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DSGE Model tell us?**

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# MACRO-PRUDENTIAL POLICY ON LIQUIDITY: WHAT DOES A DSGE MODEL TELL US?\*

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## **Abstract**

The financial crisis has led to the development of an active debate on the use of macro-prudential instruments for regulating the banking system, in particular for liquidity and capital holdings. Within the context of a micro-founded macroeconomic model, we allow commercial banks to choose their optimal mix of assets, apportioning these either to reserves or private sector loans. We examine the implications for quantities, relative non-financial and financial prices from standard macroeconomic shocks alongside shocks to the expected liquidity of banks and to the efficiency of the banking sector. We focus on the response by the monetary sector, in particular the optimal reserve-deposit ratio adopted by commercial banks over the business cycle. Overall we find some rationale for Basel III in providing commercial banks with an incentive to hold a greater stock of liquid assets, such as reserves, but also to provide incentives to increase the cyclical variation in reserves holdings as this acts to limit excessive procyclicality of lending to the private sector.

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# 1 Introduction

The recent turmoil in financial and credit markets along with the decoupling of market interest rates from the central bank policy rate has reawakened a latent interest in the connection between money, bank lending, the price of credit and the conduct of monetary policy,<sup>1</sup> particularly as there been an active and ongoing debate about the appropriate regulatory framework for commercial banking. What might ultimately be termed the first generation of micro-founded monetary policy models have had little to say on these questions, as money was determined by consumption and investment plans and there was no explicit banking sector creating credit, or broad, money at variable interest rate mark-ups to the policy rate. In this paper, we seek to address the regulatory question by considering the role of commercial banks and their reserves by using a model in which commercial banks create loans actively and so any regulatory constraints will have clear macroeconomic consequences.

The Goodfriend-McCallum (2007) model is a standard Calvo-Yun monopolistically competitive production economy with sticky prices where households respect their budget constraint in formulating consumption plans. But under a cash-in-advance constraint, households must hold bank deposits to effect transactions. A loans technology for the banking sector is adopted,<sup>2</sup> which meets the requirements of the private sector subject to screening and monitoring constraints. Households can work either in the goods producing sector or in the banking sector monitoring loans quality. But in order to consider the implications of liquidity, this version of the model, banks also have to make a choice on their asset mix between reserves with the central bank or as loans with the private sector. The central bank in this model holds commercial bank reserves and sets the interest rate paid on those reserves. Finally, the government budget constraint is modified to include claims from reserves, as well as standard issuance of public debt to meet excess of expenditures over taxes. We also examine the alternate case in which banks maintain a fixed reserves-deposit ratio.

A banking sector of this form can both amplify and add persistence to a standard macroeconomic set-up. This is because decision rules for consumption are shown to incorporate the equilibrium level of liquidity provision and the price (or spread) of that provision. The recent boom and bust in advanced country debtor economies would seem to confirm the relevance of this insight. And so we consider what role cyclical variation in commercial bank reserves - and in particular the payment of interest on these reserves - might play in improving the conduct of monetary policy, the stabilization of output and prices and the stability of the financial system, in the face of financial shocks to collateral and to monitoring. A key insight is that reserves increase the degree of freedom for commercial banks who can control their profitability and need for contingent planning with another tool, liquid assets, as well as *ex ante* screening of liquidity constrained households and requirements for posted collateral. Providing a cyclical incentive to hold reserves by paying policy interest rates also seems to increase the efficacy of

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<sup>1</sup>The 2008 US Monetary Policy Forum, for example, discussed at length the need for policy makers to consider the level of credit extended to non-financial agents when setting monetary policy.

<sup>2</sup>See Goodfriend and McCallum (2007) and Chadha, Corrado and Holly (2008) for an outline of this modelling device and its implications for commercial bank asset creation. Other devices have been used to understand capital requirements.

monetary policy.

The payment of interest on reserves has long been an issue of contention in academic and public debate, Milton Friedman (1960) argued for the need to close the interest gap between different forms of government liabilities. There has been some discussion on reserves as a policy instrument. For example, Hall (2002) develops a model in which the payment of interest on reserves can be used to control the price level as a mechanism to implement monetary policy in a world without money. And periodically the Federal Reserve formally asked Congress for authority to pay interest on bank reserves held with the Federal Reserve Banks (Meyer, 2001; Kohn, 2004). Permission to pay interest was granted in 2006 under the Financial Services Regulatory Relief Act, but because of the implication for fiscal policy, the effective date of the legislation was originally postponed until 2011.<sup>3</sup>

However, following the adoption of TARP,<sup>4</sup> the Federal Reserve indicated to Congress that permission be granted immediately. The Bank of England has, within certain limits, paid interest on bank reserves since 2006. And the suggested rules for global regulatory standards under Basel III has made recommendations, inter alia, for a transition to a minimum liquidity coverage ratio by 2018. And so given the regulatory trend towards more liquidity provision, we ask whether a move towards equalizing the returns from different forms of government liabilities induces a stabilizing response from the holders of those liabilities as procyclical demand for liquid assets is induced that limits the growth (reduction) into (in) risky assets and lending classes during an expansion (contraction). As well as the regulatory question, we also consider whether procyclical variation helps the implementation of monetary policy by allowing commercial banks to have a greater incentive to hold liquid as well as illiquid assets. As the Turner Review put it: ‘at the macroeconomic and macro-prudential level, there is a tradeoff to be struck. Increased maturity transformation delivers benefits to the non bank sectors of the economy and produces term structures of interest rates more favourable to long-term investment. But the greater the aggregate degree of maturity transformation, the more the systemic risks and the greater the extent to which risks can only be offset by the potential for central bank liquidity assistance’.

Compared to a model where the commercial banks are silent partners, commercial banks in this model are able to deliver an endogenous dynamic response for various risk premia and for the supply of liquidity. Using standard methods, we can also compare the responses of our artificial economy with and without an endogenous choice on asset allocation, that is, on the one hand where the reserve to deposit ratio is fixed by fiat (or custom and practice) or, on the other hand, is the result of the choice

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<sup>3</sup>Estimates of the cost of paying interest made by the Congressional Budget Office suggest the cost in the first year would be \$253 million, rising to \$308 million by the fifth year, with a total over 5 years of \$1.4 billion over five years. This is based on the assumption that the federal funds rate would average 4.5% from 2008 to 2016 and the Fed would pay interest at a rate 0.1 to 0.15 percentage points below that. It projected required reserves of about \$8.3 billion. If the Fed only paid interest on excess reserves held then the cost would be considerably smaller. Though that would rise if commercial banks took up more use of the facility. See Goodfriend (2002) for a recent survey.

<sup>4</sup>The Troubled Asset Relief Program (TARP) is the US program to purchase assets and equity from financial institutions in order to strengthen its financial sector. It is the largest component of the government’s measures in 2008 to address the subprime mortgage crisis.

of the commercial banks in balancing the risks of a liquidity shortfall against the higher returns from lending to the private sector. We find that the economy where commercial banks have an endogenous choice over reserve holdings performs better in welfare terms than when commercial banks do not have such an incentive. The holding of reserves over the business cycle acts as a substitute for more costly employment of monitoring workers and thus reduces the volatility of interest spreads to shocks and increases the holding of liquid assets during an expansion and reduces such holdings over a contraction, which acts to help stabilize the impulse from the monetary sector.

The structure of this paper is as follows. Section 2 outlines a simple framework for understanding a stylized flow of funds and the role of commercial banks in the monetary system. We also set-up the government's budget constraint in this section, showing that the payment of the policy rate on bank reserves will mean that there will be a direct impact on the equation of motion for government debt. Section 3 outlines the implications of the loans production function approach for key macroeconomic decision rules, outlines the determination of key market interest rates and then derives the commercial banks' decision rule over reserve holding. Section 4 considers the implications of commercial banks asset management in terms of reserve holdings to account for the relative returns from holding reserves or producing loans and liquidity concerns. Section 5 explains the standard calibration techniques used. Section 6 outlines the results of the impulse response analysis and undertakes some welfare analysis of some key results. Section 7 concludes and offers some final observations.

## 2 Monetary Analysis

### 2.1 Reserves and the flow of funds

We introduce a simple framework for analyzing bank reserves on the monetary balance sheet. For simplicity, since we abstract from other forms of central bank money and concentrate on bank reserves alone in our model, high powered money is identical to reserves, as there is no outside money. More traditionally the central bank controls the stock of fiat money (outside money) and financial intermediaries create other forms of money, which are claims on the private sector. As financial intermediation allows alternative assets to serve as money, it offers a close substitute to (outside) fiat money and the ability of the central bank to determine the overall nominal level of expenditure depends on the relationship between outside and inside money. The central bank has a powerful tool to regulate financial intermediaries and to affect the quantity of money in circulation: reserves, which may be either or both of fractional and or voluntary.<sup>5</sup>

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<sup>5</sup>See Freeman and Haslag (1996) and Sargent and Wallace (1985).

Private Sector		Government	
Assets	Liabilities	Assets	Liabilities
Deposits $D$	Loans $(D - r)$	Tax $\sum_{i=0}^{\infty} \beta^i t_i$	Bonds $B$
Bonds $\gamma B$	Tax $\sum_{i=0}^{\infty} \beta^i t_i$		

Commercial Banks		Central Bank	
Assets	Liabilities	Assets	Liabilities
Reserves $r$	Deposits $D$	Bonds $(1 - \gamma)B$	Reserves $r$
Loans $(D - r)$			

We first look at the private sector's balance sheet. The private sector has two forms of assets: deposits,  $D$ , held at banks and some fraction of bonds,  $\gamma B$ , issued by the government.<sup>6</sup> Their liabilities are loans,  $D - r$ , provided by banks and the present value of tax payments. The government sector has liabilities in the form of outstanding public debt,  $B$  and assets given by the present discounted value of future taxation. The commercial banks' balance sheet liabilities are deposits,  $D$ . Some fraction of liabilities,  $r$ , is held as reserves and the rest,  $D - r$ , is available to be lent to the private sector. The central bank holds assets in the form of some fraction of government bonds  $(1 - \gamma)B$  with liabilities determined by central bank money, which are reserves in this model.<sup>7</sup> The net assets of commercial banks and of the central bank are both zero. The private sector has net assets given by  $D + \gamma B - (D - r + \sum_{i=0}^{\infty} \beta^i t_i)$  and so because  $r = (1 - \gamma)B$  and  $\sum_{i=0}^{\infty} \beta^i t_i = B$ , we can note that the net private sector assets are also zero.

We can see that from this flow of funds, when the private sector demands a higher level of deposits to fund a level consumption then it is the job of commercial banks to match a higher level of  $D$ . Commercial banks then decide upon the allocation between reserves and loans, the former are lodged with the central bank and backed by some issuance of short term debt,  $(1 - \gamma)B$ . The private sector net asset position is invariant to the proportion of reserves held as increases in  $r$  increase gross assets and liabilities by the same amount and the government finances its issuance of any short term debt with claims on tax payers.

### 2.1.1 The loan-deposit ratio and reserves

The ratio of loans to deposits,  $\frac{L}{D}$ , is a measure of the bank multiplier and can be expressed as  $1 - \frac{r}{D}$ . Therefore the higher is the level of reserves, the lower is the amount of loans relative to deposits and the lower is the level of amplification by the banking sector of any given level of reserves. It has been documented by Adrian and Shin (2008) that the rate of growth in assets relative to liabilities

<sup>6</sup>In this example we assume that the private sector is represented by households.

<sup>7</sup>If we operate in an open economy, central bank assets would also include foreign exchange reserves  $r^f$ .

for commercial bank holding companies over the business cycle has a strong procyclical element, which would imply here an increase in  $\frac{L}{D}$  over the business cycle. But this procyclical aspect might be mitigated if commercial banks are given an incentive to hold reserves during business cycle expansions and shed reserves during contractions.

Furthermore, as commercial banks have sought to minimize their holdings of liquid reserves on a secular basis, the importance of the banks' behavior towards asset accumulation for the overall economy may have increased. Figure 1 shows the secular decline in the holdings of liquid assets by UK banks since 1980 over a number of measures.<sup>8</sup> And Figure 2 shows the ratio of Sterling Reserves Balances held at the Bank of England to eligible Liabilities from July 2006 to June 2008 (and from 1997 to 2006 the ratio of Bankers' operational balances to Eligible Sterling Liabilities).<sup>9</sup> Note that, following the adoption of money market reforms, there is a price and a compositional effect, as both the return to reserves is higher and more financial institutions became part of the reserve scheme. Initially, UK banks as a whole chose to hold an average of almost 1.5% of eligible liabilities at the Bank of England. However, this was steadily run down to only 0.88 percent in August of 2007, as alternative investments became available. But, possibly as a consequence of global money market meltdown from August 2007, by October reserves jumped to 1.25 percent and, with some seasonal variation, they stayed at this higher average level up to June 2008. In the face of turmoil in financial markets and doubts about the soundness of other borrowers in the interbank market, earning a risk free return at the repo rate by raising reserve deposits at the Bank of England arguably became an attractive proposition.

## 2.2 Reserves and the fiscal position

How might paying interest on reserves change matters? If reserves do not attract a nominal rate of return, the incentive for banks to increase lending may be substantial when demand for loans increases or the loans production technology improves. Clearly one possibility to induce commercial banks to increase the quantity of reserves in their balance sheet is to pay an interest rate on reserves lodged with the central bank. But this will have a fiscal consequence as the central bank's ability to pay interest rates on reserves will require some funding from the government. So ultimately paying interest rates on reserves will rely on public sector's budget constraint.

The per period government budget constraint means that any excess of government expenditure,  $G_t$ , over tax receipts,  $T_t$ , and payment of interest on debt,  $R_{t+1}^B \gamma B_{t+1}$ , and/or reserves,  $R_t^{IB} r_t$ , will be financed by the issuance of bonds or central bank money given the consumption good price index,  $P_t^A$ . Note that the interest paid to the private sector is  $R^B$  and to commercial banks is  $R^{IB}$ , which is the policy rate in our model. Hence if we look at the consolidated budget identity for the government sector

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<sup>8</sup>The narrow ratio corresponds to cash plus Bank of England balances plus eligible bills. The reserve ratio is proxied by Bank of England balances plus money at call plus eligible bills. Finally the broad ratio is the reserve ratio plus cash plus UK gilts. Similar stories can be shown for most advanced economies.

<sup>9</sup>Note that Sterling Reserve Balances can be held up to maximum of £1bn or 2% of Sterling Eligible Liabilities, whichever is higher, see Andrews and Janssen (2005).

we note that:<sup>10</sup>

$$g_t - tax = \frac{r_t}{P_t^A(1 + R_t^{IB})} - \frac{r_{t-1}}{P_t^A} + \frac{\gamma B_{t+1}}{P_t^A(1 + R_{t+1}^B)} - \frac{\gamma B_t}{P_t^A} \quad (1)$$

so the government can finance its net expenditure by issuing government debt,  $\gamma B$ , or by issuing reserves,  $r_t$ . However if interest rate are paid on reserves they will become interest bearing and therefore comparable to government debt. Clearly any excess government expenditure can be financed by issuing bonds to the private sector or by supplying reserves to commercial banks at a differentiated interest. We leave the determination of the relative interest rates to section 3.1. As we assume a stationary level of debt in this model there are not implications for fiscal solvency in this set-up as all deviations from steady state debt to GDP are strictly temporary.

### 3 The General Equilibrium Monetary Model

As pointed out by Kiyotaki and Moore (2001) money aggregates should be reconnected to general equilibrium models as they affect consumption decisions of liquidity constrained households and the spreads across several financial instruments and assets. Similarly optimal reserve management by banks will affect loans and therefore consumption. A simple way to incorporate money and spreads into a general equilibrium setting is to study the banking sector proposed by Goodfriend and McCallum (2007). The model by GM complements the traditional accelerator effect (Bernanke *et al.*, 1999) with an attenuator effect, which is present in the model because monitoring effort is drawn into the banking sector in response to the expansion of consumption, which is accompanied by an expansion of bank lending that raises the marginal cost of loans and the external finance premium (EFP). Figure 3 describes the timeline of events. The main feature of the model is the inclusion of a banking sector alongside households, production and the monetary authority.

#### 3.1 Households and the Production Sector

Households are liquidity constrained and decide the amount of consumption and the amount of labour they wish to supply to the production sector and to the banking sector according to the following utility function:

$$U = E_0 \sum_{t=0}^{\infty} \beta^t [\phi \log(c_t) + (1 - \phi) \log(1 - n_t^s - m_t^s)], \quad (2)$$

where  $c_t$  denotes real consumption,  $n_t^s$  is supply of labour in goods sector,  $m_t^s$  is the supply of monitoring work in the banking sector and  $\phi$  denotes the weight of consumption in the utility function. They are subject to the budget constraint:

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<sup>10</sup>In this setting the government sector includes both the government and the Bank of England. We also assume that high powered money comprises only reserves not coins.



$$\begin{aligned}
& q_t(1 - \delta)K_t + \frac{\gamma B_t}{P_t^A} + \frac{D_{t-1}}{P_t^A} + w_t(n_t^s + m_t^s) + c_t^A \left(\frac{P_t}{P_t^A}\right)^{1-\theta} + \Pi_t \\
& - w_t(n_t + m_t) - \frac{D_t}{P_t^A} - tax_t - q_t K_{t+1} - \frac{\gamma B_{t+1}}{P_t^A(1 + R_t^B)} - c_t,
\end{aligned} \tag{3}$$

where  $q_t$  is the price of capital,  $K_t$  is the quantity of capital,  $P_t$  is the price of households' produced good,  $P_t^A$  is the consumption good price index,  $n_t$  is the labour demanded by households as producer,  $m_t$ , is the labour demanded by household's banking operation,  $w_t$  is the real wage,  $D_t$  is the nominal holding of broad money,  $tax_t$  is the real lump-sum tax payment,  $R_t^B$  is the nominal interest rate on government bonds purchased in  $t + 1$ ,  $B_{t+1}$ . We also assume that any profit from the banking sector,  $\Pi_t$ , goes to the households' sector. The Lagrange multiplier of this constraint is denoted as  $\lambda_t$  and  $\theta$  is the elasticity of household demand. Household choose the level of monitoring work,  $m_t$ , and the level of employment work,  $n_t$ , they wish to offer to the production and the banking sector.

At the same time households' consumption, given the cash-in-advance constraint, is affected by the amount of loanable funds they can obtain:

$$c_t = v_t D_t / P_t^A \tag{4}$$

where  $v_t$  denotes velocity and  $D_t$  are deposits.

The production sector, characterized by monopolistic competition and Calvo pricing, adopts a standard Cobb-Douglas production function with capital,  $K_t$ , and labour,  $n_t$ , subject to productivity shocks. Firms decide the amount of production they wish to supply and the demand for labour by equalizing sales to net production:

$$K_t^\eta (A1_t n_t)^{1-\eta} - c_t^A (P_t / P_t^A)^{-\theta} = 0, \tag{5}$$

where  $\eta$  denotes the capital share in the firm production function,  $A1_t$  is a productivity shock in the goods production sector whose mean increases over time at a rate  $\gamma$  and  $\theta$  denotes the elasticity of aggregate demand,  $c_t^A$ . The Lagrange multiplier of this constraint is denoted as,  $\xi_t$ . By clearing the household and production sectors,<sup>11</sup> we can define the equilibrium in the labour market and in the goods market. Specifically the demand for monitoring work:

$$m_t = \left( \frac{\phi}{\lambda_t c_t} - 1 \right) \frac{1 - \alpha}{w_t} c_t \tag{6}$$

depends negatively on wages,  $w_t$ , and positively on consumption,  $c_t$ , and where  $1 - \alpha$  is the share of monitoring in the loan production function. These two sectors also provide the standard relationship for the riskless interest rate and the bond rate.

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<sup>11</sup>For details on the model set-up, derivation and notation see the technical appendix, available on request.

### 3.2 Banking Sector

We now turn to the analysis of how the banking sector affects the economy. The production function for the quantity of loans is given by:

$$L_t/P_t^A = F(\gamma b_{t+1} + A3_t k q_t K_{t+1})^\alpha (A2_t m_t)^{1-\alpha} \quad 0 < \alpha < 1, \quad (7)$$

where  $A2_t$  denotes a shock to monitoring work,  $A3_t$  is a shock on capital as collateral and  $b_{t+1} = B_{t+1}/P_t^A(1 + R_{t+1}^B)$  the real value of bonds. The parameter  $k$  denotes the inferiority of capital as collateral in the banking production function, while  $\alpha$  is the share of collateral in the loan production function. Increasing monitoring effort is achieved by increasing the number of people employed in the banking sector and therefore reducing the employment in the goods production sector.

While in standard Calvo-Yun models nominal consumption plans pin down the demand for money, in this model with banking, money is produced by banks, so any shift in the supply of loanable funds generated by shocks to monitoring effort or collateral also affect consumption. Specifically the banking sector matches deposit demand from liquidity constrained consumers with a technology to produce loans by substituting monitoring work for collateral in supplying loans. Also, we assume that loans are affected by the reserve/deposit ratio,  $rr_t$ :

$$L_t = (1 - rr_t)D_t. \quad (8)$$

Note that while Goodfriend and McCallum (2007) assume a fractional reserve requirement where the reserve-deposit ratio is given, we analyze the implications of an endogenous bank choice of reserves holdings, which is derived in Section 4.

Simple substitution of the bank's loan production function in the household's cash in advance constraint (4) leads to:

$$c_t = v_t \frac{F(\gamma b_{t+1} + A3_t k q_t K_{t+1})^\alpha (A2_t m_t)^{1-\alpha}}{P_t^A(1 - rr_t)}. \quad (9)$$

The differentiation of (9) with respect to  $K_{t+1}$  gives an expression  $\Omega_t A3_t k q_t$  which is a function of the marginal value of collateralized lending:

$$\Omega_t = \frac{c_t \alpha}{\gamma b_{t+1} + A3_t k q_t K_{t+1}}. \quad (10)$$

which depends on consumption,  $c_t$ , and on the value of the collateral,  $q_t$  and  $b_t$ . This expression also enters in the asset price equation:

$$q_t = \frac{\left( E_t \frac{\lambda_{t+1}}{\lambda_t} q_{t+1} (1 - \delta) \beta + E_t \beta \eta \left[ \frac{\lambda_{t+1}}{\lambda_t} \frac{\xi_{t+1}}{\xi_{t+1}} \left( \frac{A1_t n_t}{K_t} \right)^{1-\eta} \right] \right)}{\left( 1 - \left( \frac{\phi}{c_t \lambda_t} - 1 \right) \Omega A3_t k \right)} \quad (11)$$

Finally the Central Bank sets the policy rate which affects the incentives of banks to hold reserves.

### 3.3 Consumption, monitoring work and asset prices

We now describe in more detail the main log-linear relationships which characterize the model. In our notation variables without time subscript denote steady-state values whereas those with a time subscript denote log-deviation from steady-state. A log-linear formulation of (9) shows how loanable funds affect the consumption of liquidity constrained consumers:

$$c_t = \left\{ \begin{array}{l} v_t c + rr_t c + (1 - \alpha)(m_t + a2_t) + \\ \alpha \left[ \frac{b}{b+k_1} b_t + \frac{k_1}{b+k_1} (q_t + a3_t) \right] \end{array} \right\} \left( \frac{b + k_1}{b(1 - \alpha) + k_1} \right). \quad (12)$$

With the presence of a cash in advance constraint, a shock to velocity,  $v_t$ , will increase consumption. Consumption,  $c_t$ , is also positively affected by the amount of monitoring work,  $m_t$ , where  $\alpha$  is the share of collateral in the loans production function and  $(1 - \alpha)$  represents the share of monitoring costs. It is also affected by the amount of collateral represented by bonds,  $b_t$ , and capital whose value is given by  $q_t$ . A positive shock to monitoring,  $a2_t$ , by increasing the efficiency with which banks produce loans, increases the supply of loans and therefore consumption. Similarly a negative shock to collateral,  $a3_t$ , by reducing the price of capital,  $q_t$ , will negatively affect consumption. The parameters  $c$ ,  $b$  and  $k_1$  represent the steady-state fraction of consumption in output, the holding of bonds and a composite parameter reflecting the inferiority of capital compared to bonds as liquidity.<sup>12</sup>

The demand for monitoring work, which derives from (6), is given by:

$$m_t = -w_t - \frac{(1 - \alpha)c}{mw} \left( c_t + \frac{\phi}{\lambda} \lambda_t \right). \quad (13)$$

A higher wage,  $w_t$ , will reduce the resources devoted to monitoring. Similarly monitoring will be affected by the marginal utility of consumption and the marginal value of households' funds,  $\lambda_t$ . The steady state parameters,  $m$ ,  $w$ , and  $\frac{\phi}{\lambda}$  represent the steady-state proportions of employment in the banking sector, the level of the real wage, and the ratio of the weight of consumption in the utility function relative to the steady-state shadow value of consumption.

With a banking sector of this type in the model, we can link money and asset prices directly to output and inflation, as consumption, which accounts for most of the fluctuations in output in this model, is closely dependent on money market perturbations, the development of banking technology and asset prices outcomes. Now money and lending affect consumption, the level of economic activity and will also have implications for asset prices.

A key term here is the marginal value of collateralized lending,  $\Omega_t$ , from (10), which increases as consumption rises and falls as collateral becomes more widely available:

$$\Omega_t = \frac{k_2}{b + k_2} (c_t - q_t - a3_t) - \frac{b}{b + k_2} b_t. \quad (14)$$

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<sup>12</sup>The parameter  $k_1 = \frac{(1+\gamma)kK}{c}$  is a function of consumption,  $c$ , of the parameter reflecting the inferiority of capital as collateral,  $k$ , of steady-state capital,  $K$ , and of the trend growth rate,  $\gamma$ .

$\Omega_t$  depends on the value of the collateral,  $q_t$  and  $b_t$ , on a collateral shock,  $a3_t$ , and on consumption,  $c_t$ . Higher levels of consumption increase the marginal value of capital and hence the collateral value,  $q_t$ . The increase in collateral value leads to more borrowing and more consumption. The parameter  $k_2$  is again a composite coefficient similar to  $k_1$ .<sup>13</sup>

The marginal value of collateralized lending also feeds back into the capital asset price equation,  $q_t$ , derived from (11):

$$\begin{aligned} q_t = & (\delta_1 + \gamma_1)(E_t \lambda_{t+1} - \lambda_t) + \delta_1 E_t q_{t+1} - \frac{k\Omega\phi}{c\lambda}(c_t + \lambda_t) + \\ & k\Omega\left(\frac{\phi}{c\lambda} - 1\right)(\Omega_t + a3_t) + \gamma_1 E_t [mc_{t+1} + (1 - \eta)(n_{t+1} + a1_{t+1})]. \end{aligned} \quad (15)$$

In (15) the marginal value of collateralized lending,  $\Omega_t$ , potentially can amplify asset price volatility and magnify the response of the economy to both real and financial shocks. Both real,  $a1$ , and financial shocks,  $a3$ , directly feed back into asset prices alongside the expected marginal productivity of capital  $[mc_{t+1} + (1 - \eta)(n_{t+1} + a1_{t+1})]$  where  $mc_{t+1}$  denotes marginal cost in period  $t + 1$ ,  $\eta$  is the share of capital in the goods production function and  $n$  is employment in the goods production sector. Similarly expected asset prices,  $E_t q_{t+1}$ , the change in the shadow value of households' funds ( $E_t \lambda_{t+1} - \lambda_t$ ) alongside the wedge between the marginal utility of consumption and the shadow value of funds also affect the value of capital,  $q_t$ . The parameter  $\delta_1$  is a composite function of the depreciation rate of capital while the parameter  $\gamma_1$  is a composite function of steady-state marginal costs, of steady-state employment in the goods sector and of the capital share in the production of goods.<sup>14</sup>

### 3.4 Market Interest Rates

The decision of the banking sector is articulated in two stages. In the first one interest rates are determined and then, given the constellation of spreads, banks decide the optimal level of reserves and assets in order to maximize expected returns. The benchmark theoretical interest rate  $R^T$  is simply a standard intertemporal nominal pricing kernel, priced off real consumption and inflation. Basically it boils down to a one-period Fisher equation:

$$R_t^T = E_t(\lambda_t - \lambda_{t+1}) + E_t \pi_{t+1}. \quad (16)$$

The interbank rate or policy rate is set by a standard feedback rule responding to inflation,  $\pi_t$ , and output,  $y_t$ , with parameters,  $\phi_\pi$  and  $\phi_y$ , respectively. Policy rates are smoothed by  $1 > \rho > 0$ .

$$R_t^{IB} = \rho R_{t-1}^{IB} + (1 - \rho)(\phi_\pi \pi_t + \phi_y y_t) \quad (17)$$

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<sup>13</sup>The parameter  $k_2 = \frac{kK}{c}$  is a function of  $k_1$ , of steady-state capital,  $K$ , and of the steady-state ratio of consumption,  $c$ .

<sup>14</sup>The parameter  $\delta_1 = \frac{\beta(1-\delta)}{1+\gamma}$  is a function of the discount factor,  $\beta$ , of the depreciation rate of capital,  $\delta$ , and of the trend growth rate,  $\gamma$ . The parameter  $\gamma_1 = \frac{\beta\eta mc}{1+\gamma}(\frac{n}{K})^{1-\eta}$  is function of steady-state employment in goods sector,  $n$ , of steady-state marginal costs,  $mc$ , of steady-state capital,  $K$ , and of the parameter reflecting the capital share in the production function of the goods sector,  $\eta$ . Details of the derivation are reported in the technical appendix, equation (A.12), available on request.

To find the interbank rate  $R^L$  we must equate the marginal product of loans per unit of labour  $(1 - \alpha) \frac{L_t}{m_t}$  to their marginal cost  $\frac{w_t}{P_t^A}$  with loans defined by the following relationship  $L_t = D_t(1 - rr_t) = \frac{c_t P_t^A}{v_t}(1 - rr_t)$ . Therefore in log-linear form the interest rate on loans,  $R_t^L$ , is greater than the policy rate by the extent of the external finance premium.

$$R_t^L = R_t^{IB} + \underbrace{[v_t + w_t + m_t + rr_t - c_t]}_{EFP_t}. \quad (18)$$

The external finance premium,  $EFP_t$ , is the real marginal cost of loan management, and it is increasing in velocity,  $v_t$ , real wages,  $w_t$ , monitoring work in the banking sector,  $m_t$ , and reserve requirements,  $rr_t$ , and decreasing in consumption,  $c_t$ . The yield on government bonds is derived by maximizing households' utility with respect to bond holdings,  $R_t^T - R_t^B = \left[ \frac{\phi}{c_t \lambda_t} - 1 \right] \Omega_t$ . In its log-linear form it is the riskless rate,  $R_t^T$ , minus the liquidity service on bonds, which can be interpreted as a liquidity premium (LP):

$$R_t^B = R_t^T - \underbrace{\left[ \left( \frac{\phi}{c_t \lambda_t} - 1 \right) \Omega_t - \frac{\phi \Omega}{c_t \lambda_t} (c_t + \lambda_t) \right]}_{LP_t}, \quad (19)$$

where  $(c_t + \lambda_t)$  measures the household marginal utility relative to households shadow value of funds while  $\Omega_t$  is the marginal value of the collateral. It is in fact these key margins - the real marginal cost of loan management versus the liquidity service yield - that determine the behavior of spreads. In the above expression,  $\phi$  denotes the consumption weight in the utility function whereas  $\lambda_t$  is the shadow value of consumption,  $c_t$ . The interest rate on deposits is the policy rate,  $R_t^{IB}$ , minus a term in the reserve deposit ratio:

$$R_t^D = R_t^{IB} - \frac{rr}{1 - rr} rr_t \quad (20)$$

## 4 Commercial Banks Asset Management

Monetary policy operates through the manipulation of short-term interest rates as the policy instrument, which affects the market clearing level of high powered money, or reserves. The previous section shows that this short term rate also impacts on other interest rates spreads via the external finance premium and/or the liquidity premium by changing the path of aggregate private or public demand. In this section, we develop an approach for considering the implications of introducing an incentive for commercial banks to hold reserves to account for the relative returns from holding reserves or producing loans and to deal with liquidity concerns.

Commercial banks may decide to vary the liquidity mix of their assets. One problem that has emerged in this business cycle is that insufficient attention was paid to the following problem that banks perform extensive maturity transformations which expose them to liquidity risks as they continually rollover of credit supply to the real economy and so procyclical swings in price of credit may then

shift the loans function in a procyclical manner. We therefore feel it is important in this model to allow commercial bank reserves to be endogenously chosen in this model and respond to varying incentives for banks to hold liquid assets. In this respect our model differs from Goodfriend and McCallum's (2007) benchmark model where commercial banks operate under a fixed fractional reserve requirement but that case will also be explored in comparison.

We adopt a simple expression for the commercial bank's within period expected returns. Given the constellation of interest rates as defined in the previous section, the bank's problem (see Baltensperger, 1980) is to maximize total returns within period subject to the returns from loans,  $L_t$ , which are lent out at the collateralized interest rate of  $R_t^L$ , reserves held at the central bank,  $r_t$ , which are assumed to pay the interbank (policy) interest rate,  $R_t^{IB}$ , and the payment of deposit interest,  $R_t^D$ , to deposits:

$$\max_{r_t} \Pi_t = R_t^L L_t + R_t^{IB} r_t - R_t^D D_t, \quad (21)$$

$$\text{s.t. } C_t = \frac{1}{2} R_t^T (\bar{r} - r_t)^2 + \tau_t (\bar{r} - r_t). \quad (22)$$

Here commercial banks' profits are subject to a side-constraint motivated by concerns about the management of liquid reserves. Note that reserves are returned at the end of the period but loans at the beginning of the next period. We assume that there is an exogenous target for the level of reserves,  $\bar{r}$ , perhaps set by custom and practice or by legislation.<sup>15</sup> The costs of reserve management,  $C_t$ , are then modelled in two parts: banks wish to smooth reserves and face a penalty rate of an uncollateralized external finance premium,  $R_t^T$ , in deviations of reserves from target and are also subject to a liquidity preference term,  $\tau_t$ , which we can think of as an *ex ante* probability of a liquidity shortfall. The first term will imply that reserves are likely to be smoothed over time because banks may not wish to implement large-scale changes in their asset allocation from period to period, as these may signal mismanagement of previous asset allocations or run reputational risks. The cost of deviation from target is the penalty interest rate, which is symmetric in this set-up. This is because if  $r_t < \bar{r}$ , the commercial banks will fund its shortfall at the penalty rate, and if  $r_t > \bar{r}$  the commercial banks will not be paid interest on excess reserves and pays the opportunity cost of lending its assets out at  $R_t^T$ . The liquidity preference term represents shifts in the commercial banks' chosen level of reserves and reflects an exogenous probability of a liquidity shortfall and so an increase in  $\tau_t$  corresponds to a fall in bank liquidity below the minimum required level  $\bar{r}$ .

Note that by choosing the reserve level, the asset side of the commercial banks balance sheet,  $L_t + r_t$ , is now fully determined and so by construction are liabilities, that is deposits,  $D_t$ . From the balance

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<sup>15</sup>In the Eurozone, for example, 2% of commercial bank reserves are lodged with central banks. In the UK, up to 2% of eligible reserves can be lodged with the Bank of England as interest bearing accounts. Basel III's liquidity coverage ratio will seek to increase the holdings of liquid assets.

sheet of the banking sector, discussed in the previous section,  $L_t = D_t - r_t$  so we can substitute and write the Lagrangian as:

$$\Pi_t = R_t^L(D_t - r_t) + R_t^{IB}r_t - R_t^D D_t + \lambda_t^r \left( C_t - \frac{1}{2}R_t^T(\bar{r} - r_t)^2 - \tau_t(\bar{r} - r_t) \right). \quad (23)$$

For which the first order conditions are:

$$\frac{\partial \Pi_t}{\partial r_t} = -R_t^L + R_t^{IB} + \lambda_t^r (R_t^T(\bar{r} - r_t) + \tau_t) = 0. \quad (24)$$

The Lagrange multiplier is the shadow value of reserve management and is given by the ratio of profits on reserves to the ‘precautionary’ motives for holding reserves:

$$\lambda_t^r = \frac{R_t^L - R_t^{IB}}{R_t^T(\bar{r} - r_t) + \tau_t}, \quad (25)$$

If  $\lambda_t^r$  is set to one as to reflect the equal relative importance of the two arguments, we can solve for the optimal level of bank reserves:

$$r_t = \frac{\tau_t + R_t^{IB} - R_t^L}{R_t^T} + \bar{r}. \quad (26)$$

Hence at the optimal profit rate the reserve ratio is determined by the interbank loan rate (the return on reserves) minus the returns on collateralized loans,  $R_t^{IB} - R_t^L$ , scaled by the penalty uncollateralized loan rate if reserves are different from target,  $R_t^T$ . Because the loan rate is higher than the interbank loan rate there is an incentive for banks to hold reserves below the target level,  $\bar{r}$ , and that helps us understand the secular tendency to hold fewer reserves. But with a sufficiently high preference for liquidity,  $\tau_t$ , then reserves will be held in excess. Another way to think about this expression is that the deviation of reserve requirements from steady-state is the ratio of the cost of a liquidity shortfall to the opportunity cost of holding further deposits.

Now let us examine the reserve choice by commercial banks in terms of market interest rates. Given (18) we can re-write (26) as:

$$\begin{aligned} r_t &= \frac{\tau_t}{R_t^L} + \frac{R_t^{IB} - R_t^L}{R_t^L} + \bar{r} \\ &= \frac{\tau_t}{R_t^{IB} + EFP_t} - \frac{EFP_t}{R_t^{IB} + EFP_t} + \bar{r}, \end{aligned} \quad (27)$$

which, introduces the trade-off between reserves being driven down (up) by higher (lower) external finance premia and the need to offset changes in the probability of a liquidity shortfall. Let us also

note that that the responsiveness of reserves to either  $\tau_t$  or  $EFP_t$  will be higher if the external finance premium is a lower fraction of the overall loan rate. This is because an increase in the costs of providing loans (e.g. which may result from an increase in real wages and/or extent of monitoring required) will directly reduce the supply of loans and hence increase the external finance premium for a given level of loans and raise the opportunity cost of holding reserves, which will then fall.

Figure 4 illustrates this key result. The two axes show reserves ( $r_t$ ) and loans ( $L_t$ ). For a given level of deposit creation,  $D_1$ , which depends on the nominal consumption expenditure and the velocity of circulation, the slope of the Asset Allocation Curve (AAC) is  $-\frac{1+R_t^L}{1+R_t^{LB}}$  and intercepts are set by  $\frac{D_1}{1+R_t^{LB}}$  and  $\frac{D_1}{1+R_t^L}$ . The ray  $OR$  draws the set of feasible equilibria under a fixed reserve ratio system for commercial banks. At the steady-state,  $A$ , the reserve ratio is at its long-run level, the slope of the AAC reflects the steady-state ratio of policy rates to lending rates. Around this steady state are concentric circle indifference curves reflecting the liquidity target of commercial banks. And we can see that when deposits increase to  $D_2 > D_1$  commercial banks have an incentive to increase the proportion of reserve holdings,  $B'$ , over and above the level implied by a fixed reserve ratio,  $B$ . This is because reserves are preferred to loans when there is a target level of reserves required to deal with expected liquidity shocks,  $\tau_t$ . Furthermore, if relative interest rates change when  $D_2 > D_1$  such that the policy rate rises relative to the loan rate the slope of the AAC curve becomes flatter and there is an even greater demand for reserves,  $B''$ . The argument is symmetric with a fall in deposits,  $D_3 < D_1$ , inducing reserves to fall by more than the reduction in loans. We can thus trace a slope for the endogenous reserve-deposit ratio, which is steeper than that for fixed reserve deposits and means that that commercial banks are induced to hold a higher fraction of reserves to deposits during an expansion and a lower fraction during a contraction. We will return to the policy implications of this result in the conclusion.

In the simulation exercise of the following sections we will consider two possible scenarios. In the first the reserve/deposit ratio,  $rr_t$ , is determined by the following relationship:

$$rr_t = \frac{r_t}{D_t} = \frac{1}{D_t} \left( \frac{\tau_t}{R_t^L} + \frac{R_t^{LB} - R_t^L}{R_t^L} + \bar{r} \right) \quad (28)$$

In the second we adopt a fixed reserve system where the reserve-deposit ratio is fixed:

$$rr = \frac{r_t}{D_t} \quad (29)$$

In the following paragraphs we introduce the calibrating assumptions of our exercise and we will then proceed to simulate the full model reported in the Technical Appendix (available on request).

## 5 Calibration

Table 1 provides a complete list of the endogenous and exogenous variables of the model and their meaning while Table 2 reports the values for the parameters and steady-state values of relevant



variables.<sup>16</sup> Following Goodfriend and McCallum (2007) we choose the consumption weight in utility,  $\phi$ , to yield 1/3 of available time in either goods or banking services production. We also set the relative share of capital and labour in goods production  $\eta$  to be 0.36. We choose the elasticity of substitution of differentiated goods,  $\theta$ , to be equal to 11. The discount factor,  $\beta$ , is set to 0.99 which is the canonical quarterly value while the mark-up coefficient in the Phillips curve,  $\kappa$ , is set to 0.05. The depreciation rate,  $\delta$ , is set to be equal to 0.025 while the trend growth rate,  $\gamma$ , is set to 0.005 which corresponds to 2% per year. The steady-state value of bond holding level relative to GDP,  $b$ , is set to 0.56 as of the third quarter of 2005.<sup>17</sup>

The parameters linked to money and banking are defined as follows. Velocity at its steady state level is set at 0.276 which is close to the ratio between US GDP and M3 at fourth quarter 2005, yielding 0.31. The fractional reserve requirement,  $rr$ , is set at 0.1 which is higher than the value of 0.005 assumed by Goodfriend and McCallum (2007) to allow for more symmetric fluctuation in reserves. The fraction of collateral,  $\alpha$ , in loan production is set to 0.65, the coefficient reflecting the inferiority of capital as collateral,  $k$ , is set to 0.2 while the production coefficient of loan,  $F$ , is set to 9.14. The low value of capital productivity reflects the facts that usually banks use higher fraction of monitoring services and rely less on capital as collateral.

With these parameters values we see that the steady state of labour input,  $n$ , is 0.31 which is close to 1/3 as required. The ratio of time working in the banking service sector,  $\frac{m}{m+n}$ , is 1.9% under the benchmark calibration, not far the 1.6% share of total US employment in depository credit intermediation as of August 2005. As the steady-states are computed at zero inflation we can interpret all the rates as real rates. The riskless rate,  $R^T$ , is 6% per annum. The interbank rate,  $R^{IB}$ , is 0.84% per annum which is close to the 1% per year average short-term real rate. The government bond rate,  $R^B$ , is 2.1% per annum. Finally the collateralized external finance premium is 2% per annum which is in line with the average spread of the prime rate over the federal funds rate in the US.<sup>18</sup> The model is solved using the solution methods of King and Watson (1998) who also provide routines to derive the impulse responses of the endogenous variables to different shocks, to obtain asymptotic variance and covariances of the variables and to simulate the data.<sup>19</sup> For the impulse response analysis and simulation exercise we consider the real and financial shocks described in Table 3, which reports the volatility and persistence

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<sup>16</sup>The full set of derivation of the model with a detailed description are reported in the Appendix, which is available on request.

<sup>17</sup>The steady state of the transfer level, the Lagrangian of the production constraint and base money depend on the above parameters. The steady state of the marginal cost is  $mc = \frac{\theta-1}{\theta}$ .

<sup>18</sup>The equations for the steady-states are listed in Section B of the Technical Appendix, available on request. The solution for the steady-states uses a nonlinear routine in Maple and the file is also available on request.

<sup>19</sup>The log-linearized equations for the model are listed in Section C of the Technical Appendix. King and Watson's MATLAB code is generalized in that for any model, we adapt three MATLAB files. The three files for the solution of our benchmark model `gmrsys.m`, `gmrdv.m` and `gmrcn.m` are available on request. King and Watson's package includes standardized auxiliary programs `impkw.m` to generate the impulse responses to different shocks to the endogenous variables and the program `fdfkw.m` to obtain the filtered autocovariances and the filtered second moments from the model solution. The program `impkwsimu.m` simulates the artificial series and allows to generate HP filtered data.

parameters chosen for the calibration and simulation exercise. These are standard parameters in the literature.

## 6 Model Results: Impulse Response Analysis

To understand the dynamics of this model, in this section we outline the impact of shocks to goods productivity, the policy rate and to an example of a financial sector shock. Figures 5-8 plot the log deviation from steady state responses of employment in the goods sector, monitoring employment, real wages, the asset price, real consumption, inflation, real deposits, real loans, real reserves, the external finance premium, the reserve deposit ratio, the interest rate on bonds, the policy rate, the loan rate and the liquidity premium. For each set of impulse responses two lines are drawn, one (solid) corresponding to the model where interest is paid on reserves and banks choose to optimize over reserve and loans in their asset portfolio and one (dotted) corresponding to a fixed fractional reserve system.

To fix some ideas let us explain that a key role is played by the external finance premium as a regulator of demand and by reserves as a regulator for the supply of loans. For example, a shock that raises the value of collateral held by households will tend to increase the availability of loans. But at the same time the collateral shock will increase the demand for deposits and the amount of monitoring work that needs to be carried out by banks. This increase in the employment of monitoring workers will be reflected in higher real marginal costs of loans and so there will be some pressure on the external finance premium to rise. The actual path of the external finance premium will depend upon the relative importance of these two effects. The former financial accelerator and latter attenuator is well explored in Goodfriend and McCallum (2007) but to which we add a further dimension. In our set-up, commercial banks use their reserves as a substitute for employing monitoring workers. If the external finance premium jumps in response to dominance in either the financial accelerator or attenuator effect and as a result of the signal from the policy rate, which provides a rate of return for reserves, commercial banks can substitute reserves and this acts to attenuate the fluctuations in the external finance premium and the liquidity premium; in this case monitoring costs are less sensitive to the shock and the marginal value of collateral, which determines the liquidity premium, becomes less important.

### 6.1 Endogenous Reserves

Figure 5 describes the effects of a shock to goods productivity. On impact a persistent shock to goods productivity raises consumption, and given the cash in advance constraint this will drive up the demand for deposits. Deposits are a function of loans provided by the banking sector, which requires monitoring work from the fixed supply of labour and thus a switch from work in goods production to work in banks monitoring loans. The productivity shock also raises the marginal productivity of capital and hence increases the asset price. The increase in the value of collateral that the asset price represents is long-lived and so eventually reduces the need for more monitoring work. But nevertheless the external finance premium increases, initially because the marginal costs of loan production have risen with the

level of monitoring work. When banks can choose their level of reserves directly, the fall in inflation brings down the policy rate and reduces the incentive to hold reserves compared to the returns from making loans and so monitoring employment rises. But once the policy rate starts to head back to its steady-state, reserves become more attractive and banks start to increase the reserve-deposit ratio, which leads to a shake-out in monitoring employment and a quicker return in the external finance premium to its base level. One important difference between the fixed reserve-deposit case and where reserves are endogenous is that in the longer run the persistent increase in wages, which are the same in the two sectors, means that there is an incentive for banks to hold reserves rather than employ monitoring workers and so the reserve-deposit ratio rises with the increase in economic activity.

Figure 6 reports the effects of a shock to collateral, which increases the asset price. On impact, a positive shock to collateral that increases the efficiency of producing loans induces banks to switch from the use of monitoring work and also reduces the need to hold reserves; consumption and therefore deposits increase. This also acts to increase the hours worked in goods production. The initial reduction in monitoring reduces the external finance premium and therefore increase the return on reserves with respect to the return on loans; hence reserves initially are lower but then start to increase as their return is higher. The main difference with the fixed reserve-deposit scenario is that when reserves are endogenous the reduction in real wages means that banks have the incentive to employ more monitoring work and economize on reserve holdings, so loans and deposits will expand. Given that deposits increase by more than reserves the reserve-deposit ratio falls more on impact when reserves are endogenous.

Figure 7 reports the effects of a positive shock on the policy rate. On impact reserves are higher as their return (the policy rate) is higher than the opportunity cost represented by the return on loans and the penalty rate. Lower real loans and deposits through the cash in advance constraint lead to a reduction in consumption, in the employment in the goods sector, in real wages and in the price level. Given the higher level of reserves monitoring work initially falls. The fall in monitoring work coupled with lower real wages causes a fall in the external finance premium which in turn affects the benchmark rate and the loan rate which are also lower on impact. In the second stage when the policy rate falls in response to falling prices and consumption the situations reverts. Consumption starts rising alongside employment in the goods sector and real wages. Because of the higher level of consumption, deposits start to rise and this leads to an increase in the supply of loans which can now be produced replacing monitoring work to collateral whose price in the meantime has risen in response to the lower capital-labour ratio. The main difference with the fixed reserve-deposit scenario is that when reserves are endogenous a shock on the policy rate, on impact, also increases the return on reserves. Initially banks have the incentive to increase their reserve holdings, except that in a second stage lower real wages induce banks to replace their reserve holdings with monitoring work, so loans and deposits expand. Given that reserves increase by more than deposits the reserve-deposit ratio is higher with endogenous reserves than with fractional reserves.

## 6.2 Bank Liquidity

Figure 8 shows the effect of a positive shock to the probability of a liquidity shortfall. What we can show is that rather than impacting on activity in a negative manner, if banks are able to increase reserves in response to such a shock, they can shed some of their loan production costs and mitigate the impact of a such a shock on the wider macroeconomy. Referring back to Figure 4, if liquidity through reserves are available, rather than rationing loans with a higher external finance premium, they can substitute some reserves for loans to match the required level of deposits and thus shed some costly monitoring workers and help prevent the external finance premium from rising.

On impact banks choose to have higher reserves and this compresses loans, monitoring work and the external finance premium. The reduction in the level of monitoring work and the parallel increase in the employment level in the goods production sector has a positive effect on asset prices, real wages and consumption. Given the cash in advance constraint this leads to an increase in deposits. In a second phase the higher capital-labour ratio decreases the asset price and this drives up the amount of monitoring work in the banking sector and the external finance premium so both the loan rate and the riskless rate increase. With a lower employment level in the goods production sector, wages and consumption also fall. There is a temporary deflation and the policy rate drops. As the return on reserves is now lower than their opportunity cost, the reserve level falls and as it falls by more than deposits the reserve-deposit ratio also declines leading to an increase in gearing.

## 6.3 Welfare analysis

Table 4 shows the asymptotic standard deviation and the correlation with consumption from a simulation of the benchmark model under two cases: one corresponding to the shock parameters given in Table 3 and one where the shocks to collateral and to monitoring efficiency are 10 times as large under each of two scenarios, when reserves are endogenous and when there is a fixed reserve ratio. The simulation here is designed to capture what might happen in an economy when shocks to the bank technology function dominate those to the real economy. In the benchmark case, we find that inflation and employment in the goods sector are more stable and real wages exhibit slightly less volatility in the model with endogenous reserves at the cost of some small induced volatility in the loan rate, bond rate and the liquidity premium. But when banking shocks are dominant, the employment of endogenous reserves reduces markedly the standard deviation of all endogenous variables.

Figures 9 and 10 show the middle segment, as an illustration, from a simulation of 10,000 data points, discarding the first 500 observations, of the benchmark model under two cases. The simulated data are HP filtered ( $\lambda = 1600$ ). The top panel in both cases is the model with endogenous reserves and the lower panel with fixed fractional reserves. The EFP, asset prices and inflation are more volatile under fractional reserves. But we note that when banks can choose their optimal level of reserves, they vary positively with asset prices. These Figures also shows that under simple fractional reserves both loans and inflation display more volatility than when banks have the ability to alter their reserves.

Figures 11 and 12 replay Figures 9 and 10 with heavily dominant shocks to loans supply. And we find that endogenous reserves do much to militate against the excessive fluctuations that would obtain when the reserve deposit ratio is fixed. The argument here is that reserves are accumulated when policy rate rises which act to reduce fluctuations in market risk premia and hence in activity and inflation. Finally, Figures 13 and 14 compare the asymptotic standard deviation of output and inflation in an endogenous reserves model and one where fixed fractional reserves are maintained. The x-axis in both cases is the relative weight on shocks to monitoring and to collateral, which drive the supply of loans and when they increase welfare declines under a simple inflation targeting rule. But we show here that the range and scale of decline is significantly less when reserves are endogenous.

## 7 Conclusions

This paper is among the first of a new generation of micro-founded macroeconomic models to consider the implications of bank lending and interest rate spreads on macroeconomic behavior and hence on macro-prudential policy. To the model of Goodfriend and McCallum (2007), we append shifts in velocity in the demand for money function (see Chadha, Corrado and Holly, 2008) and also a liquidity shock emanating directly from the banking sector's need to ensure that it holds sufficient reserves (liquid assets) to guard against a notional probability of shortfall in the ability to refinance its loanbook. We then find that an incentive to hold liquid assets attenuates the excessive increase in the external finance premium that would otherwise ensue. We also solve for commercial banks' optimal levels of illiquid (loans) and liquid asset (reserves) holdings and for the government's budget position by allowing two forms of debt liabilities to be issued: one-period debt to finance any excess in government expenditures over tax receipts and debt to finance the issuance of reserves. These innovations to a more or less standard sticky price setting allow us to consider and speak on a number of important current policy issues.

We first examine the ability of this model economy to amplify and propagate macroeconomic shocks. Second, we consider the role of reserve accumulation in the commercial bank balance sheet and gauge the extent to which it acts to help stabilize this monetary economy - our key finding is that allowing commercial banks another way to adjust their assets (rather than just re-pricing loans), can help attenuate fluctuations. Third, we can measure directly the impact on the macroeconomy of a change in commercial banks' optimal liquidity mix in their assets. In the simple case, increases (or decreases) in loans alone act like a demand shock and can lead to excessive fluctuations in the levels of household consumption but under endogenous reserves, banks can substitute liquidity for changes in the employment of monitoring workers, which limits the variance in loan costs and acts to mitigate the impact on demand. Naturally we do not consider all the possible channels for the transmission of monetary policy, as the model has no investment sector nor does it have an open economy, but it does help us understand the relationship between liquidity-constrained consumption and bank lending.

Over the past decade or so interest rate rules have regularly been shown to deliver stable outcomes in microfounded models, in spite of earlier concerns about their efficacy (see Sargent and Wallace (1985)

and Smith (1991)) but the extension of these models with a banking sector leaves open the possibility the earlier generation of interest rate rules will turn out to be problematic, as we have discovered to some extent since the inception of the financial crisis of 2007. The addition of banks, credit and financial spreads in micro-founded macroeconomics is still in its infancy but we believe the contribution here is an important step. This model of loans supply to liquidity constrained consumers ultimately offers some rationale for commercial banks to have an incentive to hold liquid as well as illiquid assets and it would appear that holding such assets, which are sensitive to policy rates, will help stabilize a monetary economy. We have explored the role of paying interest on reserves in our model but it could be that other mechanisms to ensure procyclical liquidity provisioning may exist, perhaps related to cyclical minimum requirements. What is clear is that such procyclical variation is preferable in our model to simply steady state targets.

And so we feel able to make some suggestions in light of the proposals for macroprudential liquidity arrangements. The question of commercial bank liquidity has often gone hand in glove, with proposals for additional capital adequacy, for example, from the Financial Stability Board. The main proposals from the G20 leaders involve increasing the quantum of capital and liquidity held by commercial banks over the business cycle, which will require agreement on measurement, standards and monitoring of standards. It seems unlikely thought that any quantity of capital or liquidity held in steady-state is likely to be sufficient to fund bank's losses in the event of a financial collapse. So we maintain that more dynamic provisioning of liquidity over the business cycle is required. Our results on reserves suggests that they can be treated as an additional argument in the loans production technology of commercial banks and so it can substitute for both the value of collateral and the costs of monitoring employment. Encouraging banks to increase reserves holdings in a boom acts to limit the expansion in loans and in a recession helps to prevent too rapid a fall and so will not only increase the efficacy of standard interest rate policy but also help prevent the excesses of financial intermediation. As an aside there does not seem to be a great deal of transparency about banks' levels of liquidity, which perhaps should also be addressed. Let us also not forget that the financial crisis was triggered by a liquidity drought and so encouraging banks to hold reserves, especially at a business cycle peak by linking the return on reserves to policy rates, may ultimately prevent this kind of drought.

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Table 1: The Variables	
$c$	Real consumption
$n$	Labour input
$m$	Labour input for loan monitoring, or ‘banking employment’
$w$	Real wage
$q$	Price of capital goods
$P$	Price level
$\pi$	Inflation
$mc$	Marginal cost
$r$	Reserves
$rr$	Reserves/Deposit Ratio
$D$	Deposits
$L$	Loans
$P^A$	Aggregate Prices
$b$	Real bond holding
$\Omega$	Marginal value of collateral
$EFP$	Uncollateralized External Finance Premium ( $R^T - R^{IB}$ )
$LSY^B$	Liquidity Service on Bonds
$LSY^{KB}$	Liquidity Service on Capital ( $kLSY^B$ )
$R^T$	Benchmark risk free rate
$R^B$	Interest rate for bond
$R^{IB}$	Interbank rate
$R^L$	Loan rate
$R^D$	Deposit rate
$\lambda$	Lagrangian for budget constraint (shadow value of consumption)
$\xi$	Lagrangian for production constraint
$T$	Real transfer (%)



Table 2 Calibration		
Parameter	Description	Value
$\beta$	Discount factor	0.99
$\kappa$	Coefficient in Phillips curve	0.05
$\alpha$	Collateral share of loan production	0.65
$\phi$	Consumption weight in utility	0.4
$\eta$	Capital share of firm production	0.36
$\delta$	Depreciation rate of capital	0.025
$\gamma$	Trend growth rate	0.005
$rr$	Reserve ratio	0.1
$\rho$	Interest rate smoothing	0.8
$\phi_\pi$	Coefficient on Inflation in Policy	2.5
$\phi_y$	Coefficient on Output in Policy	0.5
$F$	Production coefficient of loan	9.14
$k$	Inferiority coefficient of capital as collateral	0.2
$\theta$	Elasticity of substitution of differentiated goods	11
Steady-States	Description	
$m$	Steady state of banking employment	0.0063
$n$	Steady state of labour input	0.3195
$R^T$	Steady state of benchmark risk free rate	0.015
$R^{IB}$	Steady state of interbank rate	0.0021
$R^L$	Steady state of loan rate	0.0066
$R^B$	Steady state of bond rate	0.0052
$b/c$	Steady state of bond holding over consumption	0.56
$c$	Steady state of consumption	0.8409
$T/c$	Steady state of transfers over consumption	0.0126
$w$	Steady state of real wage	1.9494
$\lambda$	Steady state of shadow value of consumption	0.457
$v$	Steady state level of velocity	0.276
$\Omega$	Steady state of marginal value of collateral	0.237
$K$	Steady state of capital	9.19
$r/c$	Steady state of reserves over consumption	0.58
Note: The deep parameters are explained in section 5. The steady-states have been solved by solving the set of simultaneous equations described in Section C of the technical appendix. The code is available on request.		

Table 3 Calibration of exogenous shocks		
	Description	Value
<b>Persistence</b>		
$\rho_{a1}$	productivity	0.95
$\rho_{a2}$	banking productivity	0.95
$\rho_{a3}$	collateral shocks	0.9
$\rho_{\epsilon}$	monetary policy	0.3
$\rho_u$	mark-up	0.74
$\rho_{\varepsilon}$	government debt	0.9
$\rho_v$	velocity	0.33
$\rho_{\tau}$	liquidity	0.33
<b>Volatility</b>		
$\sigma_{a1}$	productivity	0.72%
$\sigma_{a2}$	banking productivity	1.00%
$\sigma_{a3}$	collateral	1.00%
$\sigma_{\epsilon}$	monetary policy	0.82%
$\sigma_u$	mark-up	0.11%
$\sigma_{\varepsilon}$	government debt	1.00%
$\sigma_v$	velocity	1.00%
$\sigma_{\tau}$	liquidity	1.00%
Source: Chadha, Corrado and Holly (2008).		

Table 4 Impact on economy of endogenous reserves

Reserve/Deposit	(%) Benchmark		Calibration		Dominant		Banking Shocks	
	Fixed		Endogenous		Fixed		Endogenous	
	<i>S.D.</i>	<i>Corr</i>	<i>S.D.</i>	<i>Corr</i>	<i>S.D.</i>	<i>Corr</i>	<i>S.D.</i>	<i>Corr</i>
Consumption	1.52	1	1.52	1	11.71	1	5.37	1
Inflation	0.91	0.67	0.37	0.48	7.51	0.95	2.05	0.95
Employment in Monitoring	2.78	-0.64	4.48	-0.80	17.6	-0.81	5.61	-0.55
Employment in the Goods Sector	2.49	0.91	2.37	0.91	19.4	0.99	8.78	0.99
Real Wages	2.60	0.98	2.44	0.98	20.5	0.99	9.54	0.99
Bonds	1.28	0.14	1.28	0.06	1.28	0.01	1.28	0.01
Asset Price	1.82	0.98	1.50	0.98	15.1	0.99	6.50	0.99
Real Loans	1.40	0.77	0.79	0.71	11.70	0.99	5.81	0.97
Real Reserves	1.40	-0.77	1.60	-0.82	11.70	-0.99	3.74	-0.06
Policy Rate	1.83	0.14	1.45	0.23	14.91	0.52	5.27	0.63
Deposit Rate	1.83	0.14	1.39	0.18	14.91	0.52	5.83	0.66
Loan Rate	0.66	-0.10	0.90	-0.89	3.57	0.65	0.98	-0.23
Bond Rate	0.65	-0.08	0.87	-0.89	3.73	0.67	0.93	-0.17
External Finance Premium	1.65	-0.20	1.90	-0.60	11.89	-0.45	5.57	-0.64
Liquidity Premium	0.02	-0.55	0.04	-0.77	0.21	-0.71	0.11	-0.68

Note: *S.D.* denotes the asymptotic standard deviation of the relevant variables derived from the filtered second moments of the solution obtained from the model. *Corr* denotes the contemporaneous cross-correlation with consumption derived from the filtered autocovariance of the solution obtained from the model.

The benchmark scenario corresponds to the shock parameters given in Table 3 while in the scenario where the banking shocks are dominant the shocks to collateral and to monitoring efficiency are 10 times as large as in the benchmark scenario.

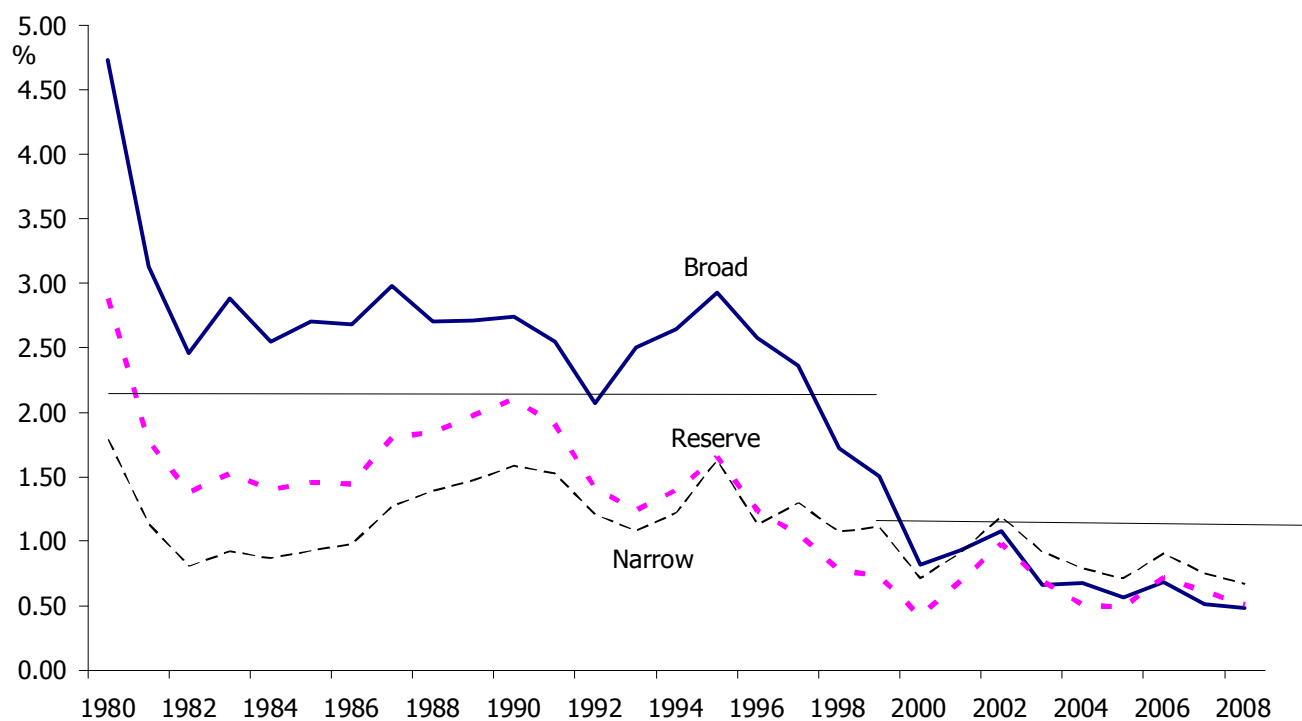


Figure 1: UK Eligible Sterling Liabilities at the Bank of England

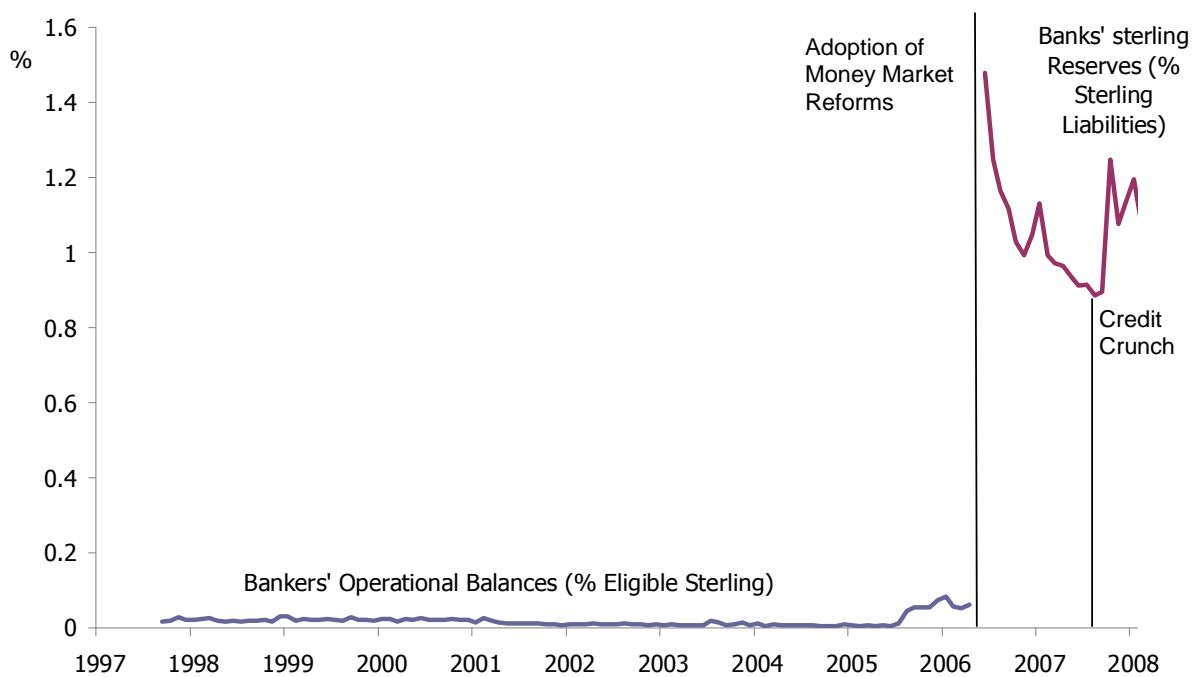


Figure 2: Liquid Assets Relative to Total Assets (Source: Bank of England)

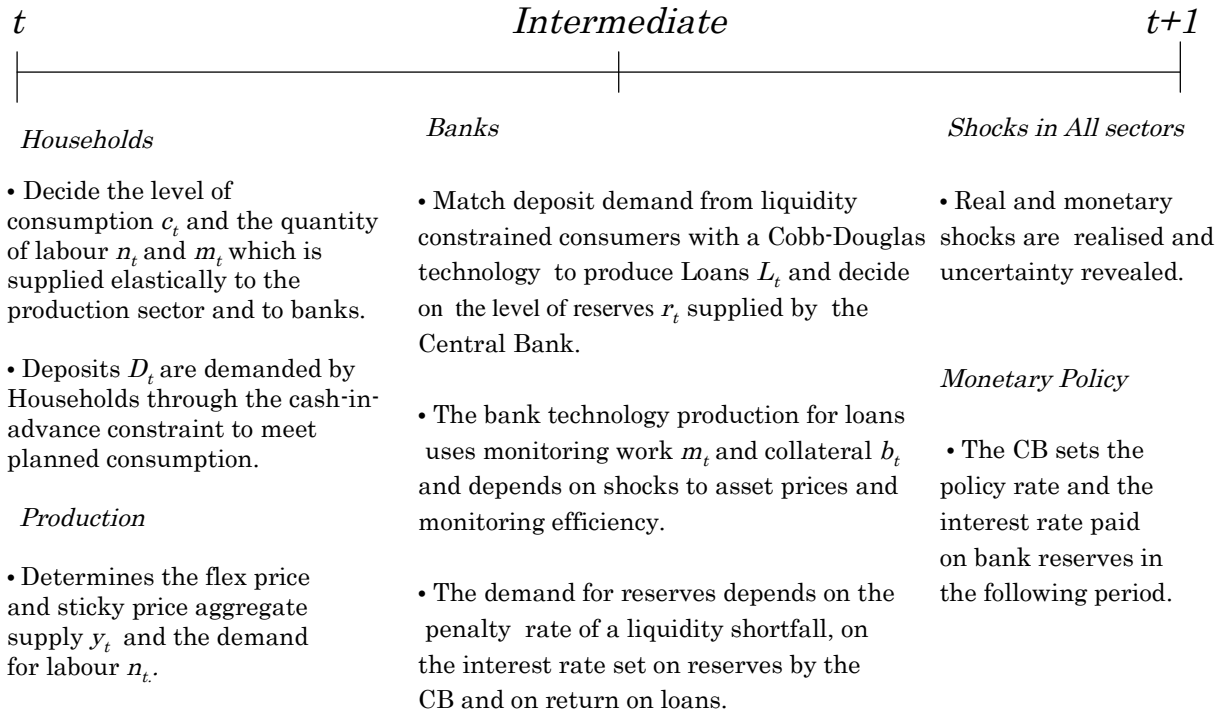


Figure 3: Timeline of Events.

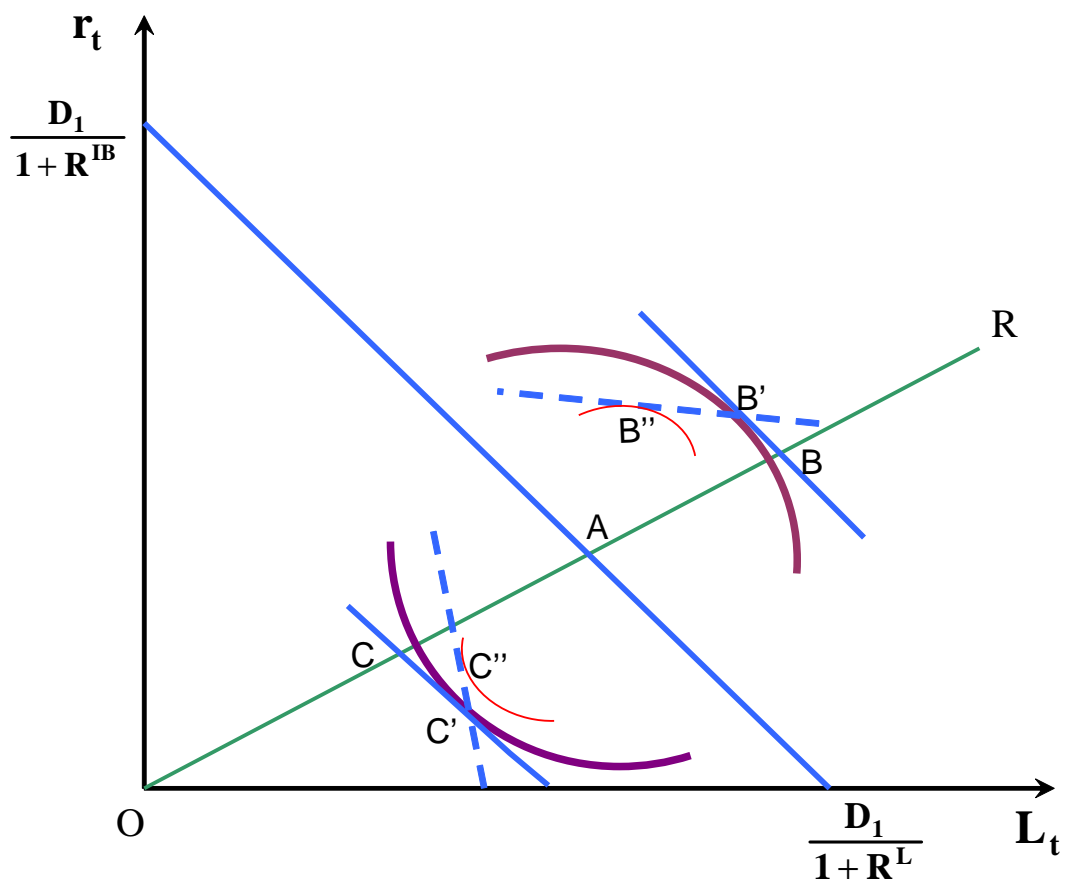


Figure 4: The Bank's Optimal Choice between Loans and Reserves.

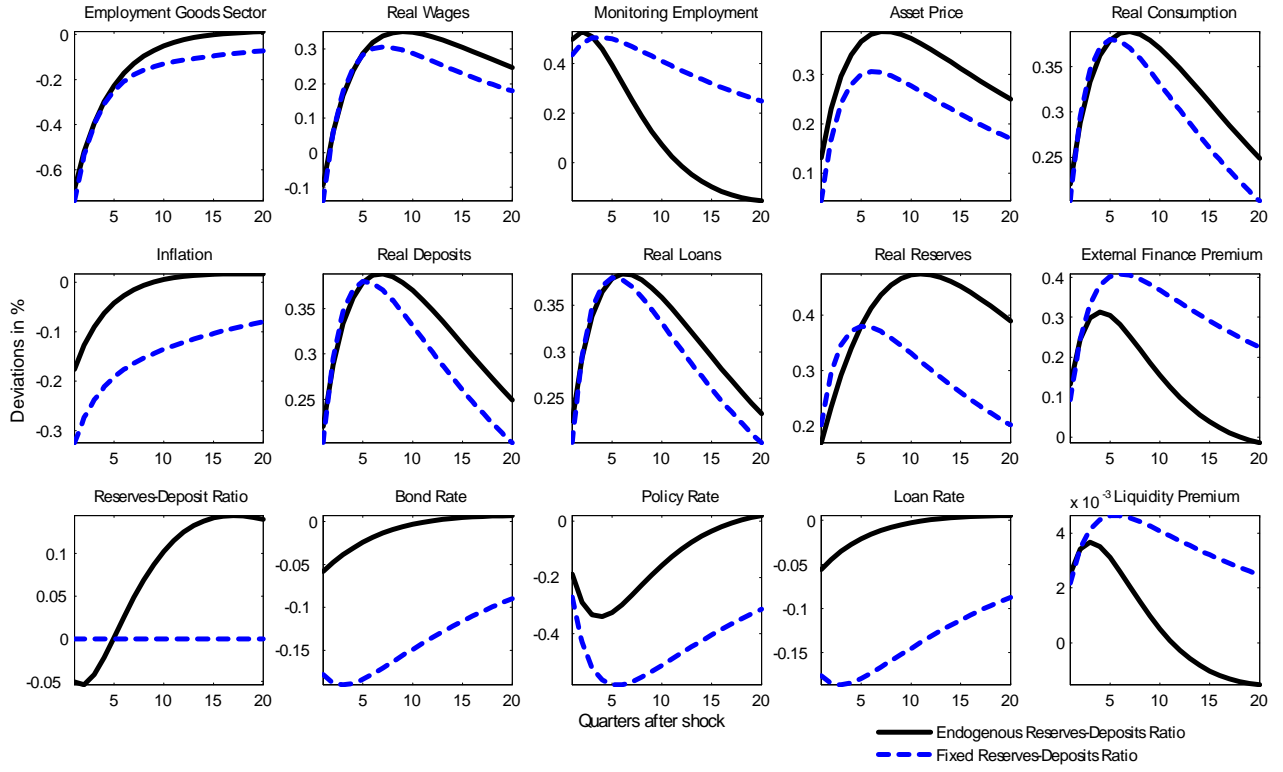


Figure 5: Impulse Responses to Productivity Shock. Note: In Figures 5-8 we report impulse responses of key variables under a benchmark calibration of exogenous shocks and policy rates for a fixed fractional reserve system and for endogenous reserves. Please refer to Tables 2 and 3 for calibration values of parameters and shocks. The impulse responses show percentage deviation from steady state from period 1 when there is a 1% shock of magnitude to specific source of fluctuation.



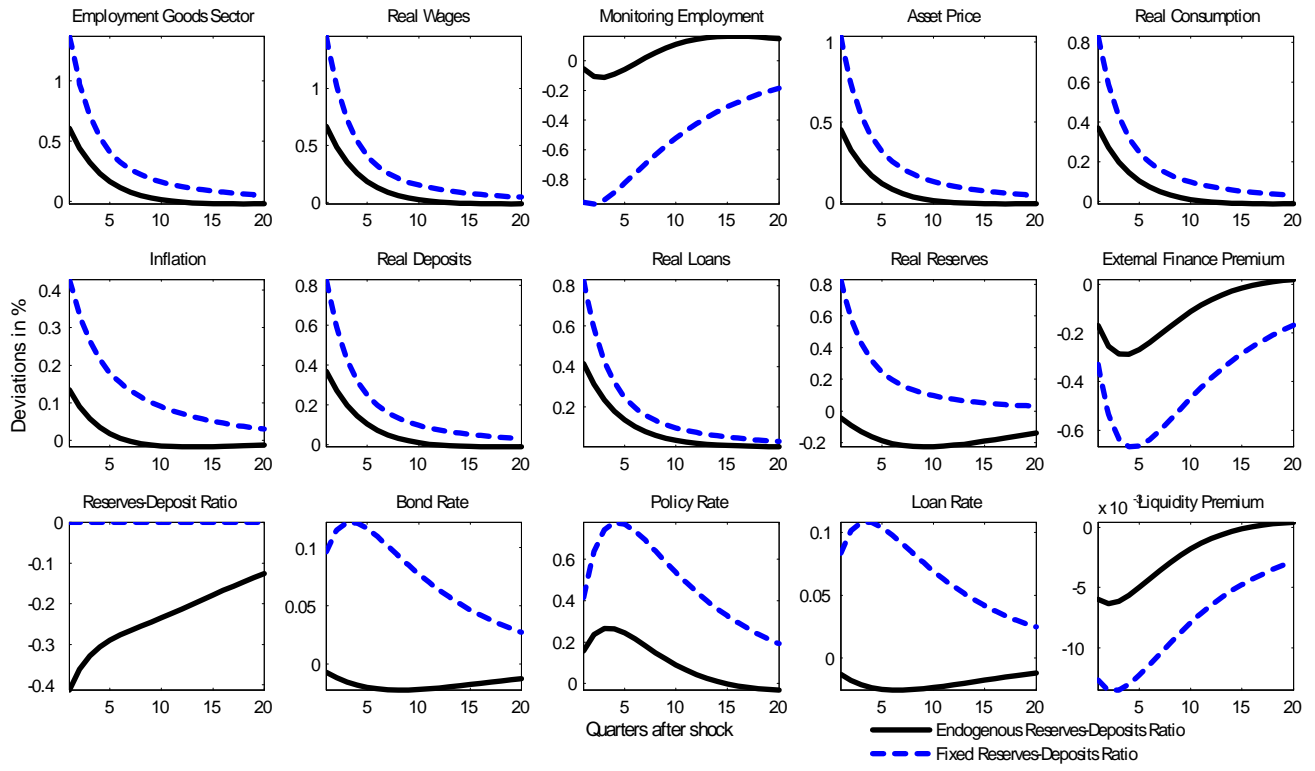


Figure 6: Impulse Responses to Positive Collateral Shock

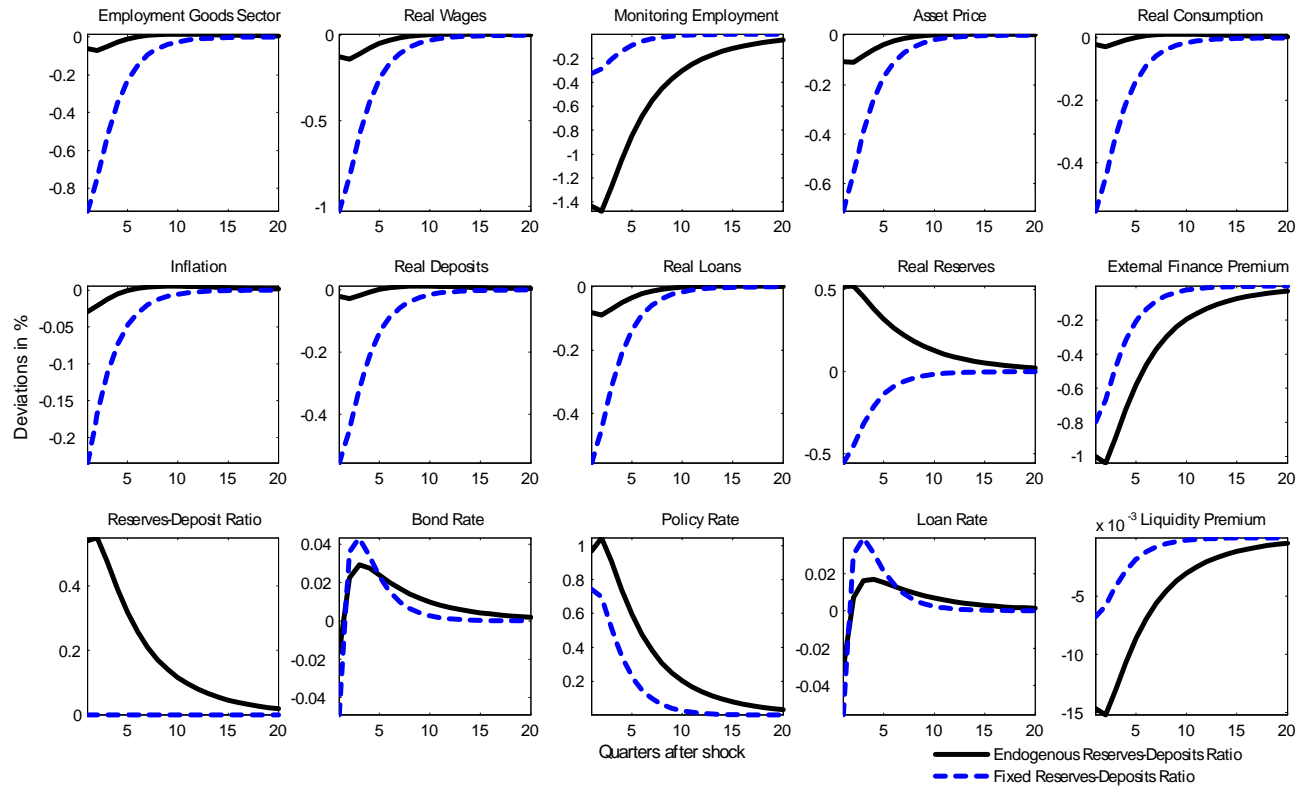


Figure 7: Impulse Response to Monetary Policy Shock

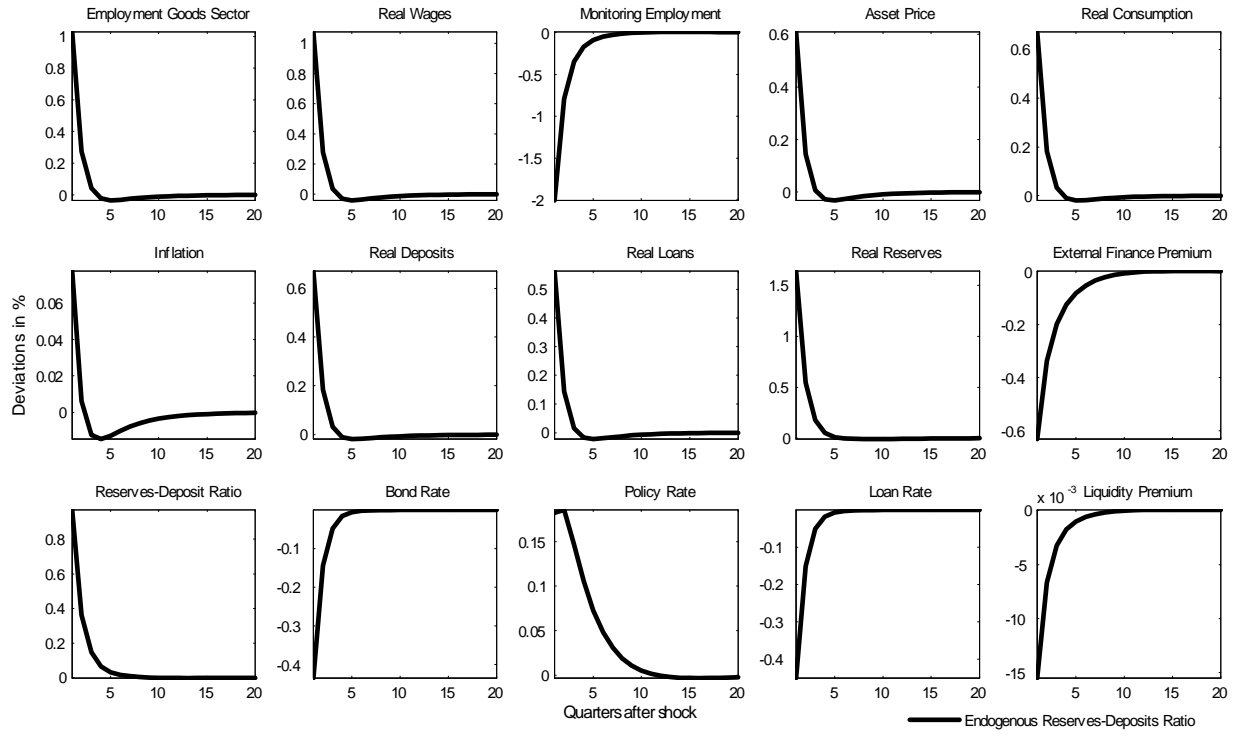


Figure 8: Impulse Response to Liquidity Shock.

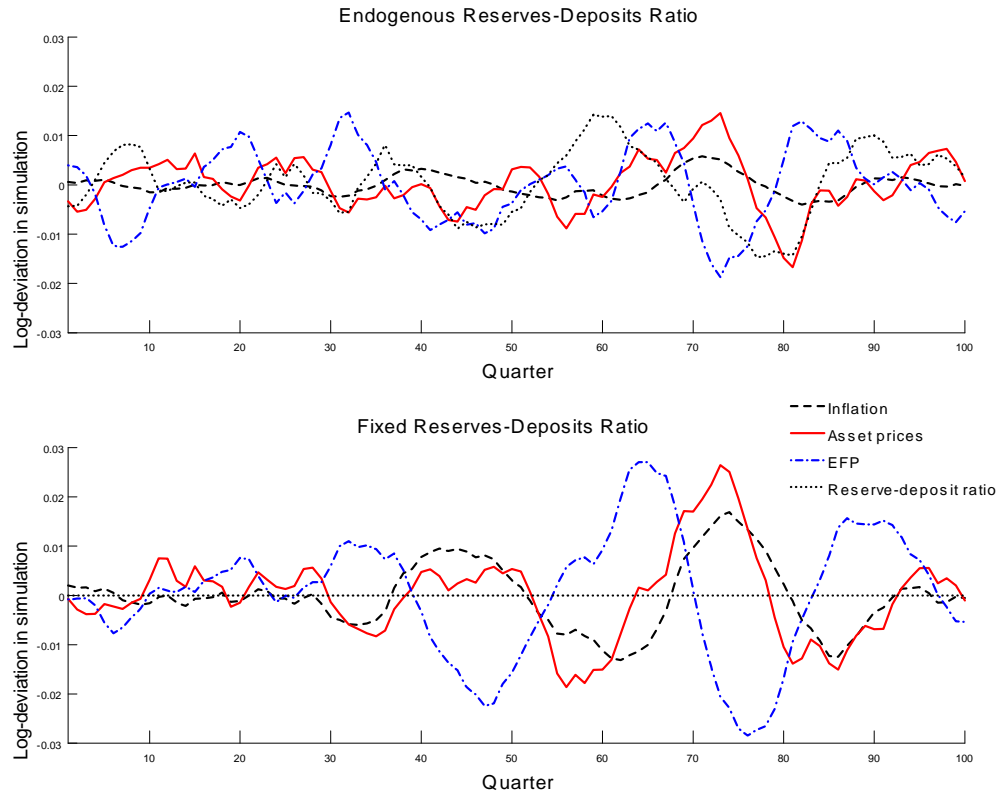


Figure 9: Simulation of Two-Year Moving Average Series of HP Filtered Monetary Variables. Note: Figures 9 and 10 show the middle segment of a simulation of 10,000 data points from a standard calibration of this model. The simulated data are HP filtered ( $\lambda = 1600$ ).

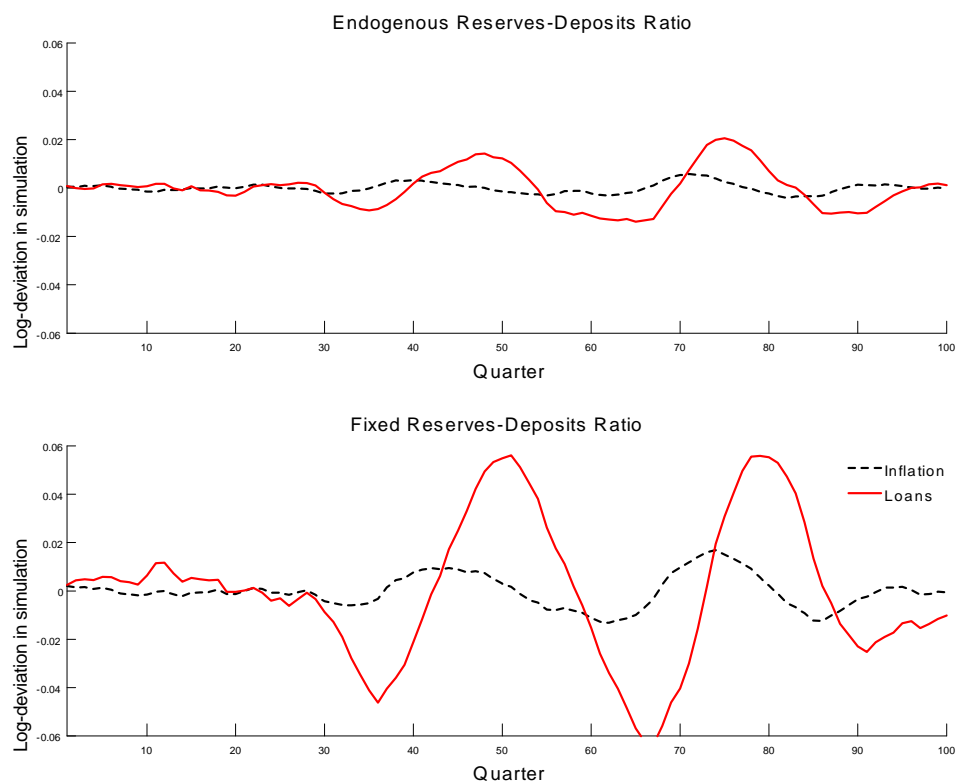


Figure 10: Simulation of Two-Year Moving Average Series of HP Filtered Key Variables.

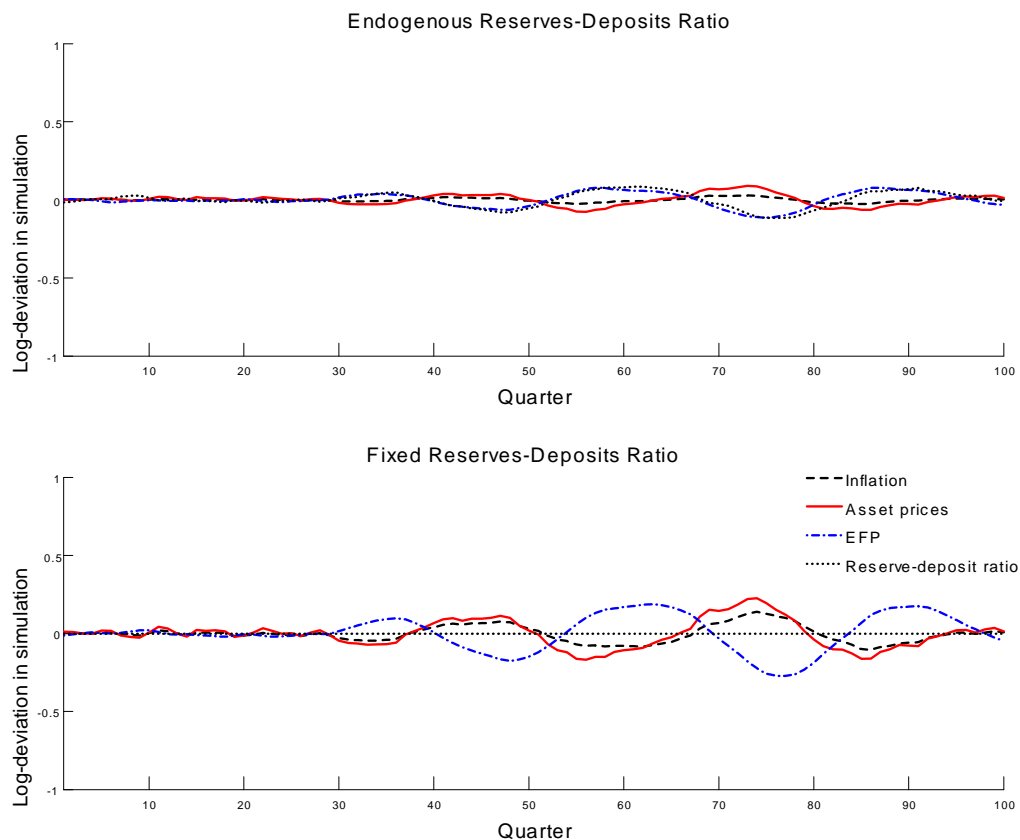


Figure 11: Simulation of Two-Year Moving Average Series of HP Filtered Monetary Variables under Dominant Banking Shocks. Note: Figures 11 and 12 show the middle segment of a simulation of 10,000 data points from a calibration where the standard deviation of banking shocks is 10 times higher than in the benchmark calibration. The simulated data are HP filtered ( $\lambda = 1600$ ).

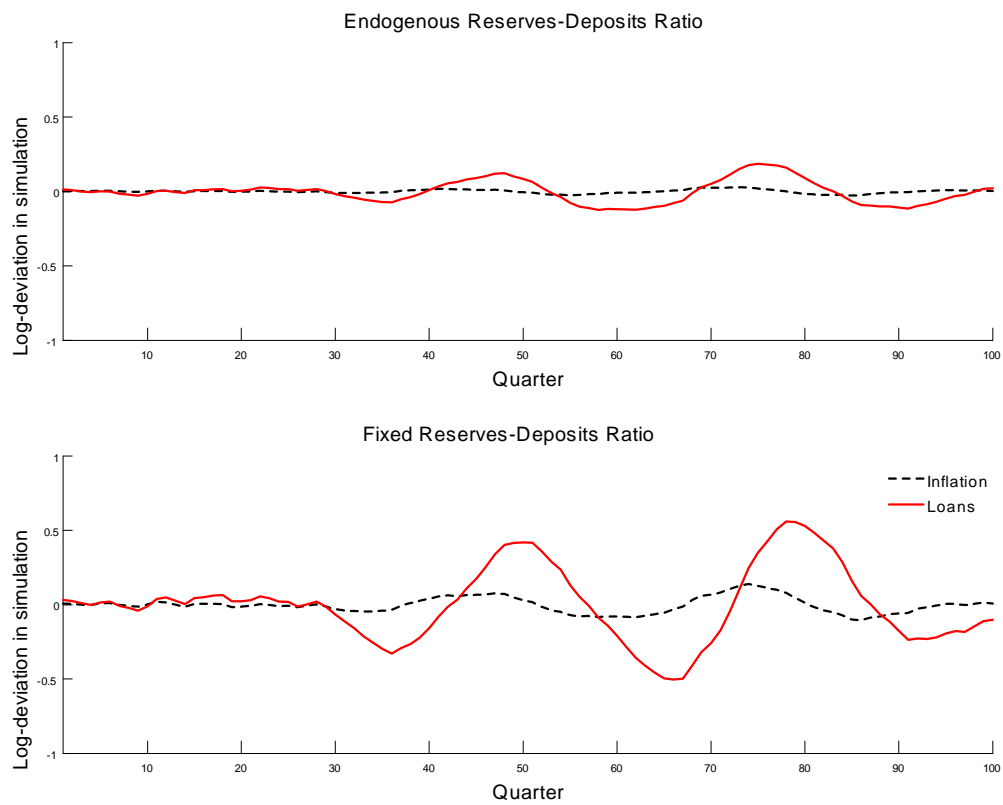


Figure 12: Simulation of Two-Year Moving Average Series of HP Filtered Key Variables under Dominant Banking Shocks

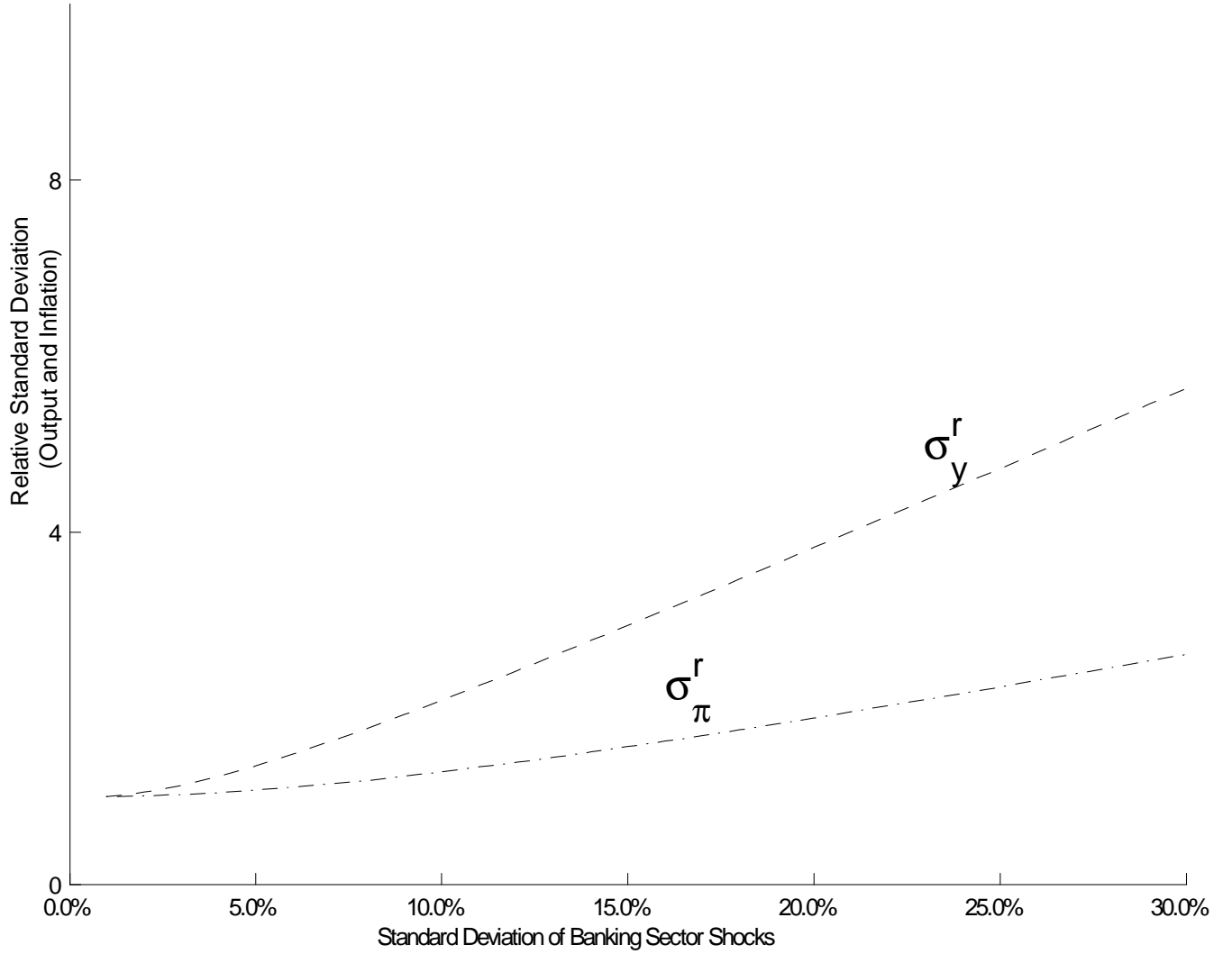


Figure 13: Macroeconomic Volatility as a Function of Banking Sector Shocks in Endogenous Reserve-Deposit Ratio Model. Note: On x-axis we allow various calibration of banking sector shocks (the monitoring productivity shock or collateral shock).  $\sigma_i^r, i = y, \pi$  denotes relative standard deviation of output or inflation to the initial case with fractional reserves (banking shocks are not dominant).



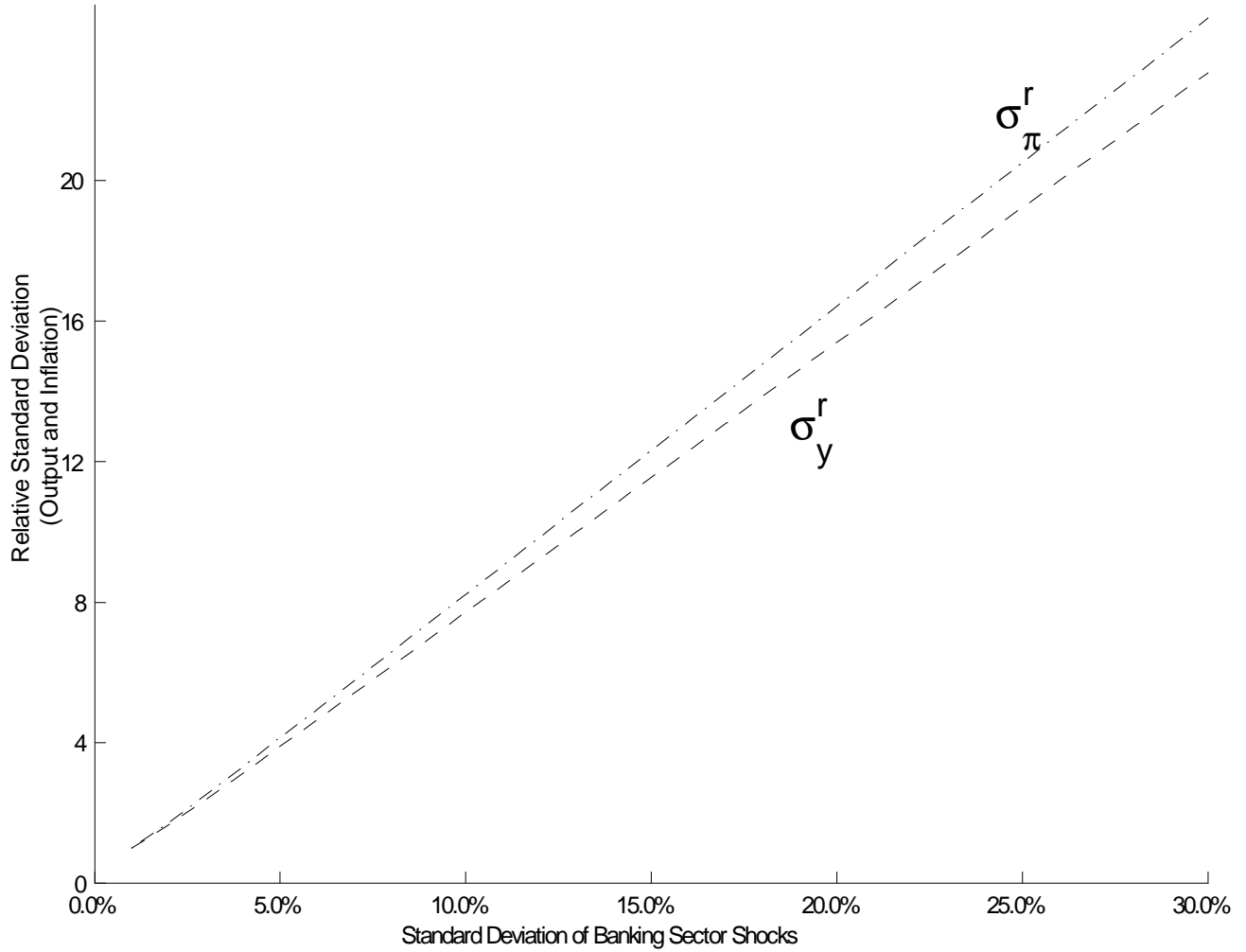


Figure 14: Macroeconomic Volatility as a Function of Banking Sector Shocks in Fixed Reserve-Deposit Ratio Model. Note: On x-axis we allow various calibration of banking sector shocks (the monitoring productivity shock or collateral shock).  $\sigma_i^r, i = y, \pi$  denotes relative standard deviation of output or inflation to the initial case with endogenous reserves (banking shocks are not dominant).