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**Accounting for Japanese Business Cycles:  
a quest for labor wedges**

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## Abstract

The Japanese business cycle from 1980-2007 portrays less contemporaneous correlation of labor with output than in the U.S. and also tends to lead output by one quarter. A canonical real business cycle model cannot account for these facts. This paper uses the business cycle accounting method a la Chari, Kehoe and McGrattan (2007) and shows that efficiency and labor market distortions are important in accounting for the quarterly business cycle fluctuation patterns in Japan. Fiscal and monetary variables such as labor income tax, money growth and interest rates cannot fully account for the distortions in the Japanese labor market.

Keywords: Business cycle accounting; Japanese labor market

JEL classification: E13; E32

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# 1 Introduction

The Japanese business cycle from 1980-2007 portrays less contemporaneous correlation of labor with output than in the U.S. and also tends to lead output by one quarter. A canonical real business cycle model cannot account for these facts. A canonical real business cycle model cannot account for these facts. This paper uses the business cycle accounting method of Chari, Kehoe and McGrattan (2007) and shows that efficiency and labor market distortions are important in accounting for the quarterly business cycle fluctuation patterns in Japan. I assess the impacts of fiscal and monetary shocks as possible candidates for the labor wedges.

The business cycle accounting model consists of the representative household, firm, and government. The government affects markets by exogenously purchasing goods while charging distortionary labor and investment taxes. The firm faces exogenous shocks to the production process. These exogenous variables are called resource wedges, labor wedges, investment wedges, and efficiency wedges. The wedges are computed as residuals in the equilibrium conditions using the data of output, consumption, investment, and labor supply. Finally, the model is simulated using the computed wedges.

Several studies use the business cycle accounting method to analyze the medium term business cycle fluctuations during recession periods. Chari, Kehoe and McGrattan (2007) find that labor and efficiency wedges are important in accounting for the US great depression and its 1981 recession. Kersting (2008) finds that the labor wedges are important in accounting for the early 1980s UK recession. Cociuba and Ueberfeldt (2008) finds that labor and efficiency wedges are important in accounting for the Canadian business cycles over the 1961-2005 period. Lama (2010) uses a small open economy version of business cycle accounting and finds that labor and efficiency wedges are important in accounting for output drops in Argentina, Brazil, Chile, Colombia, Mexico and Peru. Otsu (2010) also uses a small open economy version and finds that efficiency wedges are important in accounting

for the 1998 crises in Hong Kong, Korea, and Thailand.

There are also several studies that analyze the Japanese economy using the business cycle accounting method. Kobayashi and Inaba (2006) use a deterministic version of the business cycle accounting model and show that efficiency and labor wedges are important in accounting for the lost decade. Chakraborty (2009) shows that efficiency and investment wedges are important in accounting for the boom in the 1980s while labor wedges are important in accounting for the recession during the 1990s. Otsu and Pyo (2009) show that the positive contemporaneous correlation between efficiency and investment wedges may have amplified the effect of financial frictions especially during the boom in the 1980s. Saijo (2008) finds that efficiency wedges are important in accounting for the output drop while labor and capital wedges are important in accounting for the slow recovery of output during the great depression in Japan. Unlike these studies, I focus on the HP filtered high frequency fluctuation of the Japanese economy and show that labor wedges are important in accounting for the correlation between labor and output.

Related studies investigate Japanese labor market issues using dynamic general equilibrium models. Braun, Esteban-Pretel, Okada and Sudo (2006) assess the cyclical features of Japan and the U.S. labor market variables over the 1960-2000 period. They find that the adjustment of labor supply is made mostly in the intensive margin in Japan whereas it is done in the extensive margin in the U.S. and that fluctuations in hours worked per worker leads the business cycles while that of employment lags the business cycle in Japan. They show that the first fact can be accounted for by the differences in the elasticities of hours across gender and economies. They show that it is difficult to account for the second fact using a variation of the real business cycle model and conjecture that there was some distortion in the labor market that is independent from productivity shocks. Otsu (2011) shows that shocks to the intensity of labor can account for the lead in hours and the lag of employment. In this paper, I focus on total hours worked

following the business cycle accounting method and show that labor wedges are important in accounting for the fluctuations in aggregate labor supply.

The business cycle accounting model does not specify the underlying source of the labor wedge. In fact, many different shocks can manifest themselves as labor wedges. Labor income tax is the usual suspect, however, its fluctuation is not observed on a quarterly basis. I also show that interest rate shocks in a working capital model, money growth shocks in a cash in advance constraint model, and money growth shocks in a labor union sticky wage model can all be mapped into a prototype business cycle accounting model with labor wedges. I simulate these monetary models and show that monetary shocks cannot fully account for the labor market distortions.

The remaining of the paper is organized as follows. In section 2, the Japanese business cycle data is presented in comparison to the U.S. data. In section 3, the business cycle accounting model is described. In section 4, the quantitative results are shown. In section 5, possible sources of the labor wedges are assessed. Section 6 concludes the paper.

## 2 Data

This section presents the quarterly fluctuations of key macroeconomic variables in Japan over the 1980-2007 period<sup>1</sup>. Output is defined as gross domestic product plus the imputed flow of service from consumer durables<sup>2</sup>. Consumption is defined as the consumption expenditure on nondurables and services and the imputed flow of services from consumer durables<sup>3</sup>. Investment is defined as the gross capital formation and the expenditure on household

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<sup>1</sup>I choose this time period to remove the large effects of the oil shocks in the 1970s and the recent crisis in 2008.

<sup>2</sup>The trade balance is not reported in this section for simplicity as we will be constructing a closed economy model. In the model section, the sum of government purchases and the trade balance is defined as a wedge in the resource constraint following Chari, Kehoe and McGrattan (2007).

<sup>3</sup>Imputed rent is already included in the service consumption data.

durables. Government expenditures include only government consumption while government investment is included in gross capital formation. Capital stock is defined as the sum of residential capital stock, non-residential capital stock, government capital stock and the stock of consumer durables. The quarterly stock data for each asset is estimated by interpolating the annual data for the beginning of 1980 and 2000 using the quarterly investment data for each asset<sup>4</sup>. Labor supply is defined as the number of workers times the number of hours worked per workers, i.e. total hours worked. The source of the Japanese data is the SNA data offered by the Cabinet Office ESRI website for national accounts and the Labor Force Survey for labor statistics.

Figure 1a plots the fluctuations of HP filtered per capita output, consumption investment and government expenditures. The output series shows the recessions in the early 90s after the so called ‘bubble burst’, in the mid 90s after the Asian crisis, and in the early 2000s after the IT bubble burst<sup>5</sup>. Consumption and investment are both procyclical. Investment is much more volatile than output while consumption is less volatile than output. The fluctuation of government expenditure does not seem to have a clear relationship with output. Figure 1b plots the fluctuations of HP filtered per capita output, capital stock and labor. Labor is procyclical for most of the period except for the early 1980s. The fluctuation of capital clearly lags that of output.

Table 1a presents the statistical features of the data presented in Figures 1a and 1b. The first column reports the standard deviation of each variable while the second column reports the ratio of the standard deviation of each

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<sup>4</sup>The stock data for residential, non-residential, and government capital are from Hayashi and Prescott (2002) while the stock data for durable goods are from the ESRI website. Since durable stock data is not available for 1980, I assume that the ratio of durable stock to other stock in 1980 is the same as that in 2000.

<sup>5</sup>The 1990s is known as the lost decade where the average growth rate (0.89%) was substantially lower than that of the previous decade (2.27%). This medium term fluctuation disappears once the data is HP filtered. The main objective of this paper is to analyze the high frequency business cycle fluctuation. The non filtered data and results are presented in the appendix.

variable relative to that of output. The remainder of the table reports the cross-correlation of each variable with output. For comparison, the U.S. data is presented in Table 1b. The source of the US data is the BEA website for national accounts and the BLS website for Labor statistics.

The business cycle features of Japan are similar to those of the U.S. economy in several aspects. First, consumption, government purchases, and capital stock are less volatile than output while investment is far more volatile than output. Second, all variables are procyclical except for government purchases, which are acyclical in both countries. Finally, the fluctuation of capital lags that of output.

The key difference between Japan and the U.S. is the fluctuation of labor supply. First, labor supply in Japan is much less volatile relative to output than in the U.S. The standard deviation of labor relative to output is 0.69 in Japan while it is 1.07 in the U.S. Second, the contemporaneous correlation of consumption and labor supply to output is much lower in Japan than in the U.S. The correlation coefficients are 0.38 and 0.45 in Japan while they are 0.84 and 0.89 in the U.S. respectively. Finally, the fluctuation of labor supply leads output in Japan while in the U.S. the fluctuations of them are coincident. The second and third points are important as it turns out that they cannot be replicated by a standard real business cycle model<sup>6</sup>.

### **3 Business Cycle Accounting Model**

The business cycle accounting model of Chari, Kehoe and McGrattan (2007) is based on a neoclassical closed economy model which consists of a representative firm, household and the government. The firm produces a final good from capital and labor using a constant returns to scale production technology which faces exogenous disturbances in production efficiency. The

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<sup>6</sup>The standard deviation of investment relative to that of investment in Japan is lower (2.7) than that in the U.S. (3.57). This is also an interesting difference between the two economies which is left as a future research agenda.

infinitely-lived representative household gains utility from consumption and leisure. The household owns capital stock and labor endowment and decides how much to consume, invest and work. The government imposes distortionary labor and investment taxes on the household, spends on government purchases, and rebates the remaining to the household via lump-sum transfers. Labor and investment taxes, government purchases and production efficiency are computed as “wedges” in equilibrium conditions and are taken as exogenous. The detailed description is as follows.

### 3.1 Firm

The firm produces a final good with a Cobb-Douglas production function,

$$Y_t = z_t K_t^\theta (\Gamma_t l_t)^{1-\theta},$$

where  $Y_t$  is output,  $z_t$  is production efficiency, i.e. TFP,  $K_t$  is capital stock,  $l_t$  is labor,  $\theta$  is the income share of capital and  $\Gamma_t$  is the labor augmented technical progress. We assume that the labor augmenting technical progress grows at a constant rate  $\gamma$  such that  $\Gamma_t = (1 + \gamma)\Gamma_{t-1}$ . Labor is defined as

$$l_t = h_t * e_t,$$

where  $e_t$  is the number of workers employed per the number of the adult population and  $h_t$  is the average weekly hours worked per worker divided by the maximum possible hours available<sup>7</sup>. In a standard neoclassical growth model, real per capita macroeconomic variables such as output, consumption, investment, government expenditure and capital stock grow at the rate of the labor augmenting technical progress  $\gamma$  along the the balanced growth path. Therefore, I detrend these growing variables with the trend  $\Gamma_t$  so that all variables are stationary. The detrended variables are denoted by small class

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<sup>7</sup>I assume that the maximum hours available for work is 14\*7 hours taking into account sleeping and other activities necessary to maintain minimum standards of living.

letters.

The firm maximizes its profit defined by the value of production net of costs of hiring labor and renting capital stock from the household. That is,

$$\max \pi_t = y_t - w_t l_t - r_t k_t, \quad (1)$$

where  $w_t$  is the real wage and  $r_t$  is the real capital rental rate and the detrended production function is

$$y_t = z_t k_t^\theta l_t^{1-\theta}. \quad (2)$$

Profits will always equal to zero due to perfect competition.

## 3.2 Household

The lifetime utility for the representative household depends on consumption  $c_t$  and labor  $l_t$ :

$$\max U = E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, l_t), \quad (3)$$

where  $\beta (0 < \beta < 1)$  is the subjective discount rate. For the periodical preference function,  $u(\bullet)$ , I assume Cobb-Douglas preferences:

$$u(c_t, l_t) = \Psi \log(c_t) + (1 - \Psi) \log(1 - l_t),$$

which are commonly used in the macroeconomic literature<sup>8</sup>.

The representative household maximizes the lifetime utility (3) subject

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<sup>8</sup>This is a special case of a general form

$$u = \frac{(c_t^\Psi (1 - l_t)^{1-\Psi})^{1-\sigma}}{1 - \sigma}$$

with  $\sigma = 1$ .

to the budget constraint

$$(1 - \tau_t^l)w_t l_t + r_t k_t + \pi_t + \tau_t = c_t + (1 + \tau_t^x)x_t, \quad (4)$$

and the capital law of motion

$$\eta k_{t+1} = x_t + (1 - \delta)k_t, \quad (5)$$

where  $\eta$  is the constant growth trend of the labor augmented technical progress and population,  $x_t$  is investment,  $\tau_t^l$  and  $\tau_t^x$  are labor income and investment tax rates,  $\tau_t$  is the lump sum government transfer and  $\delta$  is the depreciation rate of capital stock.

### 3.3 Government

The government collects distortionary taxes, spends on exogenous government purchases  $g_t$  and rebates the remaining to the household using lump-sum transfer. Thus, the government budget constraint is

$$\tau_t + g_t = \tau_t^l w_t l_t + \tau_t^x x_t. \quad (6)$$

Note that the transfer can be negative in which case the government collects lump-sum taxes from the household.

Combining the household budget constraint (4), the firms profits (1) and the government budget constraints (6), we get the resource constraint

$$y_t = c_t + x_t + g_t. \quad (7)$$

### 3.4 Wedges

There are 4 exogenous variables in this model: resource wedges  $g_t$ , labor wedges  $\tau_t^l$ , investment wedges  $\tau_t^x$  and efficiency wedges  $z_t$ . Although the

wedges are defined as government purchases and taxes in the model, we do not use data of these. Instead, the wedges are measured using the following equilibrium conditions.

Resource wedges are measured using the resource constraint (7). Notice that they include all GDP expenditure components net of consumption and investment, which is government purchases and the trade balance. Labor wedges are measured using the labor first order condition:

$$\frac{1 - \Psi}{\Psi} \frac{c_t}{1 - l_t} = (1 - \tau_t^l)(1 - \theta) \frac{y_t}{l_t}. \quad (8)$$

Notice that labor wedges need not be labor taxes. Any other distortion in the labor market that creates a wedge between the marginal rate of substitution of labor to consumption and the marginal product of labor. Investment wedges are measured using the capital Euler equation:

$$\eta \frac{\Psi}{c_t} (1 + \tau_t^x) = \beta E_t \left[ \frac{\Psi}{c_{t+1}} \left( \theta \frac{y_{t+1}}{k_{t+1}} + (1 - \delta)(1 + \tau_{t+1}^x) \right) \right]. \quad (9)$$

Finally, efficiency wedges are measured using the production function (2).

The economic interpretation is as follows. An increase in resource wedges is considered as a loss of resources for the economy. Through a pure negative income effect, consumption and investment decrease while labor and output increases. An increase in labor wedges is considered as a decrease in the real wage that the household receives. Investment decreases through a negative income effect while consumption, labor and output decrease predominantly through a substitution effect<sup>9</sup>. An increase in investment wedges reduces the return on investment the household receives. Consumption increases while investment decreases due to the substitution effect. The increase in consumption causes an income effect which increases leisure. As a result,

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<sup>9</sup>Consumption and labor both decrease because consumption becomes expensive relative to leisure. This effect dominates the negative income effect which tends to increase labor.

labor and output decrease. An increase in efficiency wedges directly increases output and the labor demand<sup>10</sup>. The increase in income leads to an increase in consumption and investment.

The wedges are assumed to follow the following stochastic process

$$\tilde{s}_t = P_{4 \times 4} \tilde{s}_{t-1} + \varepsilon_t, \varepsilon_t \sim N(0_{4 \times 1}, Q_{4 \times 4}), \quad (10)$$

where  $\tilde{s}_t = (\tilde{g}_t, \tau_t^l, \tau_t^x, \tilde{z}_t)'$  while variables with “ $\sim$ ” indicate the log deviation of them from their steady state values, and  $\varepsilon_t = (\varepsilon_{gt}, \varepsilon_{lt}, \varepsilon_{xt}, \varepsilon_{zt})'$ .

### 3.5 Competitive Equilibrium

The competitive equilibrium is,  $\{c_t, l_t, k_{t+1}, y_t, x_t, \tau_t, w_t, r_t, g_t, \tau_t^l, \tau_t^x, z_t\}_{t=0}^{\infty}$  such that;

1. Households optimize given  $\{w_t, r_t, \tau_t, \tau_t^l, \tau_t^x\}$  and  $k_0$ .
2. Firm optimizes given  $\{w_t, r_t, z_t\}$ .
3. Markets clear and the government budget constraint (6) holds.
4. The resource constraint (7) holds.
5. Shocks follow the process (10).

## 4 Quantitative Analysis

The quantitative analysis is carried out in four steps. First we obtain the parameter values by calibration and estimation. Next we solve for linear decision rules of the endogenous variables. Then we back out the shocks

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<sup>10</sup>The increase in labor demand drives up the real wage. The substitution effect tends to increase labor supply. On the other hand, the increase in income tends to decrease income. In an infinite horizon model, it is usually the case that the substitution effect dominates the income effect.

using data and these decision rules. Finally we plug the shocks one by one into the decision rules and compare their impacts on the economy.

## 4.1 Parameters

This section describes the procedure of how we obtained the values of the parameters. The calibrated parameter values are listed in Table 2.

The income share of capital  $\theta$  is computed directly from data using the definition

$$\theta = \frac{\text{capital income} + \text{flow income from consumer durables}}{\text{GNP} + \text{flow income from consumer durables}}.$$

Population growth rate  $n$  is computed directly from data of the population of people older than fifteen years old.

The growth rate of labor augmenting technical progress is the trend growth rate of Solow residuals estimated with ordinary least squares as. The log of Solow residuals are defined as

$$\ln SR_t = \ln \Gamma_t^{1-\theta} + \ln z_t = \ln \Gamma_0 + (1 - \theta)t \ln(1 + \gamma) + \ln z_t, \quad (11)$$

from (2) and is directly computable using data of output, capital and labor. Thus, we can estimate  $\gamma$  from a regression of Solow residuals on a linear trend  $t$  and a constant:

$$\ln SR_t = a + bt + u_t. \quad (12)$$

That is, from (11) and (12),  $\gamma \approx \ln(1 + \gamma) = \frac{b}{1-\theta}$ . The growth trend  $\eta$  is computed as

$$\eta = (1 + \gamma)(1 + n),$$

where the constant population growth rate  $n$  is computed directly from data.

The depreciation rate  $\delta$  is computed directly from (5) using capital and

investment data. Then from (9), the discount factor  $\beta$  is computed as

$$\beta = \frac{\eta}{\theta \frac{y}{k} + 1 - \delta},$$

where the capital output ratio  $\frac{k}{y}$  is computed directly from data and I assume that investment taxes are zero in the steady state. Also, from (8), the utility parameter  $\Psi$  is computed as

$$\frac{1 - \Psi}{\Psi} = (1 - \theta) \frac{y}{l} \frac{1 - l}{c}$$

where the consumption share of output  $\frac{c}{y}$  is computed directly from data and I assume that labor taxes are zero in steady state.

Finally, the parameters in the stochastic process (10) are estimated with Bayesian estimation using the data of output, consumption, labor and investment. I rely on structural estimation because investment wedges are not directly observable. The estimated values are

$$P = \begin{vmatrix} 0.81 & 0.19 & 0.31 & 0.02 \\ 0.14 & 0.88 & -0.33 & -0.10 \\ 0.02 & -0.01 & 0.95 & -0.04 \\ 0.07 & -0.04 & -0.08 & 0.92 \end{vmatrix}$$

$$Q = \begin{vmatrix} 0.42 & 0.16 & -0.00 & 0.10 \\ 0.16 & 0.35 & 0.02 & 0.17 \\ -0.00 & 0.02 & 0.01 & 0.02 \\ 0.10 & 0.17 & 0.02 & 0.12 \end{vmatrix} * 1.00E - 03.$$

## 4.2 Computing Wedges

Given all parameters values, the model can be solved quantitatively. I use the solution method à la Uhlig (1999) to solve for linear decision rules. Having obtained the decision rules, I compute the entire series of exogenous vari-

ables  $\{\tilde{g}_t, \tau_t^l, \tau_t^x, \tilde{z}_t\}$  using data of  $\{\tilde{y}_t, \tilde{c}_t, \tilde{l}_t, \tilde{x}_t\}$ . In specific, the procedure of computing wedges is as follows:

1. Compute linear decision rules

$$\begin{aligned}\tilde{k}_{t+1} &= A_{1 \times 1} \tilde{k}_t + B_{1 \times 4} (\tilde{g}_t, \tau_t^l, \tau_t^x, \tilde{z}_t)' \\ (\tilde{y}_t, \tilde{c}_t, \tilde{l}_t, \tilde{x}_t)' &= C_{4 \times 1} \tilde{k}_t + D_{4 \times 4} (\tilde{g}_t, \tau_t^l, \tau_t^x, \tilde{z}_t)'\end{aligned}$$

while  $A, B, C,$  and  $D$  are matrixes containing the corresponding linear decision rule coefficients.

2. Assume  $\tilde{k}_0 = 0$ .

3. Given  $\tilde{k}_0$ ,  $(\tilde{g}_0, \tau_0^l, \tau_0^x, \tilde{z}_0)' = D^{-1} (\tilde{y}_0, \tilde{c}_0, \tilde{l}_0, \tilde{x}_0)'$

4. Given  $\tilde{k}_0$  and  $(\tilde{g}_0, \tau_0^l, \tau_0^x, \tilde{z}_0)'$ ,  $\tilde{k}_1 = B (\tilde{g}_0, \tau_0^l, \tau_0^x, \tilde{z}_0)'$

5. Given  $\tilde{k}_1$ ,  $(\tilde{g}_1, \tau_1^l, \tau_1^x, \tilde{z}_1)' = D^{-1} (\tilde{y}_1, \tilde{c}_1, \tilde{l}_1, \tilde{x}_1)' - D^{-1} C \tilde{k}_1$  and so on.

Figure 2 plots the estimated HP filtered wedges<sup>11</sup>. Resource, labor and efficiency wedges move along together with output where investment wedges moves to the opposite direction. The volatility of resource wedges is much larger than that of output while the volatility of investment wedges is much smaller than that of output.

The cyclical properties of the wedges are summarized in Table 3. Resource, labor and efficiency wedges are procyclical and their fluctuations are coincident with output fluctuation, while investment wedges are countercyclical and coincident. The correlation of efficiency wedges with output is much higher than that of other wedges.

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<sup>11</sup>The linearly detrended (non HP filtered) wedges are presented in the appendix.

### 4.3 Simulation

The model is simulated by plugging in the computed wedges into the linear decision rules of output, consumption, investment and labor supply. In order to focus on the impact of each wedges on high frequency cyclical comovements of these variables in Japan, I present the HP filtered results. Non HP filtered results are presented in the appendix.

Figure 3 shows the cyclical features of the HP filtered simulation results using each wedges one by one. The results show that efficiency wedges are important in accounting for the output fluctuation, investment wedges are important in accounting for fluctuation in investment, and labor wedges are important in accounting for the fluctuation in labor, while both labor and efficiency wedges are important in accounting for the fluctuation in consumption.

Table 4 reports the results of the simulation with only efficiency wedges<sup>12</sup>. There are several important discrepancies between the results and the data. First, the persistence of output and investment are too low. Second the contemporaneous correlations of consumption and labor with output are too high. Third, the volatility of labor is too low. Finally, the model cannot account for the lead in labor.

Table 5 reports the results with both efficiency and labor wedges. The results show that all three discrepancies mentioned above are improved. The persistence of output and investment increased, the contemporaneous correlations of consumption and labor with output are closer to the data, the volatility of labor is equivalent to data, and the model reproduces the lead in labor. Therefore, we conclude that labor wedges are important in accounting for the output and labor correlation pattern in Japan. The remaining question is, what is the fundamental source of labor wedges.

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<sup>12</sup>This is almost equivalent to the real business cycle model. The only difference is that in this simulation efficiency wedges affect the expectation on the future values of other wedges. Although the realization of other wedges are always zero, expectations of them to be non-zero has some effect on the outcome.

## 5 Sources of Labor Wedges in Japan

As shown in Chari, Kehoe and McGrattan (2007), there are several fundamental sources of labor wedges. In this section, I show that labor income taxes, money growth shocks and real interest rate shocks can be mapped into the prototype model with labor wedges. I assess whether these monetary and fiscal variables can account for the fluctuation pattern of the labor wedges in Japan.

### 5.1 Labor Tax

Since labor wedges are defined as taxes on labor income in the model, I will investigate the data of taxes. Not only labor income taxes, but also consumption taxes must be considered as they affect the relative price of labor to consumption, i.e. the effective real wage<sup>13</sup>. Therefore, the effective tax rate on labor is the sum of labor income and consumption tax rates. The effect of labor taxes on the long run declining trend on Japanese labor input is discussed in Otsu (2009). In this section, I focus on the impacts of them on the labor market in the business cycle frequency<sup>14</sup>.

Unfortunately, the tax data is only available in the annual frequency. Therefore, I compare them with the annual average of the labor wedges. Figure 4 plots the HP filtered annual tax data from McDaniel (2007) along with the labor wedges. The figure shows that the fluctuation of the annual labor wedges are highly correlated to the fluctuation of labor taxes in the early 1980s, the late 1980s and the early 2000s. Most of the movement in effective labor tax during the early 1980s is coming from consumption tax

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<sup>13</sup>Time varying consumption taxes also affect intertemporal choices of consumption, and thus can also be considered contributing to investment wedges.

<sup>14</sup>Braun (1994) and McGrattan (1994) show that fluctuations in distortionary taxes are important in accounting for the postwar US business cycle fluctuations. Braun (1994) focuses on the annual impacts of distortionary taxes while McGrattan (1994) estimates the quarterly stochastic process of taxes from annual tax data with maximum likelihood estimation.

while those in the late 1980s and early 2000s are coming from labor income tax. However, since we do not observe tax changes every quarter, we cannot directly infer their contributions to the quarterly fluctuations of labor wedges in Japan.

## 5.2 Monetary Shocks

Monetary shocks such as money growth or nominal interest rate shocks can create distortions in the labor market<sup>15</sup>. In this section, I show that a working capital model with nominal interest rate shocks, a cash in advance model and a sticky wage model with money growth shocks can be mapped into the business cycle accounting model with labor wedges. I simulate each model with the observed monetary shocks and discuss their impacts on the Japanese business cycle over the 1980-2007 period.

### 5.2.1 Working Capital Model

Labor working capital models such as Christiano and Eichenbaum (1992), assume that the firm has to borrow in order to pay for the wage bill. I assume a simple setting in which firms borrow cash from the government in the beginning of the period and pay it back at the end of the period. The government exogenously sets the interest rate on the loans to the firm  $Q_t$  based on its monetary policy stance. Since the borrowing cost is included in the labor cost, shocks to the interest rate can be interpreted as labor wedges.

The firm's problem will change accordingly as follows.

$$\max \pi_t = y_t - Q_t w_t l_t - r_t k_t$$

where  $Q_t$  is the interest payment the firm has to make. The firm's first order

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<sup>15</sup>In a Keynesian view, optimal monetary policy can reduce the inefficiency in the labor market through this distortionary effect.

condition for labor will be

$$Q_t w_t = (1 - \theta) \frac{y_t}{l_t}.$$

If  $\frac{1}{Q_t} = (1 - \tau_{lt})$  the labor working capital model is observationally equivalent to the prototype model with labor wedges. Therefore, shocks to nominal interest rates operate as labor wedges. That is, a rise in the interest rates will increase the cost of labor and reduce the labor supply.

### 5.2.2 Inflation Tax Model

Cash in advance models such as Cooley and Hansen (1989) gives rise to labor wedges from money growth shocks. With a cash in advance constraint, the consumer must hold cash in order to purchase consumption goods. Since labor income cannot be used to buy goods in the current period, money growth shocks affect the effective price of labor relative to consumption. Therefore, money growth shocks create labor wedges in the form of inflation tax.

The consumer's budget constraint with money without wedges is

$$w_t l_t + r_t k_t + \frac{M_{t-1}}{P_t} + \pi_t + \tau_t = c_t + x_t + \frac{M_t}{P_t} \quad (13)$$

Assume that the consumer holds money due to a standard cash in advance constraint

$$\frac{M_{t-1}}{P_t} + \tau_t \geq c_t \quad (14)$$

where  $M_t$  is the money supply and  $P_t$  is the price of consumption goods relative to money. The amount of cash held to purchase goods in the current period is predetermined during the previous period. The labor first order condition will be

$$\frac{1 - \Psi}{\Psi} \frac{c_t}{1 - l_t} = \frac{\lambda_t}{\lambda_t + \mu_t} w_t$$

where  $\lambda$  and  $\mu$  are the Lagrange multipliers on the constraints (13) and (14), respectively. If  $\frac{\lambda_t}{\lambda_t + \mu_t} = (1 - \tau_{lt})$  the cash in advance model is observationally equivalent to the prototype model with labor wedges. An increase in money growth  $M_t/M_{t-1} = \varphi_t$  that raises the expected inflation reduces the expected return on labor income and thus should reduce the labor supply.

### 5.2.3 Sticky Wage Model

Chari, Kehoe and McGrattan (2007) show that a model with sticky wages due to labor unions a la Cole and Ohanian (2002) can be mapped into a prototype model with labor wedges. Given the predetermined wage contract, an unexpected change in the inflation rate will create a discrepancy between the marginal product of labor and the marginal rate of substitution of leisure to consumption, which can be interpreted as an increase in labor wedges.

In this model, there is a continuum of identical labor unions  $j$  who have monopolistic power on differentiated labor supply. the aggregate labor is defined as

$$l_t = \left[ \int (l_t^j)^\nu dj \right]^{\frac{1}{\nu}}$$

where  $\nu$  is the markup of the labor union. The final good firm hires differentiated labor from each union  $j$  according to a labor demand function

$$l_t^j = \left[ \frac{w_{t-1}}{w_{t-1}^j} \right]^{\frac{1}{1-\nu}} l_t$$

where  $w_{t-1}$  is the aggregate wage for labor at period  $t$  which is predetermined in period  $t - 1$ . The labor first order condition is

$$(1 - \theta) \frac{y_t}{l_t} = \frac{w_{t-1}}{i_t}.$$

An unexpected increase in the money growth rate causes inflation which will lower the labor cost and raise labor demand whereas an expected increase

in inflation will be factored in  $w_{t-1}$  by the union and will not affect the real cost of labor<sup>16</sup>.

#### 5.2.4 Quantitative Analysis

Table 6 summarizes the behavior of monetary variables over the 1980-2007 period. The call rate is positively correlated to output and lags it in terms of the timing of fluctuation. Since Japan adopted the so-called zero interest rate policy from 1999, the moments over the 1980-1998 period are separately reported in the fourth row. I find that the fluctuation and the correlation with output is higher than the sample including the zero interest rate period. According to the working capital model, the procyclical interest rates should reduce the procyclicality of labor supply. The level of M1 is positively correlated to output and leads it in terms of fluctuation while the growth rate of M1 is negatively correlated to output and lags it in terms of fluctuation. According to the inflation tax model, the countercyclicality of the money growth should increase the correlation between output and labor. On the other hand, according to the sticky wage model, the countercyclicality of the money growth should decrease the correlation of labor with output. Therefore, interest rate shocks with working capital and money growth shocks with staggered price settings might be able to account for the labor wedges while inflation tax does not seem to be a plausible candidate for that.

In order to investigate the quantitative impacts of monetary shocks on the Japanese economy, I simulate each model with monetary shocks and efficiency wedges, i.e. total factor productivity. The deep parameters in each

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<sup>16</sup>The wage rate is set according to the optimality condition:

$$w_{t-1} = \frac{1}{\nu} \frac{1 - \Psi}{\Psi} \frac{E_{t-1} \left[ \frac{l_t}{1-l_t} \right]}{E_{t-1} \left[ \frac{l_t}{c_t} \frac{1}{i_t} \right]}.$$

This shows that only expected inflation will be incorporated in the predetermined wage rate. See the appendix for details of the unions problem.

model are calibrated in standard fashion. In order to conduct a stochastic simulation, I assume the following stochastic processes:

$$\begin{aligned}\tilde{z}_t &= \rho_z \widetilde{z}_{t-1} + \varepsilon_{zt}, \varepsilon_{zt} \sim N(0, \sigma_z^2) \\ \widetilde{Q}_t &= \rho_Q \widetilde{Q}_{t-1} + \varepsilon_{Qt}, \varepsilon_{Qt} \sim N(0, \sigma_Q^2) \\ \widetilde{\varphi}_t &= \rho_\varphi \widetilde{\varphi}_{t-1} + \varepsilon_{\varphi t}, \varepsilon_{\varphi t} \sim N(0, \sigma_\varphi^2).\end{aligned}$$

The estimated values are  $(\rho_z, \rho_Q, \rho_\varphi) = (0.98, 0.97, -0.02)$ . Total factor productivity is included in all exercises because we have already seen that they are important in accounting for the fluctuation in output. By adding monetary shocks, we can investigate their contribution to the labor wedges.

Table 7 reports the simulation results of each model. The first and second rows report the moments of labor computed from data and the business cycle accounting model with only efficiency wedges. The main discrepancies are the low contemporaneous correlation of labor and output and the lead in labor from output.

The third row reports the results from the working capital model. The contemporaneous correlation of labor with output falls from 0.94 to 0.81 by adding the interest rate shock. This is due to the depressing effect and procyclicality of the interest rates. If we disregard the post zero interest rate policy period after 1999, the volatility of labor increases from 0.41% to 0.46%, while other correlation patterns are not affected. Therefore, interest rate shocks with working capital does contribute to the labor wedge in terms of contemporaneous correlation between labor and output. However, the model cannot replicate the lead in labor.

The fourth row reports the results of the inflation tax model. The lead in labor cannot be replicated with this model. Furthermore, the contemporaneous correlation of labor with output is higher than the model without monetary shocks. This is due to the countercyclicality of the money growth shock. However, the inflation tax does improve the model in terms of the

volatility of labor relative to that of output.

The fifth row reports the results of the sticky wage model. The volatility of labor and the contemporaneous correlation of labor with output are too high. Moreover, there is no persistence in labor. In fact, the impact of money growth shocks dominate that of efficiency wedges and output loses persistence in the sticky wage setting. Chari, Kehoe and McGrattan (2000) shows that monetary shocks with staggered price setting cannot replicate the persistence in aggregate variables. This is why monetary models introduce habit preferences and investment adjustment costs in order to replicate the persistence of output, consumption, the price level and so on.

### 5.3 Other Candidates

Miss-specification of the preference function can result in labor wedges in the business cycle accounting model. For instance, preferences with habit persistence:

$$u(c_t, c_{t-1}, l_t) = \Psi \log(c_t - bc_{t-1}) + (1 - \Psi) \log(1 - l_t),$$

and preferences without income effects on labor as in Greenwood, Hercowitz and Huffman (1988):

$$u(c_t, l_t) = \log(c_t - \chi l_t^\nu),$$

imply different marginal rates of substitution

$$\frac{1 - \Psi}{\Psi} \frac{1}{1 - l_t} \left( \frac{1}{c_t - bc_{t-1}} - E_t \left[ \frac{\beta b}{c_{t+1} - c_t} \right] \right)^{-1}, \quad (15)$$

and

$$\frac{\chi l_t^{\nu-1}}{c_t - \chi l_t^\nu},$$

respectively<sup>17</sup>. If either of these preferences were the true preference function, labor wedges will arise simply because the marginal rate of substitution in the business cycle accounting model is false. The obvious challenge in assessing this issue is to claim these alternative preferences as “true” and the log preferences as “false”. A similar discussion can be made for the production function.

Variable preference weights  $\Psi$ , i.e. preference shocks, are observationally equivalent to labor wedges. By definition, anything that increases the household’s subjective value of consumption relative to leisure can be considered a preference shock. Nonetheless, preference weights are not directly observable unless the model endogenously links them to observable shocks. In Braun, Ikeda and Joines (2009), the preference weights on consumption and leisure depends on the family size. Their quantitative analysis shows that the drop in birth rate led to a long run decline in labor supply through this endogenous shift in preference weights<sup>18</sup>. Obviously, however, demographics do not fluctuate in the short run. It is not clear whether other underlying shocks are causing preference weights to fluctuate quarterly.

Fluctuations in the markup to monopolistically competitive firms, often defined as cost push shocks in New Keynesian models, are also observationally equivalent to labor wedges. By definition, the markup is the ratio of the optimal price to marginal cost where the marginal cost is the ratio of nominal wages to the marginal product of labor. Therefore, in equilibrium

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<sup>17</sup>Note that non-separable preferences

$$u(c_t, l_t) = \frac{(c_t^\Psi (1 - l_t)^{1-\Psi})^{1-\sigma}}{1 - \sigma},$$

has the same marginal rate of substitution as the log utility. Therefore, the computed labor wedges will be the same as the benchmark case. nonetheless, their impacts will be different as the Euler equation will be different due to the non-separability of consumption and leisure in the marginal utility of consumption.

<sup>18</sup>The head of the household maximizes the utility of the family. Every family member consumes while only the head of the household goes to work. An decrease in family members increases the preference weight of leisure and hence leads to a decrease in labor.

the markup is equal to the ratio of the marginal product of labor to the real wage. Thus, stochastic markup shocks are observationally equivalent to labor wedges. There are several sources of stochastic markup fluctuations. For instance, changes in the bargaining power of labor unions can cause a fluctuation in the markup. Steinsson (2003) shows that stochastic shocks to the household elasticity of substitution among the differentiated goods manifest themselves as markup shocks. However, it is not clear how to measure the union bargaining power or the curvature on the preference function from data.

Finally, expectational shocks can also generate labor wedges. Consider the habit preferences discussed above that gives the marginal rate of substitution as (15). Clearly, a shock to expected future consumption can affect the marginal rate of substitution, which is observationally equivalent to labor wedges in the benchmark model. Fujiwara, Hirose and Shintani (2011) estimate multi-period expectational shocks to future total factor productivity in a DSGE model using Bayesian estimation. This will enable us to investigate whether expectational shocks can account for the correlation pattern of labor with output. However, the expectational shocks cannot be directly observed.

## 6 Conclusion

This paper applies the business cycle accounting method to the Japanese economy and shows that the fluctuations in labor wedges are important in order to replicate the low contemporaneous correlation between labor and output and the lead of labor to output. Fiscal and monetary variables such as labor income tax, money growth and interest rates cannot fully account for the distortions in the Japanese labor market. Further understanding of the quarterly Japanese labor market fluctuations requires a deeper assessment on additional sources of labor wedges such as preference shocks, markup shocks and expectational shocks as well as the functional form of aggregate

preferences.

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## A Non-Filtered Results

Figure A1 presents the data of the observable variables used for the estimation and simulation. Output, consumption and investment are detrended by the growth trend  $\eta$ . The non-HP filtered data clearly shows the abrupt growth in the late 1980s and the lost decade in the 1990s. Consumption has been declining relative to output implying a decline in the consumption share of output<sup>19</sup>. Labor is declining throughout the entire period.

Figure A2 plots the non-filtered wedges. Resource, labor and efficiency wedges have growing trends. Government expenditure is growing relative to output in Japan over the entire period, which explains the growth in resource wedges. The growth in labor wedges reflects the declining trend in labor. The growth in efficiency wedges reflects the fact that labor is declining in data whereas it is assumed to be stationary in the model. Although this discrepancy is unpleasant, it does not have a major impact on the estimation or simulation results in the paper.

Figure A3 plots the non-filtered simulated output. Resource and investment wedges do not have much impact on output fluctuation. Efficiency wedges account for most of the ups and downs of output, however, it over estimates the growth throughout the entire period. Labor wedges cause output to decline throughout the entire period. The combination of labor and efficiency wedges reproduces output almost perfectly. This result is consistent with the finding of Kobayashi and Inaba (2006).

## B Monetary Models

The following describes the detail of the monetary models introduced in section 5.

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<sup>19</sup>This is mainly due to the increase in government expenditures over the entire period.

## B.1 Working Capital Model

### B.1.1 Firm's Problem

$$\begin{aligned} \max \pi_t &= y_t - Q_t w_t l_t - r_t k_t \\ \text{subject to } y_t &= z_t k_t^\theta l_t^{1-\theta} \end{aligned}$$

### B.1.2 Household Problem

$$\begin{aligned} \max U &= E_0 \sum_{t=0}^{\infty} \beta^t [\Psi \log(c_t) + (1 - \Psi) \log(1 - l_t)] \\ \text{subject to } w_t l_t + r_t k_t + \pi_t + \tau_t &= c_t + x_t \\ \eta k_{t+1} &= x_t + (1 - \delta) k_t \end{aligned}$$

### B.1.3 Government Budget Constraint

$$(Q_t - 1)w_t l_t = \tau_t + g$$

### B.1.4 Equilibrium Conditions

We have 5 equations, 5 endogenous variables  $\{y, c, x, l, k\}$  and 2 exogenous variables  $\{z_t, Q_t\}$ .

$$\begin{aligned} y_t &= z_t k_t^\theta l_t^{1-\theta} \\ y_t &= c_t + x_t + g \\ \eta k_{t+1} &= x_t + (1 - \delta) k_t \\ \eta \frac{\Psi}{c_t} &= \beta E_t \left[ \frac{\Psi}{c_{t+1}} \left( \theta \frac{y_{t+1}}{k_{t+1}} + 1 - \delta \right) \right] \\ (1 - \theta) \frac{y_t}{l_t} &= Q_t \frac{1 - \Psi}{\Psi} \frac{c_t}{1 - l_t} \end{aligned}$$

## B.2 Inflation Tax Model

### B.2.1 Firm's Problem

$$\max P_t \pi_t = P_t y_t - W_t l_t - R_t k_t$$

$$\text{subject to } y_t = z_t k_t^\theta l_t^{1-\theta}$$

### B.2.2 Household Problem

$$\max U = E_0 \sum_{t=0}^{\infty} \beta^t [\Psi \log(c_t) + (1 - \Psi) \log(1 - l_t)]$$

$$\text{subject to } W_t l_t + R_t k_t + M_{t-1} + P_t \pi_t + P_t \tau_t = P_t c_t + P_t x_t + M_t$$

$$\eta k_{t+1} = x_t + (1 - \delta) k_t$$

$$\frac{M_{t-1}}{P_t} \geq c_t$$

### B.2.3 Government Budget Constraint

$$M_t - M_{t-1} = P_t \tau_t + P_t g$$

$$M_t = \varphi_t M_{t-1}$$

### B.2.4 Change of Variables

$$m_t = \frac{M_t}{P_t}, w_t = \frac{W_t}{P_t}, r_t = \frac{R_t}{P_t}, i_t = \frac{P_t}{P_{t-1}}$$

### B.2.5 Equilibrium Conditions

We have 9 equations, 9 endogenous variables  $\{y, c, x, l, k, m, i, w, \lambda\}$ , where  $\lambda$  is the Lagrange multiplier on the household budget constraint, and 2 exogenous variables  $\{z_t, \varphi_t\}$ .

$$\begin{aligned}
y_t &= z_t k_t^\theta l_t^{1-\theta} \\
y_t &= c_t + x_t + g \\
\eta k_{t+1} &= x_t + (1 - \delta)k_t \\
\eta \lambda_t &= \beta E_t \left[ \lambda_{t+1} \left( \theta \frac{y_{t+1}}{k_{t+1}} + 1 - \delta \right) \right] \\
\lambda_t &= \frac{1}{w_t} \frac{1 - \Psi}{1 - l_t} \\
\lambda_t &= \beta E_t \left[ \frac{1}{i_{t+1}} \frac{\Psi}{c_{t+1}} \right] \\
w_t &= (1 - \theta) \frac{y_t}{l_t} \\
m_t &= c_t + g \\
i_t &= \frac{P_t}{P_{t-1}} = \frac{m_{t-1}}{m_t} \frac{M_t}{M_{t-1}} = \frac{m_{t-1}}{m_t} \varphi_t
\end{aligned}$$

## B.3 Sticky Wage Model

### B.3.1 Firm's Problem

#### Profit Maximization

$$\max P_t \pi_t = P_t y_t - W_{t-1} l_t - R_t k_t$$

$$\text{subject to } y_t = z_t k_t^\theta l_t^{1-\theta}$$

#### Labor Cost Minimization

$$\min \int W_{t-1}^j l_t^j dj$$

$$\text{subject to } l_t = \left[ \int (l_t^j)^\nu dj \right]^{\frac{1}{\nu}}$$

### B.3.2 Union's Problem

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \left[ \Psi \log(c_t^j) + (1 - \Psi) \log(1 - l_t^j) + \chi \log\left(\frac{M_t}{P_t}\right) \right]$$

$$\text{subject to } W_{t-1}^j l_t^j + R_t k_t^j + M_{t-1}^j + P_t \pi_t^j + P_t \tau_t^j = P_t c_t^j + P_t x_t^j + M_t^j$$

$$\eta k_{t+1}^j = x_t^j + (1 - \delta) k_t^j$$

$$l_t^j = \left[ \frac{W_{t-1}}{W_{t-1}^j} \right]^{\frac{1}{1-\nu}} l_t$$

### B.3.3 Government Budget Constraint

$$M_t - M_{t-1} = P_t \tau_t + P_t g$$

$$M_t = \varphi_t M_{t-1}$$

### B.3.4 Change of Variables

$$m_t = \frac{M_t}{P_t}, w_t = \frac{W_t}{P_t}, r_t = \frac{R_t}{P_t}, i_t = \frac{P_t}{P_{t-1}}$$

### B.3.5 Equilibrium Conditions

All unions are identical so we can drop the super-scripts  $j$ . We have 9 equations, 9 endogenous variables  $\{y, c, x, l, k, m, w, i\}$  and 2 exogenous variables  $\{z_t, \varphi_t\}$ .

$$y_t = z_t k_t^\theta l_t^{1-\theta}$$

$$\eta k_{t+1} = x_t + (1 - \delta) k_t$$

$$y_t = c_t + x_t + g$$

$$\eta \frac{\Psi}{c_t} = \beta E_t \left[ \frac{\Psi}{c_{t+1}} \left( \theta \frac{y_{t+1}}{k_{t+1}} + 1 - \delta \right) \right]$$

$$w_t = \frac{1}{\nu} \frac{1 - \Psi}{\Psi} \frac{E_t \left[ \frac{l_{t+1}}{1 - l_{t+1}} \right]}{E_t \left[ \frac{l_{t+1}}{c_{t+1}} \frac{1}{i_{t+1}} \right]}$$

$$i_t (1 - \theta) \frac{y_t}{l_t} = w_{t-1}$$

$$\frac{\chi}{m_t} = \frac{\Psi}{c_t} - \beta E_t \left[ \frac{\Psi}{c_{t+1}} \frac{1}{i_{t+1}} \right]$$

$$i_t = \frac{m_{t-1}}{m_t} \varphi_t$$

## C Tables and Figures

Table 1a. Business Cycle Features of the Japanese Economy (1980-2007)

$v$	$std\ of\ v$		Correlation of Output with						
	%	$rel.y$	$v(-3)$	$v(-2)$	$v(-1)$	$v(0)$	$v(1)$	$v(2)$	$v(3)$
Output	1.06	1.00	0.46	0.61	0.79	1.00	0.79	0.61	0.46
Consumption	0.60	0.56	0.15	0.11	0.18	0.38	0.28	0.29	0.36
Investment	2.87	2.70	0.58	0.67	0.80	0.89	0.72	0.54	0.35
Government	0.94	0.88	0.14	0.10	0.06	0.00	-0.07	-0.12	-0.19
Capital	0.45	0.42	-0.25	-0.11	0.03	0.18	0.36	0.50	0.60
Labor	0.74	0.69	0.36	0.47	0.55	0.45	0.40	0.24	0.12

Table 1b. Business Cycle Features of the US Economy (1980-2007)

$v$	$std\ of\ v$		Correlation of Output with						
	%	$rel.y$	$v(-3)$	$v(-2)$	$v(-1)$	$v(0)$	$v(1)$	$v(2)$	$v(3)$
Output	1.22	1.00	0.50	0.68	0.86	1.00	0.86	0.68	0.50
Consumption	0.74	0.60	0.56	0.68	0.78	0.84	0.73	0.61	0.47
Investment	4.37	3.57	0.54	0.69	0.84	0.94	0.76	0.54	0.33
Government	0.94	0.77	-0.07	-0.12	-0.12	-0.03	-0.02	0.01	0.06
Capital	0.33	0.27	-0.04	0.07	0.21	0.43	0.55	0.64	0.71
Labor	1.31	1.07	0.36	0.55	0.75	0.89	0.87	0.78	0.66

Table 2. Parameter Values

$\theta$	Capital Share	0.388
$\delta$	Depreciation	0.022
$\beta$	Discount Factor	0.987
$\Psi$	Preference Weight	0.228
$\gamma$	Technology Growth	0.0035
$n$	Population Growth	0.0019

Table 3. Business Cycle Features of Estimated Wedges

$v$	<i>std of v</i>		Correlation of Output with						
	%	<i>rel.y</i>	$v(-3)$	$v(-2)$	$v(-1)$	$v(0)$	$v(1)$	$v(2)$	$v(3)$
Output	1.06	1.00	0.46	0.61	0.79	1.00	0.79	0.61	0.46
Resource W.	2.40	2.25	0.01	0.22	0.36	0.54	0.43	0.31	0.24
Labor W.	1.05	0.99	0.05	0.13	0.19	0.37	0.27	0.23	0.16
Investment W.	0.25	0.24	-0.14	-0.30	-0.38	-0.24	-0.37	-0.32	-0.17
Efficiency W.	0.92	0.87	0.40	0.50	0.64	0.90	0.65	0.50	0.38

Table 4. Business Cycle Accounting Results with Efficiency Wedges

$v$	<i>std of v</i>		Correlation of Output with						
	%	<i>rel.y</i>	$v(-3)$	$v(-2)$	$v(-1)$	$v(0)$	$v(1)$	$v(2)$	$v(3)$
Output	1.25	1.00	0.34	0.47	0.57	1.00	0.57	0.47	0.34
Consumption	0.63	0.50	0.22	0.36	0.50	0.92	0.64	0.59	0.51
Investment	3.57	2.86	0.37	0.49	0.58	0.99	0.53	0.41	0.27
Labor	0.53	0.42	0.41	0.50	0.57	0.94	0.44	0.30	0.15

Table 5. Business Cycle Accounting Results with Efficiency and Labor Wedges

$v$	<i>std of v</i>		Correlation of Output with						
	%	<i>rel.y</i>	$v(-3)$	$v(-2)$	$v(-1)$	$v(0)$	$v(1)$	$v(2)$	$v(3)$
Output	0.98	1.00	0.43	0.51	0.67	1.00	0.67	0.51	0.43
Consumption	0.49	0.50	0.25	0.35	0.54	0.81	0.67	0.59	0.58
Investment	2.95	3.01	0.46	0.52	0.66	0.98	0.62	0.44	0.35
Labor	0.64	0.65	0.30	0.29	0.33	0.28	0.13	-0.01	-0.04

Table 6. Business Cycle Features of Monetary Variables

$v$	<i>std of v</i>		Correlation of Output with						
	%	<i>rel.y</i>	$v(-3)$	$v(-2)$	$v(-1)$	$v(0)$	$v(1)$	$v(2)$	$v(3)$
Output	1.06	1.00	0.46	0.61	0.79	1.00	0.79	0.61	0.46
Labor Wedges	1.05	0.99	0.05	0.13	0.19	0.37	0.27	0.23	0.16
Call Rate	0.21	0.20	0.22	0.30	0.34	0.38	0.40	0.39	0.38
Call Rate (-98)	0.26	0.22	0.26	0.34	0.38	0.41	0.44	0.45	0.46
M1	4.10	3.86	0.21	0.17	0.12	0.04	-0.06	-0.10	-0.11
M1 Growth	2.82	2.65	0.07	-0.04	-0.06	-0.10	-0.13	-0.04	0.01

Table 7. Simulation Results of Monetary Models

$l$	<i>std of l</i>		Correlation of Output with Labor						
	%	<i>rel.y</i>	$l(-3)$	$l(-2)$	$l(-1)$	$l(0)$	$l(1)$	$l(2)$	$l(3)$
Data	0.74	0.69	0.36	0.47	0.55	0.45	0.40	0.24	0.12
Efficiency W.	0.53	0.42	0.41	0.50	0.57	0.94	0.44	0.30	0.15
Work. Cap	0.41	0.36	0.30	0.37	0.44	0.81	0.29	0.13	-0.01
Inflation Tax	1.08	0.67	0.33	0.51	0.59	0.99	0.53	0.41	0.27
Sticky Wage	6.48	1.59	-0.09	0.02	-0.17	0.97	-0.23	-0.01	-0.16

**Figure 1a. Japanese Business Cycles (Expenditure)**

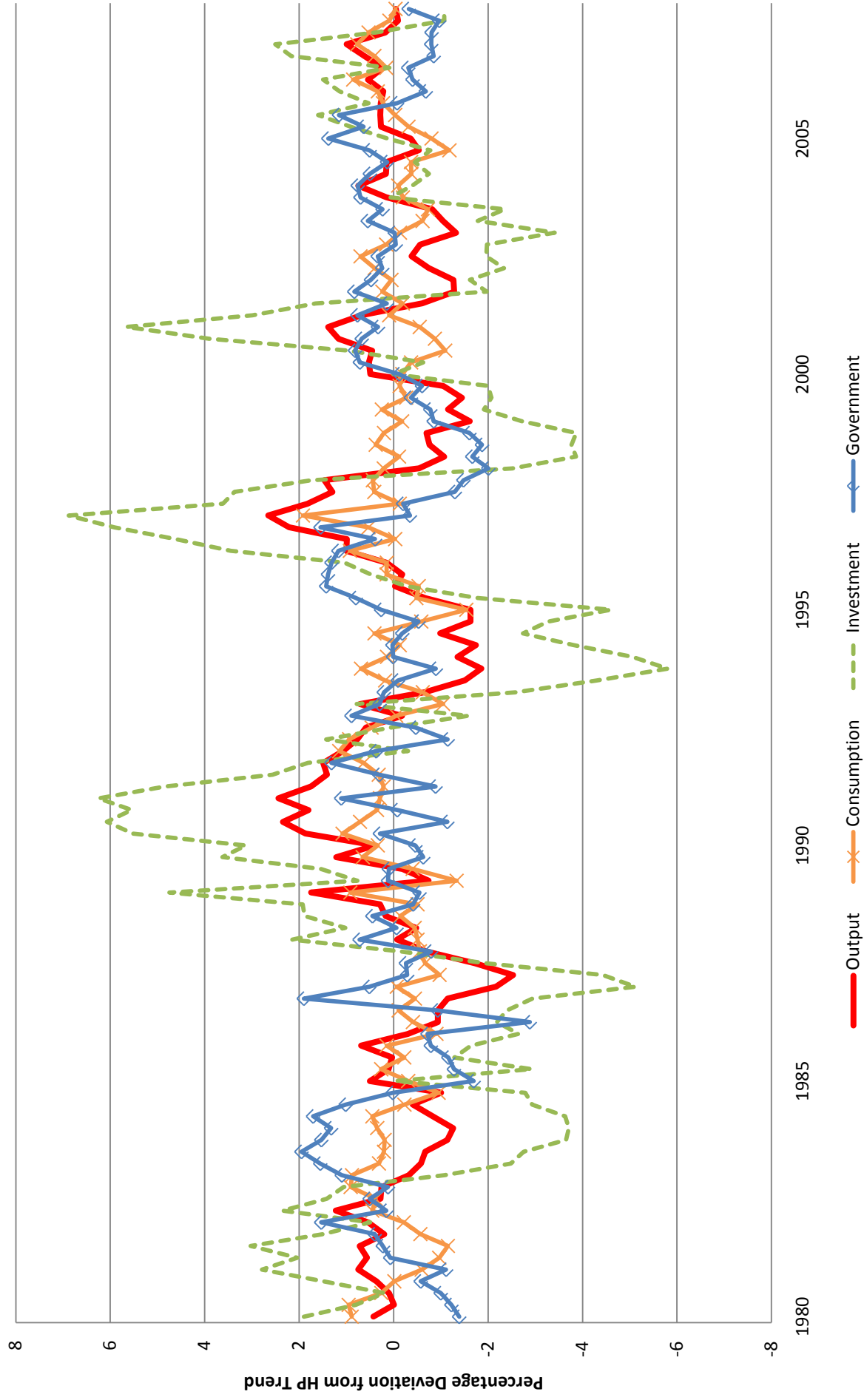


Figure 1b. Japanese Business Cycle (Production)

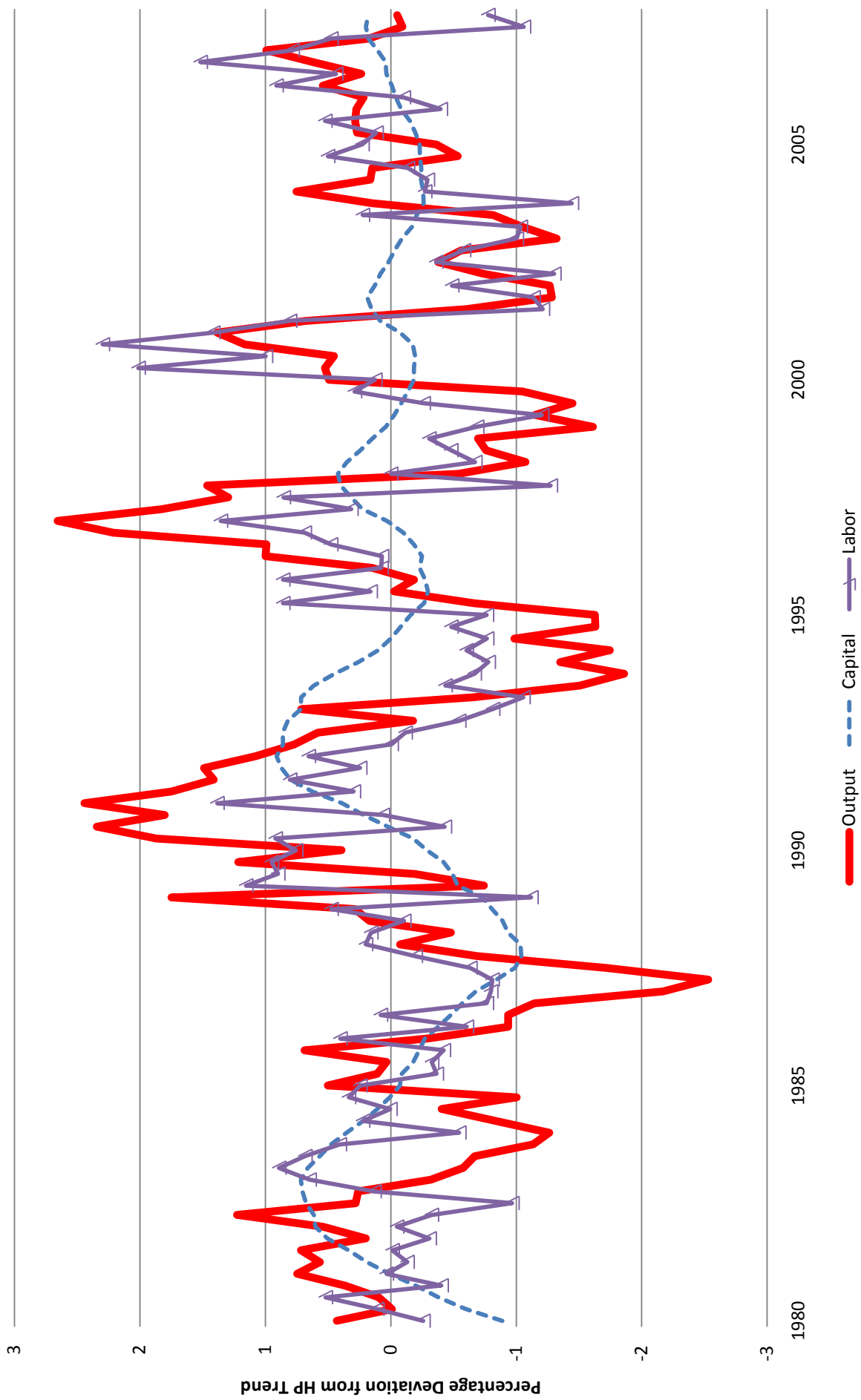


Figure 2. Wedges

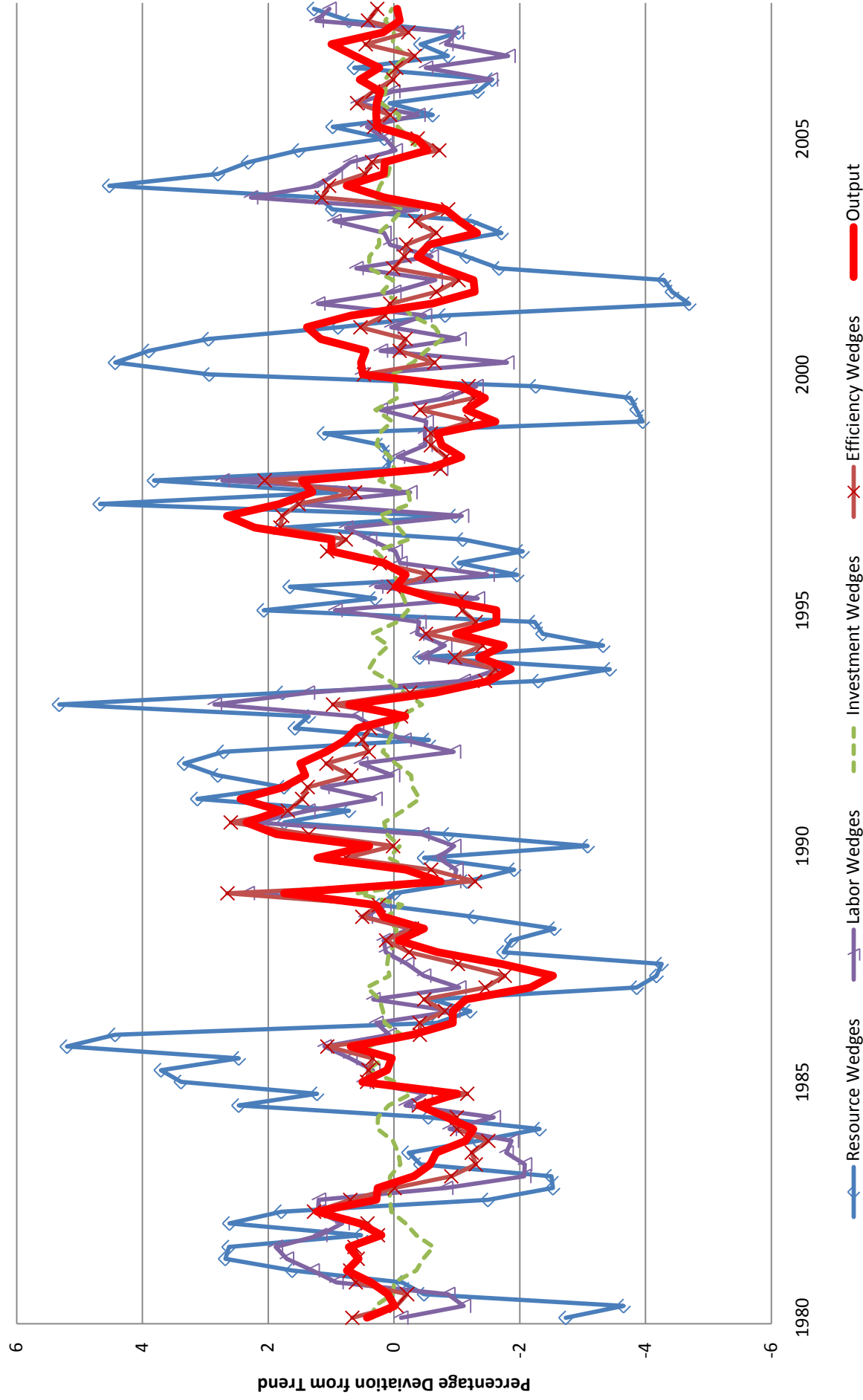


Figure 3. Simulated Output

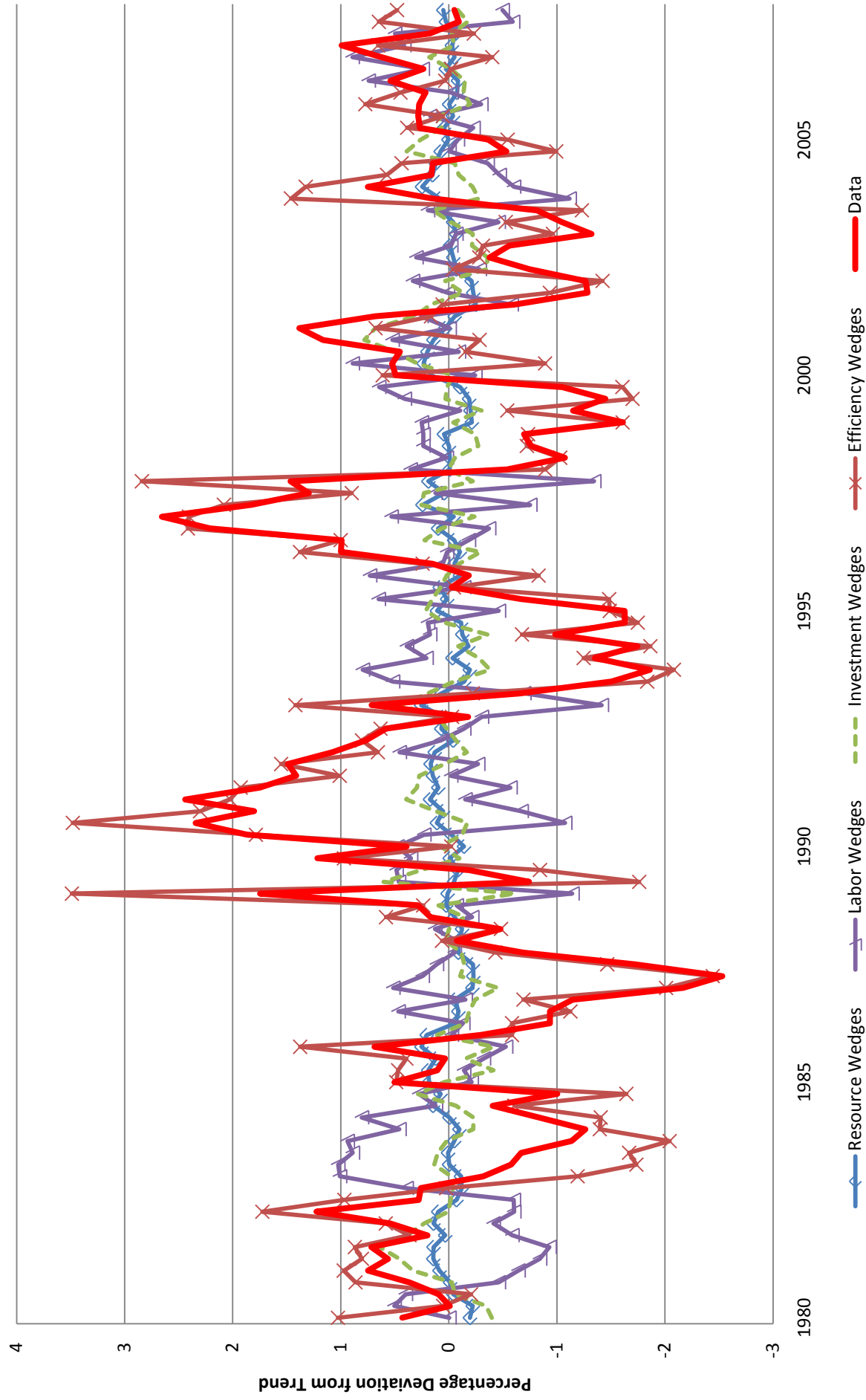
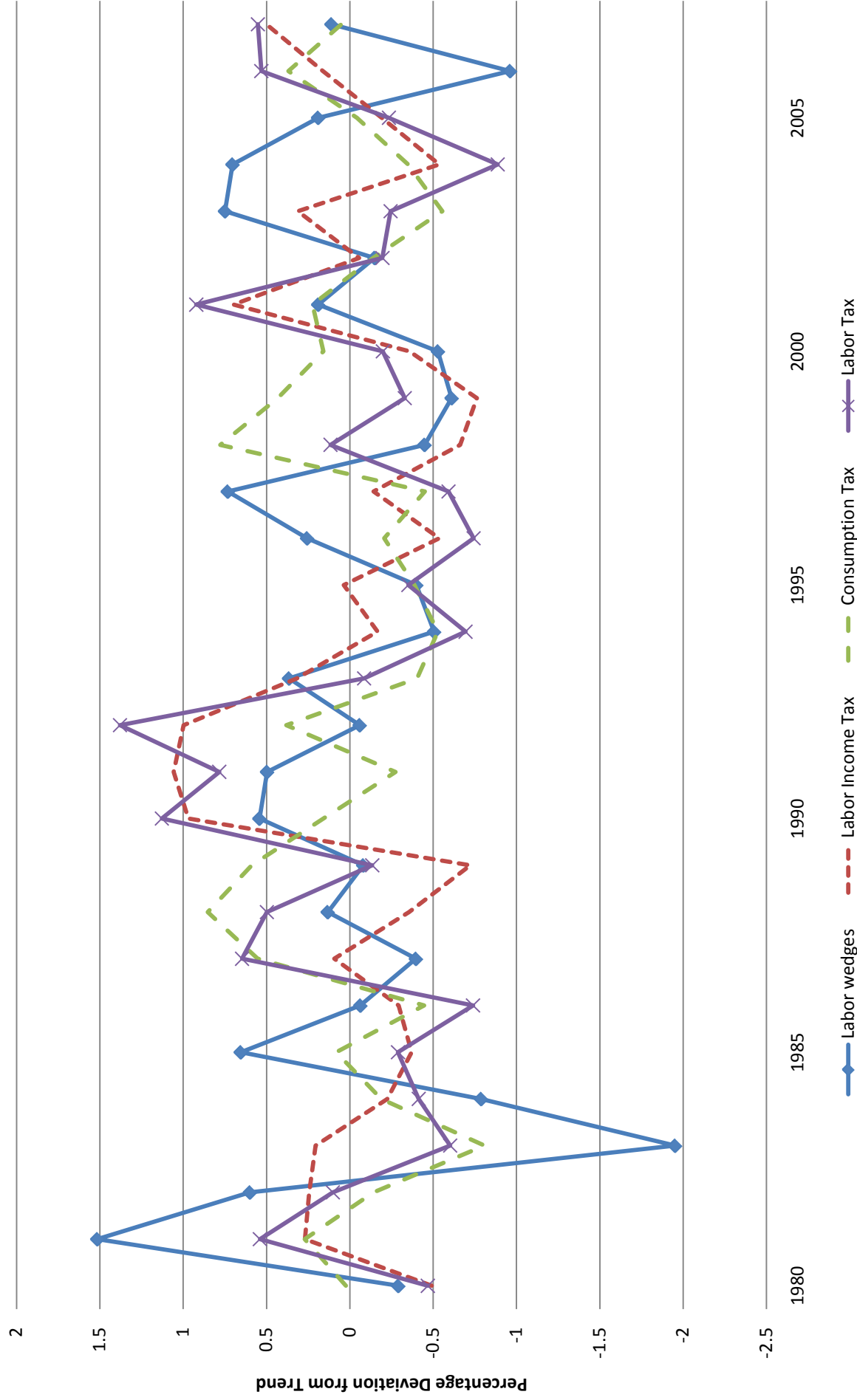


Figure 4. Labor Wedges and Labor Tax



**Figure A1. Japanese Business Cycles (Non-Filtered)**

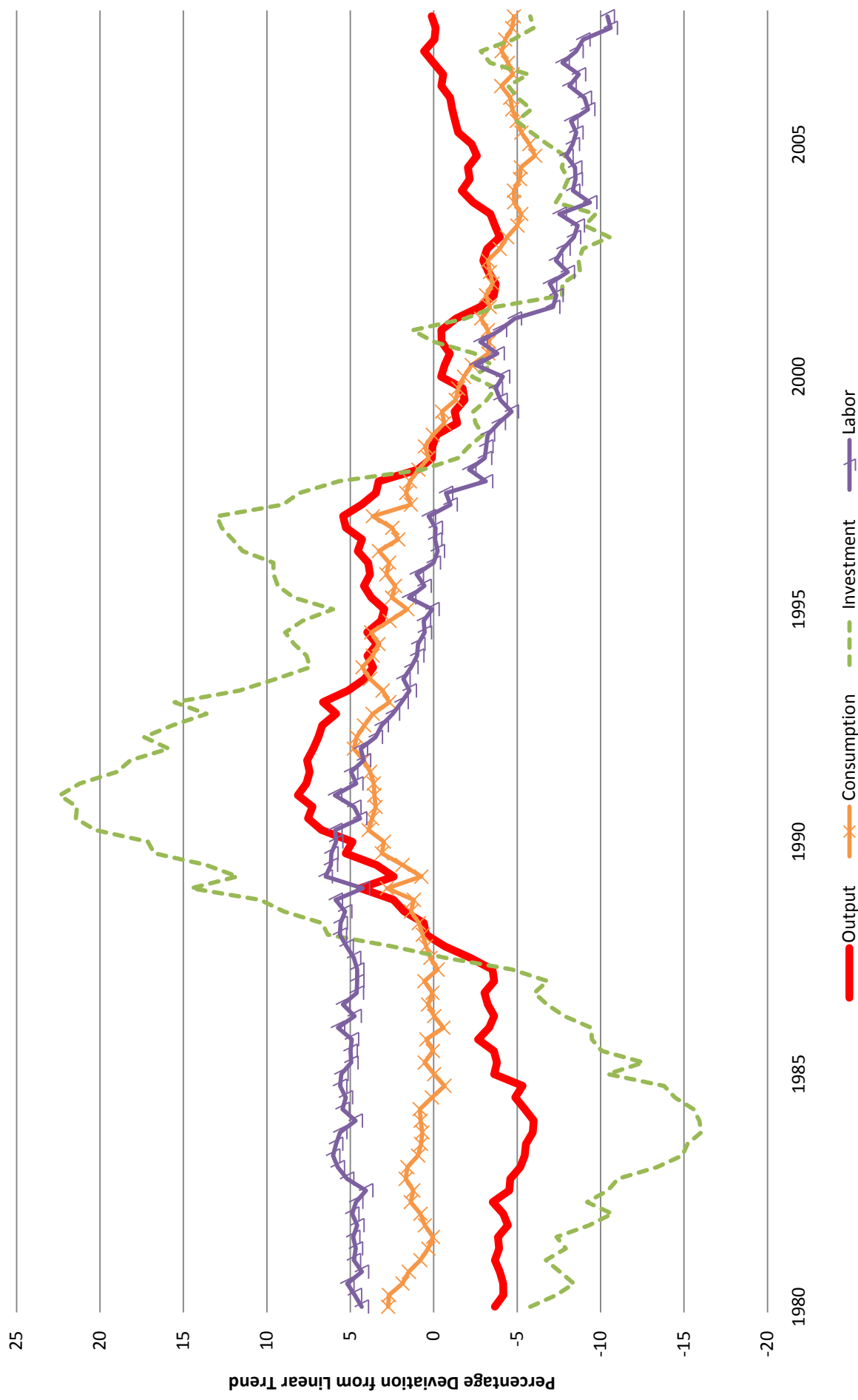


Figure A2. Wedges (Non-Filtered)

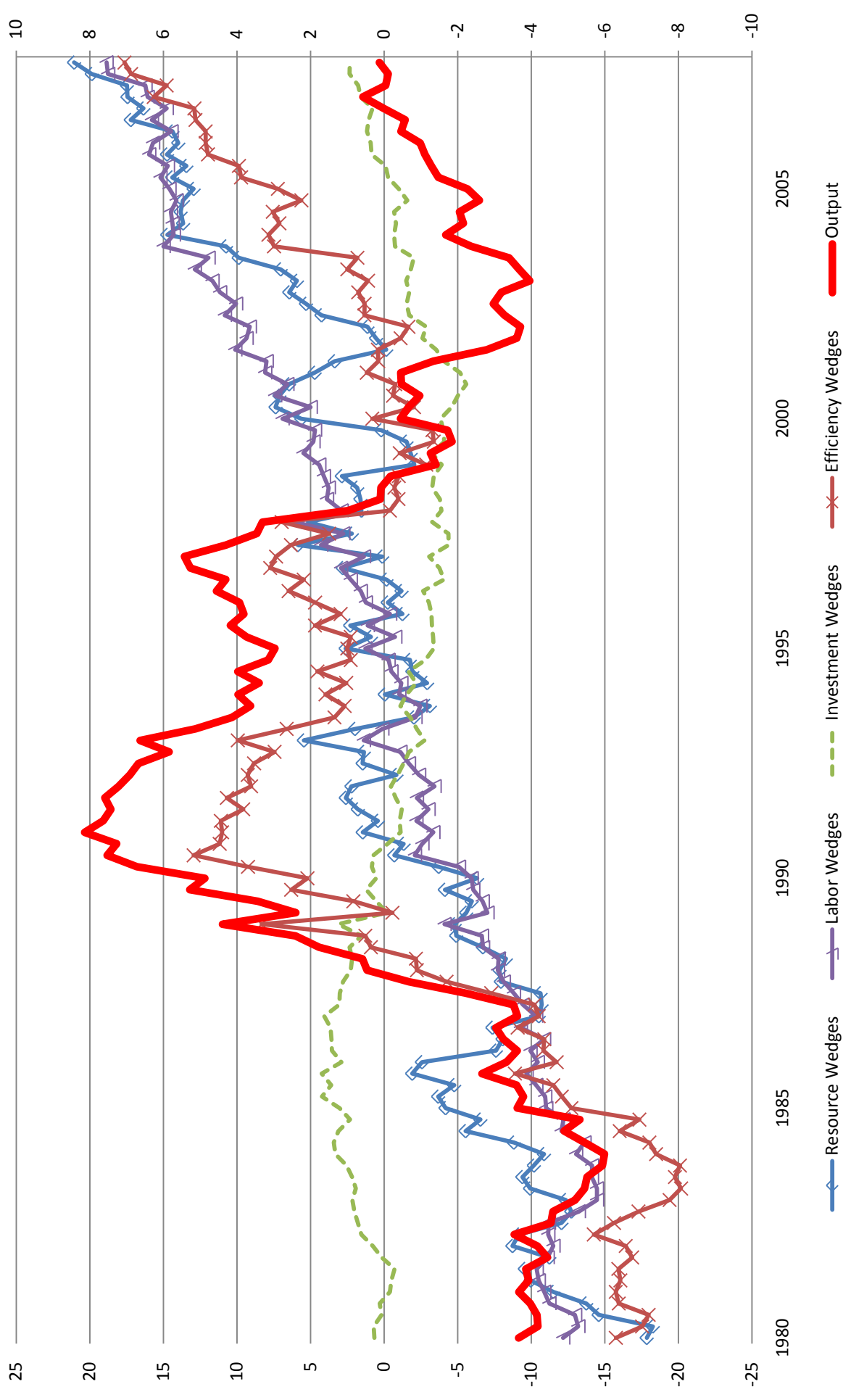


Figure A3. Simulated Output (Non-Filtered)

