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Ignorance is bliss: rationality, information and equilibrium

Sylvain Barde

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Ignorance is bliss: rationality, information and equilibrium[*]

Sylvain Barde[a,b]

[a]*School of Economics, Keynes College, University of Kent, Canterbury, CT2 7NP, UK. tel :
+44 (0)1 227 824 092, email: s.barde@kent.ac.uk*
[b]*OFCE - Department for Research on Innovation and Competition*

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Abstract

An information-theoretic thought experiment is developed to provide a methodology for predicting endowment distributions in the absence of information on agent preferences. The allocation problem is first presented as a stylised knapsack problem. Although this knapsack allocation is intractable, the social planner can nevertheless make precise predictions concerning the endowment distribution by using its information-theoretic structure. By construction these predictions do not rest on the rationality of agents. It is also shown, however, that the knapsack problem is equivalent to a congestion game under weak assumptions, which means that the planner can nevertheless evaluate the optimality of the unobserved allocation.

JEL classification: C02, C62, D51, D83.

Keywords: Information theoretic measure, knapsack problem, congestion game, potential function.

1 Introduction

Economics is traditionally based on the observation of two central stylised facts. The first is the existence of stable economy-wide characteristics, such as stable prices, demands etc. The second is the existence of systematic behaviour from agents, which broadly consist of rational and optimal actions with respect to selfish interests. Standard economic models show that this systematic behaviour of agents can lead to such stable aggregate

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characteristics. However, the reverse link is also of interest: Does the observation of stable aggregate characteristics imply that agents are rational ? In other words, while systematic agent behaviour might be a sufficient condition for stable aggregate characteristics, is it also necessary one?

The existence of this reverse link was raised in particular by Becker (1962), who shows that downwards sloping market demands can occur independently of agent rationality, simply through changes in the opportunity set of agents following changes in prices. Gode and Sunder (1993) analyse this further and show that market discipline can lead to efficient outcomes even with zero-intelligence traders that perform no optimisation. Their conclusion is in fact that "learning, intelligence, or profit motivation is not necessary" (Gode and Sunder, 1993, p134). Finally, the nature of the link between these two stylised facts is also put into question by the result of Sonnenschein (1972, 1973), Mantel (1974) and Debreu (1974), as their central theorem shows that "the utility hypothesis tells us nothing about market demand unless it is augmented by additional requirements" (Sonnenschein and Shafer, 1992, p672).

The relation between these two stylised facts is investigated using a thought experiment in which the problem of allocating resources between agents is presented as a variant of the knapsack problem. This is a well known combinatorial optimisation problem where one has a set of objects with given values and weights and the objective is to pick the combinations of objects with the highest value without exceeding fixed a weight limit, i.e. the capacity of the knapsack. The purpose of this thought experiment is to clarify the sequence of steps required to solve the problem in theory, rather than to provide a practical solution to allocation problems.

The first key finding of the thought experiment is that information-theoretic considerations strongly constrain the feasible endowment distribution. The second is that under standard assumptions on preferences the knapsack allocation problem is equivalent to a congestion game, a form of game identified by Rosenthal (1973) in which a single potential function encodes changes in payoffs when agents switch strategies and attains an extremum for Nash equilibria. Overall, this implies that the endowment distributions

depend only on the constraints of the knapsack problem while the conditions under which the knapsack problem is equivalent to a congestion game only relate to the preferences of agents.

The rest of the paper is structured as follows. Section 2 presents the the knapsack framework used to model the allocation problem facing a social planner. Section 3 then uses an information-theoretic methodology to show how the predicted aggregate distributions depend on the constraints only and how the similarity between knapsack problems and congestion games depends only on assumptions relating to preferences. Section 4 discusses the implications of these findings and concludes.

2 A thought experiment: allocation as a knapsack problem

The allocation problem facing a social planner is modeled using the multichoice multidimensional variant of the knapsack problem (MMKP). Compared to a standard knapsack problem the MMKP enlarges both the number of choices and constraints, making the framework more general. In this variant several groups of objects are available, with each object providing a specific value and requiring distinct resources. The objective is to pick a single object from each of the groups, maximising their aggregate value while ensuring the multi-dimensional resource constraint is met.[1] The MMKP has already been used in the operational research literature to model practical allocation problems, for example allocating nurses with different skills and time preferences to different types of shifts (Dowsland and Thompson, 2000), or allocating distinct computing resources such as memory and CPU cycles to several networked users with different session preferences (Khan, Li, Manning, and Akbar, 2002).

There are N agents in the economy, labeled $i \in \{1, 2, ..., N\}$, and the social planner has to allocate Q different units amongst those agents. Although this does not influence

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[1]For instance, in the allocation problem each agent is faced with a group of bundles and the optimisation requires picking a single bundle for each agent.

the general problem, it will be convenient in the discussion to distinguish K types of commodities, labeled $k \in \{1, 2, ..., K\}$ for which $q_k \in \mathbf{N}$ units are available, in which case $Q = \sum_k q_k$. The allocation problem can be solved, in principle with the following four steps.

- Step 1: The social planner labels all the possible bundles that can be built with the Q units available and lists them in a $2^Q \times Q$ binary identifier table B, shown in table (1). The binary string formed by each row provides a unique identifier for the bundle as well as the bundle's composition. [2]

Table 1: Binary bundle identifiers

B	$j = 1$	$j = 2$	$j = 3$	$j = 4$	...	$j = Q$
$b = 1$	0	0	0	0	...	0
$b = 2$	1	0	0	0	...	0
$b = 3$	0	1	0	0	...	0
...	...	...	...	...	...	...
$b = 2^Q$	1	1	1	1	...	1

- Step 2: The social planner sends the B-table to the N agents who rank the 2^Q bundles according to their preference. The rankings are returned to the social planner who then builds a $2^Q \times N$ ranking table U, shown in table (2). Under the usual assumptions of transitivity and monotonicity, all agents will rank the full bundle first and the empty bundle last.

Table 2: Bundle preference ranking

U	$i = 1$	$i = 2$	$i = 3$	$i = 4$	...	$i = N$
$b = 1$	2^Q	2^Q	2^Q	2^Q	...	2^Q
$b = 2$	...	...	...	...	...	...
...	...	...	...	...	...	...
$b = 2^Q$	1	1	1	1	...	1

- Step 3: The social planner must pick a bundle for each agent, using a $2^Q \times N$ choice matrix X, where the choice variables are $X_{b,i} \in \{0, 1\}$. Importantly, each agent only

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[2]As a matter of convention, the first bundle listed is the empty bundle and the last one is the full bundle.

receives a single bundle, i.e. $\sum_{b=1}^{2^Q} X_{b,i} = 1 \quad \forall i \in N.$[3] The goal of the social planner is to minimise the sum of the ranks over agents while remaining within the resource constraint. Formally, this can be expressed as the following MMKP:

$$
\begin{aligned}
&\min \operatorname{tr}\left(U X'\right), \\
&\text{s.t. :} B' X 1_N = 1_Q .
\end{aligned}
\tag{1}
$$

Here 1_N and 1_Q are the N and Q-length unit vectors respectively. Choosing an objective function for the MMKP is directly related to the problem of choosing a social welfare function. Given the rankings, one can select any bijective function to transform ranks into cardinal values, with the lowest numeral rank producing the highest utility. Minimising the sum of the ranks is therefore equivalent to a maximisation of the sum of the utilities, which is a standard Benthamite social welfare function.[4] The constraint ensures that the sum of the binary identifiers for each selected bundle equals the unit vector, i.e. each unit in Q is selected only once. Expressed in scalar notation, this corresponds to the MMKP as presented by Hifi, Michrafy, and Sbihi (2004); Sbihi (2007). The only differences compared to the more general framework in the operational research literature is that the program uses minimisation rather than maximisation, the resource requirement per bundle in B is the same for all i agents, and the available capacity is restricted to one for all dimensions in Q:

$$
\begin{aligned}
&\min \sum_{i=1}^{N} \sum_{b=1}^{2^Q} U_{b,i} X_{b,i}, \\
&\text{s.t. :} \sum_{i=1}^{N} \sum_{b=1}^{2^Q} B_{b,j} X_{b,i} = 1 \quad \forall j \in Q .
\end{aligned}
$$

- Step 4: Once the optimal choice table X^* is obtained, the social planner can build a $Q \times N$ allocation table $A^* = B' X^*$, shown in table (3). This table uniquely assigns every unit in Q to an agent in N, and can therefore be used for the purpose of

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[3]One can see that even if the agent is allocated two bundles a and b from the B-table 1, then $a + b$ is also a bundle in B.

[4]Any bijective functions is fine, as a utility function is never uniquely defined. Choosing a different function for different agents is equivalent to choosing different weights for the agents in the linear sum.

selecting goods one by one and dispatching them to their allocated owner.

Table 3: Allocation table

A^*	$i = 1$	$i = 2$	$i = 3$	$i = 4$	...	$i = N$
$j = 1$	0	0	1	0	...	0
$j = 2$	1	0	0	0	...	0
$j = 3$	0	1	0	0	...	0
...	...	...	...	...		...
$j = Q$	0	0	0	1	...	0

There are two main advantages to presenting an allocation problem in this way. The first is that it provides a stylised model that neatly separates the types of hurdles facing an economic planner, starting with the choice of the correct social welfare function, then, as pointed out by Hayek (1945), the presence of a high and potentially unfeasible informational requirement (Step 2), followed by a large computationally complex combinatorial optimisation (Step 3).[5] Given these difficulties, the second advantage of the thought experiment is the fact that the structure of the MMKP problem and its solution provides a framework within which the state space, and the ignorance of the location within it, can be measured and used for analytical purposes. This is examined in the next section.

3   An information-theoretic prediction methodology

In theory all four steps of the MMKP are feasible and A^* exists. However given that it is not tractable in practice for the social planner to perform either Step 2 or 3 of this procedure, the optimal allocation A^* is unknown. As a first step, we show that this ignorance can not only be precisely measured, but also used to obtain a prediction of the stable distribution of the Q units over the N agents that relies only on the constraint of the MMKP. In a second step, we then show that whether or not the stable distribution is linked to the existence of a decentralised equilibrium depends on characteristics of the objective function of the MMKP only.

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[5] The knapsack problem is known to be NP-complete, in other words solutions to the problem can be verified efficiently (in polynomial time), but there is no known algorithm for calculating the solutions efficiently in the first place.

3.1 Information-theoretic constraints to aggregate behaviour

Shannon (1948) shows that the information content of an uncertain message M, where the message is X characters long and each character can take Y values, is given by:

$$I(M) = X \sum_{y=1}^{Y} p_y \log p_y \qquad (2)$$

Here p_y is the frequency of the yth character. If one has a frequency distribution $\{p_y\}$, equation (2) gives the amount of bits required to code the message, as is done in Shannon (1948) with the average information content of English text.[6] Alternatively, as explained by Jaynes (1957a,b), if one can obtain the information content $I(M)$ directly, equation (2) can be used in the opposite direction, in order to predict the frequency distribution consistent with the information content and the length of the message. The information content can be derived either from measurement or from theoretical considerations, as Theil (1967) and Jaynes (1989) show that the information content of a message is simply the logarithm of its state space.

This has given rise to the maximum entropy (MaxEnt) methodology, which is used to describe the aggregate behaviour of a system in situations where there is very little detailed information available. In an economic setting it has been used by Foley (1994) and Toda (2010) to prove the existence of a statistical market equilibrium when agents have "offer sets" of transactions they are willing to accept and interact in a random fashion. In their framework the uncertainty comes from the sequence of transactions, while in the thought experiment used here the uncertainty is much more fundamental, as the social planner is unable to even obtain the preference rankings of agents.

The MaxEnt approach can be applied to the allocation table A^* obtained from the MMKP, which has a known state space and can be coded efficiently into a message with a measurable information content. This information content then strongly constrains the observable frequencies of the events. The most efficient way of coding the information in table A^* (3) would be to use a Q-length output message, where each entry can take N

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[6]The information unit will be the bit only if $\log_2$ is used. The natural logarithm produces an information measure in "nats" with 1 nat = $\log_2(e)$ bit (approximately 1.4427 bit).

possibilities.

Units:	$j=1$	$j=2$	$j=3$	$j=4$	$j=5$	...	$j=Q$
Agents:	3	4	1	6	1	...	5

As the social planner reads each entry, the allocated owner of the unit is given by the label uniquely identifying each agent. The state space of this message is simply N^Q. As a result, the detailed information content of the allocation table A^* is given by:

$$I\left(A^*\right) = Q \log N \tag{3}$$

If the social planner knows that the Q units can be grouped into K types of commodities, then this full description of the allocation contains redundancy. Let a K-section of the allocation table A_k^* be the group of rows in the allocation table (3) that contain the allocation of the Kth commodity. Because the q_k units are undistinguishable, many bundles in the binary identifier table B are for all purposes identical, and any permutation of the rows within a K-section A_k^* leads to an identical allocation. This redundant information can be removed to obtain a more efficient coding of the message. Given q_k units per commodity K, this allows for $q_k!$ permutations per given K-section. Taking logs, using Stirling's approximation and subtracting from (3) gives the corrected information measure for A_k^*:

$$I\left(A_k^*\right) = q_k \log N - \left(q_k \log q_k - q_k\right) \ ,$$

$$I\left(A_k^*\right) = q_k \left(1 + \log \frac{N}{q_k}\right) \ . \tag{4}$$

This is the information entropy for a q_k length message using an exponential distribution with mean N/q_k. The intuition behind this is that permuting rows in a particular K-section A_k^* is equivalent to permuting cells in the output message. Out of all the possible permutations a particular one of interest is the one where the q_k entries are sorted by ascending agent label:

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Units:	$j = 1$	$j = 2$	$j = 3$	$j = 4$	$j = 5$	...	$j = q_k$
Agents:	1	1	2	2	2	...	N

Each agent is expected to be selected q_k/N times meaning that new agents arrive in the string at a rate N/q_k, thus explaining the exponential form. Furthermore, the permutation also reveals the information redundancies mentioned above. Sending a message in this format is clearly inefficient, particularly if the number of units to be allocated q_k is much greater than the number of agents N. It would be better to convert the message into an N-length message where each entry simply indicates the number of units to be allocated to the ith agent.

Agents:	$i = 1$	$i = 2$	...	$i = N$
Units:	2	3	...	4

Performing this change in variable provides the following information content for the shorter message. The underlying distribution remains exponential, with a message length of N and an expected endowment of q_k/N. This is in line with the result of Foley (1994) which shows that the MaxEnt prediction for an exchange economy is an exponential distribution.

$$I\left(A_k^*\right) = N\left(1 + \log\frac{q_k}{N}\right) \tag{5}$$

This illustrates the information-theoretic interpretation of the MaxEnt approach and the endowment distribution it predicts. Given a message length of N, should the social planner predict a distribution of endowments that is not exponential, this would imply an information measure different from (5). This in turn requires the allocated quantities to be either greater or smaller than q_k, both of which violate the MMKP constraint in (1). Therefore, the only consistent predicted endowment distribution is the one that produces the same information content as the allocation table (3), subject to known information redundancies. Any difference between observed and predicted distributions can then be used to identify the remaining information redundancies in the structure of the allocation table A^* that the social planner was initially unaware of.

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This methodology shows that only a single predicted endowment distribution is consistent with a given information structure for the allocation matrix A^*. Crucially, however, all the allocation tables allowed by the MMKP constraint in (1) carry the same information content. The existence of the stable distribution therefore has no connection to objective function of the MMKP. We now move to investigating under which conditions the social planner can be confident that this distribution describes and underlying equilibrium if the allocation problem is solved in a decentralised but unobserved manner.

3.2   Knapsacks, congestion games and convergence to equilibrium

As explained in section 2, the operational research literature has used the MMKP to model resource allocation on a network. Similar network allocation frameworks also serve as illustrations of congestion games, for example the road congestion setting presented by Rosenthal (1973), where road users attempt to select routes so as to minimise the congestion they experience. A network congestion framework will therefore be used here to illustrate the conditions under which the two settings are equivalent. The congestion game that will be examined here uses an N-edge multigraph, corresponding to N routes between two points, as illustrated by figure 1. This choice of graph implies that distinct routes follow separate edges, and avoids the interdependence of costs between edges.[7] Each edge is assumed to have a controller who can work out the congestion cost for any given number of users.

There are $q_k \in \mathbf{N}$ users for each of the K types, who all need to get from the start point s to the finish point f.[8] In terms of the congestion game, there are N routes/strategies that can be chosen by each user, and the cost to the user is simply the cost of using the selected edge.

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[7]In other words, this simple example avoids the presence of externalities between routes, where the cost of a number of users choosing a route depends on the number of users choosing another route that shares a common edge.

[8]Following the road congestion example of Rosenthal (1973), one could imagine that the K types represent different categories of vehicles, such as cars, trucks, etc. who each generate different congestion costs.
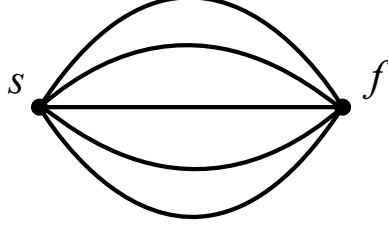
Figure 1: Multigraph congestion game

The different sequences of this allocation problem can be set up using the MMKP thought experiment presented in section 2. The only difference is that at step 2, the social planner sends the B-table to the N edge controllers, who each return the congestion cost that will arise on their edge for all the possible bundles. As a result, a $2^Q \times N$ cost table C is obtained.[9] This implies that at step 3, the MMKP program uses $\operatorname{tr}(CX')$ as the objective function to be minimised.

Two assumptions are required to show the equivalence between the MMKP and congestion game solutions to this framework. In order do so, $\Delta_k C_{b,i} = C_{b,i} - C_{b-,i}$ is defined as the marginal cost of an extra k-type user, where $b-$ is a bundle obtained by removing a k-type user from bundle b. Similarly, in the following, $b+$ refers to a bundle obtained by adding a k-type user to bundle b.[10]

Monotonicity: $\Delta_k C_{b,i} > 0 \ \forall k, i, b$ i.e. congestion costs on an edge are increasing with the number of users.

Global convexity: Given two bundles a and b, $\forall i, x \in N$ if $C_{b,i} > C_{a,x}$ then $\Delta_k C_{b,i} > \Delta_k C_{a,x}$

Monotonically increasing costs are a standard assumption for congestion games. Convexity intuitively means that congestion exhibits increasing marginal values. Furthermore global convexity implies that removing a k-type user from a heavily congested road brings a larger reduction in congestion than removing the same user from another less congested road. As will become apparent in the following proofs, this is needed because the objective

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[9] The difference with the ranking table U in section 2 is that the null bundle is ranked last and the full bundle is ranked highest. In the cost table C, assuming monotonicity, the null bundle produces the lowest cost and the full bundle the highest cost.

[10] $\Delta_k C_{b,i}$ is of course undefined for the empty bundle.

function used in the MMKP is not the same as the one used in the basic congestion game framework of Rosenthal (1973). We now prove that if the cost table C satisfies these two assumptions, the MMKP and congestion game formulations are equivalent.

Proposition: *If the cost table displays monotonicity and global convexity, the optimal solution to the MMKP problem is a Nash equilibrium for the corresponding multigraph congestion game.*

Proof: By contradiction. Let X^* be the decision table that satisfies the MMKP (1) and $A^* = B'X^*$ the corresponding allocation of the Q users over the N edges of the multigraph. Let us assume that A^* is not a Nash equilibrium for the Q users. Then there exists a k-type user $j \in Q$ whose cost is reduced by switching from edge i to edge x. If b and a are the bundles allocated to edges i and x respectively by X^*, this requires $C_{b,i} > C_{a+,x}$. Given that the allocated bundles to all the other edges are unchanged, the change in the objective function of the MMKP is:

$$C_{b-,i} - C_{b,i} + C_{a+,x} - C_{a,x} \Leftrightarrow -\Delta_k C_{b,i} + \Delta_k C_{a+,x}$$

Global convexity of costs implies that this term is negative. Following the switch in edge by the jth user the objective function is smaller, therefore contradicting the fact that X^* satisfies the MMKP. ∎

Corollary: *If the cost table displays monotonicity and global convexity, the objective function of the MMKP is an ordinal potential function for the corresponding multigraph congestion game.*[11]

Proof: Immediate from the definition of convexity and the previous proof. The change in cost to a k-type user for switching from edge i to x is $C_{a+,x} - C_{b,i}$. The corresponding change in the objective function of the MMKP is $-\Delta_k C_{b,i} + \Delta_k C_{a+,x}$. One has:

$$\text{sgn}\,(C_{a+,x} - C_{b,i}) = \text{sgn}\,(-\Delta_k C_{b,i} + \Delta_k C_{a+,x})$$

---

[11] An ordinal potential occurs when the changes in the potential function and the changes in the payoffs have the same sign. An exact potential further requires they also have the same value.

The objective function of the MMKP is an ordinal potential for the corresponding congestion game. ■

Monderer and Shapely (1996) show that all ordinal potential game possess the finite improvement property (FIP), meaning that even a simple a myopic best response path is enough to lead to the optimal equilibrium in a finite number of steps. Combined with the proposition shown above, the central implication of this result is that if the preferences expressed by the N agents/edges are globally convex, then the social planner can be confident that the predictions obtained using the information-theoretic methodology outlined in section 3.1 will describe the aggregate properties of an underlying optimal outcome.

An important note is that in the U-table (2), preference rankings are monotonically decreasing with units allocated, while in the C-table costs increase monotonically with users allocated. The two problems are therefore slightly different, but the equivalence proposition nevertheless carries across. This formally requires transforming the rankings in the U-table into proxy utility values by applying a concave inverse mapping, hence changing the rank-minimisation problem into a utility-maximisation. The global convexity requirement for preferences when minimising overall rank thus becomes a global concavity requirement for utility, which is a standard assumption in economics.

4   Discussion and Conclusion

The purpose of the thought experiment and the justification for using the MMKP to model the allocation problem is twofold. First of all, because it provides a clear state-space, it allows for the use of information theory to obtain the specification for the information content of the solution, and hence a constraint on predicted distributions at the aggregate level. The second is that the structure of the MMKP is very similar to that of congestion games. In fact, the only requirement for the optimal MMKP allocation to also be a Nash equilibrium and the objective function to be an ordinal potential is the existence of convexity (or concavity for a maximisation) in the objective function.

As a result, the thought experiment sheds light on the link between rationality and

aggregate characteristics mentioned in the introduction. As was explained at the beginning of section 3.2, the predictions obtained with the information-theoretic methodology do not depend on the rationality of agents: all that is required is that the information content of the message containing the allocation not violate the structure of the allocation matrix A^*, which depends only on the MMKP constraint and the known redundancies in the information structure. This is in line with the findings presented in the introduction, particularly Becker (1962), who shows that changes in the budget constraint of agents suffice to generate observable demand curves. However, the equivalence between the MMKP and congestion games when preferences are convex reveals that if the allocation problem is solved in a decentralised manner, the distribution predicted by the social planner at the aggregate level will indeed coincide with optimum state at the decentralised level.

More generally, the knapsack thought experiment rationalises the use of information-theoretic methods in economics: useful predictions can be made about endowment distributions even in the absence of detailed information on the consistency of agent preferences. As pointed by Hayek (1945), we are all observers whose knowledge of the state of the economy is vanishingly small. Even for the case where some data is accessible, it does not completely describe the state of the economy. In such a context, the MaxEnt methodology developed by Jaynes (1957a,b) uses the information-theoretic Shannon entropy as a measure of the ignorance of an observer. As a result, following this methodology provides the best prediction of the state of an economy in the absence of all other knowledge.

The use of this methodology, however, goes beyond simple prediction, as it also allows for successive improvements in the predictions. As stated above, what can the social planner conclude if empirical frequency data does not confirm her predicted distribution? The answer to this question is provided by Jaynes (1989): any significant difference between the predicted and empirical information measures represents the amount of information that can be extracted from the frequency data in order to improve the knowledge of the information redundancies in the system, hence improving further the prediction, and more importantly, improving the understanding of the underlying data-generating mechanisms.

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