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Interpreting the Hours-Technology time-varying relationship

Cristiano Cantore, Filippo Ferroni and Miguel Léon-Ledesma

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Cristiano Cantore   Filippo Ferroni   Miguel A. León-Ledesma

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Abstract

We investigate the time variation in the correlation between hours and technology shocks using a structural business cycle model. We propose an RBC model with a Constant Elasticity of Substitution (CES) production function that allows for capital- and labor-augmenting technology shocks. We estimate the model using US data with Bayesian techniques. In the full sample, we find (i) evidence in favor of a less than unitary elasticity of substitution (rejecting Cobb-Douglas) and (ii) a sizable role for capital augmenting shock for business cycles fluctuations. In rolling sub-samples, we document that the impact of technology shocks on hours worked varies over time and switches from negative to positive towards the end of the sample. We argue that this change is due to the increase in the elasticity of factor substitution. That is, labor and capital became less complementary throughout the sample inducing a change in the sign and size of the the response of hours. We conjecture that this change may have been induced by a change in the skill composition of the labor input.

JEL classification: E32, E37, C53

Keywords: Real Business Cycles models, Constant Elasticity of Substitution production function, Hours worked dynamics.

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*Cantore: Department of Economics, University of Surrey, Surrey GU2 7XH UK (e-mail:c.cantore@surrey.ac.uk). Ferroni: Banque de France (e-mail:filippo.ferroni@banque-france.fr). León-Ledesma: School of Economics, University of Kent, Canterbury, Kent, CT27NP UK (e-mail:M.A.Leon-Ledesma@kent.ac.uk). The views expressed in this paper do not necessarily reflect those of the Banque de France. We thank seminar participants at University of Surrey, Banque de France, University of Kent, European Central Bank, University of Lancaster, the Structural Change and Macroeconomic Performance workshop at Universita' di Cagliari, and the 17^{th} International Conference on Computing in Economics and Finance. We thank Patrick Feve for his excellent discussion, and Fabio Canova, Vasco Carvalho, Christian Hellwig, Alex Klein, Alessio Moro, Keisuke Otsu, Alessia Paccagnini and Mathan Satchi for helpful comments.

1 Introduction

One of the most controversial issues in business cycle theory regards the impact of technology shocks on hours worked. The sign and size of the hours response to a productivity shock can have important consequences for policy analysis. The estimated response has also been interpreted as shedding light on the ability of contrasting macro models to explain features of the business cycle. The focus of most of this literature has been on the analysis of the response of hours in full samples.[1] However, recent business cycle literature has shifted attention to the changing nature of some key data moments since the works of Kim and Nelson (1999), McConnell and Perez-Quiros (2000), and Stock and Watson (2003). Specifically concentrating on the time-varying relationship between productivity and hours worked, Gambetti (2006), Stiroh (2009), and Galí and Gambetti (2009) unveil important changes in the sign and size of these responses in the US economy since the post-war era. Technology shocks appear to have a strong negative effect before the 1980s and positive or non significant afterwards, although this increase is not monotonic. Fernald (2007) also finds that, after allowing for trend breaks in productivity, hours tend to fall when technology improves.[2] Hence, time-varying structures have been considered as a possible statistical explanation for the instability of the full sample SVAR estimates. Most of this literature, however, focuses on reduced form representations that allow for limited structural interpretations in terms of deep model parameters.

In this paper, we propose a structural explanation for the time-varying nature of the reaction of hours to technology shocks. We first provide further evidence on the changes in the impulse-response of hours to technology shocks for the US economy using a standard SVAR with long-run restrictions. We then propose and estimate a parsimonious model that is potentially able to capture this observed time variation. Specifically, we propose a simple RBC model where we introduce a Constant Elasticity of Substitution (CES) production function. As shown by Cantore, León-Ledesma, McAdam and Willman (2010), the sign of the response of hours to a technology shock depends crucially on the relative magnitude of the

[1]There is a large literature on this issue that we do not aim to survey here. For comprehensive reviews, see Galí and Rabanal (2005) and Whelan (2009).

[2]Kahn and Rich (2007) and Roberts (2001) amongst other document two changes in labor productivity in US. One in early 70's and one during the mid 90's. Fernald (2007) finds two breaks in private-business labor productivity growth: 1973:2 and 1997:2. He finds also that the mean growth is similar before 1973 and after 1997. Hansen (2001), using a simple first-order autoregressive model finds a break in February 1992.

elasticity of capital-labor substitution and the capital intensity in production. The model contains a preference shock and two technology shocks: labor- and capital-augmenting. These shocks can be distinguished when the elasticity of capital-labor substitution differs from one (the Cobb-Douglas case). We first study the properties of our specification over the full sample. Several results stand out. First, we show that the proposed specification, despite its parsimony, fits the postwar US data on productivity and hours worked reasonably well, especially when compared to a standard Cobb-Douglas specification. Second, the elasticity of substitution between capital and labor is statistically well below unity, supporting the increasing consensus in the empirical literature (see Chirinko (2008)). Third, by looking at the historical decomposition of hours worked, we find a sizable role for capital augmenting shocks in explaining business cycles fluctuations. In particular, the level of productivity is mostly explained by the labor augmenting shock, and the level of hours worked is mostly explained by the capital augmenting shock.

We then estimate the model on rolling samples of the same length as our SVAR, and find that there is a significant sign variation of the response of hours worked to a positive technology shock. We also find that the time-varying impulse responses to a labor-augmenting shock obtained from the estimated model track satisfactorily the changes observed in the data-based SVAR. Such variation is driven by a change in the magnitude of the elasticity of factor substitution which, in our model, governs the sign of the hours response. In particular, we observe an increase in the degree of factor substitution along the sample. That is, labor and capital became less complementary through time. We conjecture that these changes may be associated to the changing skill composition of the labor force.[3] With heterogeneous labor, an increase in the share of skilled workers or their relative productivity can lead to an increase in the aggregate elasticity of substitution. We further explore the robustness of our claim that the time varying response of hours crucially depends on the magnitude of the elasticity of capital-labor substitution. Following Chari, Kehoe and McGrattan (2008), we study whether SVAR estimates on data simulated from our structural model would lead to impact responses similar to the ones obtained using actual data. We find little support for a significant difference between the two. Finally, we complement our analysis by analyzing the robustness of the results to alternative data construction and the introduction of investment adjustment costs. It is also important to highlight that, to the best of our knowledge, this is the first attempt at directly estimating the

[3]An equivalent argument is structural change towards more skill-intensive sectors.

(time-varying) elasticity of capital-labor substitution in a fully fledged DSGE model accounting for both supply and demand blocks.[4]

It is worth emphasizing, however, that we do not view our interpretation as excluding other potential sources of structural changes that may have led to time-variation in the hours-technology correlation. One explanation that has received much attention is the well known change in monetary policy at the beginning of the 80's.[5] However, this explanation is not free from criticism. For instance, Canova and Gambetti (2009) find little support for the role of monetary policy changes in driving output and inflation dynamics and point towards the potential importance of changes in private sector behavior. Changes in the labor market can be another important source of time-variation. Along these lines, Nucci and Riggi (2009) attribute changes in the response of hours to an increase in performance-related pay schemes during the 1980s. Their model, however, can account for a reduction in the negative response of hours to a technology shock but not for a sign switch. In parallel to increased labor market flexibility we also observe another important change in the labor market that may have shaped aggregate hours responses. As reported by Acemoglu (2002) and Acemoglu and Autor (2011), the US labor market experienced significant changes in its skill composition. These changes can affect the elasticity of capital-labor substitution and hence the response of hours to technology shocks.[6] Equivalently, changes in the composition of output towards sectors with higher skill requirements may have contributed to a change in the aggregate elasticity of substitution. These effects, however, have received little attention as potential sources of time-variation in labor market data moments. Our setup is deliberately parsimonious since the time variation of the response of hours can be seized by the change in the relative magnitude of the parameters entering the production function. For this reason, we analyze how far changes in few crucial parameters can go to explain the time-variation of hours responses. We do not go as far as claiming, however, that frictions and macroeconomic policies cannot potentially play an important role.

The paper in organized as follows. Section 2 presents some empirical evidence.

[4]The literature on the estimation of CES parameters has focused almost exclusively on supply side static models as in León-Ledesma, McAdam and Willman (2010).

[5]See amongst other Clarida, Galí and Gertler (2000), Galí, Lopez-Salido and Valles (2003) and Cogley and Sargent (2005).

[6]During this period, we can also observe an important process of de-unionization, although this may well be the consequence of changes in skill composition of the labor force due to the introduction of skill-biased technologies as argued by Acemoglu, Aghion and Violante (2001).

Section 3 presents the model and study the response of hours with a sensible calibration exercise. Section 4 describes the estimation strategy and presents the full sample estimates. The dynamics of hours worked and productivity are reported in Section 5. Section 6 offers a theoretical discussion of potential sources of changes in capital-labor substitutability. Finally, Section 7 concludes.

2 Empirical Evidence

While there is a large literature documenting the changes in the second moments of various US times series, here we focus on response of hours worked to a technology shock. Data ranges from 1948:Q1 until 2006:Q1 and were obtained from the FRED database. The times series include output in the non-farm business sector (OUTNFB), and hours of all persons in the non-farm business sector (HOANBS). Both series are normalized by the the civilian non-institutional population of 16 years and over (CNP16OV). Labor productivity is computed as the ratio between the measure of output and hours, and we take logarithms of both series. We indicate with p_t labor productivity and with $hobs_t$ hours.[7]

To identify a technology shock we adopt the long-run restriction proposed by Blanchard and Quah (1989) where we assume that only the technology shock has a permanent effect on the level of productivity (as in Galí (1999)). We estimate the structural VAR (SVAR) model on rolling windows of fixed length, starting from the sample [1948Q1,1967Q4], and repeating the estimation moving the starting date by one year. We obtain 39 estimates of the coefficients of the reduced from VAR and of the identified impact matrix (one for each window) and compute the impulse response of hours to a technology shock. We considered different lag lengths for the VAR and rolling windows sizes and the results remained unchanged.[8] We report here the results with 80 quarters and four lags in the VAR. More formally, the reduced form VAR of can be represented as

$$x_t = A_0 + A_1 x_{t-1} + ... + A_p x_{t-p} + u_t$$

where u_t are i.i.d. zero mean normal shocks with covariance matrix Σ. We assume that $u_t = K\epsilon_t$ where $\epsilon_t = [\epsilon_t^s, \epsilon_t^d]$ is a normal i.i.d. shock with $E(\epsilon_t \epsilon_t') = I$, and

[7]In appendix A we carry out a comprehensive analysis of the robustness of the results to alternative data definitions. More details on data construction are also available there.

[8]We used rolling windows of 60, 70, 80, and 90 quarters and four lag lengths.

where ϵ_t^s is the technology shock and ϵ_t^d a non technology shock. It follows from the assumptions that $\Sigma = KK'$.

We consider $x_t = [\Delta p_t, hobs_t]$ in estimation.[9] For exposition purposes it is more convenient to rewrite the system in a companion form

$$z_t = \mu + Bz_{t-1} + e_t$$

where $z_t = [x_t', x_{t-1}', ..., x_{t-p+1}']'$, $e_t = [u_t', 0, ..., 0]'$, $\mu = [A_0', 0, ..., 0]'$, and B is the companion form matrix. The long run restriction implies that the impact matrix of cumulative effects of the shock on labor productivity has a Cholesky factor, i.e. the matrix $F = \sum_{k=0}^{\infty} S_{2,2}(B^k)\ K$ has a lower triangular structure where $S_{2,2}(.)$ is a selection matrix that picks the first two rows and columns of matrix B^k.

Figure 1 plots the response of hours worked to a technology shock. The response

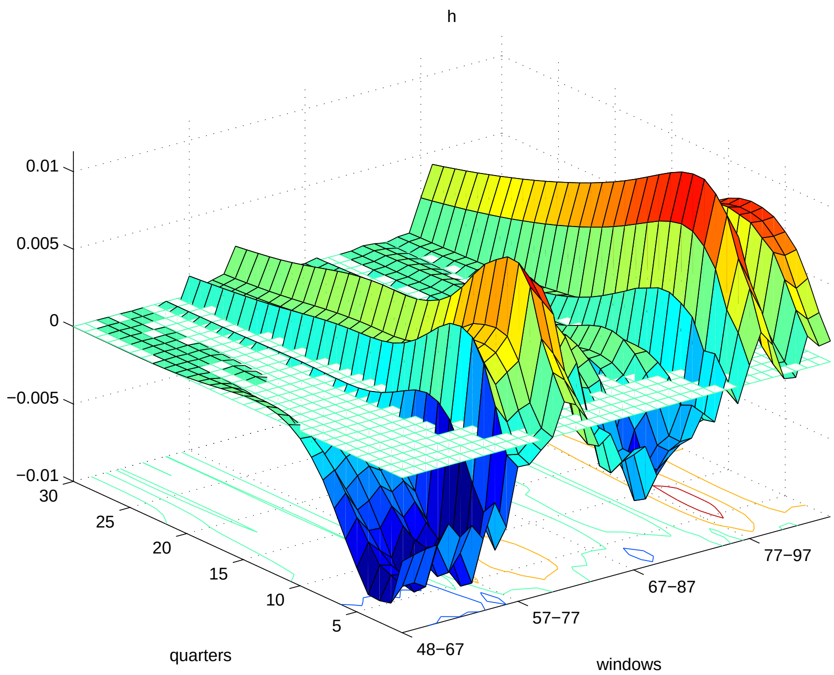


Figure 1: Response of hours worked to a technology shock. The level of hours is used in estimation.

of hours worked displays significant time variations. In fact, the impact response is

[9] We also considered hours in first difference, i.e. $x_t = [\Delta p_t, \Delta hobs_t]$. While we find time variations, we do not detect any sign switch. This result is due to the fact that first differencing removes the long run frequencies of hours worked. As shown in Canova, Lopez-Salido and Michelacci (2010), if secular cycles are removed from the raw series of hours worked, hours respond negatively to technology.

negative in early samples, increases up until the mid-1970s, then falls, and then increases steadily thereafter. These results are similar to those of a more parameterized set up, as in Galí and Gambetti (2009), using a VAR with time-varying coefficients and stochastic volatility, the same specification of hours, and the same identification scheme.[10] To ease the visual analysis, figure 2 reports the impulse responses

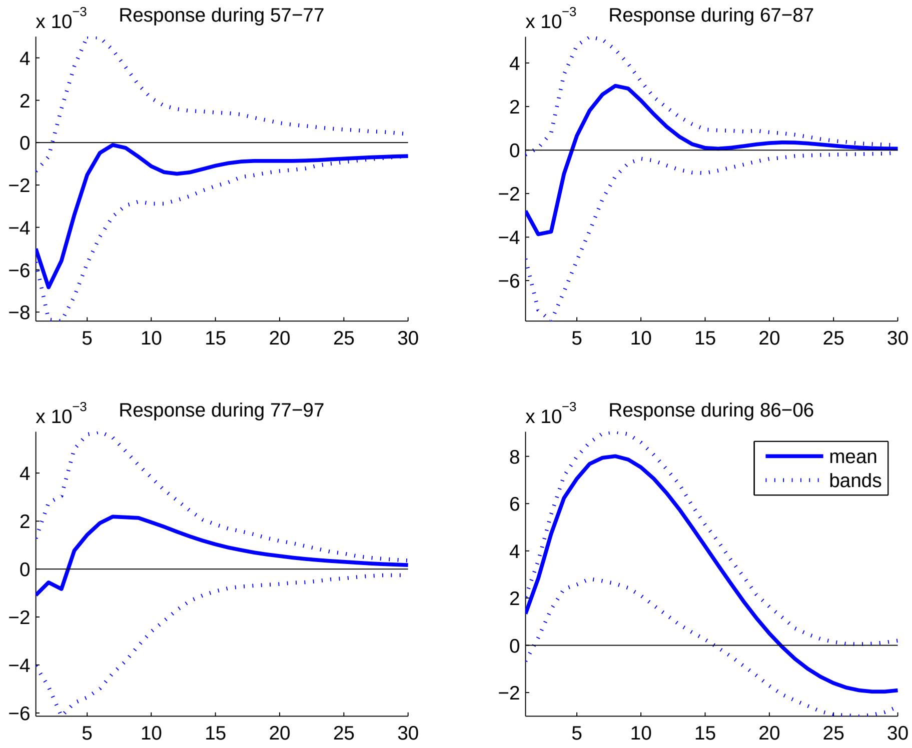


Figure 2: Response of the growth rate of productivity and of hours worked to a technology shock for selected sub-samples. The level of hours is used in estimation.

for selected sub-samples. As it clearly stands out, the response of hours worked to an identified technology shock has changed over time. In particular, while it was negative during the 60s on impact, hours increase following a technology shock if we consider the sample including the 1990s for estimation. In all, these results confirm the existence of important changes in the short-run technology-hours correlations in the US over the post-war period.

[10]We replicated the time-varying coefficients model of Galí and Gambetti (2009) on our data and it yielded almost identical outcomes. Results are available on request.

3 The Structural Model

We consider a closed economy Real Business Cycles (RBC) model. The novelty is that it features a Constant Elasticity of Substitution (CES) production function, which is characterized by two sources of fluctuations, a labor- and a capital-augmenting stochastic shift to the production frontier. The model is otherwise standard, it is a single good optimizing agent framework. The advantage of this model is that, with an elasticity of capital-labor substitution that differs from unity (the Cobb-Douglas case), even in the canonical RBC model the response of hours to a labor-augmenting technology shock can be positive or negative. Cantore et al. (2010) show analytically that the sign of the response depends on the relative magnitudes of the elasticity of substitution and the capital share.[11]

The representative household is characterized by the following preferences[12]

$$U_t = \ln C_t - V_t \xi \frac{H_t^{1+\gamma}}{1+\gamma}, \tag{1}$$

where C_t denotes consumption, H_t hours worked, β is the discount factor, γ is the inverse of the Frisch elasticity, ξ affects the marginal rate of substitution between consumption and leisure and determines the steady state hours and V_t is a preference shock process that has an AR(1) representation, i.e. (in log deviations from the steady state)

$$v_t = \rho_v v_{t-1} + \eta_t^v \quad \eta_t^v \sim N(0, \sigma_v). \tag{2}$$

The production is CES and presented in *normalized* form as in Cantore et al. (2010)[13]

$$Y_t = y \left[\alpha \left(\frac{Z_t^k K_{t-1}}{k} \right)^{\frac{\sigma-1}{\sigma}} + (1-\alpha) \left(\frac{Z_t^h h_t}{h} \right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \tag{3}$$

where, as usual, output is produced by a combination of two factors: K_{t-1}, the installed physical capital at time t, and h_t, the number of hours worked. y and k are the steady state values of output and capital re-scaled by the labor augmenting

[11]The response also depends on the reaction of consumption. Cantore et al. (2010) also show that a similar change in the sign of responses can occur in a New Keynesian model, but in this case for a capital-augmenting shock.

[12]We assume a log preference in consumption to guarantee a balanced growth path.

[13]Normalization is required to compare responses when we change the elasticity of substitution. Also, it allows us to interpret directly the share parameter α as the capital income share at the point of normalization (the steady state in this case).

process, and h is the steady state value for hours. α and σ are parameters controlling the capital intensity in production and the degree of substitutability between factors. As $\sigma \to 0$, factors are net complements, and the production function is Leontief. If $\sigma \to \infty$ factors are net substitutes and the production function is linear. As σ approaches 1, we have a Cobb-Douglas production function. The CES production function encompasses two types technological change, i.e. the capital augmenting, Z_t^k, and the labor augmenting technological process, Z_t^h. We assume that capital-augmenting technology has an AR(1) representation, i.e. (in log deviations from the steady state)

$$z_t^k = \rho_k z_{t-1}^k + \eta_t^k \quad \eta_t^k \sim N(0, \sigma_k), \tag{4}$$

where $\rho_k < 1$ to ensure the existence of a balanced growth path. For the labor-augmenting shock we adopt a flexible specification following an AR(2) process, i.e.[14]

$$z_t^h = \psi_{1,h}(1 - \psi_{2,h}) z_{t-1}^h + \psi_{2,h} z_{t-2}^h + \eta_t^h, \tag{5}$$

with $z_t^h = \ln Z_t^H - \ln Z_0^H$ and the original autoregressive processes is rewritten in terms of partial autocorrelations $\psi_{1,h}$ and $\psi_{2,h}$.[15] If $\psi_{1,h} = 1$, then labor-augmenting technology shocks have a permanent effect and the labor-augmenting technology process is stationary in first differences with autoregressive coefficient $-\psi_{2,h}$. If $0 < \psi_{1,h} < 1$ and $\psi_{2,h} = 0$, then the labor-augmenting technology process is persistent but stationary and follows an AR(1) process. The model is then closed by assuming that capital depreciates at rate δ and that the economy's resource constraint is given by:

$$Y_t = C_t + K_t - (1 - \delta) K_{t-1}. \tag{6}$$

As mentioned, this model has the property that the capital intensity in production and the elasticity of factor substitution, α and σ, are the main drivers of the dynamics

[14] See Ríos-Rull, Schorfheide, Fuentes-Albero, Kryshko and Santaeulália-Llopis (2009).

[15] By assuming

$$(\ln Z_t^H - \ln Z_0^H) = \rho_{1,h}(\ln Z_{t-1}^H - \ln Z_0^H) + \rho_{2,h}(\ln Z_{t-2}^h - \ln Z_0^h) + \eta_t^h$$

if $\rho_{1,h} + \rho_{2,h} = 1$, then technology has a unit root and the serial correlation of its growth rates is $-\rho_{2,h}$. We can re-parameterize them in terms of partial autocorrelations $\psi_{1,h}$ and $\psi_{2,h}$ by setting:

$$\begin{aligned} \rho_{1,h} &= \psi_{1,h}(1 - \psi_{2,h}) \\ \rho_{2,h} &= \psi_{2,h} \end{aligned}$$

of output and hours worked conditional on a labor augmenting technology shock. The intuitive reason for this is that α determines the output effect of a labor augmenting shock on labor demand, whereas σ determines the substitution effect. Depending on their relative magnitudes, the shock can increase or decrease labor demand.

By means of a sensible calibration exercise, we can study the impact of a labor augmenting technology shock to hours worked for different values of the capital-labor elasticity. Without loss of generality, we assume that the labor augmenting technology process is stationary, i.e. $\psi_{2,h} = 0$ and $\psi_{1,h} = 0.8$. Moreover, we set the time discount factor, β, to 0.99, and the depreciation rate, δ, to 0.025, and the inverse of the Frisch elasticity, γ, to 1. We let the capital-labor elasticity vary between 0.1 and 1, and we fix the capital intensity in production to 0.33. Figure 3 (left panel) reports the impulse response of hours worked to a labor-augmenting technology shock for different values of σ and keeping the value of α fixed at 0.33. Approximately, when $\sigma > \alpha$ the response of hours to a labor augmenting technology shock is positive. However, hours worked decrease if $\sigma < \alpha$, which essentially replicates the results of Cantore et al. (2010). The right panel of Figure 3 displays the instantaneous response of hours worked to a labor-augmenting technology shock for different values of σ and α. We let the value of capital intensity vary between 0.2 to 0.6. Thus, for values of σ larger than 0.7 and close to the Cobb-Douglas specification, the response of hours is positive regardless of the values of α.

As we are not aware of previous work attempting to estimate σ within a dynamic general equilibrium model, we first study whether the parameter is empirically identifiable. To this end, we perform a controlled simulation experiment in appendix B. Our results show that the information contained in hours worked and productivity is sufficient to identify σ in estimation.

4 Full sample estimates with a CES production technology.

We now analyze the behavior of the model when confronted with observed data on US productivity and hours worked. In particular, we are interested on verifying that the model fits the data reasonably and that its performance is comparable with the fit of a more standard specification. Hence, we confront two specifications: an RBC model with a CES production function and an RBC model with a Cobb-Douglas production technology (i.e. $\sigma = 1$ and only the labor-augmenting technological process). We

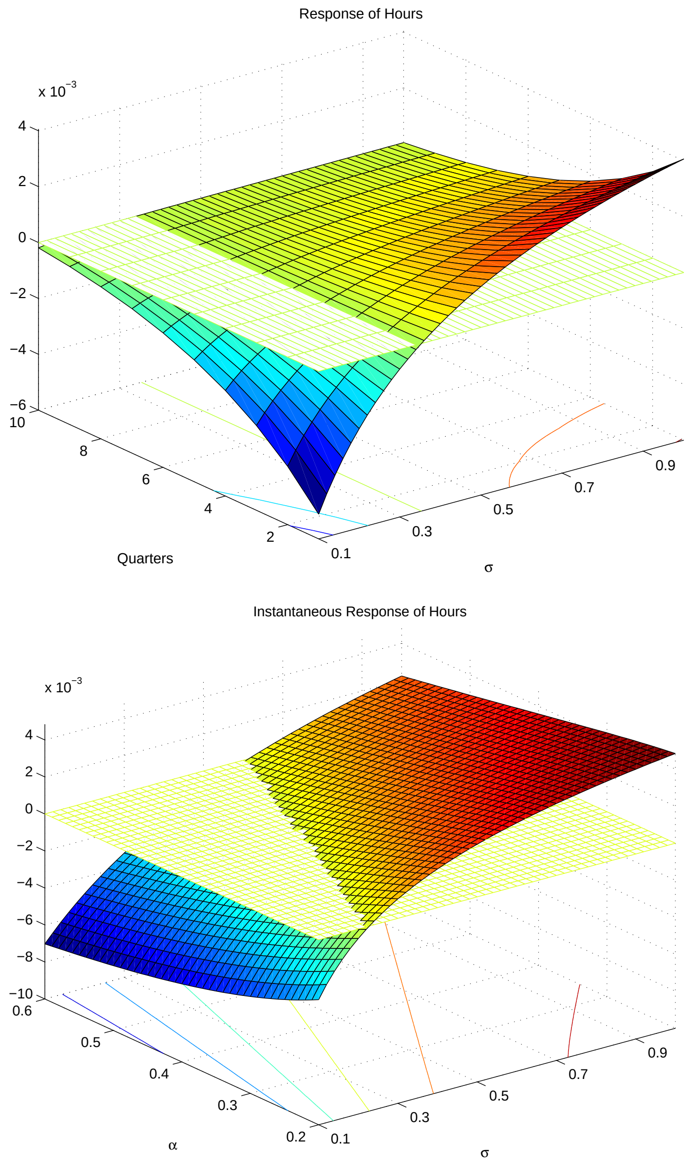


Figure 3: Impulse response of hours worked to a labor-augmenting technology shock for different values of σ and $\alpha = 0.33$ (Top panel). Instantaneous response of hours worked to a labor-augmenting technology shock for different values of σ and α (Bottom Panel).

verify whether data favors a less parameterized model and thus the CES specification is redundant, or whether the latter helps characterize the data better.

Since the raw series of labor productivity displays a clear upward trend, we bridge the model to the data by imposing a permanent labor-augmenting technology shock. Hence, real variables grow at the rate of the technological process and hours worked are stationary.[16] These assumptions imply that $\psi_{1,h} = 1$ and that the following measurement equations hold[17]

$$\Delta p_t = \Delta(y_t - h_t) + \Delta z_t^h$$
$$hobs_t = h_t$$

Table 1 reports prior and posteriors statistics for the full sample. The choice of the priors is standard. We assume inverse gammas for standard deviations, beta distributions for the autoregressive parameters, a normal distribution for the inverse of the Frisch elasticity, γ, and for the capital intensity in production, α. The prior for σ follows a gamma distribution centered around one and with a loose precision. While posterior distributions of σ are very similar using a flat prior (i.e. the posterior mean is centered around 0.15 and has a tight credible set), we prefer to use a proper priors for marginal likelihood comparisons.

A few things are worth noting. First, for many parameters, posterior distributions have different locations, spread and shape relative to the priors. This is indicative that data provide relevant information for estimation. Moreover, in most cases, the mean and median coincide ruling out asymmetric posterior distributions (not shown here). Third, the standard deviations of technology shocks are a posteriori significant implying that data favor the mechanisms induced by the CES production function.

Concerning the parameters of interest, the posterior median of the elasticity of factor substitution is centered around 0.13 and the posterior distribution is quite tight in absolute terms and relative to the prior. This suggests that the data favor a more general specification for the production function. The capital share is estimated around the standard value in the RBC literature, i.e. 0.34, thus larger than the elasticity of substitution. This implies that, assuming no time-variation along the full sample, the point estimate of the correlation between hours worked and productivity is negative conditional on a labor augmenting technology shock. A formal comparison between the two models is reported at the bottom of Table 1 where we contrast the log of the marginal likelihood using the modified harmonic mean (see Geweke (1999)). If the two sources of technological progress and a non-unitary elasticity of substitution

[16] If we assume that innovations to the labor-augmenting technology process have a permanent effect on the economy, we need to generate stationary variables in the model using the following transformations: $\frac{Y_t}{Z_t^H}$ $\frac{K_t}{Z_t^H}$ $\frac{C_t}{Z_t^H}$ $\frac{W_t}{Z_t^H}$ H_t R_t where W_t is the real wage and R_t is the rental price of capital.

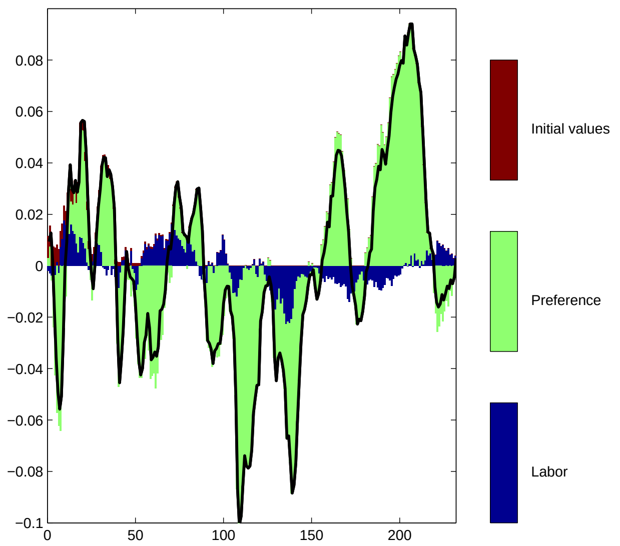
[17] Both series are demeaned to guarantee consistency with the log-linearized variables in the model that fluctuate around a value of 0 in steady-state.

	Prior			CD		CES 3		CES 2	
	Distr	mean	sd	median	sd	median	sd	median	sd
α	Normal	0.30	0.05	0.29	0.0373	0.33	0.051	0.35	0.041
σ	Gamma	1.00	1.00	-	-	0.14	0.031	0.13	0.023
γ	Normal	1.00	0.10	1.04	0.0949	1.00	0.100	0.98	0.100
ρ_v	Beta	0.50	0.20	0.97	0.0113	0.95	0.019	-	-
ρ_k	Beta	0.70	0.20	-	-	0.96	0.016	0.96	0.013
$\psi_{2,h}$	Beta	0.50	0.20	0.04	0.0220	0.07	0.037	0.05	0.027
σ_h	Igamma	0.010	2.00	0.01	0.0010	0.010	0.001	0.010	0.001
σ_k	Igamma	0.010	2.00	-	-	0.019	0.002	0.018	0.001
σ_v	Igamma	0.010	2.00	0.01	0.0010	0.012	0.004	-	-
Log ML				1424		1430		1432	

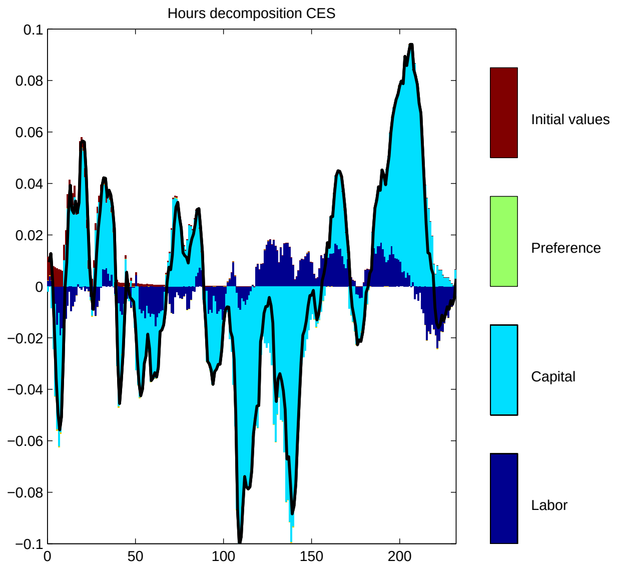
Table 1: Prior, posterior statistics and marginal likelihoods across specifications. Igamma stands for the inverse gamma distribution. CES 3 and 2 refers to the number of shocks.

between inputs were not important to characterize the dynamics of output and hours, a more parsimonious model would be preferred by means of marginal likelihood. In order to favor a Cobb-Douglas production function we need a prior probability for the model with Cobb-Douglas 403 ($= e^6$) times larger than the one associated with a CES production function (in other words, CES beats the CD production function with posterior model probabilities of 0.9975:0.0025). Moreover, we find that, regardless of the number of shocks, the CES structure is preferred to the Cobb-Douglas production specification.[18] Given the feeble role of preference shocks in our CES setting, we expect to observe a completely different historical decomposition of the observable variables among specifications. Figure 4 reports the decomposition of hours worked in terms of structural residuals. Under the Cobb-Douglas specification, where the capital augmenting shock is absent, the preference shock plays the most important role in the historical evolution of hours worked. When we turn to the CES, the contribution of the preference shock vanishes and the capital-augmenting shock contributes significantly to the observed levels of hours worked. A reason for this change is that, when σ is constrained to unity, the preference shock, which directly enters the labor supply equation, has to vary more to capture the variation of hours.

[18] We notice that the difference in terms log marginal likelihood is not sufficient to strictly prefer the CES specification with two shocks to the specification with three shocks. The literature adopts as a cutoff value 3, see amongst others Jeffries (1996) and Kass and Raftery (1995)).



(a) Cobb-Douglas



(b) CES

Figure 4: Historical decomposition of hours. Top panel CES specification, bottom panel Cobb-Douglas specification.

When σ is unconstrained and we introduce a capital augmenting shock, there is more variability in the labor demand equation, which now captures most of the variation in hours. The decomposition of productivity is similar across the two settings (not shown here), where the labor-augmenting shock represents the dominant source of the observed fluctuations of productivity. Hence, if we adopt a more general specification of the production function, we obtain that the full set of technological shocks account for the entire portion of historical fluctuations of productivity and hours experienced by the US economy within this RBC setting.

5 Time-varying dynamics

We want to investigate the dynamics of hours and technology over time through the lens of the structural model. To this end, we estimate the model on rolling windows of the same fixed length of our SVAR and we look closely at propagation mechanism of the structural shocks. Let the solution of the DSGE model be of the from,

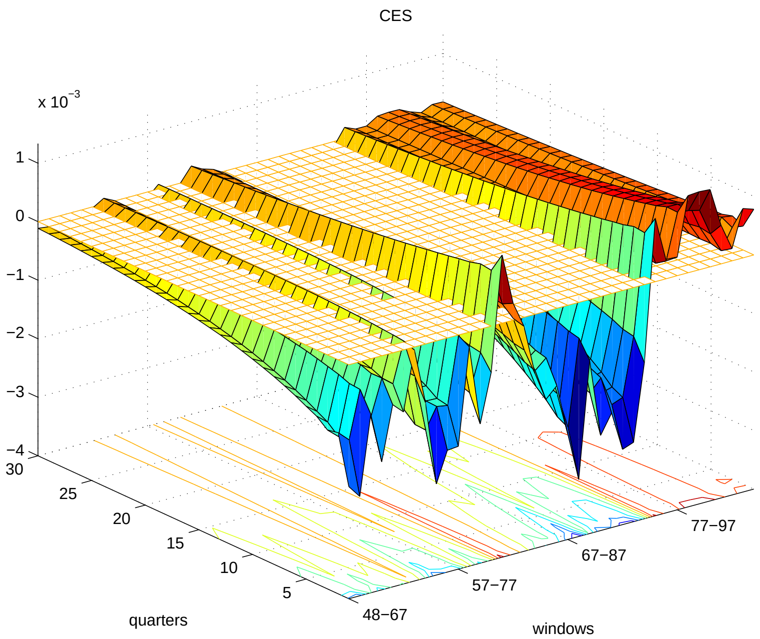
$$y_{t+1}^{\dagger} = \Phi(\vartheta) y_t^{\dagger} + \Psi(\vartheta) \eta_{t+1},$$

where the the vector $y_t^{\dagger}$ contains the endogenous variables of the model and η_t the structural vector of innovations with zero mean and diagonal covariance matrix Σ_η. Φ and Ψ are matrices which are non-linear functions of the structural parameters of the model, ϑ. Since we have a unique mapping from the structural parameters of the model to the reduced form matrix, we can back out the 'deep' parameters responsible for the changes (if any) in the transmission of shocks. Then, we look closely at the time pattern of the estimated structural parameters and try to provide intuition for such changes. Finally, we perform a 'reverse' exercise in the same spirit of Chari et al. (2008). We ask whether the estimates of the SVAR on data simulated from our structural model are in line with the impact results of the SVAR on actual data. We find little support for a difference between the two.

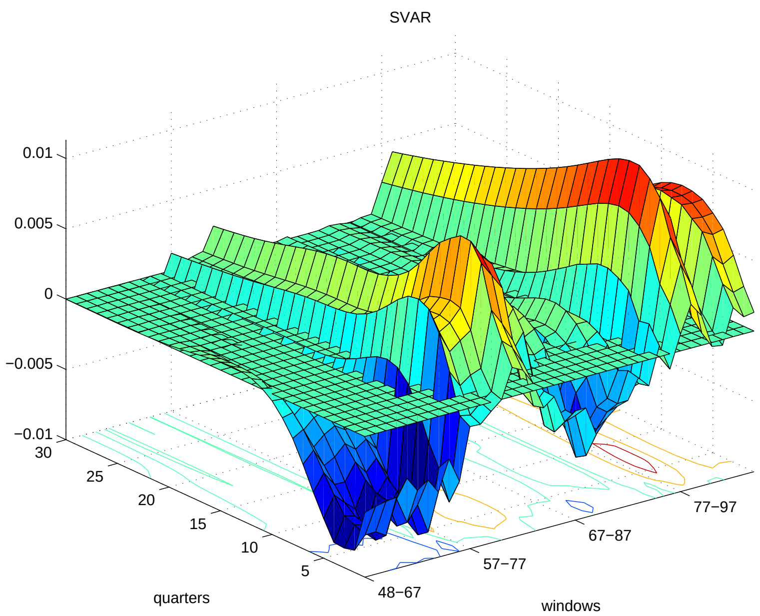
5.1 The transmission of technology shocks

One key fact that our setup would like to explain is the time varying relationship between hours worked and technology shocks and, in particular, if the model is able to reproduce the patterns found using the SVAR model. Figure 5 plots the response of hours worked to a labor augmenting technology shock.[19] The response of hours

[19] While there are variations in the level of the response, we do not detect any changes in the pattern of the response of hours to a capital augmenting technology shock (not shown here). Thus, we do not report it.



(a) CES



(b) SVAR

Figure 5: Impulse responses of hours to a positive labor augmenting technology shock.

worked shows clear shape and sign variations along the sample. Taken literally, the very early samples are characterized by a negative response. Then, for samples that include mainly the 1970s hours react positively to technology shocks. Then, the reaction of hours turns negative and positive again in the last ten rolling windows.

On impact, the resemblance with the SVAR evidence is striking. The signs of the response of hours appear to be correctly identified. Figure 6 plots the 68% credible

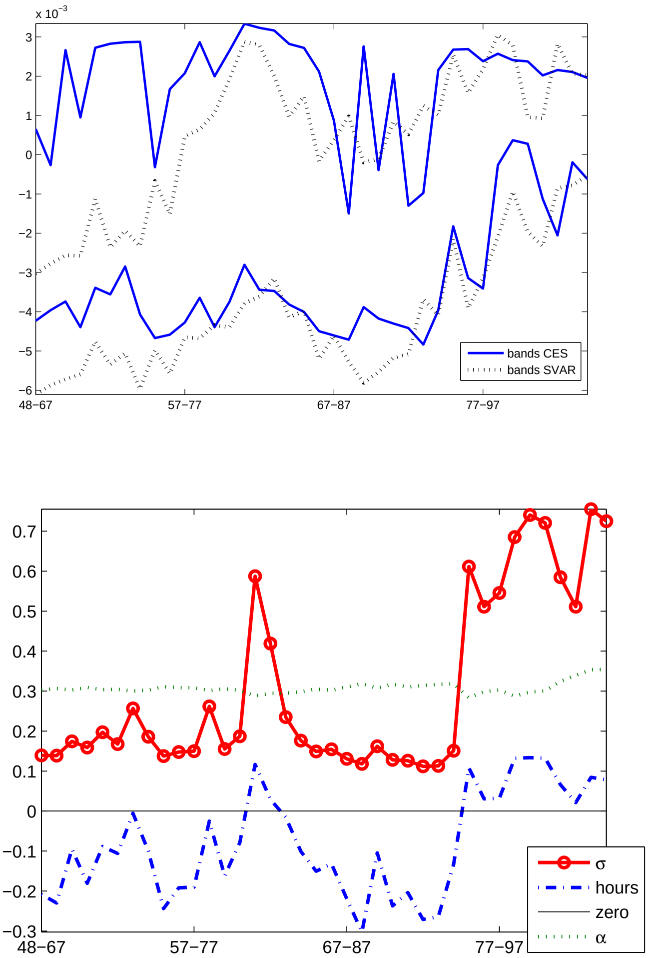


Figure 6: On the top panel credible sets of the contemporaneous impact of hours to a technology shock in the SVAR (solid line) and in the RBC with CES (dotted line). On the bottom panel, the median estimates of σ, α and the instantaneous impact of technology shock on hours across windows.

sets around the instantaneous response of hours with the SVAR estimates and the RBC-CES. If the instantaneous response of hours were different in the two settings, we would observe windows with non overlapping bands. Looking at Figure 6, we detect no significant difference of the contemporaneous response of hours between the estimates of the SVAR and the estimates of the RBC model with CES production function.[20]

The natural question that follows is what are the driving parameters behind the change in the propagation mechanism. Since the impulse response is computed as the marginal impact of a structural innovation to a variable, we can rule out changes in the standard deviations of the structural shocks as responsible for such variations. Even if the model is very stylized, the CES production function allows us to disentangle the scenarios where hours increase (decrease) in response to a technology innovation because the degree of factor substitution is larger (smaller) than the capital share in production. We find that there are large variations in the posterior estimates of the elasticity of factor substitution, in absolute terms and relative to the capital share parameter in production, α which remains stable throughout. Figure 6 (right panel) plots the posterior mean of the elasticity of substitution and the capital share in each of the sub-samples. Changes in the hours-technology conditional correlation on impact are associated with changes in the elasticity of capital-labor substitution, which varies between a low value close to 0.1 to a high 0.8 but always below 1.

Two things are worth mentioning. First, changes in the estimate of σ are significant but abrupt. This is partly due to the non parametric approach we adopt and to the uniform weighting scheme we impose on each window. One way to smooth the estimates of σ is to downsize the impact of sub-sample endpoints. As in the sample spectrum estimation (see Priestley (1982), Ch.7), we could design a bell shape distribution so that break points would have milder impact on structural estimates. However, we preferred to be agnostic and to give priority to the observables without imposing any ad hoc weighting scheme. The other approach is to parameterize the changes in σ by assuming that the capital-labor elasticity follows a slow moving exogenous process (i.e. an autoregressive process). Since first order approximations are insufficient to capture such process, higher order approximations are required. With higher order solutions, the implied state space system is neither linear nor gaussian, and we need to move to particle filters to extract the likelihood. Despite important advances in this direction (see Fernández-Villaverde and Rubio-Ramírez (2008)), the estimation of time-varying structures is still computationally burdensome and difficult to handle. Given these constraints, and for comparison with our SVAR results, we study what a computationally less intensive yet intuitively appealing structural

[20] It is worth noting, however, that the lack of persistence of the structural model has to do with the lag structure of the solution of the DSGE model, which makes it difficult to replicate the hump shaped response of a four lags VAR.

method could tell us about the time varying relation between hours and productivity.[21]

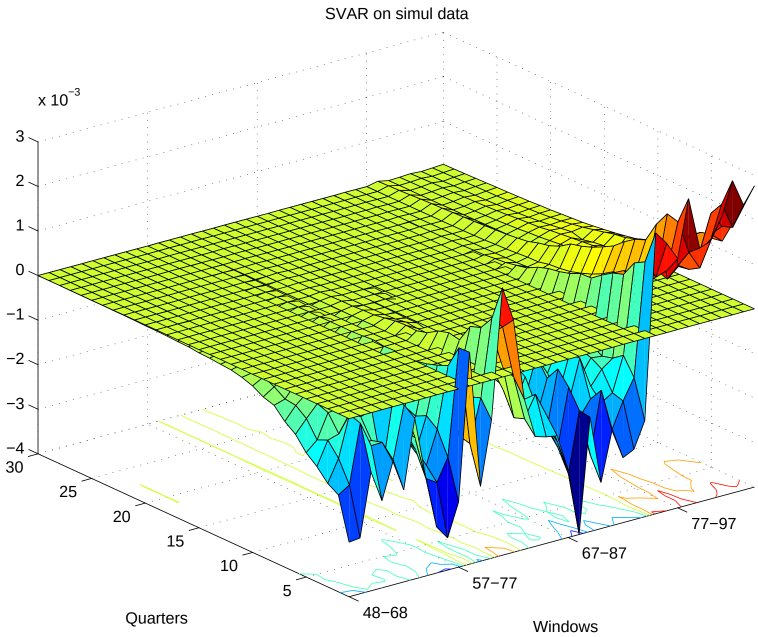
The second observation has to do with the interpretation of the changes in σ. In our estimates, we find periods where the sign of the response of hours switches sing. These periods coincide with episodes of relatively large estimates of σ. We observe a spike in two windows that includes the 1970s (i.e. the window from 1962 to 1982 and the one from 1963 to 1983) and a protracted period in the final part of the sample, i.e. the last 10 windows with data starting since the mid-1970s. The two sub-samples that include the 1970s contain very eventful years. In fact, during the 1970s the US economy was hit by a sequence of negative oil price shocks. And the beginning of the 1980s is characterized by the change in the monetary policy stance. As a consequence, we suspect that the variation in σ during these windows is contaminated by the turbulence of the seventies. After 1982, however, the US economy entered a relatively quiet period, where either 'good policy' or 'good luck' (or both) contributed to render the macroeconomic environment less volatile and more predictable. We thus believe that neither policy nor changes in the structure of the shocks are corrupting the estimated changes in the capital-labor elasticity in the latest samples. However, during this period, the US was experiencing important changes in the labor market and the sectoral structure of production. The literature has documented a sizable increase in the relative supply of skilled workers over time as well as a decline in the importance of manufacturing. We are inclined to interpret these changes as the source of the observed changes in the estimates of the capital-labor elasticity of substitution. We will return to this issue in Section 6.

5.2 Is the story of change in capital-labor elasticity consistent with a SVAR ?

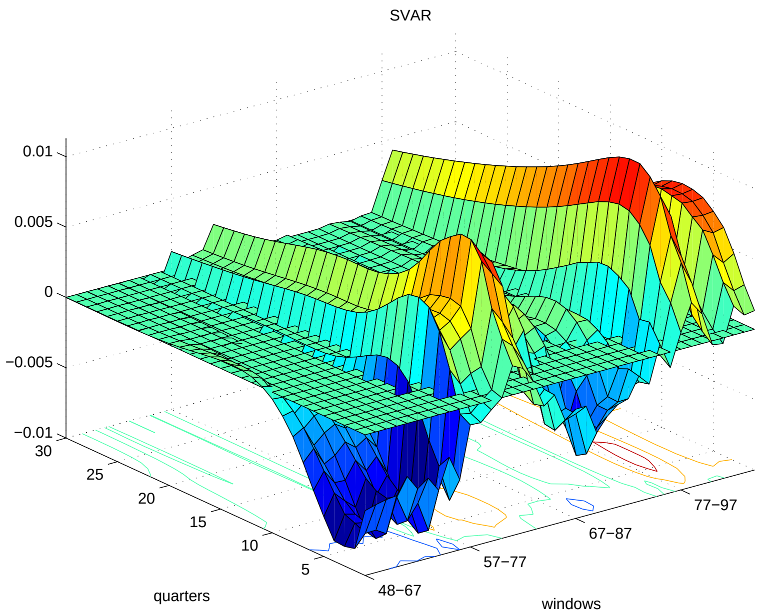
The time-varying relationship between hours and technology identified by a SVAR with long-run restrictions is very similar to the one obtained from our RBC model with CES production function. However, Chari et al. (2008), amongst others, express concerns about the ability of SVARs with long-run restrictions to identify model shocks. This may then cast doubts about whether comparisons of model-based and SVAR-based impulse-responses constitute a reliable way to evaluate our model. To address this issue, we follow Chari et al. (2008) and simulate 50 sets of data of 100

[21]See Canova and Ferroni (2012) for further discussion on the advantages of rolling subsample estimates of DSGE models.

observations from the RBC model with CES production function using the mean parameter estimates in each window. For each simulated dataset we estimate a SVAR with 4 lags and compute the impulse response. We then compare the data-based SVAR with the SVAR with model-simulated data.



(a) SVAR with simulated data



(b) SVAR with actual data

Figure 7: Impulse responses of hours to a positive labor augmenting technology shock.

Figure 7 reports the median impulse responses of hours for the SVAR (on the left panel) with simulated data, and those obtained by a SVAR with actual data. A visual inspection reveals that the instantaneous response of hours obtained with a SVAR on simulated data is similar to the one obtained with SVAR using actual data.[22] Figure

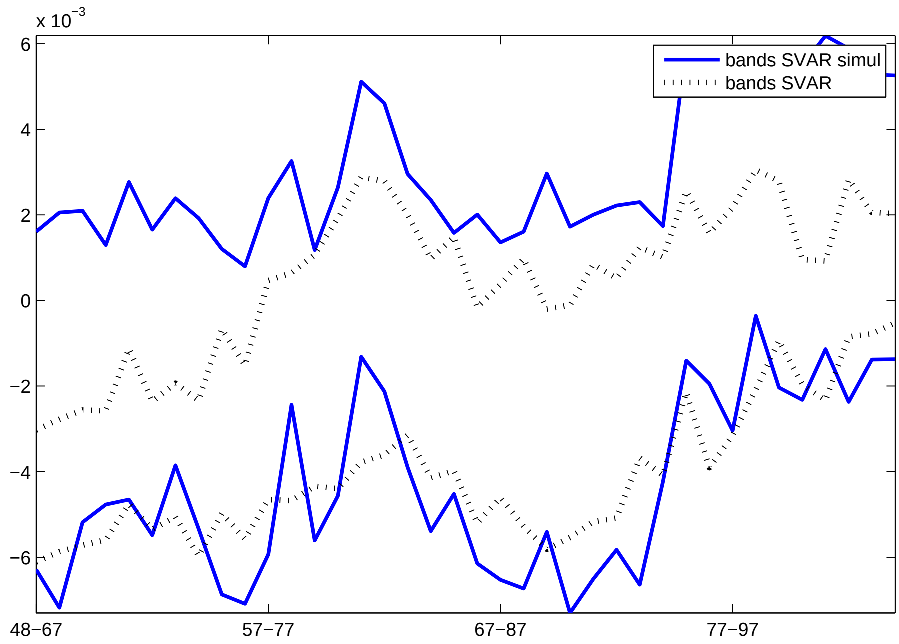


Figure 8: Credible sets of the contemporaneous impact of hours to a technology shock in the SVAR on actual data (dotted line) and in the SVAR with simulated data (solid line).

6 plots the credible sets around the instantaneous response of hours in the SVAR on actual data (solid line) and in the SVAR with simulated data (dotted line). As in the previous case, we detect no significant difference of the contemporaneous response of hours between the estimates of the SVAR and the estimates of the RBC with CES production function. Overall, the evidence supports the hypothesis that changes in the elasticity of capital-labor substitution are able to generate the observed time varying path of a SVAR with long-run restrictions.

[22]Note, as commented in a previous footnote, that the persistence properties are not satisfactorily captured because of the lag structure of the solution to the model.

5.3 Robustness: adjustment costs

As a robustness exercise, we also studied the role of investment adjustment costs. As adjustment costs can generate negative hours responses, changes in these could also be a driving force behind the changing response of hours. We thus analyzed a version of the model with CES production technology and adjustment costs. The model and calibration results are presented in appendix C. The results show that changes to investment adjustment costs are a priori unable to track the observed changes in the response of hours.

6 Rationalizing changes in the elasticity of substitution

Our estimation results suggest that the driving factor behind the change in the response of hours is the increase in the elasticity of capital-labor substitution σ. Changes in deep parameters, such as e.g. the degree of risk aversion, are commonly used to explain the existence of instabilities in macroeconomic relationships. However, we devote further attention to the observed evolution of σ by analyzing some conjectures about the driving forces behind this change. We leave detailed testing strategies for future research while we keep here the focus on the change in the hours-technology correlation.

Changes in the elasticity of substitution have been associated with economic growth since La Grandville (1989). Parameter σ, nevertheless, was treated as exogenous in that context. Hicks (1932), however, hypothesized that the elasticity of substitution may be variable and a by-product of economic development. Along these lines, Miyagiwa and Papageorgiou (2007) present a multisector growth model where σ is endogenously determined and positively related to economic development. Similarly, Álvarez-Cuadrado and Van Long (2011) present a multisector model of structural change where the aggregate elasticity of substitution is endogenous as capital intensity increases in the more flexible sectors (i.e. those with higher elasticity of substitution). Since the aggregate elasticity is a weighted average of sectoral elasticities, growth and structural change can lead to changes in aggregate σ.

These forces would naturally lead to slow and protracted increases in σ, contrasting with the more pronounced changes we observe in our estimates. The 1970s and 1980s, however, witnessed an accelerated process of technological change as reported by Greenwood and Yorukoglu (1997). This process was also associated with rapid structural change, with a fast decline in manufacturing and an increase in the share of business services. The structural change hypothesis has gained relevance in recent years as an explanation of changes in output volatility in the US, as reported in Carvalho and Gabaix (2010) and Moro (2011). These changes are also closely associated with the important changes occurring in the US labor market, especially since the mid-1970s, that could potentially drive the increase in σ observed in the latter parts of our rolling sample.[23]

One such relevant change is the increased importance of skilled workers in production. The evolution of skilled to unskilled employment and wages has been widely documented in papers such as Acemoglu (2002) and Acemoglu and Autor (2011). Figure 9 reproduces the observed trends by level of skills in the US economy. It displays the share of skilled workers as a percentage of all workers using two measures. The first is the share of non-production workers in US manufacturing for the 1958-2005 period from the Annual Survey of Manufactures. The second is the share of hours of workers with college education or above, as a percentage of total hours by workers with at least high school education coming from Autor, Katz and Kearney (2008) for the whole economy and the 1963-2005 period. Although both measures differ substantially, they both show positive trends. In the case of manufacturing, however, the share falls towards the end of the sample. As mentioned above, this is not independent from the process of structural change in the US economy, as sectors using skilled workers more intensively tend to grow faster since the 1970s.[24]

The question is then whether these changes in the composition of the labor force could have affected the aggregate elasticity of substitution. In a two-factor CES production function, σ is constant. However, in the presence of heterogeneous labor

[23] Galí and van Rens (2010) also point towards changes in the labor market to explain the decreased procyclicality of labor productivity and the increased volatility of the real wage. They, however, focus on improved matching due to increased labor market flexibility. We note that Rotemberg (2008) shows that the volatility of wages is a positive function of the elasticity of capital-labor substitution within a search and bargaining model. Also, Sargent and Wallace (1974) show that the elasticity is a key parameter to understand the cyclical behavior of productivity and wages.

[24] Using the EU-Klems Growth and Productivity database (www.euklems.net), we decomposed changes in the share of skilled workers in employment for 54 SIC sectors in the US for the 1970-2004 period. We found that around 20% of the increase in the share of skilled workers is due to structural change alone. For the 1970s, however, the contribution of structural change is around 30%. This is, however, bound to be a very low estimate, as it does not take into account inter-sectoral linkages and the level of disaggregation is relatively small. Nevertheless, it is consistent with the results of Hendricks (2010).

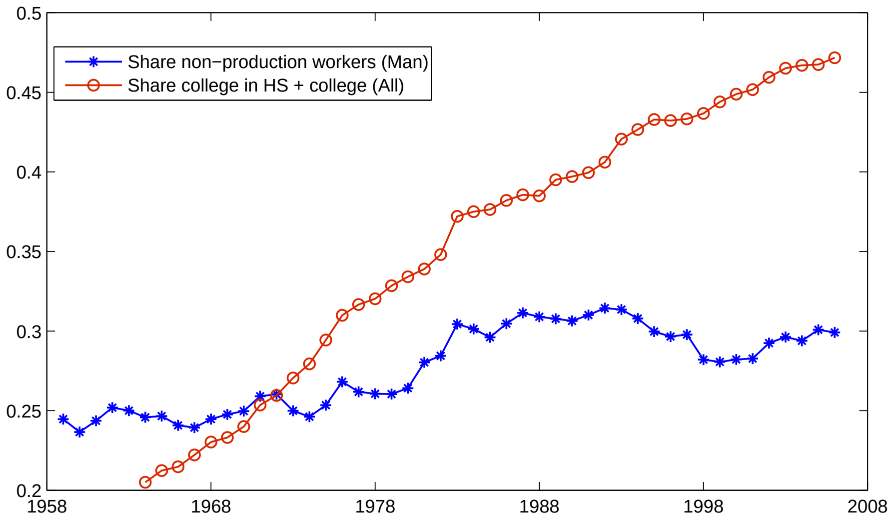


Figure 9: Shares of non-production workers (manufacturing) and college-plus hours in high-school-plus hours (aggregate economy).

(i.e. skilled and unskilled), the aggregate capital-labor elasticity of substitution is not constant and will depend, among other things, on the share of skilled labor hours in total hours input. We focus here on the case of a CES with three factors of production (capital, skilled labor, and unskilled labor) and use the common specification of a two-level nested CES function. Here, the effects of changes in the share of skilled workers will depend on the (constant) elasticities of substitution between the three factors, and the type of nesting specified for the CES.[25] Thus, we analyze the effect of changes in the proportion of skilled workers under three possible nestings.[26]

Without loss of generality, and for simplicity, we ignore technological process terms and time subscripts and assume all variables are measured at the normalization point. We denote skilled labor as S and unskilled labor as U. The first nesting corresponds

[25] Papageorgiou and Saam (2008) also show that, within this kind of CES specification, the aggregate elasticity is a negative function of capital intensity. This may also help explain some of the shorter-run changes observed in our estimates.

[26] Note that, as will be apparent below, analyzing the effect of changes in the proportion of skilled workers on σ is equivalent to analyzing the effects of changes in skilled-saving relative to unskilled-saving technical change (which is skill-biased technical change if both are gross substitutes in production). Under equal elasticities of substitution, it would also be equivalent to a change in the proportion of workers towards skill-intensive sectors.

to:

$$Y = \left[\pi_X X^{\psi} + (1 - \pi_X) U^{\psi}\right]^{1/\psi} \tag{7}$$
$$X = \left[\pi_K K^{\theta} + (1 - \pi_K) S^{\theta}\right]^{1/\theta}, \tag{8}$$

where ψ and θ are the inter- and intra-class substitution parameters, π_X is the income share parameter for aggregator X at the point of normalization, and π_K is the share parameter of capital in X (also at the normalization point). The corresponding elasticities of substitution are $\sigma_{K,S} = \frac{1}{1-\theta}$ and $\sigma_{K,U} = \sigma_{S,U} = \frac{1}{1-\psi}$ with $-\infty < \theta < 1$ and $-\infty < \psi < 1$. It is worth noting that the Cobb-Douglas case occurs when ψ (θ) $= 0$, the Leontief case when ψ $(\theta) = -\infty$, and the perfect substitutes case when ψ $(\theta) = 1$. The second nesting is:

$$Y = \left[\pi_X X^{\psi} + (1 - \pi_X) S^{\psi}\right]^{1/\psi} \tag{9}$$
$$X = \left[\pi_K K^{\theta} + (1 - \pi_K) U^{\theta}\right]^{1/\theta}, \tag{10}$$

where parameters have the same interpretation as in (7)-(8), but now $\sigma_{K,U} = \frac{1}{1-\theta}$ and $\sigma_{K,S} = \sigma_{S,U} = \frac{1}{1-\psi}$. And the third nesting is:

$$Y = \left[\pi_X X^{\psi} + (1 - \pi_X) K^{\psi}\right]^{1/\psi} \tag{11}$$
$$X = \left[\pi_S S^{\theta} + (1 - \pi_S) U^{\theta}\right]^{1/\theta}, \tag{12}$$

where we have $\sigma_{S,U} = \frac{1}{1-\theta}$ and $\sigma_{K,S} = \sigma_{K,U} = \frac{1}{1-\psi}$.

The nestings differ in terms of the assumptions imposed about the value of the elasticity of substitution across factors. While in the first nesting both K and S are equally substitutable for U but not between them, in nesting two both K and U are equally substitutable with S but not between them. Nesting (7)-(8) has been widely used in the capital-skill complementarity literature as discussed in Krusell, Ohanian, Ríos-Rull and Violante (2000). Capital-skill complementarity in this nesting simply implies that $\psi > \theta$. In nesting two, however, capital-skill complementarity implies that $\theta > \psi$ such that capital is more substitutable with U than with S. Note, however, that the third nesting does not allow for capital-skill complementarity as both skilled and unskilled workers are assumed to substitute capital the same way. In fact, it is easy to show analytically that, in this case, the *aggregate* elasticity of substitution between labor and capital is simply $\frac{1}{1-\psi}$ which is constant. Hence, we leave aside the third nesting as, by construction, it cannot generate time-variation of σ.

In order to analyze the effect of changes in the proportion of skilled workers in the first two nestings, we define $n = \frac{U}{U+S}$ as the fraction of unskilled workers. Since total labor input is $H = U + S$, we can write $U = nH$ and $S = (1-n)H$. Now, we use the definition of the *aggregate* elasticity of substitution σ:

$$\sigma = \frac{\frac{w/r}{K/H}}{\frac{\partial(w/r)}{\partial(K/H)}}, \tag{13}$$

where r is the rental price of capital. Note also that, at the normalization point, $\frac{w}{r} = \frac{1-\pi_X\pi_K}{\pi_X\pi_K}\frac{K}{H}$. Using this and expression (13), Papageorgiou and Saam (2008) show that the aggregate elasticity of substitution between H and K is a harmonic mean of the elastcities of substitution in the nested CES functions that can be expressed as:

$$\sigma = \frac{1}{(1-\theta)+(\theta-\psi)g}, \tag{14}$$

$$g = \frac{\pi_K}{\frac{1-\pi_K}{1-\pi_X}+\pi_K}. \tag{15}$$

Since θ and ψ are constants, we can analyze the effect of a change in $(1-n)$ on σ by obtaining the derivative of g with respect to $(1-n)$. We are then in a position to state the following lemma:

Lemma 1 *The aggregate capital-labor elasticity of substitution σ is a positive function of the share of skilled workers $(1-n)$ (and the productivity of skilled relative to unskilled workers) if:*

1. *$|\theta| > |\psi|$ for the first three-factor CES nesting (X, U);*
2. *$|\theta| < |\psi|$ for the second three-factor CES nesting (X, S).*

Proof. See Appendix D. ■

Take the first nesting. This condition would imply that if capital and skills are complements (within the X aggregator), i.e. $\theta < 0$, and unskilled workers and K and U substitutes ($\psi > 0$), the degree of complementarity between K and S has to be stronger than the degree of substitutability between U and the other two factors. On the other hand, this would also be the case if all factors are substitutes ($\theta > 0$ and $\psi > 0$) but U is less substitutable for X than S and K are between each other. The same conclusions apply for the other nesting bearing in mind that, in this case, $\theta > \psi$ implies capital-skill complementarity.

The question is, of course, how likely is this to be the case? Estimates of the skilled-unskilled workers substitution parameter ψ usually range between 0.25 and 0.5.[27] Regarding substitution between capital and skilled workers, estimates differ by study and are less abundant. Krusell et al. (2000) find and elasticity of 0.67 ($\theta \simeq -0.5$). However, given that aggregate σ is estimated to be substantially below unity (see Chirinko (2008)) and our estimates for the full sample are below 0.2, this elasticity is likely to be even lower. Hence, the conditions for a positive effect of $1-n$ on σ are plausible.

Based on this, we carry out a simple numerical exercise. We calibrate ψ to a value of 0.33 (corresponding to an elasticity of 1.5). Baseline values for the shares are $\pi_X = 0.6$ and $\pi_K = 0.5$, corresponding to a an aggregate capital income share of 0.27 and a skilled income share of 0.33. The initial share of skilled workers as a proportion of total workers is 20% ($n = 0.2$). To be compatible with our low σ estimate, we then set $\theta = -3$ corresponding to a plausible elasticity of 0.25. The value of the aggregate elasticity of substitution yields 0.32. We then analyze the impact of an increase of the share of skilled workers of 0.25 (25 percentage points) similar to that observed in the data. The corresponding new value for σ is almost 0.9. This large change is thus compatible with that observed in our estimates.[28]

Within reasonable bounds, hence, the effect of the change in the relative proportion of skilled workers is compatible with our conjecture and may have driven the change in the response of hours to technology shocks observed in the data. Similar conclusions could be drawn by considering changes in the skill-bias content of technical change or structural change towards skill-intensive sectors. Indeed, these well documented changes in the US labor market can plausibly have an important effect on how shocks are transmitted into the economy.

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[27] For evidence on the elasticity of substitution between workers by skill level see, amongst many others, Katz and Murphy (1992), Autor, Katz and Krueger (1998), Ciccone and Peri (2005) and Autor et al. (2008). Most of these estimates range between 1.3 and 2.5, with consensus estimates around 1.5, corresponding to $\psi = 0.33$.

[28] Recently, Balleer and van Rens (2009) analyze the effect of skill-biased technology shocks on the labor market using a SVAR identification scheme. Their findings show that the response of the wage premium to investment-specific shocks is incompatible with capital-skill complementarity. Their preferred model would display a strong capital-skill substitutability such that $\theta > \psi > 0$. This would also be compatible with the results from Lemma 1. Nevertheless, we note that this would imply an aggregate σ much larger than 1, which clashes with a large body of evidence for the US where $\sigma \ll 1$. Also, this would imply a strongly pro-cyclical aggregate labor share. The correlation of the private sector labor share with output growth in the data, however, is about -0.4.

7 Conclusions

We analyze the time variation of the response of hours worked to technology shocks observed in the US economy over the last 60 years. We first report evidence based on a SVAR model with long-run restrictions estimated on rolling samples. Consistent with previous results, the correlation between hours and the technological process conditional on technology shocks increases over the sample in a non-monotonic fashion. We then propose a structural interpretation of this time variation using a parsimonious RBC model with a Constant Elasticity of Substitution production function. Within this setting, the sign of the response of hours crucially depends on the relative magnitudes of the elasticity of capital-labor substitution and the capital income share.

We estimated the model using Bayesian methods. For the whole sample, the proposed specification fits the postwar US data on productivity and hours worked reasonably well, especially when compared to a standard Cobb-Douglas specification. We then estimate the model on rolling samples of the same length as our SVAR and find that there is a significant sign variation in the response of hours worked to a positive labor-augmenting technology shock. We find that the time-varying impulse responses to a labor-augmenting shock obtained from the estimated model track satisfactorily the changes observed in the data-based SVAR in spite of its parsimonious nature. Such variation is driven by a change in the magnitude of the elasticity of factor substitution: we observe an increase in the elasticity of capital-labor substitution towards the end of the sample that leads to a change in the sign and size of the response of hours.

We conjecture that the observed increase in the aggregate elasticity of substitution driving our results may be associated to the changing skill composition of the labor force, a change in the skill content of technological change, or structural change towards skill-intensive sectors. With heterogeneous labor, an increase in the share of skilled workers or their relative productivity can lead to an increase in the aggregate elasticity of substitution that is quantitatively compatible with that observed in the time-varying estimates. This highlights the importance of further research on the role of changes in the skill composition of the labor force and skill-biased technical change for the transmission of macroeconomic shocks.

Our analysis also brings two other important byproducts. First, as a first attempt to estimate the elasticity of factor substitution in a general equilibrium setup, our findings show a low estimated value around 0.15 over the full sample. We thus find little support for the Cobb-Douglas aggregate production function. A more general specification of the production side is preferred to better characterize the evolution of hours and productivity in the US economy. Second, capital-augmenting technology shocks are found to be the main driving force of the fluctuations of hours over the full sample.

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A Data construction

We explore the robustness of the empirical estimates to alternative databases. For all databases the time span covers the period from 1948:Q1 until 2006:Q1 and the series were obtained from the FRED database. Labor productivity is computed as the ratio between the measure of output and hours, and we take logarithms of both series. We indicate with p_t^j labor productivity and with $hobs_t^j$ hours of database j. In parenthesis, we indicate the ID series in the FRED database of the Federal Reserve Bank of Saint Louis.

The first database (GG) follows closely the data construction in Galí and Gambetti (2009) which is used in the main body of the paper. We consider output in the non-farm business sector (OUTNFB), and hours of all persons in the non-farm business sector (HOANBS) and the civilian non-institutional population of 16 years and over (CNP16OV). We thus have

$$\begin{aligned} \Delta p_t^{GG} &= \Delta \left(\ln \frac{OUTNFB_t}{CNP16OV_t} \right) \\ hobs_t^{GG} &= \ln \frac{HOANBS_t}{CNP16OV_t} \end{aligned}$$

An second database is considered following the work in Chang, Doh and Schorfheide (2007). We employ Average Weekly Hours of the non-farm Business Sector (PRS85006023), total non-farm employees (PAYEMS), Civilian non institutional population of 20 years and over (CNP20OV = LNU00000025 (men) + LNU00000026(women)), real GDP (GDPC96). The database (CDS) is

$$\begin{aligned} \Delta p_t^{CDS} &= \Delta \left(\ln \frac{GDPC96_t}{CNP20OV_t} \right) \\ hobs_t^{CDS} &= \ln \frac{PRS85006023_t * PAYEMS_t}{CNP20OV_t} \end{aligned}$$

The third data set is constructed following Ríos-Rull et al. (2009), where the series are similar to CDS but normalized by a different population structure, i.e. the civilian non-institutional population of 16 years and over (CNP16OV). The database (RR) is

$$\begin{aligned} \Delta p_t^{RR} &= \Delta \left(\ln \frac{GDPC96_t}{CNP16OV_t} \right) \\ hobs_t^{RR} &= \ln \frac{PRS85006023_t * PAYEMS_t}{CNP16OV_t} \end{aligned}$$

Finally, we borrow the last dataset form the work by Francis and Ramey (2009), where they propose a new measure of hours per capita and a new measure of productivity. Both series are adjusted for sectoral shifts and for changes in the composition of the age structure of the working population. The authors have kindly shared the data and are available at: http://weber.ucsd.edu/∼vramey.

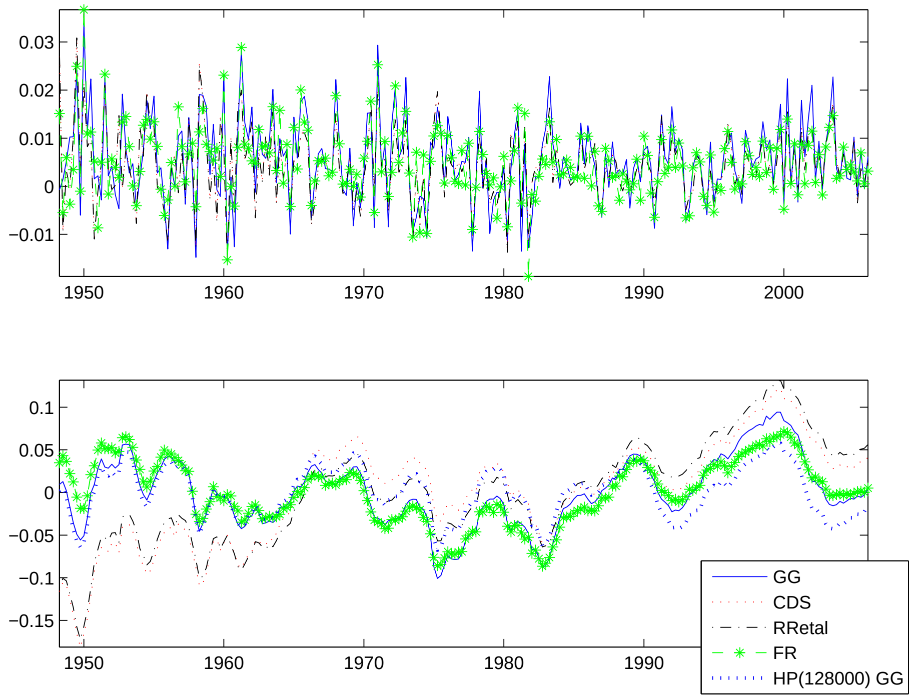


Figure 10: Growth rate of productivity and hours worked in different databases

Figure 10 reports the growth rate of productivity and the evolution of hours worked across different data sets. While there are minor differences in the growth rate of productivity, the pattern of hours worked looks distinct across measures. In particular, the series constructed using Average Weekly Hours and total non-farm employees (as in CDS and RR) display a more pronounced upward trend then the one constructed in GG or in FR. This is clearly visible at the beginning and at the end of the sample. This suggests that the series display different properties at long run frequencies. Table 2 presents some sample moments. All measures of hours worked display very similar autoregressive properties. However, there are important differences in the volatility both in terms of magnitude and in terms of location across frequencies of the spectrum. While the measures built with Average Weekly Hours and total non-farm employees are more volatile, most of their volatility is located outside business cycle frequencies, which is not the case for the series of hours worked

Database	ar	sd	% of vol at BC freq	% of vol at medium term freq
GG	0.97	0.039	36	47
CDS	0.98	0.059	15	20
$RRetal$	0.99	0.057	16	21
FR	0.98	0.036	27	36

Table 2: AR, Standard deviations and Percentage of volatility at selected frequencies for different measures of hours worked. BC fluctuations are obtained by carving out fluctuations with a periodicity less then 32 quarters. Medium term fluctuations are obtained by carving out fluctuations with periodicity less then 48 quarters.

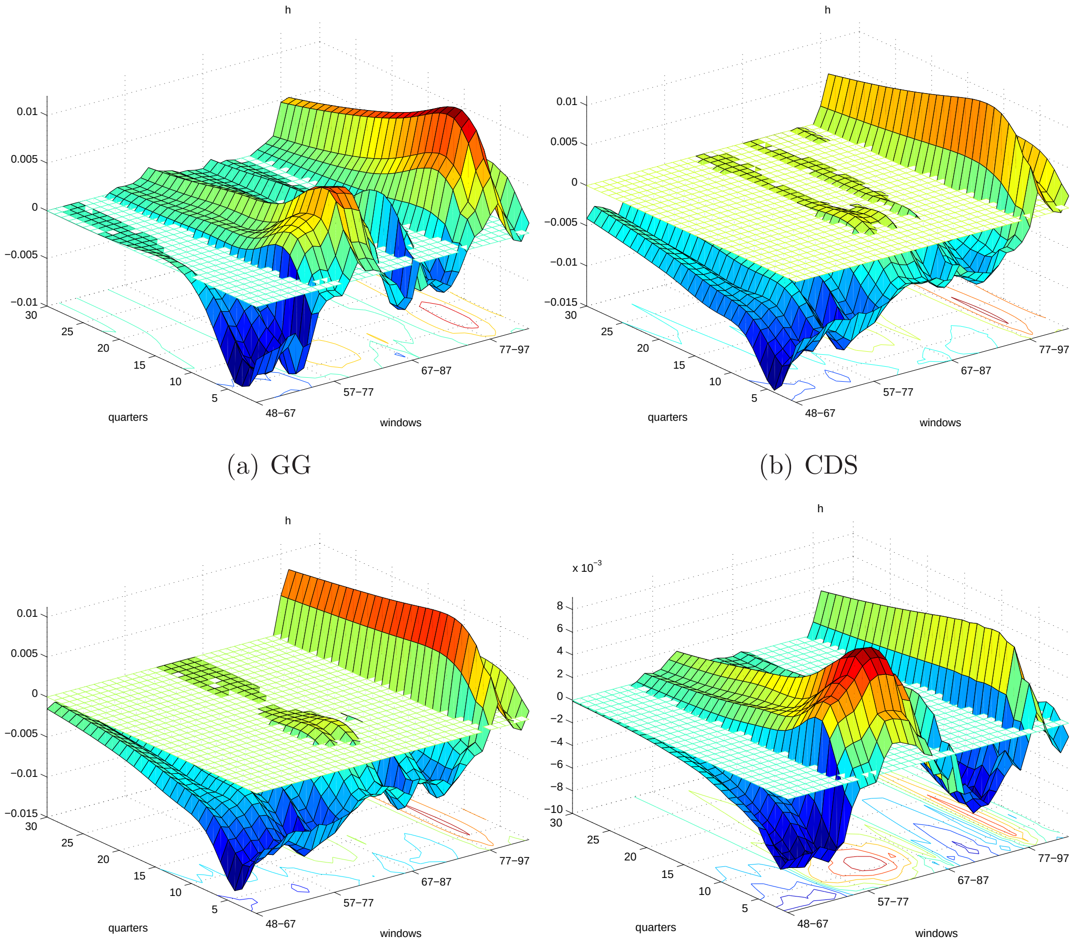
constructed in GG or FR. Moreover, the data where most of the volatility is located at typical business cycles frequencies is the measure used in GG. Hence, without a strong a priori preference for a particular measure we tend to prefer the measure where most of the power spectrum is located between 2 and 32 quarters.

Despite these differences, the (time varying) response of hours worked to an identified technology shock looks similar across data series. Figure 11 reports the response of hours across different settings on 39 overlapping windows of 20 years length and with the long run restriction identification scheme. While there are differences in selected sub samples, the broad picture that hours worked responded to technology negatively in early samples and positively in recent samples is consistent across different measures of hours worked.

B Identification of σ

This section verifies whether data can carry enough information to pin down σ in estimation.[29] Without loss of generality, we assume that the model is stationary, i.e. $0 < \psi_{1,h} < 1$ and $\psi_{2,h} = 0$. We simulate 100 observations for output and productivity assuming that $\alpha = 0.4$ and $\sigma = 0.2$ in one case (Case A) and that $\alpha = 0.4$ and $\sigma = 0.99$ in the other case (Case B). We then estimate the structural parameters of the model using Bayesian techniques. Prior elicitation is pretty standard. We assume inverse gamma for standard deviations, beta distributions for the autoregressive parameters, a normal distribution for the inverse of the Frisch elasticity, γ and for the capital intensity in production, α. All priors are centered at the true values. For the capital-labor elasticity of substitution, σ, we assume a uniform prior with 0 and 1.5 as

[29]The estimation of σ presents some econometric challenges, especially when combined with estimates of factor-augmenting technical change. See León-Ledesma et al. (2010).



(a) GG (b) CDS

(c) RR et al (d) FR

Figure 11: Impulse responses of hours to a positive labor augmenting technology shock using different database.

boundaries. Posterior medians and credible sets are reported in Table 3. Data on productivity and hours worked appear informative about the parameters and shocks of interest. Typically, posterior credible sets include the parameter value used to simulate data. Even for σ, where we postulate a flat prior, the likelihood peaks very close to the true population value meaning that, if the model is correctly specified, we are able to pin down in estimation the parameters governing the CES production function.

Since the estimate of these parameters are not far off the 'true' population value, and since their relative magnitude determines the sign of the conditional and unconditional correlations, we expect to be able to track the correct sign of such correlations. In particular, the last row of Table 3 displays the unconditional correlation between

	true	Case A	Case B
α	0.40	0.39[0.31,0.47]	0.41 [0.35,0.48]
σ	0.20/0.99	0.27 [0.18,0.37]	1.13 [0.86,1.48]
γ	1.00	1.01 [0.85,1.18]	1.00 [0.85,1.15]
ρ_v	0.60	0.61 [0.43,0.77]	0.52 [0.42,0.62]
ρ_k	0.60	0.49 [0.34,0.63]	0.60 [0.36,0.83]
$\psi_{1,h}$	0.60	0.65 [0.56,0.75]	0.61 [0.46,0.76]
σ_v	0.01	0.0110 [0.0095,0.0125]	0.0093 [0.0064,0.012]
σ_k	0.01	0.0090 [0.0066,0.0102]	0.0108 [0.066,0.0149]
σ_h	0.01	0.0096 [0.0061,0.0128]	0.0094 [0.008,0.0108]
$corr(p_t, h_t)$	-0.15/0.15	-0.30 [-0.06,-0.49]	0.14 [-0.04,0.29]

Table 3: Prior and Posterior estimates with simulated data. Median and the credible sets in parenthesis.

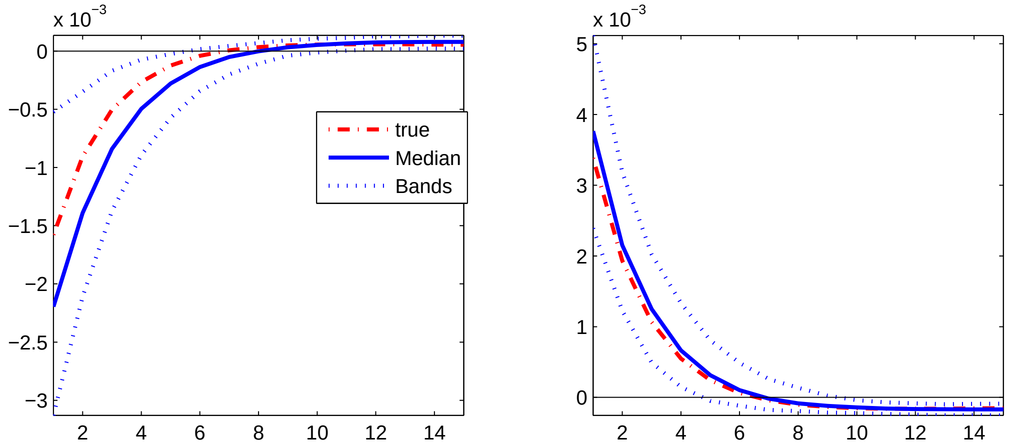
hours and productivity and its estimates.[30] On average, the signs are correctly identified for both cases. Similarly, Figure 12 reports the (true and estimates) impulse response of hours worked to a labor-augmenting technology shock. The estimated impulse response correctly captures the the sign and the persistence of the response. Regardless of the relative magnitude of σ and α, the response of productivity to a labor-augmenting technology shocks is positive and correctly estimated (not shown here). Hence, the sign of the correlation of hours and productivity conditional on an labor-augmenting technology shock crucially depends on the estimated relative magnitude of σ and α. We conclude that data on productivity and hours worked contain enough information to correctly capture conditional and unconditional moments of productivity and hours worked in our model.

C RBC model with investment adjustment costs

We consider a Real Business Cycles (RBC) model with Constant Elasticity of Substitution (CES) production function and with investment adjustment costs. We model those cost so that

$$K_t = (1-\delta)K_{t-1} + (1 - s(X_t))I_t$$

[30]The bands of the estimated correlation are obtained by simulating 50 times the model using the mean, and for each simulated data sets we compute the correlation.



(a) Case A

(b) Case B

Figure 12: Impulse response (true and estimates) of hours worked to a labor-augmenting technology shock. Left panel Case A, where $0.2 = \sigma < \alpha = 0.4$ and right panel Case B, where $0.4 = \alpha < \sigma = 0.99$.

where $s(1) = s'(1) = 0$ and $s''(1) \neq 0$ and $X_t = I_t/I_{t-1}$; K_t is capital and I_t is investment. The rest of the model follows the one presented in the text.

The system of equilibrium condition is given by

$$
\begin{aligned}
V_t h_t^{\gamma} &= \frac{W_t}{C_t} \\
1 &= q_t \left(1 - s(X_t) - s'(X_t)X_t\right) + \beta E_t \left(q_{t+1} \frac{C_t}{C_{t+1}} s'(X_{t+1}) X_{t+1}^2 \right) \\
q_t &= E_t \beta \frac{C_t}{C_{t+1}} \left(r_{t+1} + q_{t+1}(1-\delta) \right) \\
Y_t &= y \left[\alpha \left(\frac{Z_t^k K_{t-1}}{k} \right)^{\frac{\sigma-1}{\sigma}} + (1-\alpha) \left(\frac{Z_t^h h_t}{h} \right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \\
W_t &= (1-\alpha) \left(Z_t^h \frac{y}{h} \right)^{\frac{\sigma-1}{\sigma}} \left(\frac{Y_t}{h_t} \right)^{\frac{1}{\sigma}} \\
r_t &= \alpha \left(Z_t^k \frac{y}{k} \right)^{\frac{\sigma-1}{\sigma}} \left(\frac{Y_t}{K_{t-1}} \right)^{\frac{1}{\sigma}} \\
K_t &= (1-\delta)K_{t-1} + (1 - s(X_t))I_t \\
Y_t &= C_t + I_t \\
X_t &= I_t/I_{t-1}
\end{aligned}
$$

where q_t is Tobin q, or equivalently the ratio between the multipliers of the household constraints. Assuming $\psi_1 = 1$, Z_t^h is not stationary and the dynamics of the model are explosive. In order to have a well defined steady state, we need to re-scale the real variables by the non stationary process.

At the non stochastic steady state we have $h^\gamma = \frac{w}{c}$, $1 = q$, $1 = \beta(r + 1 - \delta)$, $y = y$, $w = (1-\alpha)\frac{y}{h}$, $r = \alpha\frac{y}{k}$, $i = \delta k$, $y = c + i$, $x = 1$. The log linearized equilibrium conditions, around the non-stochastic steady-state, of the variables rescaled by the non stationary process are (for simplicity, we indicate with small case letters the log deviation of a variable from its steady state)

$$y_t = i/y\ i_t + c/y\ c_t \tag{C.1}$$

$$k_t = (1-\delta)(k_{t-1} - z_t^h + z_{t-1}^h) + \delta i_t \tag{C.2}$$

$$y_t = \alpha k_{t-1} + \alpha z_t^k + (1-\alpha)h_t - \alpha(z_t^h - z_{t-1}^h) \tag{C.3}$$

$$q_t = c_t - c_{t+1} - z_{t+1}^h + z_t^h + \beta r r_{t+1} + \beta(1-\delta)q_{t+1} \tag{C.4}$$

$$q_t = s''(1)(x_t + (z_t^h - z_{t-1}^h)) - \beta s''(1)(x_{t+1} + (z_{t+1}^h - z_t^h)) \tag{C.5}$$

$$w_t = v_t + \gamma h_t + c_t \tag{C.6}$$

$$w_t = 1/\sigma(y_t - h_t) \tag{C.7}$$

$$r_t = (\sigma - 1)/\sigma z_t^k + 1/\sigma(y_t - k_{t-1}) + 1/\sigma(z_t^h - z_{t-1}^h) \tag{C.8}$$

$$x_t = i_t - i_{t-1} \tag{C.9}$$

C.1 A priori sensitivity analysis

By means of a sensible calibration exercise, we can study the impact of a labor augmenting technology shock on hours worked for different values of the elasticity of capital-labor substitution and investment adjustment costs, the parameters of interest. Without loss of generality, we assume that the labor augmenting technology process is non stationary, i.e. $\psi_{1,h} = 1$. The rest of the parameters are calibrated as in section 3. We considered first the case where the production function is Cobb-Douglas ($\sigma \to 1$) and the capital adjustment cost varies from 0 to 20 (Figure 13 left panel). We then fix the adjustment cost parameter to a value of 2 and vary σ between 0.1 and 2 (Figure 13 right panel). A few things are worth noting. First, there is a sign switch in the response of hours due to a change in investment adjustment cost. However, this change in sign occurs only on impact. Indeed, regardless of the value of the investment adjustment cost, the response of hours turns positive after few quarters. Hence, while investment adjustment costs are able to generate negative hours responses on impact, they are unable to produce a long-lasting negative response of hours worked to a technology shock.[31] Second, with positive adjustment costs the elasticity of substitution is the crucial parameter which generates a long lasting positive or negative response of hours. However, the threshold is no longer uniquely determined by value of α. Third, the support of σ able to generate positive and negative response of hours has opened.

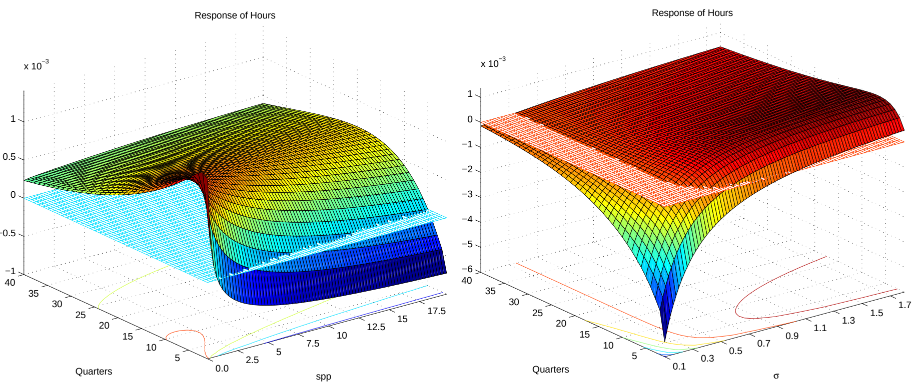


Figure 13: Impulse response of hours worked to a labor-augmenting technology shock for different values of s'' with $\sigma \to 1$ and $\alpha = 0.33$ (Left panel). Instantaneous response of hours worked to a labor-augmenting technology shock for different values of σ and $s'' = 2$ and $\alpha = 0.33$ (Right Panel).

D Proof of Lemma 1

We prove Lemma 1 for the first nesting corresponding to the Krusell et al. (2000) two-level CES, which we denote as (X, U) nesting. The results for the second nesting easily follow through from these. We first need to use the following results:

$$\frac{\partial \pi_X}{\partial X} = \frac{\psi}{X}\pi_X(1-\pi_X), \tag{D.1}$$

$$\frac{\partial X}{\partial(1-n)} = \frac{1-\pi_K}{1-n}X, \tag{D.2}$$

$$\frac{\partial \pi_K}{\partial(1-n)} = -\theta\frac{\pi_K(1-\pi_K)}{1-n}, \tag{D.3}$$

which, since, for any variables (z, q, s), $\partial z/\partial q = (\partial z/\partial s)(\partial s/\partial q)$, immediately implies

$$\frac{\partial \pi_X}{\partial(1-n)} = \psi\frac{\pi_X(1-\pi_X)(1-\pi_K)}{1-n}. \tag{D.4}$$

[31]This result is insensitive to different calibration of the remaining structural parameters and also introducing endogenous persistence with habits in consumption. We obtain similar results for $\gamma = \{0.5, 1.0, 1.5, 2, 2.5, 3.0\}$ and for different parameterizations of the exogenous processes.

With these results we can then calculate the partial derivative of $g = \frac{\pi_K}{\frac{1-\pi_K}{1-\pi_X}+\pi_K}$. After some tedious algebra, we can write this expression as:

$$\left.\frac{\partial g}{\partial(1-n)}\right|_{X,U} = \frac{-\frac{\pi_K(1-\pi_K)}{(1-n)(1-\pi_X)}(\theta+\psi)}{\left[\frac{1-\pi_K}{1-\pi_X}+\pi_K\right]^2}, \tag{D.5}$$

Since $\frac{\partial\sigma}{\partial(1-n)} = \frac{\partial\sigma}{\partial g}\frac{\partial g}{\partial(1-n)}$ we can derive, again after some algebra, the expression:

$$\left.\frac{\partial\sigma}{\partial(1-n)}\right|_{X,U} = -\Pi\frac{(\psi^2-\theta^2)}{[(1-\theta)+(\theta-\psi)g]^2}, \tag{D.6}$$

where $\Pi > 0$ is a function of share parameters:

$$\Pi = \frac{\pi_K(1-\pi_K)}{(1-n)(1-\pi_X)\left[\frac{1-\pi_K}{1-\pi_X}+\pi_X\right]^2} \tag{D.7}$$

Given that $\Pi > 0$ and that the denominator of (D.6) is positive, the effect of a change in $1-n$ will be positive if $\theta^2 > \psi^2$. Hence, in the (X, U) nesting, an increase in the share of skilled workers will increase aggregate σ if $|\theta| > |\psi|$. Following the same logic, in the (X, S) nesting, the effect will be positive as long as $|\theta| < |\psi|$.