2 with a 2σ interval of $[43.10, 131.33]$ is consistent with the theory. Furthermore, with sample averages of 8.9 and 7.3 of the yield of 10-year government bonds of the UK and the US, the average duration is only 6.8 and 7.3 years, respectively. Thus, the point estimate of 87.2 is actually very close to the predicted values of 81.6 and 87.6. According to (20), the long-term equilibrium movement in the foreign exchange rate can be traced back to the non-stationary nominal long-term interest rate differential, exhibiting long swings, and long-term exchange rate expectations.

The system estimation results for the four cointegration vectors and their interaction with the variables of the system are shown in Table 7. The three over-identifying restrictions on the cointegration space are accepted by the likelihood ratio (LR) test with a statistic of $\chi^2(3) = 3.67$ and a p-value of 0.30. The only unrestricted β-coefficient is precisely estimated. In contrast, only few α-coefficients are statistically different from zero.

Table 7 *Cointegration vectors and loadings, t-values in brackets*

	Cointegration vectors				Loadings			
	β_1^d	β_2^d	β_3^d	β_4^d	α_1^d	α_2^d	α_3^d	α_4^d
π_t^d	0	0	-1	0	0.025 (0.62)	0.118 (0.93)	0.736** (9.80)	0.0006 (0.45)
Δy_t^d	1	0	0	0	-1.230** (-12.10)	-0.159 (-0.49)	0.186 (0.98)	-0.0002 (-0.16)
i_t^d	0	1	0	0	0.004 (0.62)	-0.073** (-3.96)	0.006 (0.55)	-0.0002 (-1.26)
r_t^d	0	0	1	-87.2 (4.0)	-0.001 (-0.17)	-0.011 (-1.05)	0.004 (0.64)	0.0001 (0.83)
e_t	0	0	0	1	-0.191 (-0.67)	-0.074 (-0.08)	0.993 (1.89)	-0.031** (-3.20)

** significant at 1% level, * significant at 5% level.

Altogether we find that the nominal long-term interest rate differential, r_t^d, is of central importance to the system. It constitutes the common stochastic trend, it cointegrates with the inflation differential π_t^d to the stationary 'real' long-term rate differential, and it also drives the exchange rate $e_t = 87.2 r_t^d$, which is consistent with UIP and stable long-term exchange rate expectations $\mathsf{E}_t e_{t+120}$. The output gap y_t^d and the short-term rate differential i_t^d are both self error correcting and weakly exogenous to the other cointegration relations.

The four cointegrating relations are plotted in Figure 7. The upper panels are just the output growth and the short-term interest rate differentials. In the lower left panel the real long-term interest rate differential can be seen, which is dominated by the pattern of the inflation differential. The only new time series is the last diagram which shows the deviation of the exchange rate from its long-run equilibrium with the bond yield differential.

Also a country difference model for the UK and the US has been analysed in Heinlein and Krolzig (2012) for a very similar sample of quarterly data from 1972Q1 to 2009Q2. It is very reassuring that the same long-run cointegration relationships could be found for both studies, earlier with quarterly data and now with monthly data.

4.3 Cointegrated vector autoregression II - Country average model

The country average subsystem is a four dimensional model containing the inflation average, $\pi_t^a = 0.5(\pi_t + \pi_t^*)$, the output growth average, $\Delta y_t^a = 0.5(\Delta y_t + \Delta y_t^*)$, the short-term interest rate average,

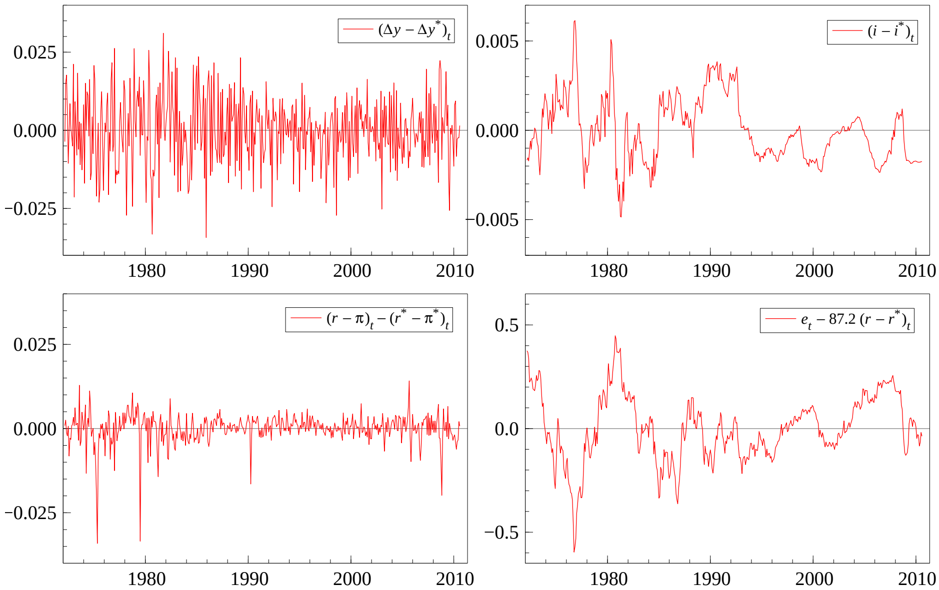

Figure 7 *The four cointegrating vectors in the country difference model*

$i_t^a = 0.5(i_t + i_t^*)$ and the long-term interest rate average, $r_t^a = 0.5(r_t + r_t^*)$. The results of Augmented Dickey Fuller tests indicate that the output growth average Δy_t^a is stationary, the inflation average π_t^a is trend stationary, also the interest rate averages are close to be marginally trend stationary. Altogether the vector process, $\tilde{x}_t^a = (\pi_t^a, \Delta y_t^a, i_t^a, r_t^a)'$ is integrated of order one: $\tilde{x}_t^a \sim I(1)$.

Again we start with the specification of the deterministic terms, selection of the lag length and misspecification test to check the validity of the assumptions made. The lag structure analysis of the unrestricted VAR, indicates a lag order of three. An unrestricted constant is included as deterministic term. A linear trend is statistically significant but excluded on economic grounds. When the trend is included, the average inflation as well as the long-term interest rate are stationary cointegration relationships on its own. This deterministic downward trend is a pure sample effect and not supported by economic theory.

The results of tests for misspecification are displayed in Table 8. The autocorrelation test with 13 lags is a demanding test to pass. Therefore we are content with a 1% significance level, which is passed in all equations but the inflation rate equation. The OLS estimator is inconsistent when there are residual autocorrelations. The LM test with 13 lags suggests that there is some seasonal autocorrelation left in the model. Having monthly data some correlation with 12 lags is expected. To mitigate the problems some data preparation has been done. The UK price index, which is provided by the Office for National Statistics (ONS) seasonally unadjusted, has been seasonally adjusted using the programme TRAMO/SEATS. Despite the seasonal adjustment there is residual autocorrelation in the inflation rate equation. Industrial production as the output measure is noisy due to the monthly frequency and has been outlier corrected using TRAMO/SEATS reducing the non-normality. Still non-normality is present in all equations.

We continue by analyzing the long-run properties of the country average system. Like before, the number of stable long-run relations $\beta' \tilde{x}_t^a$, is determined by the Johansen (1995) test for $I(1)$ cointegration. The eigenvalues and trace test results for the model with an unrestricted constant are shown in Table 9. According to this result the rank of the matrix $\mathbf{\Pi}$ is two. The constant is in the unrestricted model,

Table 8 *Misspecification tests for the unrestricted VAR(3) of the country average model*

Test		π_t^a	Δy_t^a	i_t^a	r_t^a
AR 1-13	F(13,436)	2.877**	2.142*	2.084*	0.932
		[0.001]	[0.011]	[0.014]	[0.520]
Normality	$\chi^2(2)$	254.13**	16.15**	223.49**	25.92**
		[0.000]	[0.000]	[0.000]	[0.000]
ARCH 1-13	F(13,423)	2.790**	1.127	6.007**	4.508**
		[0.001]	[0.334]	[0.000]	[0.000]
Hetero	F(24,424)	2.325**	2.525**	7.408**	6.747**
		[0.001]	[0.000]	[0.000]	[0.000]

** significant at 1% level, * significant at 5% level.

not only jointly with a F-test p-val of 0.82, but also in every equation, highly insignificant. The trace test results for a model without a constant, see Table 10, suggest a rank of three. The critical cointegration relationship, which would drop out of the three cointegration relations we present in the following if we restrict the rank to two, is the term spread relation. The term spread is a very important economic relation. Therefore for economic reasons we decide here to go with a rank of three in the country-average model. So the long-run properties of the system are characterized by three cointegration relations and one common stochastic trend.

Table 9 *Johansen likelihood ratio trace test of H_0 : rank $\leq r$. Model including a constant*

r	eigenvalue	trace test	prob
0	0.184	149.23 **	[0.000]
1	0.088	55.25 **	[0.000]
2	0.026	12.61	[0.131]
3	0.001	0.65	[0.420]

** significant at 1% level, * significant at 5% level.

Table 10 *Johansen likelihood ratio trace test of H_0 : rank $\leq r$. Model without constant*

r	eigenvalue	trace test	prob
0	0.184	148.04 **	[0.000]
1	0.087	54.10 **	[0.000]
2	0.024	11.91	[0.058]
3	0.001	0.57	[0.517]

** significant at 1% level, * significant at 5% level.

The LR test results for long-run weak exogeneity displayed in Table 11 indicate that the short-term interest rate average and the long-term interest rate average are candidates for the common stochastic trend.

A clear idea of the cointegration relationships can be drawn from tests of simple hypotheses. A combination of the three cointegration relationships is not only statistically accepted but economically reasonable.

Table 11 *Testing for weak exogeneity*

	π_t^a	Δy_t^a	i_t^a	r_t^a
$\chi^2(3)$	36.35	–	7.54	4.76
	[0.00]	[0.00]	[0.06]	[0.19]

(i) *Stationary output growth average.*

$$\Delta y_t^a = 0.5(\Delta y_t + \Delta y_t^*) \sim I(0). \tag{21}$$

The first cointegration vector is the average between the UK and US output growth rates. Stationarity is expected here due to the stationarity of the output growth rates of both countries.

(ii) *Stationary real long-term interest rate average.*

$$r_t^a - \pi_t^a = 0.5[(r-\pi)_t + (r^* - \pi^*)_t] \sim I(0). \tag{22}$$

The second cointegration vector is the average between the UK and the US real long-term interest rates. The Fisher relation suggests the stationarity of the real rates, for a country average of real rates, this is even more likely.

(iii) *Stationary term spread average.*

$$r_t^a - i_t^a = 0.5[(r-i)_t + (r^* - i^*)_t] \sim I(0). \tag{23}$$

The third cointegrating vector reflects the stationarity of the spread between long and short-term interest rate averages.

Table 12 *Cointegration vectors and loadings, t-values in brackets*

	Cointegration vectors			Loadings		
	β_1^a	β_2^a	β_3^a	α_1^a	α_2^a	α_3^a
π_t^a	0	−1	0	−0.022	0.251**	−0.055
				(−0.80)	(5.95)	(−0.54)
Δy_t^a	1	0	0	−0.619**	0.136	0.838**
				(−9.45)	(1.35)	(3.46)
i_t^a	0	0	−1	0.008	−0.006	0.023
				(1.90)	(−0.95)	(1.56)
r_t^a	0	1	1	0.002	−0.009	−0.011
				(0.70)	(−1.93)	(−1.07)

** significant at 1% level, * significant at 5% level.

The system estimation results for the three cointegration vectors and their interaction with the variables of the system are shown in Table 12. The three over-identifying restrictions on the cointegration space are accepted by a likelihood ratio (LR) test with a statistic of $\chi^2(3) = 1.54$ and a p-value of 0.67.

All β-coefficients are restricted. Only some α-coefficients are statistically different from zero.[6]

The three cointegrating relations are plotted in Figure 8. The left panel is just the output growth average. The middle panel is the real long-term interest rate average. In the right panel the spread between long and short-term interest rate average can be seen.

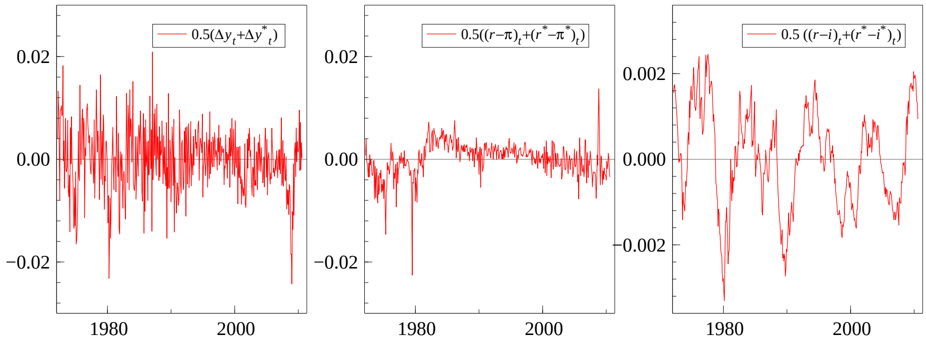

Figure 8 *The three cointegrating vectors in the country average model*

Because both interest rate averages are marginally adjusting to cointegration relationships, it is not obvious what is the common stochastic trend. In the moving average representation of the model, a normalized version of the orthogonal complement of the matrix beta, $\beta_{\perp}^{a}$, is the loading to the common stochastic trend and $\alpha_{\perp}^{a'} \sum_{i=1}^{t} u_i^a$ is the common driving trend, with u_t^a being the residuals of the system, see Johansen (1995). The matrix $\alpha_{\perp}^{a}$ is defined by $\alpha^{a'}\alpha_{\perp}^{a} = \mathbf{0}$ and looks as follows:

$$\alpha^{a'} = \begin{pmatrix} -0.022 & -0.619 & 0.008 & 0.002 \\ 0.251 & 0.136 & -0.006 & -0.009 \\ -0.055 & 0.838 & 0.023 & -0.011 \end{pmatrix} \quad \text{and} \quad \alpha_{\perp}^{a} = \begin{pmatrix} 0.037 \\ 0.006 \\ 0.340 \\ 0.940 \end{pmatrix}.$$

Thus the common stochastic trend of the country average model, CST^a, is a linear combination of the cumulated prediction errors of the long-term interest rate and the short-term interest rate equations. Overall the stochastic trend is very similar to the country average of the bond rates with a correlation of 0.94, see Figure 9 middle panel. The bond rates are pushing the system.

4.4 Testing for long-run symmetry

When deriving the cointegration vectors of the two-country model, we assumed symmetry of the UK and the US economy. To confirm this assumption we analyse the degree of separation in the cointegrated system following Konishi et al. (1993), Gonzalo and Granger (1995) and Granger and Haldrup (1997). Long-run separation is equivalent to long-run symmetry in our set-up. Only when the stochastic trends of the two subsystems do not cointegrate there exist no cointegration relationships combining variables of both subsystems. As we have seen, the two detected stochastic trends are similar to the long-term interest

[6] The long-term real interest rate average is chosen in the second cointegration relationship instead of the short-term real rate because of the following reasons. Due to the term spread cointegration relationship both options are equivalent in a way that they are two different representations of the same model with exactly the same likelihood. But in the following econometric modelling procedure the further restricting of the model leads to different selected parsimonious models for the two options and thus different final models. The decision is made upon the better Akaike and Schwarz information criteria of the final 9-dimensional model, which are both in favour of the model option with the second cointegration relationship to be the stationary long-term real interest rate average.

rate differential, r_t^d, of the country difference subsystem and to the long-term interest rate average, r_t^a, of the country average model, see Figure 9. A trace test for a system consisting of the two stochastic trends clearly does not reject a zero rank with a trace test p-value of 0.37.[7] Since the two stochastic trends do not cointegrate the separate analysis of the long-run is appropriate.

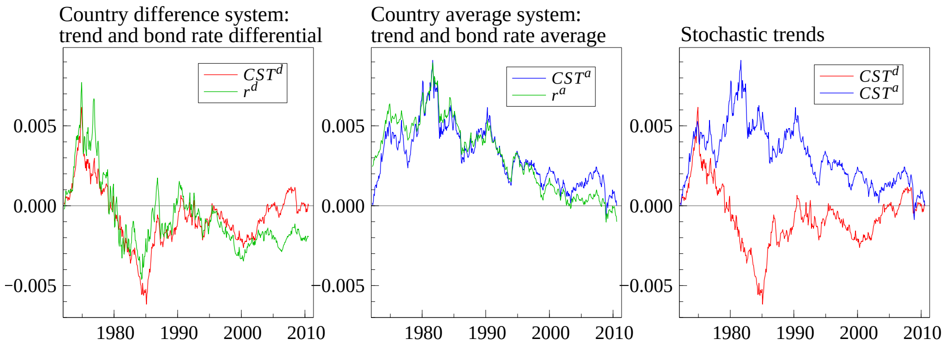

Figure 9 *Stochastic trends of the country difference and country average systems*

5 The full two-country macro dynamics

5.1 Testing for short-run symmetry

After having analysed the long-run we now focus on the contemporaneous effects and the short-run dynamics. Is the symmetry assumption also acceptable for the short-run? If yes, the country average and the country difference vector space are orthogonal to each other. Regressors from the country average system would not be significant, when regressed on country difference variables, and the other way around. Thus symmetry can be tested with overidentifying restrictions. In the combined average-difference VECM, see (24), short-run symmetry is tested by testing for significance of the off-diagonal block coefficient matrices.

$$
\begin{aligned}
\begin{bmatrix} \Delta\tilde{\boldsymbol{x}}_t^a \\ \Delta\tilde{\boldsymbol{x}}_t^d \end{bmatrix} &= \begin{bmatrix} \boldsymbol{\nu}_a \\ \boldsymbol{\nu}_d \end{bmatrix} + \begin{bmatrix} \boldsymbol{\alpha}_{aa} & \boldsymbol{\alpha}_{ad} \\ \boldsymbol{\alpha}_{da} & \boldsymbol{\alpha}_{dd} \end{bmatrix} \begin{bmatrix} \boldsymbol{\beta}_a'\tilde{\boldsymbol{x}}_{t-1}^a \\ \boldsymbol{\beta}_d'\tilde{\boldsymbol{x}}_{t-1}^d \end{bmatrix} \\
&+ \begin{bmatrix} \boldsymbol{\Gamma}_{aa,1} & \boldsymbol{\Gamma}_{ad,1} \\ \boldsymbol{\Gamma}_{da,1} & \boldsymbol{\Gamma}_{dd,1} \end{bmatrix} \begin{bmatrix} \Delta\tilde{\boldsymbol{x}}_{t-1}^a \\ \Delta\tilde{\boldsymbol{x}}_{t-1}^d \end{bmatrix} + \begin{bmatrix} \boldsymbol{\Gamma}_{aa,2} & \boldsymbol{\Gamma}_{ad,2} \\ \boldsymbol{\Gamma}_{da,2} & \boldsymbol{\Gamma}_{dd,2} \end{bmatrix} \begin{bmatrix} \Delta\tilde{\boldsymbol{x}}_{t-2}^a \\ \Delta\tilde{\boldsymbol{x}}_{t-2}^d \end{bmatrix} \\
&+ \begin{bmatrix} \boldsymbol{0} & \boldsymbol{\Gamma}_{ad,3} \\ \boldsymbol{0} & \boldsymbol{\Gamma}_{dd,3} \end{bmatrix} \begin{bmatrix} \boldsymbol{0} \\ \Delta\tilde{\boldsymbol{x}}_{t-3}^d \end{bmatrix} + \begin{bmatrix} \boldsymbol{u}_t^a \\ \boldsymbol{u}_t^d \end{bmatrix}, \\
&\text{where} \begin{bmatrix} \boldsymbol{u}_t^a \\ \boldsymbol{u}_t^d \end{bmatrix} \sim \mathsf{NID}\left(\begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{0} \end{bmatrix}, \begin{bmatrix} \boldsymbol{\Sigma}^{aa} & \boldsymbol{\Sigma}^{ad} \\ \boldsymbol{\Sigma}^{da} & \boldsymbol{\Sigma}^{dd} \end{bmatrix} \right).
\end{aligned} \tag{24}
$$

In table 13, tests of Hypotheses H_1 and H_2 show that the country average cointegration relationships have explanatory power in the country difference equation and vice versa. From this it can be followed that the adjustment processes are different in the two countries. Hypotheses H_3 and H_4 are concerned

[7] In the bivariate system of the two stochastic trends, no deterministic trend is included. As in the country average subsystem the country average bond rate would be marginally accepted as a cointegration relationship and loose the property of a stochastic trend, i.e. the country average system would become trendstaionary.

with the significance of the off-diagonal short-run dynamics. The short-run dynamics of the country average subsystem have marginally explanatory power in the country difference equations, but the short-run dynamics of the country difference model is clearly not helpful in explaining the country average variables. The rejection of Hypotheses H_5 and H_6 show that the adjustment coefficients jointly with the short-run dynamics of one subsystem do have explanatory power in the other subsystem. Symmetry in the short-run and in the adjustment to the long-run is clearly rejected. For the following analysis we return to the full model of domestic-foreign variables with the cointegration relationships §4.2 and §4.3 preserved.

Table 13 *Testing overidentifying restrictions*

$H_o^1 : \alpha_{da} = 0$	$\chi^2(15) = 27.2$	[0.027]
$H_o^2 : \alpha_{ad} = 0$	$\chi^2(16) = 47.7$	[0.000]
$H_o^3 : \Gamma_{da,1} = 0, \Gamma_{da,2} = 0$	$\chi^2(40) = 55.3$	[0.055]
$H_o^4 : \Gamma_{ad,1} = 0, \Gamma_{ad,2} = 0, \Gamma_{ad,3} = 0$	$\chi^2(60) = 72.4$	[0.130]
$H_o^5 : \alpha_{da} = 0, \Gamma_{da,j} = 0$	$\chi^2(55) = 90.4$	[0.002]
$H_o^6 : \alpha_{ad} = 0, \Gamma_{ad,j} = 0$	$\chi^2(76) = 134.8$	[0.000]

5.2 Identifying instantaneous causality

The residual correlation matrix of the full 9-dimensional VECM(3) with the cointegration relationships identified before is reported in Table 14. Clear statistically significant contemporaneous correlation of shocks is between the short and long-term interest rates of each country. Thus, in the very short term, the term spread is a strong link between the macroeconomic variables. As the dominant force in transmitting and absorbing macroeconomic shocks, it will play an important role in the transmission of monetary shocks to the exchange rate. Another large contemporaneous correlation of shocks is between the domestic and foreign bond yields, due to the strong interconnectedness of financial markets. Only weak correlation is present relative to the exchange rate, see last row of Table 14, what is of interest when studying the impact effects of monetary policy on exchange rates.

Table 14 *Contemporaneous correlation of the VECM*

	π_t	Δy_t	i_t	r_t	π_t^*	Δy_t^*	i_t^*	r_t^*	e_t
π_t	1								
Δy_t	−0.02	1							
i_t	0.00	0.07	1						
r_t	0.08	0.03	**0.52**	1					
π_t^*	0.13	−0.06	0.15	0.10	1				
Δy_t^*	0.01	0.06	0.02	0.06	0.05	1			
i_t^*	−0.02	−0.04	0.13	0.22	0.10	0.19	1		
r_t^*	0.04	0.01	0.12	**0.38**	0.17	0.17	**0.54**	1	
e_t	0.05	−0.05	−0.09	−0.10	0.12	−0.06	−0.14	−0.15	1

For further investigations of these issues, the correlation matrix in Table 14 is subjected to a graph-theoretical search for instantaneous causal relations. The Conservative PC algorithm (CPC) is applied in the following, which is a variant of the PC algorithm, designed to improve arrowpoint orientation accuracy. The CPC algorithm finds, at a 10% significance level[8] the acyclical graph shown in Figure

[8]Because of the importance of the causal structure for the issue of an impact effect on the exchange rate and hence the

10. Four directed and five undirected edges are found. The final causal structure of a preferred model is chosen in the following by comparing information criteria of final models.

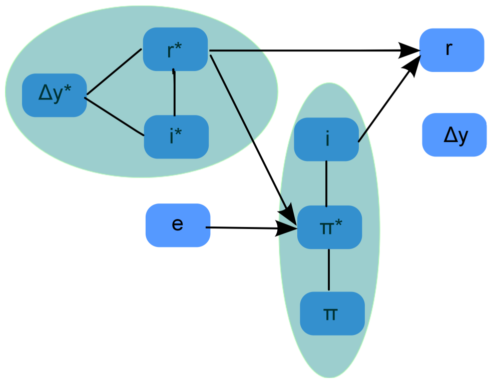

Figure 10 *Causal structure, Conservative PC algorithm* (10% *significance level)*

To direct the five undirected edges we compare all $2^5 = 32$ models associated with a DAG consistent with the Tetrad graphs. Removing the model options, leading to a directed cyclical graph, 24 model specificatios remain. Because the nine equations of the final model are estimated by OLS and having the present structure, it is possible to calculate the information criteria for three subsets and add those results together.

The first subset is including the US output growth, the US short-term interest rate and the US long-term interest rate. Here are three undirected edges to direct, when removing cyclicality, then 6 model options remain, which are compared in Table 15. The best value of Akaike and Schwarz criteria has model 3, where US output growth is driving the short- and the long-term US interest rates and the short-term rate is driving the long-term rate. All links are in line with economic theory.

Table 15 *Information criteria of final selected models with different causal structure, subset 1*

	subsystem with $(\Delta y^*, i^*, r^*)$	*AIC*	*SC*
M_1	$i^* \to \Delta y^*, i^* \to r^*, r^* \to \Delta y^*$	-43.618	-43.090
M_2	$i^* \to \Delta y^*, i^* \to r^*, \Delta y^* \to r^*$	-43.612	-43.075
M_3	$\Delta y^* \to i^*, i^* \to r^*, \Delta y^* \to r^*$	-43.647	-43.128
M_4	$\Delta y^* \to i^*, r^* \to i^*, r^* \to \Delta y^*$	-43.563	-43.035
M_5	$\Delta y^* \to i^*, r^* \to i^*, \Delta y^* \to r^*$	-43.535	-42.989
M_6	$i^* \to \Delta y^*, r^* \to i^*, r^* \to \Delta y^*$	-43.623	-43.086

The second subset to analyse consists of the two inflation rates and the UK short-term interest rate, see Table 16. Although model option 4 has a minimal advantage concerning information criteria, we decide here to proceed with model option 1. The explanation for this is a strong dummy effect in the UK short term interest rate equation on the information criteria. When increasing the information set for the automatic model selection procedure by including an additional regressor to choose from and the regressor is significantly included into the model, also the punished loglikelihood should not decrease

question of the delayed overshooting, we decide to go here with a 10% significance level.

after including the additional regressor. In the case of the UK short-term interest rate equation dummies improve the information criteria for the model with the smaller information set.

Table 16 *Information criteria of final selected models with different causal structure, subset 2*

	subsystem with $(\Delta p^*, \Delta p, i)$	*AIC*	*SC*
M_1	$\Delta p^* \to \Delta p, \Delta p^* \to i$	-39.877	-39.376
M_2	$\Delta p \to \Delta p^*, \Delta p^* \to i$	-39.827	-39.326
M_3	$\Delta p \to \Delta p^*, i \to \Delta p^*$	-39.831	-39.312
M_4	$\Delta p^* \to \Delta p, i \to \Delta p^*$	-39.899	-39.380

The final set involving the UK output growth, the UK bond yield and the exchange rate equation is directed, therefore determined with regard to contemporaneous effects.

The efficiency of the single-equation reduction procedure depends on the orthogonality of the structural shocks. In contrast to just identified Cholesky-type orthogonalisations of errors in the literature, we can use the overidentifying restrictions on $\boldsymbol{B}^r$ to test for instantaneous causal relations depicted in Figure 10 by comparing its log-likelihood to a just identified SVECM nesting our model. The LR test with a test statistic of $\chi^2(27) = 40.84$ and a p-value of 0.043 does at this stage reject the hypothesis of a diagonal covariance matrix.

Table 17 *Correlation matrix of the SVECM*

	π_t	Δy_t	i_t	r_t	π_t^*	Δy_t^*	i_t^*	r_t^*	e_t
π_t	1								
Δy_t	-0.02	1							
i_t	-0.02	0.08	1						
r_t	0.09	-0.02	0.00	1					
π_t^*	-0.01	-0.05	0.00	-0.03	1				
Δy_t^*	0.00	0.06	0.01	-0.01	0.02	1			
i_t^*	-0.04	-0.05	0.12^*	-0.02	0.02	0.00	1		
r_t^*	0.05	0.04	0.04	0.01	-0.02	0.00	0.00	1	
e_t	0.04	-0.05	-0.11^*	-0.02	0.00	-0.06	$\mathbf{-0.14}^{**}$	-0.08	1

** significant at 1% level, * significant at 5% level, with Tetrad CPC algorithm.

Secondly, after including the final causal structure, detected by Tetrad and directed with the help of information criteria, into the VECM, the resulting SVECM is estimated with OLS. The correlation matrix of this SVECM can be seen in Table 17. When running the Tetrad CPC algorithm with 1% and 5% significance level on this variance-covariance matrix, one out of the 36 entries is significant at a 1% significance level, it is between the US short-term interest rate and the exchange rate. So the inclusion of the Tetrad results has not yet fully orthogonalised the variance-covariance matrix.

To orthogonalise the variance-covariance matrix and have all contemporaneous effects considered, we include an additional undirected edge in the causal structure, connecting the US short-term interest rate and the exchange rate. The exchange rate is not significant in the US interest rate equation, with p-values between 0.13 to 0.91 depending on specification of this equation, while the US short term interest rate is highly significant in the exchange rate equation. By the inclusion of this link the number of models increase to 48. We select a best model via information criteria. The variance-covariance matrix is now orthogonal. The LR test with a test statistic of $\chi^2(26) = 31.13$ and a p-value of 0.22 does now not reject the hypothesis of a diagonal covariance matrix. Only one entry significant at 5% is left, see Table 18. After directing all six undirected edges the causal structure is fully determined like shown in Figure 11.

Table 18 *Correlation matrix of the SVECM when the additional link between US short-term interest rate and exchange rate is included*

	π_t	Δy_t	i_t	r_t	π_t^*	Δy_t^*	i_t^*	r_t^*	e_t
π_t	1								
Δy_t	−0.02	1							
i_t	−0.02	0.08	1						
r_t	0.09	−0.02	0.00	1					
π_t^*	−0.01	−0.05	0.00	−0.03	1				
Δy_t^*	0.00	0.06	0.01	−0.01	0.02	1			
i_t^*	−0.04	−0.05	0.12*	−0.02	0.02	0.00	1		
r_t^*	0.05	0.04	0.04	0.01	−0.02	0.00	0.00	1	
e_t	0.03	−0.06	−0.09	−0.02	0.00	−0.03	0.01	−0.08	1

** significant at 1% level, * significant at 5% level, with Tetrad CPC algorithm.

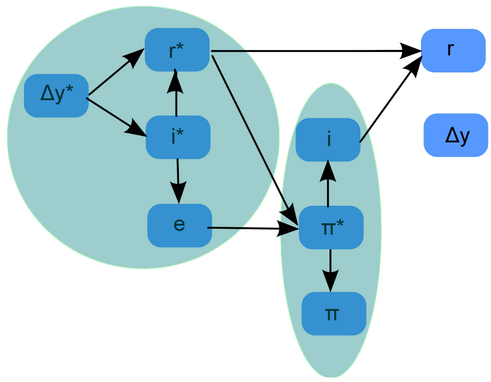

Figure 11 *Final causal structure*

The identified causal structure[9] is mapped into the contemporaneous matrix $\mathbf{B}^r$, see Table 19. Compared to a Cholesky decomposition our causal structure is highly parsimonious. Only a few non-zero lower-off-diagonal elements are present. There are two blocks in this contemporaneous matrix, one is the set of US variables together with the exchange rate, the other block is the set of UK variables. This block-recursive structure shows that in the very short-run the UK can be described by a small economy model. The interest rates are ordered before the inflation rates, what might be explained by the monetary policy authorities having to rely on historical databased nowcasts for policy making. The exchange rate being ordered after the US short-term interest rate but before the UK short-term interest rate implies a 'delayed' response of the exchange rate after a monetary policy action of the Bank of England.

[9]The presented causal order is not unique. The choice of one of the possible causal orderings does not influence the further analysis.

Table 19 *Final structure of the restricted matrix* $\boldsymbol{B}^r$

$$
\begin{array}{c}
\Delta y_t^* \\ i_t^* \\ r_t^* \\ e_t \\ \pi_t^* \\ \Delta y_t \\ i_t \\ r_t \\ \pi_t
\end{array}
\begin{pmatrix}
1 & & & & & & & & \\
b_{21} & 1 & & & & & & & \\
b_{31} & b_{32} & 1 & & & & & & \\
0 & b_{42} & 0 & 1 & & & & & \\
0 & 0 & b_{53} & b_{54} & 1 & & & & \\
0 & 0 & 0 & 0 & 0 & 1 & & & \\
0 & 0 & 0 & 0 & b_{75} & 0 & 1 & & \\
0 & 0 & b_{83} & 0 & 0 & 0 & b_{87} & 1 & \\
0 & 0 & 0 & 0 & b_{95} & 0 & 0 & 0 & 1
\end{pmatrix}
$$

5.3 The parsimonious structural vector equilibrium correction model

Having specified the SVECM in (6) with the cointegration relations found in §4.2 and §4.3 and the contemporaneous relations detected by the empirical PC causal search algorithm in §5.2, the model reduction is performed with the help of an automatic general-to-specific model reduction procedure. As the design of $\boldsymbol{B}^r$ and the parameters of $\beta^{s,r}$ are given, the model search is limited to the parameters of the short-run dynamics, $\boldsymbol{\Gamma}_1, \ldots, \boldsymbol{\Gamma}_3$, and the long-run equilibrium adjustment, $\boldsymbol{\alpha}$, while it is ensured that the rank of the long-run matrix $\boldsymbol{\Pi}$ is unaltered by the constraints on $\boldsymbol{\alpha}$. As shown in Krolzig (2003), when commencing from a structural VECM with known causal order and diagonal variance-covariance matrix, all possible reductions of the SVECM can be efficiently estimated by OLS and model selection procedures can operate equation-by-equation without a loss in efficiency. The liberal strategy of *PcGets* used here approximates in large samples the Hannan-Quinn (HQ) information criteria (for more about mapping information criteria to significance levels see Campos et al., 2003). The properties of automatic *Gets* selection are discussed in more detail in Hendry and Krolzig (2005).

The final parsimonious baseline model selected by *PcGets* and estimated with OLS is as follows: All coefficients are significant with a t-value of at least 2. The adjusted R^2 of the reduced single equations are from 27% for the exchange rate equation up to 70% for the UK inflation rate equation. Major outliers are corrected by including impulse dummies. Only the exchange rate equation is reported here:

$$
\begin{aligned}
\widehat{\Delta e_t} \;=\;& \underset{(0.57)}{1.97}\;(i-i^*)_{t-1} + \underset{(0.27)}{0.755}\;[(r-\pi)_{t-1}-(r^*-\pi^*)_{t-1}] \\
& - \underset{(0.0032)}{0.0118}\;[e_{t-1}-87.2(r-r^*)_{t-1}] + \underset{(2.4)}{5.54}\;\Delta i_{t-3} - \underset{(2.7)}{8.79}\;\Delta i_t^* \\
& + \underset{(4.0)}{9.56}\;\Delta r_{t-3}^*, \quad \hat{\sigma} = 0.0255,\; \bar{R}^2 = 0.27,\; 8 \text{ dummies.}
\end{aligned} \tag{25}
$$

The exchange rate is error correcting towards three cointegration relationships. The first two can be interpreted as a carry trade effect, high rates lead to capital inflow, hence appreciation of the currency. The third is a multi-period UIP relation. Three cointegration relationships are driving the exchange rate, with all being from the country difference system. The importance of the cointegration approach is obvious. A lot of information would have been lost if the VAR had been specified in differences. Further, one contemporaneous effect is present in this equation. An increase of the short-term interest rate in the US leads to an appreciation of the dollar on impact.

5.4 Testing for the validity and congruency of the model

Finally, we test whether the application of the automatic general-to-specific model selection has maintained the diagonal structure of the variance-covariance matrix. In the correlation matrix of the PSVECM shown in Table 20, there are no large contemporaneous correlations. Altogether a slide increase in correlation can be observed, compared to the SVECM in Table 18, with a correlation between UK inflation and US short-term interest rate significant at a 1% significance level.

Table 20 *Correlation matrix of the PSVECM*

	π_t	Δy_t	i_t	r_t	π_t^*	Δy_t^*	i_t^*	r_t^*	e_t
π_t	1								
Δy_t	0.01	1							
i_t	-0.04	0.02	1						
r_t	0.11^*	0.07	0.03	1					
π_t^*	0.00	-0.07	-0.05	-0.01	1				
Δy_t^*	0.09^*	0.02	0.03	-0.02	0.07	1			
i_t^*	-0.13^{**}	-0.11^*	0.05	-0.10^*	-0.03	-0.01	1		
r_t^*	0.02	0.02	0.01	0.07	-0.01	0.02	0.01	1	
e_t	0.00	-0.06	-0.08	-0.07	0.00	0.00	0.04	-0.08	1

** significant at 1% level, * significant at 5% level, with Tetrad CPC algorithm.

The congruency of the model is investigated in Table 21. There are some problems of autocorrelation in the US inflation rate and short-term interest rate equation. See the discussion of problems of autocorrelation in §4.3. With the inclusion of dummies and reduction of the model, there is a huge reduction in non-normality, but still some issues of non-normality and heteroscedasticity remain.

Table 21 *Misspecification tests of the parsimonious SVECM*

Test	π_t	π_t^*	Δy_t	Δy_t^*	i_t	i_t^*	r_t	r_t^*	e_t
AR 1-7	0.828 [0.564]	1.879 [0.071]	1.022 [0.415]	1.136 [0.339]	0.323 [0.943]	0.648 [0.716]	1.056 [0.391]	1.325 [0.237]	0.973 [0.451]
AR 1-13	1.391 [0.160]	3.729^{**} [0.000]	1.192 [0.282]	1.116 [0.343]	1.169 [0.299]	2.110^* [0.013]	0.804 [0.656]	1.075 [0.379]	1.174 [0.296]
Normality	42.34^{**} [0.000]	7.262^* [0.027]	0.472 [0.790]	0.934 [0.627]	67.33^{**} [0.000]	32.48^{**} [0.000]	4.944 [0.084]	10.26^{**} [0.006]	1.229 [0.541]
ARCH 1-13	8.997^{**} [0.000]	2.665^{**} [0.001]	1.965^* [0.022]	0.706 [0.758]	4.918^{**} [0.000]	13.19^{**} [0.000]	3.124^{**} [0.000]	2.462^{**} [0.003]	0.871 [0.585]
Hetero	1.703^* [0.012]	3.771^{**} [0.000]	0.698 [0.885]	1.017 [0.445]	1.769^{**} [0.008]	2.926^{**} [0.000]	2.523^{**} [0.000]	1.236 [0.212]	0.464 [0.985]

** significant at 1% level, * significant at 5% level.

6 The effects of a monetary policy shock

In this section, we consider the dynamic responses to an asymmetric monetary policy shock in form of an unpredicted one percentage-point increase of the nominal short-term interest rate of the UK and respectively of the US.

6.1 An impulse response analysis of a monetary policy shock in the UK

Figure 12 displays the responses of the system variables, i.e., the inflation rates, the output growth rates, the 3-month interest rates, the 10-year government bond yields, and the nominal exchange rate, with regard to an one-percentage point increase in the monthly 3-month treasury bill return of the UK. The 95% confidence bands are Hall (1992) bootstrap intervals with 2000 replications. The computation follows the algorithm of Benkwitz et al. (2001) in the version without reestimation of the cointegration relations. Due to the employed model selection strategy, the impulse responses are estimated precisely offering clear conclusions about the sign of responses.

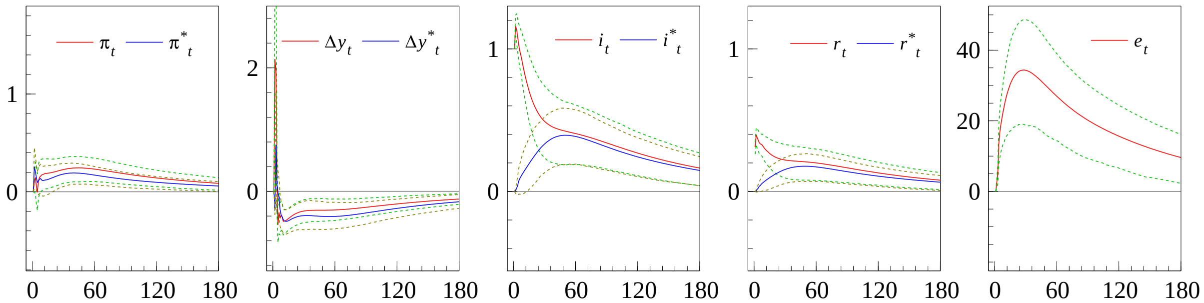

Figure 12 *Responses to an asymmetric monetary policy shock of the UK in the baseline parsimonious SVECM, 95% confidence bands*

After a UK monetary policy shock the bond rate of the UK reacts contemporaneously with 31% of the size of the shock. Both UK interest rates slowly revert to the original level. The interest rates of the US increase, not on impact but steadily, to a peak of 40% of the size of the shock after 4 years for the short-term rate and to 18% of the shock for the bond rate. The gap between the US and UK short-term rates is closed after 4 years. Taking the confidence bands into account already after 18 month. The interest rates in both countries return steadily back to the original level, although with considerable persistence, being still significantly different from zero after 15 years. After a short-lived initial rise, both output growth rates react negatively to the monetary policy shock, in line with economic theory. The inflation rates however show a positive response, the well known ‘price puzzle’. The exchange rate appreciates steadily for several months, achieving a peak after 28 months, and finally depreciates thereafter. A clear pattern of delayed overshooting is present.

6.2 An impulse response analysis of a monetary policy shock in the US

The impulse responses of the system variables with regard to an one-percentage point increase in the monthly 3-month treasury bill return of the US are plotted in Figure 13.

After a US monetary policy shock the US bond yield reacts contemporaneously with 38% of the size of the shock. So the Fed influences the long-term rates more strongly on impact than the Bank of England. The UK short-term interest rate only reacts on impact with 1% of the size of the shock, but reaches 54% after 12 month. Thereafter the UK short-term rate is in fact larger than in the US. This strong mimicry is responsible for a higher bond yield in the UK than in the US one year after the US monetary shock. Just as in the UK case also here the responses of the output growth rates are negative and of the inflation rates positive. Overall the persistence in the system is much smaller in the case of

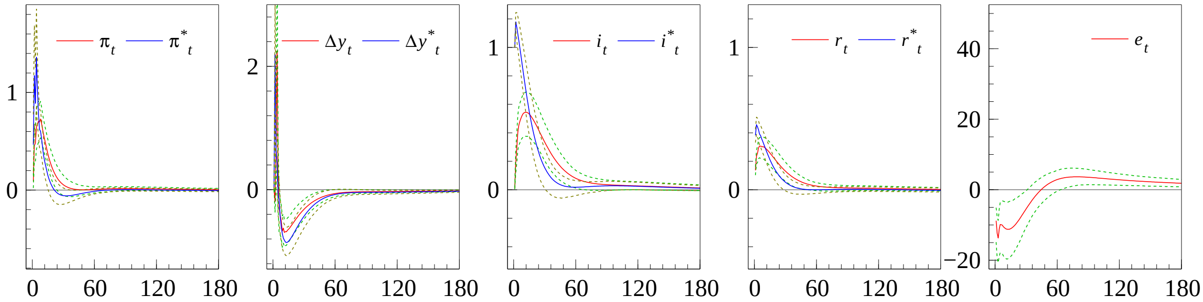

Figure 13 *Responses to an idiosyncratic US monetary policy shock in the baseline parsimonious SVECM,* 95% *confidence bands*

the US monetary policy shock, with the variables being back to origin after 5 years. The exchange rate depreciates on impact but then appreciates to a level higher than at the beginning, what is due to the higher bond rate in the UK during the adjustment process. In total the reaction of the exchange rate is much smaller in the case of a US monetary policy shock, but delayed overshooting is not present, according to the baseline model.

6.3 Robustness of the effects of a monetary policy shock

As an alternative to the choice of a best model we take in the following all 48 models, which are the outcome of the graph-theoretic approach for the identification, into account. By doing so we achieve, additional to the estimation uncertainty, knowledge about the model uncertainty present in the modelling process. In Figure 14 we present all impulse responses of the 48 models and an unweighted model average (in blue colour) for the UK case. The model simulations are quite similar to each other, with half of them having a larger degree of persistence. Due to the similarity of the models we refrain from trimming. Likewise no weighting scheme is employed due to the only minor differences in the fit of these quite similar models as measured by the usual information criteria. A weighting scheme results in a model hardly distinguishable from a simple average. With this approach we take some model uncertainty into account, yet the results expressed before do not change.

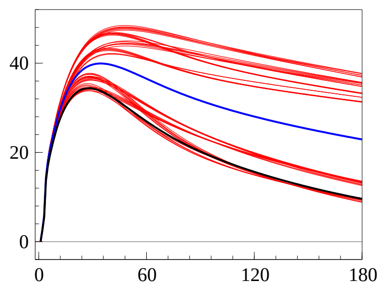

Figure 14 *Robustness of the exchange rate response to an asymmetric UK monetary policy shock in all 48 PSVECMs, in black the baseline model, in blue the average of the models*

When taking all 48 models into account in the US case, see Figure 15, half of the models display a sudden jump of the exchange rate while the rest show a delayed overshooting response. The models with the delayed overshooting response are these where the exchange rate feeds contemporaneously into the US short term interest rate equation. The exchange rate is not significant in the US interest rate equation, with p-values between 0.13 to 0.91 depending on specification of this equation, while the US short term interest rate is highly significant in the exchange rate equation. The model average shows a sudden jump with half the size of the baseline model but taking model uncertainty into account, a confidence band would include the zero line, however the average model offers the possibility of an appreciation of the Dollar on impact. The maximum peak appreciation of the Dollar in the average model is after 16 month, considerably earlier as in the case of a Bank of England shock with 34 month.

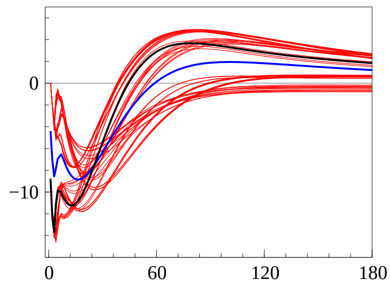

Figure 15 *Robustness of the exchange rate response to an asymmetric US monetary policy shock in all 48 PSVECMs, in black the baseline model, in blue the average of the models*

7 Conclusion

A balanced UK-US two-country model was set up to investigate the presence of a 'delayed overshooting puzzle'. The analysis found a delayed overshooting in respect to a Bank of England shock but an immediate response to a Fed surprise. Altogether the exchange rate response is larger in size in the former case. This asymmetric result shows the need for a rigorous analysis in research to this problem and contributes to an explanation of the conflicting results of the literature.

Unrestricted two-country cointegrated VAR models suffer from the curse of dimensionality and are forced to restrict the number of variables. To overcome this limitation we proposed a modelling approach allowing to split the analysis of the long-run into two subsystems: a country-difference and a country-average system. Separability requires symmetry of the two countries, see Krolzig and Heinlein (2013). For the short-run this was rejected. Thus, the full 9-dimensional system has been modelled to determine the short-run and the contemporaneous effects, using the equilibrium correction terms from the earlier two subsystems. For the econometric model selection we proposed a data-driven approach combining a likelihood based cointegration analysis with a graph-theoretic search for instantaneous causal relations and an automatic general-to-specific approach for the selection of a parsimonious structural vector equilibrium correction model. Collectively putting these elements together, we believe to have a strong programme of how to set up empirical two-country models.

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