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Abstract

In this paper, we study the impact of the economic growth on the environment. First, we show that, at each income level, η determines the direction of environmental degradation, where η is the elasticity of substitution between consumption and the environment. That is, for η large enough, as income increases people accept environmental degradation by enjoying more consumption as compensation, and vice versa. Intuitively, there are two effects operating; the income effect encourages the demand for better environmental quality simply because the environment is a normal good, whereas the substitution effect discourages it because maintaining the environment becomes more expensive as technology improvement increases the production of the general consumption good per unit of emission. The strength of the substitution effect is governed by η. Hence, the impact of economic growth on the environment crucially depends on η. Second, we demonstrate that exponential utility generates the environmental Kuznets curve (EKC) under a wide class of models without adding any other peculiar assumptions. Under exponential utility, η is decreasing in income; intuitively, when a country is poor (large η), people seek more consumption at the cost of environmental degradation, but, once it becomes rich enough (small η), they seek increased environmental quality.

KEYWORDS: Environmental Kuznets Curve, Economic Growth, Non-Homothetic Preferences, Generalized Isoelastic Preferences.

JEL CLASSIFICATION: O13, Q56

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1 Introduction

In this paper, we investigate a set of general equilibrium growth models to study the environmental Kuznets curve (EKC). Generally speaking, the EKC is the empirical observation that environmental degradation increases when the income level is low, but, after passing a certain income level, it decreases as income increases; see Dinda (2004), Stern (2004) and Brock and Taylor (2005) and Xepapadeas (2003) to name a few. The EKC is important because, if it indeed holds in reality, economic success is consistent with, rather than contradictory to, environmental sustainability.

A recent survey paper by Carson (2010) explains that the number of empirical studies far exceeds papers that have attempted to develop a theoretical basis for the existence of the EKC. We add to the literature that examines the EKC theoretically. In particular, our theoretical models reveal the importance of η, where η is the elasticity of substitution between the service flow from the environment R and general consumption goods C. Throughout this paper, we define η as the observed ratio of a percentage change of R/C to the percentage change of the (shadow) price of R (see Section 2 for more details). There are two main findings in this paper.

First, *at each income level, if η is high enough, the quality of the environment deteriorates as income increases*, and vice versa. To get the intuition, note first that η shows how the demand for R is sensitive to the change in the relative price; that is, if it is high, people accept a small amount of C as a compensation for giving up one unit of R, when the price of the environment P_R increases. Suppose for a while that P_R is unchanged at all; in this case, as income increases, people demand more R due to the standard income effect. However, as the productivity improves, C becomes cheaper, meaning that the (shadow) price P_R increases. Since R becomes more expensive relative to C, this change in the relative price discourages the demand for a good quality of R due to the substitution effect. The relative strength of these two effects hinges on η. If η is low enough, the substitution effect is weak and is dominated, and vice versa.

Second, as a natural consequence of the first finding, *if η decreases fast enough as income increases, it generates the EKC*. That is, initially (when income is low) η should be high so that the quality of R deteriorates, but, after income passing a certain level, η should be low so that R improves. As such, we show that, with exponential (DES) utility[1], η is decreasing in income, and hence it generates the

[1]Exponential utility is also known as constant absolute risk aversion (CARA) utility. In this paper, we mainly call

EKC in a wide class of model settings. In contrast, with power (CES) utility[2], η is constant, and as a result environmental degradation is either monotonically increasing or decreasing in income depending on η. While CES utility is quite popular because of its analytical tractability, DES utility also has its own appeal (see Section 4.3 for the discussion of the competitive limit). In this relation, one of the important findings lies in the fact that the only requirement to generate the EKC in our framework is preference with decreasing η. More specifically, with DES utility, the EKC naturally arises under a wide class of model specifications without adding any other peculiar assumptions. Hence, the most important empirical implication of this paper is: Is η constant or decreasing?

As a theoretical prediction (or conjecture), it has been known that some sort of non-homotheticity plays a key role in generating the EKC (see Lopez, 1994). Indeed, what we demonstrate is that homothetic preference (CES utility) cannot generate the EKC, while non-homothetic utility (DES utility) can. While the importance of non-homotheticity for the EKC has long been recognized, to the best of our knowledge, there exists only one paper other than ours that explicitly constructs a non-homothetic model to study the EKC. That is, Andreoni and Levinson (2001)[3] demonstrate that the abatement technology that exhibits increasing returns to scale (IRS) can generate the EKC. In this case, the source of non-homotheticity comes from IRS abatement technology. All other models that generate the EKC rely on a constraint that is binding only before or after the peak of environmental degradation (i.e., the bottom the environment quality R). This group of models includes Stokey (1998), and Hartman and Kwon (2005), to name a few. Though such constraint driven models are also often quite convincing, the mechanism behind this class of models is totally different from Andreoni and Levinson (2001) and ours. We discuss this in depth in Section 4.2.

Finally, this paper has one important practical implication. We argue that the inverted U-shape is perhaps only important from a theoretical perspective. Rather, policy makers and the public are interested in the fate of the environment in the long-run. Also, as discussed above, unless there are exogenous binding constraints, whatever the utility function is, η must be low enough (i.e., lower

it "decreasing elasticity of substitution (DES) utility", to emphasize the fact that η is decreasing under this utility. Although perhaps CARA is the most popular name, this paper shows that it is not the risk attitude that generates the EKC (see Model IV in Section 3). However, note also that, unlike CES utility, exponential utility is not the only utility that exhibits decreasing η.

[2]Power utility is also known as constant relative risk aversion (CRRA) utility and constant elasticity of substitution (CES) utility. For our analyses, the latter name is suggestive, because this paper mainly discuss the role of η.

[3]See also Egli and Steger (2007) as a dynamic extension of Andreoni and Levinson (2001).

than a threshold) to have improving environmental quality when income is high enough (income is presumably high in the far future). Note also that, in reality, there are many types of pollutants and many dimensions of the environmental quality, meaning that η and its threshold should take different values for different types of pollutants. In this regard, our numerical experiments suggest that pollutants such as CO_2 are likely to keep increasing in the future, because the depreciation rate of CO_2 is very low; see Stern's (2007) review. In our models, not surprisingly, pollutants with a very low depreciation rate (i.e., the speed that the nature purifies them) have a low (tight) threshold of η. Indeed, some empirical surveys find that the EKC is well observed for flow pollutants (the nature can reduce it quickly) rather than stock pollutants (the reduction by the nature is slow); see Lieb(2004) for example. Setting aside the EKC, this supports the view that, for high enough income, pollutants are likely to decrease if they have a high depreciation rate.

The plan of this paper is as follows. Section 2 investigates the role of η; at each income level, if η is low enough, R deteriorates as income increases. Section 3 first shows the Slutsky decomposition for simple models, and then numerically demonstrates that DES utility generates the EKC in a wide range of models. Section 4 discusses some additional key issues and the last section concludes.

2 A Model with Analytical Solution

We begin by describing a typical modelling convention in this literature. Next, to provide intuition, we investigate a simple general equilibrium model, where the only source of growth is an exogenous productivity change W. The main message of this section is that, at each income level, R deteriorates as income increases if η is low enough. Rephrasing this, for each W, there exists a threshold $\bar{\eta}$ such that $\mathrm{d}R/\mathrm{d}W \gtrless 0$ if $\eta \gtrless \bar{\eta}$. Note that this section focuses on the *local* behavior of R, meaning that it describes the direction of R (i.e., R deteriorates or improves) *at each income level.* Although the EKC is a *global* phenomenon *over a range of income*, naturally we conjecture that if η is decreasing in income it shows the EKC, which we confirm in the next section.

2.1 Modelling Convention in The Literature

Before going on, we would like to mention some rather technical conventions in this literature. First, throughout this paper, we assume no externality in the pollution emission. That is, the emission of pollution is properly priced. In fact, many environmental economists believe that governments have fairly successfully imposed regulations on pollution emissions at least in developed countries; see Brock and Taylor (2005) for example. Indeed, if pollution emission is totally external, there is no incentive for firms to cut their emission and hence we cannot observe the EKC. This also implies that we can solve the models below as a social planner's problem. Second, we assume that pollution emission works as if it is a production factor. Although modeling pollution as a by-product is intuitively more appealing, as Stokey (1998) shows, under reasonable regularity assumptions, modelling it as a by-product or as a production factor does not make any difference. Finally, we measure the income level by either the level of exogenous technology or accumulated capital stock, depending on the context.

2.2 The Model

Given the assumptions above, we start with a representative household. Its utility is increasing in both consumption C and the service flow from R. Here, R shows the quality of the environment and its service flow is higher for higher R. The household takes the price of the environment P_R in terms of consumption goods price as given, and its only income is the compensation for the pollution emission $P_R X$. The environmental resource constraint implies that higher emission X leads to a lower quality of R.

$$\max \quad U(C, R) \tag{1a}$$

$$\text{s.t.} \quad C = P_R X \qquad \text{(budget constraint)} \tag{1b}$$

$$X = 1 - R \qquad \text{(environmental resource constraint)} \tag{1c}$$

This environmental resource constraint (1c) implies that (i) the upper and lower limits of R is 1 and 0, respectively, (ii) the price charged for the pollution emission is equal to the price of the environment; $P_X = P_R$, and (iii) $\frac{\mathrm{d}R}{\mathrm{d}W}\frac{W}{R} = -\frac{X}{R}\frac{\mathrm{d}X}{\mathrm{d}W}\frac{W}{X}$. Solving this model, we obtain one first order condition (FOC) and two constraints, which implies that we can write R, X and C as functions of P_R (as well as parameters in utility). Recognizing that $\mathrm{d}(R/C)/\mathrm{d}P_R$ is the derivative of the ratio R/C, the elasticity of substitution η between C and R can be decomposed into the price elasticities of R and C.

$$\eta = -\frac{\mathrm{d}(R/C)}{\mathrm{d}P_R}\frac{P_R}{R/C} = \frac{\mathrm{d}C}{\mathrm{d}P_R}\frac{P_R}{C} - \frac{\mathrm{d}R}{\mathrm{d}P_R}\frac{P_R}{R} \tag{2}$$

The key trick here is that we do not solve the utility maximization explicitly. Instead, we summarize the household's optimal behavior by η.[4] Note that η is not necessarily a constant unless the household has a CES utility. Also, the definition of our η is based only on the observed changes in quantities and their relative price (below we discuss this further). Applying the chain rule,[5] we decompose the elasticity of R with respect to technology W as follows.

$$\frac{\mathrm{d}R}{\mathrm{d}W}\frac{W}{R} = \frac{\mathrm{d}C}{\mathrm{d}W}\frac{W}{C} - \eta\frac{\mathrm{d}P_R}{\mathrm{d}W}\frac{W}{P_R} \tag{3}$$

Next, we turn to the production side of the economy. To produce C, the representative firm must emit pollution X with Hicks neutral productivity W. Here, following the convention, we treat X as a production factor.

$$\begin{aligned} C &= Y && \text{(market clearing)} \\ Y &= Wf(X) && \text{(production)} \end{aligned}$$

At the firm's optimum, $P_R = \partial C/\partial X = Wf_X(X)$. Hence, it is straightforward to show that

$$\frac{\mathrm{d}C}{\mathrm{d}W}\frac{W}{C} = \bar{\eta}\frac{\mathrm{d}Y}{\mathrm{d}W}\frac{W}{Y} \qquad \text{where } \bar{\eta} = 1 \text{ in this case} \tag{4a}$$

$$\frac{\mathrm{d}Y}{\mathrm{d}W}\frac{W}{Y} = 1 + \frac{P_R X}{Y}\frac{\mathrm{d}X}{\mathrm{d}W}\frac{W}{X} \tag{4b}$$

$$\frac{\mathrm{d}P_R}{\mathrm{d}W}\frac{W}{P_R} = 1 + \varepsilon_{XX}\frac{\mathrm{d}X}{\mathrm{d}W}\frac{W}{X} \tag{4c}$$

Note that the second order optimality condition implies that $\varepsilon_{XX} = \frac{f_{XX}X}{f_X} \leq 0$. For example, $\varepsilon_{XX} = -\alpha$ under a Cobb-Douglas production function, $Y = Wf(X) = W\bar{K}^{\alpha}X^{1-\alpha}$ where $\bar{K}$ is a

[4] Note that P_R is the only signal that the households receives exogenously. Hence, we take the total derivatives with respect to it. This also implies that technology W can affect the households' behavior only through P_R.

[5] That is, $\frac{\mathrm{d}C}{\mathrm{d}P_R}\frac{P_R}{C} = \frac{\mathrm{d}C}{\mathrm{d}W}\frac{W}{C}\frac{\mathrm{d}W}{\mathrm{d}P_R}\frac{P_R}{W}$ and $\frac{\mathrm{d}R}{\mathrm{d}P_R}\frac{P_R}{R} = \frac{\mathrm{d}R}{\mathrm{d}W}\frac{W}{R}\frac{\mathrm{d}W}{\mathrm{d}P_R}\frac{P_R}{W}$.

fixed production factor. Later $\bar{\eta}$ can take different values, but it is always one in this section regardless of the exact functional forms of U and f.

Gathering both household's and firm's optimality conditions, in equilibrium,

$$\left(\frac{X}{R} + \bar{\eta} - \eta\varepsilon_{XX}\right)\frac{\mathrm{d}X}{\mathrm{d}W}\frac{W}{X} = \eta - \bar{\eta} \tag{5}$$

Because the inside of the bracket on the left hand side is positive,[6] if $\eta > \bar{\eta}$, environment quality deteriorates as W increases (i.e., $\frac{\mathrm{d}X}{\mathrm{d}W}\frac{W}{X} > 0$), and vice versa. Note, however, that $\bar{\eta} = 1$ hinges on several technical conditions such as Hicks neutral technology, and a more general model can have a different threshold value.

There are several remarks we can make about this model. First, as discussed in the Introduction, if η decreases as income grows and passes through $\bar{\eta}$ from above, we observe the EKC, which we investigate in the next section.

Second, the definition of η here is only based on the observed quantity and price changes. However, the change in P_R also implies a change in income (wealth) because, while the environmental endowment is fixed at one, which is the only wealth in this model, its price P_R changes; obviously, the value of the total wealth is P_R in this economy. However, although our η mixes up both income and substitution effects in general, η exactly corresponds to the substitution effect for homothetic utility, because the income effects on R and C are exactly the same and they offset each other in ratio R/C. In this case, η reflects the substitution effect only. For DES utility, the next section shows that the change in η is mostly driven by the substitution effects under reasonable parameter sets. In this context, note that η can be negative without violating any second order optimality conditions, especially if preferences are non-homothetic.

Third, the key equation here is (3). To avoid the ambiguity due to the gap between η and the substitution effect, assume a homothetic preference. Suppose also that counterfactually $\frac{\mathrm{d}P_R}{\mathrm{d}W}\frac{W}{P_R} = 0$ (no change in relative price) so that there is no substitution effect operating. In this hypothetical case, $\frac{\mathrm{d}R}{\mathrm{d}W}\frac{W}{R} = \frac{\mathrm{d}C}{\mathrm{d}W}\frac{W}{C} = 1$; i.e., as W increases as wealth (income), both R and C increase at the same rate as

[6] This is obvious if $\eta > 0$. For $\eta < 0$, assuming C is a normal good, $\eta = \left(\frac{\mathrm{d}C}{\mathrm{d}W}\frac{W}{C} - \frac{\mathrm{d}R}{\mathrm{d}W}\frac{W}{R}\right)\frac{\mathrm{d}W}{\mathrm{d}P_R}\frac{P_R}{W}$ takes its minimum value, when the preference is such that $\frac{\mathrm{d}C}{\mathrm{d}W}\frac{W}{C} = 0$; i.e., households choose to use all additional wealth to buy back R. Since $C = Y = P_R X$, from (4b) and (4c), we find $-1 = \frac{\mathrm{d}X}{\mathrm{d}W}\frac{W}{X} = -\frac{R}{X}\frac{\mathrm{d}R}{\mathrm{d}W}\frac{W}{R}$ and $\frac{\mathrm{d}P_R}{\mathrm{d}W}\frac{W}{P_R} = 1 - \varepsilon_{XX}$, implying $\eta = -\frac{X}{R}\left(\frac{1}{1-\varepsilon_{XX}}\right)$. Hence, $\frac{X}{R} - \eta\varepsilon_{XX} = -\eta > 0$.

W increases. This is the direct income effect (the first term in (3)). However, as W increases, P_R also increases in general. Since the production cost of C decreases as the production technology improves, it is natural to think that P_R is increasing in W.[7] Unless $\eta = 0$ (Leontief utility), an increase in P_R as a price (i.e., not as the value of the environmental endowment) induces the household to shift its demand mix from R to C. This substitution effect is stronger when η is larger (the second term in (3)).[8] All in all, as W increases, the income effect leads to an improvement in R, whereas the opposite is true for the substitution effect; and, when η is large enough, the substitution effect is dominating, and vice versa. Intuitively, if C and R are close substitutes (η large enough), as the production cost of C decreases (i.e., as W increases), people want to exploit it by tilting their demand mix toward C simply because C is now cheaper; that is, they give up the quality of R to produce more C.

Fourth, non-homotheticity on the production side can also generate the EKC. To see this, assume that $0 < \eta < \bar{\eta}$ is a constant so that the right hand side of (5) is negative. Assume also that ε_{XX} is positive and variable. While $\varepsilon_{XX} > 0$ implies the violation of the second order optimality condition for individual firms, it can be justified in aggregate if, say, there is a production externality. In this case, if $\varepsilon_{XX} > \frac{1}{\eta}\left(\frac{X}{R} + \bar{\eta}\right)$, the environment deteriorates as W increases (i.e., $\frac{\mathrm{d}X}{\mathrm{d}W}\frac{W}{X} > 0$), and vice versa;[9] obviously, the EKC requires ε_{XX} to be decreasing in income. This is quite intuitive because $-1/\varepsilon_{XX}$ is the price elasticity of X, but with $\varepsilon_{XX} > 0$, an increase in P_R does not discourage firms to cut the emission. In this way, non-homotheticity in any part of the economy can be the source of the EKC at least potentially (see also Andreoni and Levinson, 2001).

3 Models with Numerical Solution

This section provides further results with some numerical examples. First, we study Models I and II, which are special cases of the model in Section 2, by applying Slutsky decomposition. Model I (DES utility) demonstrates the importance of decreasing η in generating the EKC, whereas Model

[7] Actually, in this simple model, it is easy to show $\frac{\mathrm{d}P_R}{\mathrm{d}W}\frac{W}{P_R} \geq 0$ for a general utility function, because from (4c) and (5) we have $\frac{\mathrm{d}P_R}{\mathrm{d}W}\frac{W}{P_R} = 1 - \frac{\eta-1}{\eta-(X/R+P_RX/C)/\varepsilon_{XX}} \geq 0$, which is zero only when $\eta = \infty$ and is strictly positive otherwise.

[8] The substitution effect here should be understood as the sum of Hicks income and Hicks substitution effects. But, these statements hold even if the substitution effect is defined as the Hicks substitution effect only. This is again because the Hicks income effects on C and R exactly offset each other in η for homothetic utility.

[9] Specifically, the production function per se is still homothetic (since there is only one production factor). But, the Lagrangian of the profit maximization $\mathcal{L}(C, X; \lambda) = C - P_RX + \lambda(Wf(X) - C)$ is non-homothetic (in C and X).

II (CES utility) works as a good benchmark because, for CES utility, our η and Hicksian effects are totally consistent. Model III demonstrates that our main findings hold even in the case with capital accumulation. Model IV examines generalized preference to demonstrate that it is the substitutability between C and R, rather than other curvature parameters, that determines the fate of R.

3.1 Model I: Exponential (DES) Utility

Model I studies the property of exponential (DES) utility in the simplest static formulation; as is clear below, DES utility exhibits decreasing η in income. Here, to produce output Y, a linear production function takes only pollution emission X as a production factor, and productivity W increases exogenously. All output is consumed as C. The total environmental endowment is normalized to be one, and the quality of the environment R is one minus X. This simple model can be fully analytically solved and hence offers detailed analyses such as Slutsky decomposition of demand change. In the next subsection, we apply the same analyses to (6b).

$$\begin{aligned}
\text{DES utility} \quad &: \quad U(C,R) = -e^{-\alpha_C C} - \phi e^{-\alpha_R R} && (6a) \\
\text{CES utility} \quad &: \quad U(C,R) = \frac{C^{1-1/\eta}}{1-1/\eta} + \phi \frac{R^{1-1/\eta}}{1-1/\eta} && (6b)
\end{aligned}$$

subject to

$$\begin{aligned}
\text{production} \quad &: \quad Y = WX && (7a) \\
\text{environmental endowment} \quad &: \quad 1 = X + R && (7b) \\
\text{resource constraint} \quad &: \quad Y = C && (7c)
\end{aligned}$$

For (6a) and (7), the Slutsky decompositions of R and C are:

$$\begin{aligned}
\frac{\mathrm{d}R}{\mathrm{d}W} &= \frac{-1/W}{\alpha_R + \alpha_C W} - \frac{\alpha_C R}{\alpha_R + \alpha_C W} + \frac{\alpha_C}{\alpha_R + \alpha_C W} \\
\frac{\mathrm{d}C}{\mathrm{d}W} &= \frac{1}{\alpha_R + \alpha_C W} - \frac{\alpha_R R}{\alpha_R + \alpha_C W} + \frac{\alpha_R}{\alpha_R + \alpha_C W}
\end{aligned}$$

For both of these decompositions, the first, second and third terms show Hicks substitution, Hicks income and the direct income effects, respectively (see Table 1). The Slutsky decomposition for R shows the positive income effect and the negative two Hicks effects, and these effects all shrink in absolute term as W increases.[10]

To obtain the intuition, note first that (a) the price of the environment is equal to technology level, $P_R = W$, and (b) the value of total wealth is also W. These two are true in Model II as well; the former is because of our production function and the latter is because the only wealth in this economy is the environmental endowment, which is 1 and its price is W. The direct income effect is the effect of a change in W as wealth. This is positive because, as wealth increases, demand for R increases, simply because R is a normal good. However, P_R increases as the marginal product of pollution increases. Hence, due to negative Hicks effects, the demand for R is suppressed because it is now more expensive. Note that this substitution effect (the sum of two Hicks effects) is due to a change in W as a (shadow) price. The direct income effect and two Hicks effects offset each other, which itself is true even for CES utility (see Table 2 below). Hence, whether $\mathrm{d}X/\mathrm{d}W = -\mathrm{d}R/\mathrm{d}W$ changes its sign from positive to negative or not depends on the relative strength of these two effects.

In this respect, Figure 1 shows the key model behaviors with DES utility, where we set $\alpha_C = 1.0$, $\alpha_R = 1.0$ and $\phi = 0.5$. The upper left panel plots X, which we regard as environmental degradation, for each technology level W. As the EKC hypothesis postulates, when income is low (which is represented by low W), the economy accepts a lower environment quality. However, once W exceeds a certain level, the economy starts cutting X. The upper right panel shows that such a turning point coincides with η being 1, at which $\mathrm{d}X/\mathrm{d}W$ changes its sign. The lower right panel shows the Slutsky decomposition of $\mathrm{d}R/\mathrm{d}W$. It is now obvious that the decrease in two Hicks substitution effects in absolute term is fast enough relative to that of the direct income effect. That is, as discussed in Section 2, the substitution effect is first stronger but later weaker. The lower left panel shows the locus of the equilibrium. An increase in W (both as the shadow price and as wealth) is represented by the clockwise rotation of the budget constraint (straight lines). The optimum points are the tangency points between the indifference curves and the budget constraints for different W. As is visually clear, non-homotheticity

[10] Hicks income effect captures the effect of the change in real income because of the change in a general price level due to a change in a price. Hicks substitution effect is the effect of the change in a relative price after adjusting for the Hicks income effect. The direct income effect simply means the effect of the change in income, keeping relative prices unchanged.

of the preference is the key to generating the inverted U-curve.

Finally, asymptotically (i.e., for very large W), R approaches its upper limit 1 in this model. In a sense, the production of C is squeezed by the conservation of R. That is, C is increasing without limit but increasingly slowly.[11] Also, technically, when W is too small, the lower bound of R is binding; $R = 0$. Figure 1 plots the results only for W large enough for which the model has an interior solution.

[Figure 1: For Exponential (DES) Utility around here]

[Table 1: For Exponential (DES) Utility around here]

3.2 Model II: Power (CES) Utility

In comparison, Figures 2 and 3 show the results of the essentially same exercises as in Section 3.1 for power (CES) utility with $\eta = 3.0$ and 0.7, respectively; see (6b) and (7). Here, the two right panels show the elasticities, rather than the derivatives.[12] Different to DES utility, they do not generate an inverted U-curve. Rather, they generate a monotone improvement or deterioration of R. The Slutsky decompositions are:

$$\frac{\mathrm{d}R}{\mathrm{d}W}\frac{W}{R} = -\eta(1-R) - R + 1 \tag{8a}$$

$$\frac{\mathrm{d}C}{\mathrm{d}W}\frac{W}{C} = \eta R - R + 1 \tag{8b}$$

Again, the first, second and third terms show Hicks substitution, Hicks income and the direct income effect, respectively. The difference of these two expressions shows that, with CES utility, η captures Hicks substitution effect only; see (2). With CES utility income effect is 1 for both C and R, meaning that C and R increase at the same rate as income. For $\eta > 1$, the sum of two Hick effects decreases from -1 to $-\eta$, meaning that it is always dominating the income effect. Similarly, for $\eta < 1$, it increases from -1 to $-\eta$, implying that it is always dominated by the income effect. In this way, $\frac{\mathrm{d}R}{\mathrm{d}W}\frac{W}{R}$ is always negative (positive) for $\eta > 1$ ($\eta < 1$). From Figures 2 and 3, now it is clear that, as its name

[11] Note that, $\mathrm{d}C/\mathrm{d}W \to 0$ does not necessarily imply that there is a saturation point of C. The situation is somewhat like a logarithmic function; for $y = \ln x$, $\mathrm{d}y/\mathrm{d}x \to 0$ as $x \to \infty$, but y is unbounded.

[12] Because power utility functions show quasi-multiplicative separability, it is more natural to show the elasticities. On the contrary, since exponential utility functions show quasi-additive separability, it is more straightforward to show derivatives. We discuss this further in Section 4.3. The key reference for this is Behrens and Murata (2007); see also Barde (2010) for further discussions.

suggests, the change in the substitutability is not enough to overturn the direction of R. This is in sharp contrast with DES utility.

Setting aside the EKC, Model II shows that the key parameter is η in determining the sign of $\frac{\mathrm{d}R}{\mathrm{d}W}\frac{W}{R}$ at each income level. If utility is flexible (high η), R is a close substitute to C. In this case, the substitution effect is strong (i.e., a small change in the relative price induces a large shift in R/C) and, hence, it is dominating, which discourages the demand for R as productivity W increases. If the preference is inflexible (low η), as W increases, people do not want to switch from R to C very much, even though R becomes more expensive. In this case, the income effect is dominating.

[Figure 2: Power (CES) Utility with $\eta = 3.0$ around here]

[Figure 3: Power (CES) Utility with $\eta = 0.7$ around here]

[Table 2: For Power (CES) Utility around here]

3.3 Model III: Exponential Utility with Capital and Pollution Stocks

We now consider a model with capital and pollution stocks with two endogenous state variables $\{K_{t-1}, X_{t-1}\}$. Since we discretize the model for computation,[13] we formulate the model in discrete time from the beginning. Here we measure the income level by accumulated capital stock K_{t-1}.

$$
\begin{aligned}
V(K_t, X_t) &= \max \sum_{t=0}^{\infty} \beta^t \left\{ -e^{-\alpha_C C_t} - \phi e^{-\alpha_R R_t} \right\} && \text{(9a)} \\
\text{s.t.} \quad Y_t &= A K_t^{\alpha} Z_t^{1-\alpha} && \text{(9b)} \\
1 &= X_t + R_t && \text{(9c)} \\
K_t - K_{t-1} &= Y_t - C_t - \delta_K K_{t-1} && \text{(9d)} \\
X_t - X_{t-1} &= Z_t - \delta_X X_{t-1} && \text{(9e)}
\end{aligned}
$$

In this model, we assume that production emits flow pollution Z_t (9b), and pollution stock X_t is the accumulation of flow pollution emission Z_t, where, if there is no pollution emission, X_t decreases at rate δ_X, because of the assimilative capacity of the environment (9e). Pollution stock X_t deteriorates the

[13] We implement the standard Euler equation iteration (see Appendix for details) for Model III.
In this section, all dynamic models (Models III and IV) have the steady state at which economic growth stops. One may tempted to seek a long-run balanced growth path instead of a short-run dynamics around the steady state, but given the non-monotonic nature of the EKC it is hard to construct a model with a balanced growth path.

environmental quality R_t (9c). The parameter values are; $\delta_K = 0.1$ (10% annual capital depreciation rate), $\alpha = 0.5$ (capital share in production is one half), $\beta = 1/(1+0.06)$ (6% annual risk-free rate), $\alpha_C = \alpha_R = 1$ and $\phi = 1$. We experiment with several values for δ_X.

Figure 4 shows results for $\delta_X = 1$, where X_{t-1} is not a state variable anymore and $X_t = Z_t$ (flow pollutant). The upper two panels show that the shape of the EKC is similar to Model I. Since the peak of X_t appears to the left of the steady state, starting from a low level of capital and output, we observe the inverted U-shape of X_t. Also, as predicted, η is decreasing and shadow price P_R of the environment is increasing in K_t (lower left panel). One important difference is that the threshold value of η is now around 2; we discuss this further in Section 4. Under this parameter assumption, starting with $K_0 = 0.49$, it takes 10 to 15 years for Y_t (as it does for K_t) to arrive at the steady state, while the peak of environmental degradation X_t is reached in the third year (lower right panel). Though arriving at the peak in three years may sound a bit quick, it is quite easy to delay the peak of X_t, for example, by setting ϕ higher than 1. Note that we do not target any special peak year in this paper, because the exact shape of the EKC is different for different pollutants as discussed in the Introduction. All in all, the qualitative implication is the same as that of Model I; as η decreases the speed of the environmental deterioration decreases and at a certain point it becomes negative.

[Figure 4: With Capital Stock around here]

Figure 5 shows the equilibrium pollution emission for $\delta_X < 1$ (stock pollutant).[14] There are several observations worth mentioning. First, not surprisingly, when δ_X is close to 1, the model behavior resembles that of the flow pollutant case (Figure 4). For $\delta_X = 0.9$, pollution emission Z_t is almost unaffected by pollution stock X_{t-1} (the surface is flat along X-axis in the upper left panel). Second, as δ_X decreases, the inverted U-shape becomes weaker, and for δ_X low enough it disappears for a reasonable range of W_t. This is again not surprising, because, at the limit $\delta_X \to 0$, the behavior of the pollution stock is one sided.[15] That is, X can increase but cannot decrease; i.e., the possibility of the inverted U-shape is physically eliminated. Third, output is low when X_{t-1} is high, because there is little leeway to emit additional Z_t. In a sense, having high stock pollution is similar to having a high debt level. Fourth, capital and output in the steady state are strongly affected by δ_X. Intuitively, in

[14] The vertical lines in the three panels show the steady state, and the lines on the x, y-plane show the contour sets.

[15] This is an irreversibility case. For $\delta_X < 1$, if in addition there is uncertainty in the model, the real option kicks in.

the steady state, the economy can emit pollution at an amount equal to that which the environment can assimilate; $Z = \delta_X X$. However, the optimal size of X is limited by preferences. Hence, as δ_X decreases toward zero, allowable pollution emission Z for production decreases. In most parameter ranges, this level effect of the depreciation rate is quite strong; under our parameter assumption, moving from $\delta_X = 0.9$ to $\delta_X = 0.3$, the steady state output becomes one third.

[Figure 5: With Pollution Stock around here]

3.4 Model IV: GIE Preference

To complement the models above, we examine the generalized isoelastic (GIE) preference utility specification pioneered by Epstein and Zin (1989), Svensson (1989) and Weil (1990), which is applied to environmental issues by Smith and Son (2005). With GIE preferences we can separate the following three economic concepts; (i) η is elasticity of substitution between R and C, (ii) θ is elasticity of intertemporal substitution and (iii) γ is the coefficient of relative risk aversion.[16] In this version of the model, since period utility is homothetic as in Model II, we do not observe the EKC. Rather, our intention is to demonstrate that it is η, but not θ or γ, that determines the fate of R. To make γ and θ meaningful, the model is dynamic and stochastic.

$$V_t\left(W_t, K_{t-1}\right) = \max\left\{U\left(C_t, R_t\right)^{1-1/\theta} + \beta E_t\left[V_{t+1}\left(W_{t+1}, K_t\right)^{1-\gamma}\right]^{\frac{1-1/\theta}{1-\gamma}}\right\}^{\frac{1}{1-1/\theta}} \tag{10a}$$

$$\text{where} \qquad U\left(C_t, R_t\right) = \left(C_t^{1-1/\eta} + \phi R_t^{1-1/\eta}\right)^{\frac{1}{1-1/\eta}} \tag{10b}$$

subject to

$$Y_t = W_t K_{t-1}^{\alpha} Z_t^{1-\alpha} \tag{11a}$$

$$1 = X_t + R_t \tag{11b}$$

$$K_t - K_{t-1} = Y_t - C_t - \delta_K K_{t-1} \tag{11c}$$

$$X_t - X_{t-1} = Z_t - \delta_X X_{t-1} \tag{11d}$$

$$\ln W_t = (1-\rho_W)\ln A + \rho_W \ln W_{t-1} + \xi_t \text{ where } \xi_t \sim N\left(0, \sigma_\xi\right) \tag{11e}$$

[16] Note that, if $\theta = 1/\gamma$, GIE reduces to von Neumann-Morgenstern's expected utility (vNM). Thus, even under vNM preference, η can be set independently from $\theta = 1/\gamma$.

Most of the parameter values are the same as before; $A = 1$ (steady state technology level), $\beta = 1/(1+0.06)$, $\alpha = 0.5$, $\delta_K = 0.1$, and $\phi = 1$. For simplicity, we assume the flow pollutant; $\delta_X = 1$ (hence, X_{t-1} is not a state variable). Technology shock is fairly persistent ($\rho_W = 0.6$) but very volatile ($\sigma_\xi = 0.2$).[17] To demonstrate the importance of η we use two values for it; $\eta = 3.0$ and 0.5. We set $\theta = 2.0$ and $\gamma = 4.0$ (see Barro, 2009) as the baseline case, but we also check the result sensitivity to these parameters.

The bold lines in Figure 6 show that, measuring the income level by the capital accumulation K_{t-1}, X_t is decreasing for η low enough, and vice versa. Figure 6 also shows the results of different values of θ and γ, and it demonstrates that they have no qualitative effects. In this formulation, their effects are small even quantitatively,[18] because of the assumption $\delta_X = 1$. For γ, if $\delta_X < 1$, today's choice of Z_t depends on not only today but also future utility through the accumulation of the pollution stock X_t, and the optimal choice of Z_t is strongly affected by uncertainty in the future. Similarly, we know from the standard saving theory that, with lower θ, people strongly prefer a smooth consumption path. If $\delta_X < 1$, the household can use X_t as a (dis)saving tool like K_t, but such an effect is absent for flow pollutants. In summary, although θ and γ can have much stronger quantitative effects depending on the value of δ_X, only η changes the model behavior qualitatively.

[Figure 6: With GIE preference around here]

4 Discussions

4.1 Types of Pollutants and the EKC

As Carson (2010) and many other empirical studies report that the EKC is not a common observation for all types of pollutants (e.g., CO_2) and environmental goods and services (e.g., biodiversity). It is also the case that the exact shape and turning point of the EKC differ among

[17] This is very large compared to the convention in business cycle literature, in which often $\sigma_\xi = 0.01$ or lower is chosen (see Cooley and Prescott, 1995, for example). Also, $\gamma = 24$ is also extremely large. For example, Mehra and Prescott (1985) suggest in their seminal paper that a reasonable value for γ is 10 or less. These extreme choices are because otherwise it is hard to see the effect of changing γ visually.

[18] Note that the lines should not (and actually do not) pass through the *non-stochastic* steady state. However, since the effect of uncertainty is very small in this formulation, visually it is almost impossible to see that they do not pass through it.

pollutants as well as geographical and administrative locations. This section discusses how our results apply to various types of pollutant and the resulting optimal policy response of government.

Figure 7 shows the effect of changing parameters in model III (flow pollutant with DES utility). First of all, in this framework, threshold $\bar{\eta}$ is mostly affected by δ_K but not very much by other parameters, where $\bar{\eta}$ is such that the environment degrades if η is above it and vice versa. Indeed, some analytical exercises suggest that $\bar{\eta}$ is mainly affected by the share of investment in output. For example, in Model III, if the government (or firm) keeps K at a certain level $\bar{K}$ (i.e., it does not optimize with respect K but it keeps investment equal to depreciation to maintain the level of $\bar{K}$), the model becomes effectively static and it reduces to a variant of the model in Section 2 where the market clearing condition is $Y = C + \delta_K \bar{K}$. In this case, $\bar{\eta} = Y/C = 1 + \delta_K \bar{K}/C > 1$. Note that $\bar{\eta}$ is not necessarily an increasing function of δ_K because K is decreasing in δ_K; indeed, as shown in Figure 7, $\bar{\eta}$ is decreasing in δ_K near the benchmark parameter assumption.[19] Figure 7 shows that, as δ_X decreases from 1.0 to 0.4, $\bar{\eta}$ decreases from near 2.0 to around 1.2. Although $\bar{\eta}$ does not capture all the effects of a parameter change, together with Figure 5, we find that it is less likely to observe the EKC for a pollutant with a low δ_X.

For preference parameters α_R, α_C and ϕ, they mainly affect the level of emission, while they affect $\bar{\eta}$ very little. If society puts more weight on the environment (higher α_R, lower α_C and/or higher ϕ), the level of X tends to be higher for given capital accumulation. This is quite intuitive and not surprising. For lower ρ, as the productivity of X increases (because K increases in production), the government (or society) chooses higher X as discussed above. However, this result can be overturned for a stock pollutant. If δ_X is low enough, the quality of the environment becomes an asset and hence X can be decreasing in ρ. Finally, the effect of capital share α in production is complicated near the benchmark parameter set. As α increases, the line of X rotates anti-clockwise. However, for α large enough (say, larger than 0.65), as α increases, the level of X decreases for the whole range of K. This is again intuitive; $1 - \alpha$ is the share of a pollutant and, if it is small, the firm needs to emit less X.

[Figure 7: Sensitivity Analysis around here]

In sum, preference parameters and capital share in production mainly affect the level of emission

[19] Note that, due to similar reasoning, $\bar{\eta}$ is higher for lower ρ, because K in the steady state is decreasing in ρ. However, the effect of ρ tends to be smaller than that of δ_X for reasonable parameter ranges.

but not the threshold $\bar{\eta}$. The threshold becomes lower (i.e., tighter), if the depreciation of capital increases and/or if that of pollutant decreases. Of these two, the latter seems to produce the stronger effect. Indeed, even under DES utility, if the depreciation of a pollutant is low enough, it seems that the EKC disappears for a reasonable range of W, as shown in Model III above.

Setting aside the *level* of emission, the *direction* of the environmental degradation mainly depends on the depreciation rate of a pollutant.[20] More specifically, a pollutant with lower δ_X is less likely to decrease in the future. For example, we conjecture that CO_2 emission is going to keep increasing as discussed in the Introduction, because, as documented in Stern (2007), its depreciation rate δ_X is quite low (i.e., the nature cannot reduce it quickly). Certainly, since we do not explicitly model any catastrophic disasters that could happen for extremely high CO_2 levels, and international coordination failure (we assume that the price of the emission is efficiently imposed on firms), this prediction might be premature. However, understanding these limitations, our model still suggests that it is likely that the government *optimally* allows CO_2 to increase in the future, given its low depreciation rate. The choice of adverb *optimally* may sound wrong, but this is in a similar vein to the non-existence of involuntary unemployment. Here, *optimally* can be understood to mean that there is an incentive in the economy to allow CO_2 to keep increasing.

4.2 Antecedent Literature

Having derived our various model results we now place these findings within the context of the existing theoretical literature on the EKC. In general, we find that the existing papers about the EKC can be classified into two groups. The first group is initiated by Stokey (1998), which we call "constraint driven models". This group includes paper such as Chimeli and Braden (2002), Lieb (2004), Brock and Taylor (2005, 2010), Hartman and Kwon (2005), Smulders (2006) and Smulders et al. (2011), to name a few. The most important feature of the constraint driven models is that an exogenous constraint is binding only either before or after the peak of the environmental degradation (the equilibrium is a corner solution only before or after the peak). Perhaps, Smulders (2006) makes this point most clearly, and we basically follow his explanation here. Suppose that an economic agent has CES utility with $\eta < 1$, which implies that, as demonstrated in Model II, people demand more

[20] Note that the depreciation of capital is irrelevant to the characteristics of each pollutant.

R as they become richer. In this case, if there is no exogenously given constraint, R monotonically improves as the technology improves, which captures the decreasing right tail of the EKC. However, on top of $\eta < 1$, he assumes a technical constraint, which states that, when the technology level is too low, the economy does not have enough ability to exploit environmental quality fully. Hence, until a certain technology level is attained, the maximum possible level of destroying R is limited as an increasing function of the technology level. This binding technological constraint forces R to follow a gradual increasing path before the peak of the EKC, which generates the increasing left tail of the EKC.

Alternatively, it can be assumed that $\eta > 1$ (R deteriorates freely until W arrives at a certain point) and the regulator imposes some emission regulation once the economy reaches a certain level of technology. For the constraint driven approach, we do not need to assume non-homothetic preferences or production functions. One prominent feature of this type of model is the inverted V-shape, rather than the inverted U-shape, at the point where a constraint becomes binding, it exhibits a sharp peak. This type of model is also a powerful candidates to explain the EKC. First, though the empirical data shows an inverted U-shape, in our opinion, given the noisy nature of the data used in many empirical studies, the theoretically predicted sharp pointed inverted V-shape is not a caveat. Second, the assumption imposed in these models is often quite convincing. Smulders (2006, p.12), for example, argues that *"Loggers in a poor village simply lack the technical means to cut all trees in the rain forest surrounding the village. Prehistoric man could hunt many deer, but lacked the capacity to destroy the ozone layer."*

In the second group, which we call interior solution models there are only two types of model; one is Andreoni and Levinson (2001) (and Egli and Steger (2007) who provide a dynamic modelling extension), and the other is ours.[21] As conceptually predicted by Lopez (1994), non-homotheticity is the key for interior solution models. Since these models do not rely on an exogenously imposed constraint, they are theoretically preferable in the sense that they reveal the economic mechanism that generates the EKC internally. In the case of Andreoni and Levinson (2001) the issue reduces to whether the abatement technology exhibits increasing returns to scale (IRS) or not, while, in our case, it reduces to whether η is decreasing fast enough or not. Perhaps, some types of pollutants may follow

[21] See the Appendix for how non-homotheticity works in Andreoni and Levinson (2001)

a constraint driven model, while others are better captured by an interior solution model. It is beyond the scope of this paper to discuss which is empirically better for which pollutant.

4.3 Implication from Competitive Limit

This paper is partly motivated by the literature of competitive limit. The idea of it is that, under exponential (DES) utility family, as the number of varieties available to consumers increases, monopolistic competition approaches perfect competition. That is, as the number of varieties increases the demand elasticity increases (to positive infinity at the limit). This is perhaps intuitively convincing; for example, the firm that produces blue widgets has relatively strong monopolistic power if there is only one competitor, the red-widget producer, but if, say, a purple-widgets producer enters into the market, the blue firm's monopolistic power may be undermined. However, under power (CES) utility, this decrease in monopolistic power does not take place. The monopolistic power is constant regardless of the number of competitors. One property of the exponential utility is that, as the level of consumption of good A decreases, the price elasticity of good A increases. At the competitive limit, since, without income growth, as the number of varieties increases, money spent for each good decreases, and hence the demand for each type of goods becomes more elastic. In our case, as W increases, consumption of C and R increases, and hence the demand for each of C and R becomes less elastic.

In the growth literature, researchers almost always assume power utility, perhaps because of its tractability. However, we would like to emphasize that power utility is not necessarily empirically more plausible than other. Setting aside the analytical tractability, the choice between them is totally an empirical issue. If exponential utility is plausible to a certain degree, then the EKC is equally plausible because the only requirement for it to exist is decreasing η in income. As demonstrated above, the models do not require any other specific assumptions other than that.

5 Conclusion

This paper demonstrates that, at each income level, environmental degradation is decreasing as people become richer if η is small enough, and vice versa. Here, the key parameter η is the elasticity of substitution between consumption and the service flow from the environment. Intuitively, η shows how easily the quality of the environment can be substituted by consumption in the household's preferences. As shown above, as people become richer, there are mainly two effects; (i) given a price of the environment, people demand more environment (income effect); and (ii) the environment becomes more expensive, which induces people to accept a lower quality of the environment (substitution effect). The latter is stronger when η is higher (i.e., people's preference is more flexible). In this regard, we can understand the EKC such that the substitution effect is dominating when the economy is poor, but the income effect overwhelms the substitution effect when people are richer. Indeed, this paper demonstrates that, exponential (DES) utility, with which η is decreasing in income, generates the EKC in a wide class of model formulations.

There are, however, other economic models that generate the EKC. We find that such models can be classified into two groups. One group employs an exogenous constraint which is binding only before or after the peak of environmental degradation. The other group is such that non-homotheticity in a part of the economy changes the response of the economy as income increases. Our model belongs to the latter. Specifically, our non-homothetic preference (exponential utility) exhibits decreasing η, which is the main driver of the emergence of the EKC. Although this paper does not provide detailed analyses, non-homotheticity in any other part of the economy can, at least potentially, generate the EKC as well. However, the importance of this paper lies in the fact that the only requirement to have the EKC is that preferences are such that η is decreasing in income.

Apart from the EKC, in terms of policy implication, to predict the long-run fate of the environment, we need to know whether η is low enough or not relative to its threshold value for a large income level. The exact value of η and its threshold depend on economy wide parameters such as the discount rate as well as the characteristics of each pollutant. Not surprisingly, our study suggests that a pollutant with a low depreciation rate, such as CO_2, tends to have a low (tight) threshold. This implies that it is likely that society (or the government) will allow such a pollutant to keep increasing in the long-run (or till a catastrophic phenomenon takes place).

In this relation, the most important empirical implication of this paper is that, for each pollutant, whether η is decreasing fast enough or not is the key to determining if its emission is going to increase or not. We argue that the non-monotonic behaviour of EKC is theoretically interesting but practically it is much more important to know whether the environmental quality is improving or deteriorating as the economy grows. It is interesting to test whether η is decreasing fast or not, because, if so, economic success is consistent with the environmental conservation.

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A Appendix

A.1 Non-Homotheticity in Andreoni and Levinson (2001) and N- & M-Shaped EKCs

This section reviews Andreoni and Levinson's (2001) model *as a model building procedure.* Their model looks as follows.

$$\begin{aligned}\max U &= C - X \\ \text{s.t. } X &= C - A \\ A &= C^{\alpha}E^{\beta} \\ M &= C + E\end{aligned}$$

where utility U is increasing in consumption C and decreasing in pollution X and their elasticity of substitution is infinite (linear utility). Pollution X is proportionally increasing in C but can be reduced by employing abatement technology A, which is an increasing function of C and abatement effort E. The total resource M available, which is increasing at an exogenous rate, can be used either for consumption or abatement effort. In this model, if the abatement technology exhibits increasing returns to scale (i.e., $\alpha + \beta > 1$), the optimal X shows an inverted U-shape.

After substituting out some variables, it is easy to reformulate the model without X

$$\begin{aligned}\max U &= C^{\alpha}E^{\beta} \\ \text{s.t. } C &= Y = M - E\end{aligned} \tag{12}$$

where we can reinterpret E as environmental quality and $-E$ as pollution emission which contributes to the production of output Y. Here, obviously Cobb-Douglas utility and linear production functions both show homotheticity. One of the good points of this model is, since it has homothetic functions only, it is easy to generate the balanced growth path. Indeed, the solution to this optimization problem shows C/M and E/M are constants.

$$C = \frac{\alpha}{\alpha+\beta}M \text{ and } E = \frac{\beta}{\alpha+\beta}M$$

Non-homotheticity in this model does not enter into the core part of the model (12). Instead, it appears in X, which can be obtained after solving the core part of the model.

$$X = C - C^{\alpha}E^{\beta} = \alpha\frac{M}{\alpha+\beta} - \alpha^{\alpha}\beta^{\beta}\left(\frac{M}{\alpha+\beta}\right)^{\alpha+\beta} \tag{13}$$

For $\alpha + \beta > 1$ and $\beta < 1$, X shows an inverted U-curve in M. Up to here, we reviewed Andreoni and Levinson (2001).

Next, motivated by the fact that Andreoni and Levinson (2001) can be written as a balanced growth model, we re-define (13). Any re-definition is fine as long as it exhibits an inverted U-shape. A straightforward example is a quadratic function.

$$X = M_+ - \left(E - \frac{\beta}{\alpha+\beta}M_*\right)^2 = M_+ - \left(\frac{\beta}{\alpha+\beta}\right)^2 (M - M_*)^2 \tag{14}$$

where M_+ is a large positive number and M_* is the threshold income level; X is increasing in M for $M < M_*$, but X is decreasing in M for $M > M_*$. If we substitute $E = (M_+ - X)^{1/2} + \frac{\beta}{\alpha+\beta}M_*$ back into the original formulation, we obtain

$$\begin{aligned} \max U &= C^\alpha \left((M_+ - X)^{1/2} + \frac{\beta}{\alpha+\beta}M_*\right)^\beta \\ \text{s.t. } C &= Y = M - \left((M_+ - X)^{1/2} + \frac{\beta}{\alpha+\beta}M_*\right) \end{aligned}$$

This model shows an inverted U-curve by construction, and can be solved first for intermediate variable E and then solve for X as a function of E. If we want to have N-shaped environmental degradation, we should have a proper cubic polynomial instead of (14), and, by having X as a fourth order polynomial of E, we can even construct even M-shaped curve.

In sum, this way of constructing a model can be summarized as follows. First, write a model with homothetic preference and production; they are homothetic in "transformed" environmental quality E. Without non-homotheticity, it is relatively easy to find the balanced growth path. Second, define the "true" environmental degradation X as a function of E so that X shows an inverted U-shape. Finally, substitute E back into the original model, which shows non-homotheticity in X (but it shows homotheticity in E). Unlike Andreoni and Levinson (2001), the economic intuition of the models constructed in this way may be vague in general. However, if the main interest is not revealing the mechanism that generates the EKC but investigating the consequence of the EKC, this way of model building can be a good device, because it can generate EKC and the balanced growth path in an easy way (indeed, an analytical solution is frequently available).

A.2 Computational Details for Model III

◊ Lagrangian:

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \left\{ \begin{array}{c} -e^{-\alpha_C C_t} - \phi e^{-\alpha_R (1-X_t)} \\ +\lambda_t \left\{ A K_{t-1}^{\alpha} Z_t^{1-\alpha} - C_t + (1-\delta_K) K_{t-1} - K_t \right\} \\ +\mu_t \left\{ X_t - Z_t - (1-\delta_X) X_{t-1} \right\} \end{array} \right\}$$

$\diamondsuit$ Equilibrium Equations (FOCs and constraints):

$$\begin{aligned}
\partial C &: \lambda_t = \alpha_C e^{-\alpha_C C_t} \\
\partial K_t &: \lambda_t = \beta \lambda_{t+1} \left\{ \alpha A \left(Z_{t+1}/K_t \right)^{1-\alpha} + (1-\delta_K) \right\} \\
\partial Z_t &: \mu_t = \lambda_t (1-\alpha) A \left(Z_t/K_{t-1} \right)^{-\alpha} \\
\partial X_t &: \mu_t = \mu_{t+1} \beta (1-\delta_X) + \alpha_R \phi e^{-\alpha_R (1-X_t)} \\
lomK &: K_t = A K_{t-1}^{\alpha} Z_t^{1-\alpha} - C_t + (1-\delta_K) K_{t-1} \\
lomX &: X_t = Z_t + (1-\delta_X) X_{t-1}
\end{aligned}$$

where lom stands for the law of motion.

$\diamondsuit$ Non-Stochastic Steady State:

Defining $\rho = \frac{1-\beta}{\beta}$,

$$\begin{aligned}
\rho + \delta_K &= \alpha A (Z/K)^{1-\alpha} \\
\mu \beta (\rho + \delta_X) &= \alpha_R \phi e^{-\alpha_R (1-X)} \\
\frac{\mu}{\lambda} &= (1-\alpha) A (Z/K)^{-\alpha} \\
A K^{\alpha} Z^{1-\alpha} - C &= \delta_K K \\
Z &= \delta_X X
\end{aligned}$$

Hence, eliminating Lagrange multipliers, $(1-\alpha) A (Z/K)^{-\alpha} (\rho + \delta_X) = \frac{\alpha_R \phi e^{-\alpha_R (1-X)}}{\alpha_C e^{-\alpha_C C}}$. Taking the log of this,

$$\begin{aligned}
K &= \frac{\ln\left(\beta \frac{\mu}{\lambda} \frac{\alpha_C}{\alpha_R \phi} (\rho + \delta_X) \right) + \alpha_R}{\alpha_C \frac{C}{K} + \alpha_R \frac{X}{K}} \\
& \text{where } \frac{Z}{K} = \left(\frac{\rho + \delta_K}{\alpha A} \right)^{\frac{1}{1-\alpha}}, \ \frac{C}{K} = A \left(\frac{Z}{K} \right)^{1-\alpha} - \delta_K, \ \frac{X}{K} = \frac{1}{\delta_X} \frac{Z}{K}
\end{aligned}$$

$\diamondsuit$ Euler Equation Iteration:

Define node points on the state space (K_{t-1}, X_{t-1}). Suppose that we preliminarily know optimal C_t and Z_t as functions of states K_{t-1} and X_{t-1} from the previous iteration step; that is, we have

$C_t = C[K_{t-1}, X_{t-1}]$ and $Z_t = Z[K_{t-1}, X_{t-1}]$. Then, we sequentially obtain

$$\begin{aligned}
K_t &= AK_{t-1}^{\alpha} Z_t^{1-\alpha} - C[K_{t-1}, X_{t-1}] + (1-\delta_K) K_{t-1} \\
X_t &= Z[K_{t-1}, X_{t-1}] + (1-\delta_X) X_{t-1} \\
C_{t+1} &= C[K_t, X_t] \qquad \text{(interpolate to adjust node points)} \\
Z_{t+1} &= Z[K_t, X_t] \qquad \text{(interpolate to adjust node points)} \\
\lambda_{t+1} &= \alpha_C e^{-\alpha_C C_{t+1}} \\
\mu_{t+1} &= \lambda_{t+1} (1-\alpha) A (Z_{t+1}/K_t)^{-\alpha} \\
\lambda_t^{new} &= \beta \lambda_{t+1} \left\{ \alpha A (Z_{t+1}/K_t)^{1-\alpha} + (1-\delta_K) \right\} \\
\mu_t^{new} &= \beta \mu_{t+1} (1-\delta_X) + \alpha_R \phi e^{-\alpha_R (1-X_t)} \\
C_t^{new} &= \frac{\ln(\lambda_t^{new}/\alpha_C)}{-\alpha_C} \\
Z_t^{new} &= \left(\frac{\mu_t^{new}}{\lambda_t (1-\alpha) A} \right)^{\frac{-1}{\alpha}} K_{t-1}
\end{aligned}$$

Iterate this until $C_t^{new} = C_t$ and $Z_t^{new} = Z_t$ at each node.

A.3 Computational Details for Model IV

◊ Non-Stochastic Steady State:

Since without any stochasticity, GIE preference reduces to vNM preference, it is straightforward, though tedious, to find the non-stochastic steady state. For the value function iteration, the steady state values are not necessary but often useful in, say, determining node points. Defining $\rho = \frac{1-\beta}{\beta}$, we find the ratios.

$$\begin{aligned}
\frac{Y}{K} &= \frac{\rho + \delta_K}{\alpha} \quad , \quad \frac{C}{K} = \frac{Y}{K} - \delta_K \quad , \quad \frac{Z}{K} = \left(\frac{\rho + \delta_K}{\alpha A} \right)^{\frac{1}{1-\alpha}} \quad , \quad \frac{X}{K} = \frac{1}{\delta_X} \frac{Z}{K} \\
\frac{\mu}{\lambda} &= (1-\alpha) \frac{Y/K}{Z/K} = (1-\alpha) A^{\frac{1}{1-\alpha}} \left(\frac{\rho + \delta_K}{\alpha} \right)^{\frac{-\alpha}{1-\alpha}}
\end{aligned}$$

With these ratios on hand, we can find the steady state value of K.

$$\frac{1}{K} = \left(\frac{\mu}{\lambda} \frac{\rho + \delta_X}{\phi/\beta} \right)^{-\eta} \frac{C}{K} + \frac{X}{K}$$

◊ Value Function Iteration:

Substitute out some variables to obtain

$$
\begin{aligned}
F_t\left(W_t, K_{t-1}, X_{t-1}\right) &= \max_{K_t, X_t}\left\{U_t\left(W_t, K_{t-1}, X_{t-1}; K_t, X_t\right)+\beta E_t\left[F_{t+1}\left(W_{t+1}, K_t, X_t\right)^{1-\gamma}\right]^{\frac{1-1/\theta}{1-\gamma}}\right\}^{\frac{1}{1-1/\theta}} \\
U_t\left(W_t, K_{t-1}, X_{t-1}; K_t, X_t\right) &= \left(\begin{array}{c}\left[W_t K_{t-1}^{\alpha}\left(\left(1-\delta_X\right) X_{t-1}-X_t\right)^{1-\alpha}+\left(1-\delta_K\right) K_{t-1}-K_t\right]^{1-1/\eta} \\ +\phi\left(1-X_t\right)^{1-1/\eta}\end{array}\right)^{\frac{1-1/\theta}{1-1/\eta}}
\end{aligned}
$$

Suppose that we have the functional form of F_{t+1} from the previous Iteration step; then we can maximize the RHS with respect to K_t and X_t to obtain the functional form of F_t. Replacing F_{t+1} with F_t, and repeat this until F_t converges.

B Tables and Figures

Table 1: **Summary Table for Exponential (DES) Utility**

	analytical expression	for low W	for high W	$W \to \infty$
elas of subs η	$1 - \frac{\alpha_C C-1}{W(\alpha_R+\alpha_C W)}\left(\frac{W}{R}+\frac{W}{1-R}\right)$	>1.0	<1.0	0.0
$\mathrm{d}R/\mathrm{d}W$	$\frac{\alpha_C C-1}{R(\alpha_R+\alpha_C W)}$	-ve	+ve	0.0
Hicks Sub	$\frac{-1/W}{\alpha_R+\alpha_C W} < 0$	-ve		0.0
Hicks Income	$-\frac{\alpha_C \alpha_R}{\alpha_R+\alpha_C W}\frac{R}{\alpha_R} < 0$	-ve		0.0
Direct Income	$\frac{\alpha_C}{\alpha_R+\alpha_C W} > 0$	+ve		0.0
R	$\frac{\ln \phi \alpha_R / W \alpha_C + \alpha_C W}{\alpha_R+\alpha_C W}$	decreasing	increasing	1.0

Note: Because $\frac{W}{R} + \frac{W}{1-R} > 0$, the above analytical expressions show $\eta > 1 \Leftrightarrow \mathrm{d}R/\mathrm{d}W < 0$.

Table 2: **Summary Table for Power (CES) Utility**

	$\boldsymbol{\eta > 1}$			$\boldsymbol{\eta < 1}$		
	$W \to 0$	middle W	$W \to \infty$	$W \to 0$	middle W	$W \to \infty$
elas of subs η	constant at η			constant at η		
$\frac{\mathrm{d}R/R}{\mathrm{d}W/W}$	0.0	-ve	$1-\eta < 0$	$1-\eta > 0$	+ve	0.0
Hicks Sub	0.0	-ve	$-\eta < 0$	$-\eta < 0$	-ve	0.0
Hicks Income	-1.0	$-R < 0$	0.0	0.0	$-R < 0$	-1.0
Direct Income	constant at 1.0			constant at 1.0		
R	1.0	decreasing	0.0	0.0	increasing	1.0

Note: See equations (8) for the algebraic expression of the decomposition.

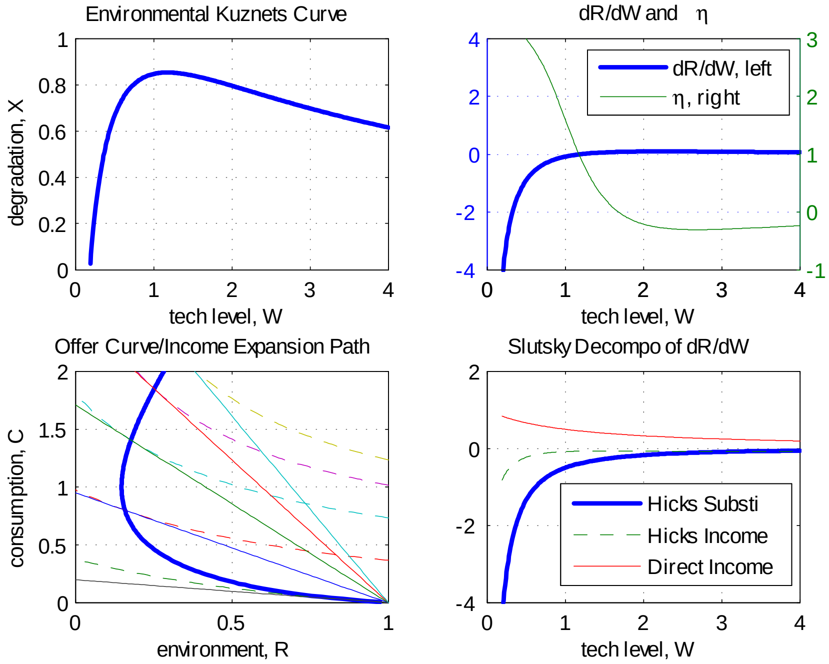

Figure 1: Results for Exponential (DES) Utility Function ($\alpha_R = \alpha_C = 1.0$ and $\phi = 0.5$)

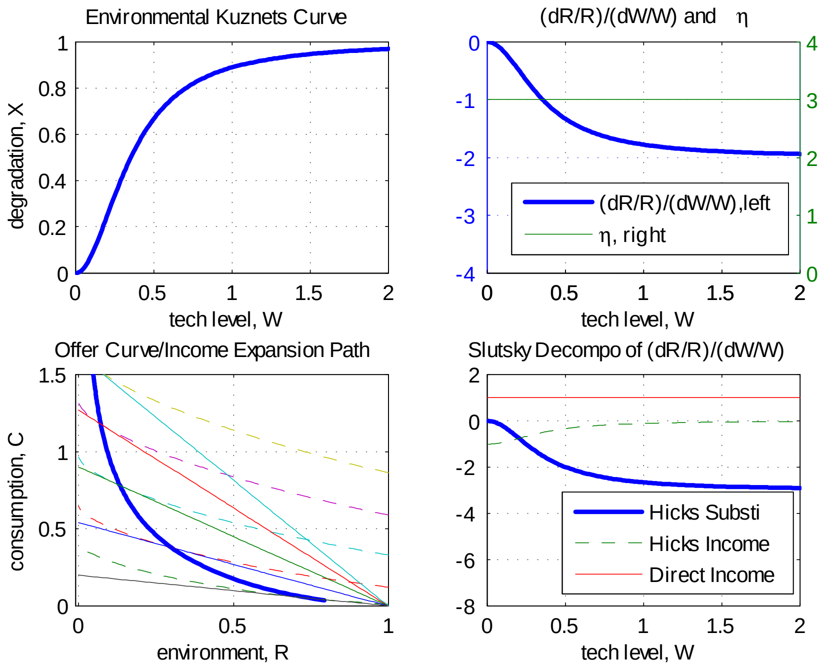

Figure 2: Results for Power (CES) Utility with $\eta = 3$ ($\phi = 0.5$)

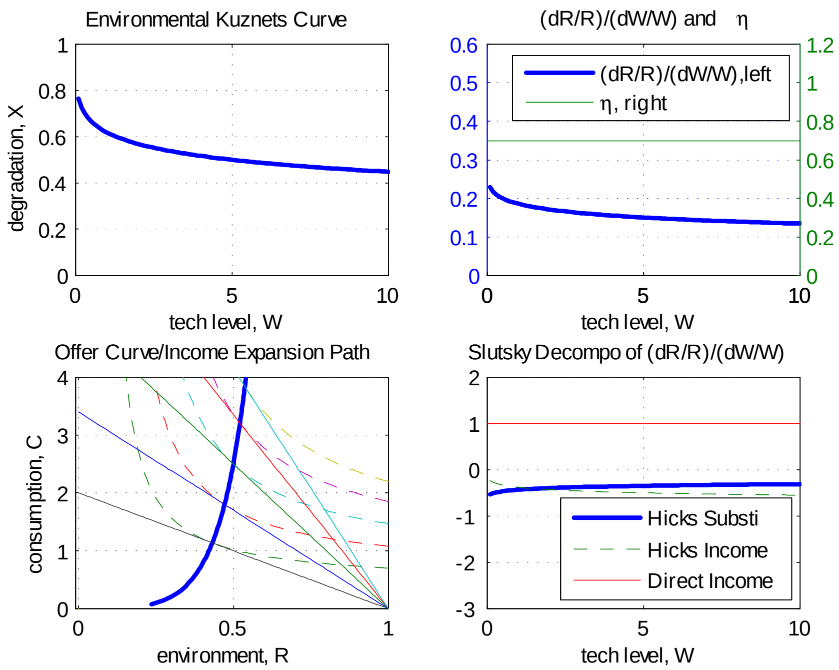

Figure 3: Results for Power (CES) Utility with $\eta = 0.7$ ($\phi = 0.5$)

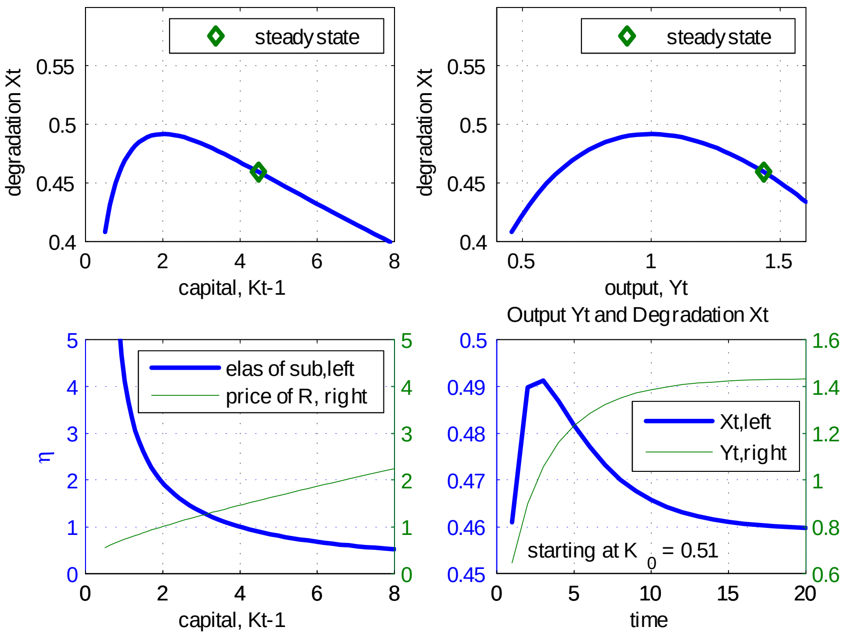

Figure 4: Environmental Kuznets Curve with Capital Accumulation

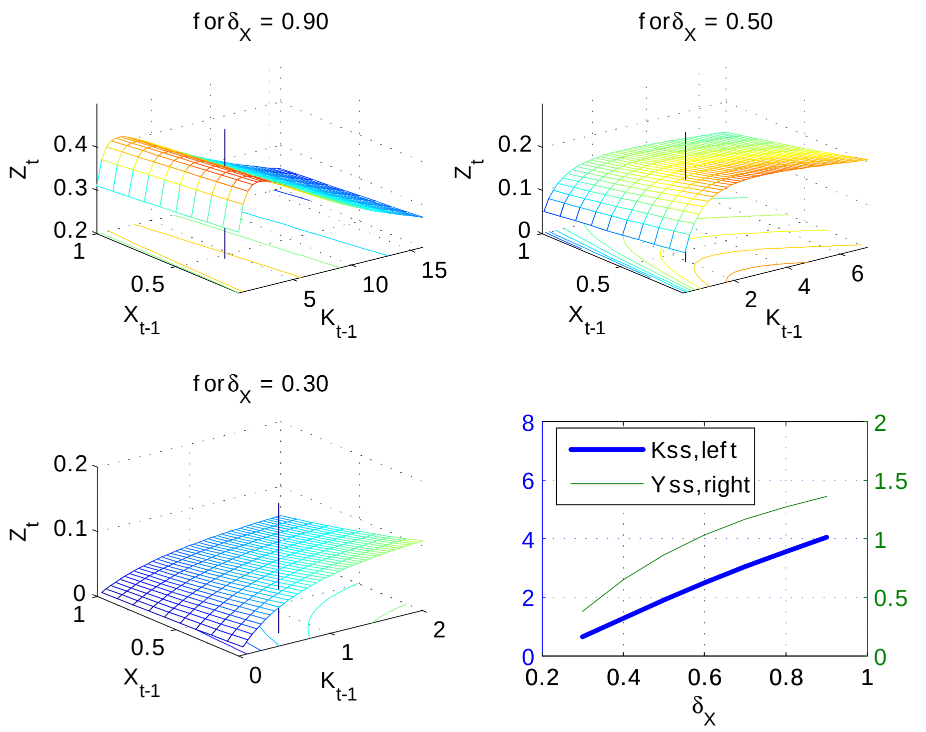

Figure 5: Environmental Kuznets Curve for Pollution Stock (Model III) for different δ_X.

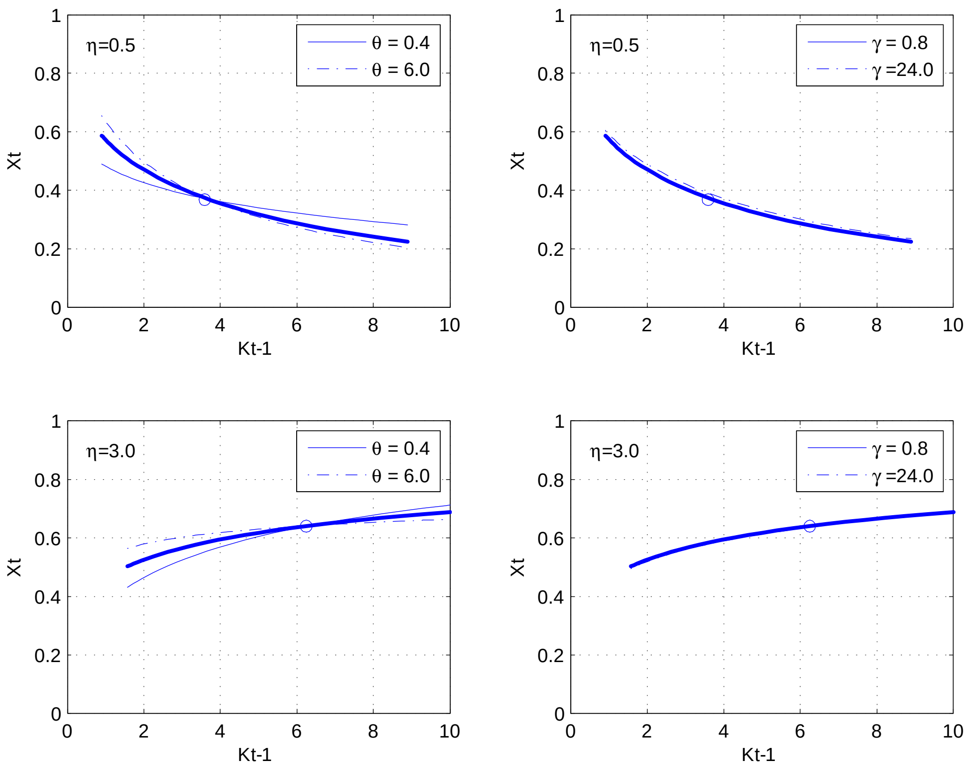

Figure 6: Optimal X_t as a function of K_{t-1} with $W_t = A$ for selected values of θ and γ. Note that the bold line and the circle in each panel show the baseline case and the steady state, respectively.

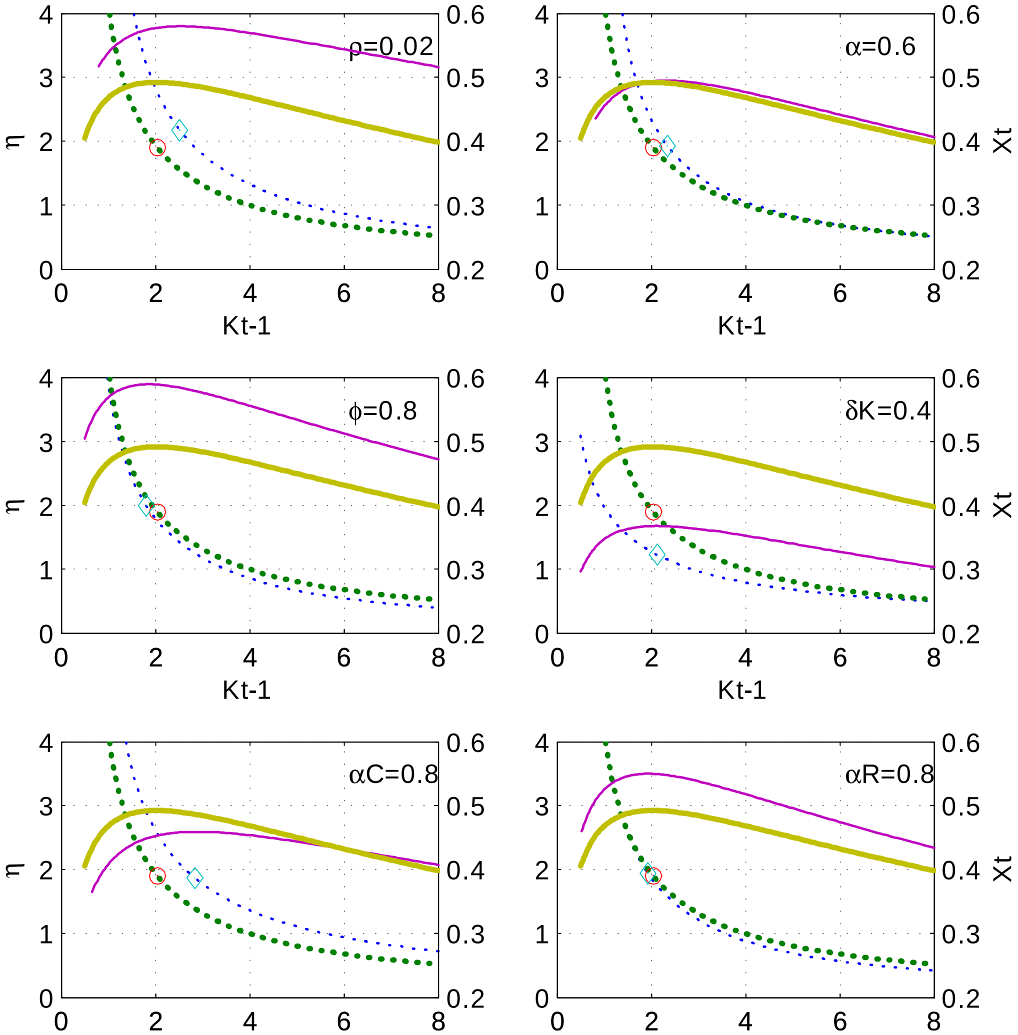

Figure 7: Sensitivity Analysis for Model III (flow pollutants). The dotted and solid lines show η_t and X_t as functions of K_{t-1}, respectively. The bold lines are the baseline case and thin lines show the effect of changing parameters. Circles and diamonds show threshold η.