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**Policy Rules Under the Monetary and the  
Fiscal Theories of the Price-Level**

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# Policy Rules Under the Monetary and the Fiscal Theories of the Price-Level\*

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## Abstract

Price-level determination requires co-ordination of monetary and fiscal policy to ensure a unique rational expectations equilibrium (REE). This paper derives a number of implications for simple interest rate rules resulting from various fiscal strategies. We show that fiscal choices under either the monetary theory of the price-level (MTPL) and the fiscal theory of the price-level (FTPL) can challenge widely accepted principles of monetary policy. Specifically, we show that a fiscal rule that responds aggressively to output and inflation may force the monetary authorities to adopt significantly more aggressive output and inflation stabilization policy than suggested by the Taylor Principle. We also show how when monetary policy is severely constrained, the fiscal policy maker can act to stabilise the economy. Some policy conclusions in light of the lower zero bound for monetary policy and debt stabilization are drawn.

**JEL Classification:** E21; E32; E52; E63.

**Keywords:** Monetary and Fiscal Policy Rules; Ricardian, non-Ricardian fiscal policy.

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## 1. Introduction

This paper examines the mutual constraints operating on monetary and fiscal policy in a standard model with sticky prices and endogenous output variation. We establish a number of propositions that will ensure a unique rational expectations equilibrium (REE) for such a model. A number of contributions (e.g., Woodford, 2003) have emphasized a set of principles for monetary policy. Notably there is the well-known proposition, linked to the name of John Taylor, which suggests that anything more than an equiproportional increase in the short-term interest rate, following an increase in inflation, is sufficient to ensure a unique REE.<sup>1</sup> We note that this proposition is subject to the lacunae that fully characterized stabilization policy requires consideration of both monetary *and* fiscal policy. This insight is of particular interest when monetary policy is constrained in some manner, for example because of a zero lower bound or when fiscal policy adopts a particularly aggressive approach to output stabilization. Accordingly, in this paper we develop versions of this proposition, extended to incorporate various roles for fiscal policy.

The joint determination of monetary and fiscal policies as a stabilization device is well recognized.<sup>2</sup> Recent work on simple policy rules has shown that an optimal monetary and fiscal policy should respond systematically to inflation and output, respectively, as this will stabilize the economy well.<sup>3</sup> But as well as establishing

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<sup>1</sup>The seminal contributions we have in mind are Taylor (1993) and (1999). In a number of important contributions Benhabib, Schmitt-Grohé and Uribe (2001, 2002) and Schmitt-Grohé and Uribe (2000) have criticized the universality of the ‘Taylor principle’ both in terms of local rather than global stability analysis, and the need to take account of structural non-linearities, such as the zero bound on nominal interest rates when formulating optimal monetary policy.

<sup>2</sup>See Persson and Tabellini (1994) for an important collection of papers.

<sup>3</sup>See, for example, Chadha and Nolan (2007), where it is shown that optimal stabilization policy involves an output stabilizing fiscal rule and inflation stabilizing interest rate rule. Such policy is shown to provide a close approximation to recent experience in the US and the UK. This is because the losses in expected utility are proportional to quadratic deviations in inflation and output from their full flex-price level. We consider the joint effort of monetary and fiscal policies, which may be used to pursue jointly the maximisation of representative household utility. Note that in this set-up tensions between monetary and fiscal policy do not result from different stabilization objectives or horizons.

principles for optimality, we would also wish to know more generally what mutual constraints apply to these two branches of stabilization policy in order to establish a unique REE. As a number of authors have noted, there is no guarantee that actual monetary and fiscal policy is conducted optimally. That being the case, understanding the wider issues discussed in this paper appear important. There are clear policy implications, for example, in both the Eurozone and in the UK, monetary and fiscal reform has tried to develop hand in glove with the joint aims of achieving price and economic stability but in the Eurozone there remain many question marks over how such coordination should proceed that may ultimately undermine the monetary union.<sup>4</sup> Furthermore if monetary policy is constrained and cannot act in a manner consistent with the Taylor Principle, for example, when facing the zero lower bound or within a monetary union, fiscal policy may be required to ensure stability. In this paper, we find that if fiscal policy tries to stabilize either or both of output or inflation aggressively there are important implications for the operating procedure of an interest rate setting authority, specifically that a significantly more active rule is required. Emerging economies may also have the problem of dealing with an aggressive fiscal policy maker and so may have to design monetary policy to consider a more active rule than that suggested by the Taylor Principle.

In a standard New Keynesian model, where both output and inflation are forward-looking, we find that there are tensions between the arms of stabilization policy because of the nature of instruments employed. The interest rate operates multiplicatively to tilt private output (consumption) whereas fiscal policy acts

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<sup>4</sup>In the UK, for example, a senior Finance ministry official observes the deliberations of the Bank of England's Monetary Policy Committee and the Committee is itself kept closely informed on fiscal choices. The model of co-operation we explore in this paper closes mirrors current UK practice. See House of Lords (2003) for a further analysis of the links between monetary and fiscal policy with concern on the Stability and Growth Pact uppermost. For the Eurozone, fiscal policy is conducted on a national, rather than supranational basis, and that complicates choices over recent aspects of policy such as how to design operations to purchases government bonds under so-called quantitative easing or, indeed, whether European wide stabilisation fund will ever be sufficient to ensure fiscal solvency.

additively. Under a monetary theory of the price level (MTPL), the economy can be well stabilized by the nominal interest rate which controls effectively the per period flow of demand by setting intertemporal prices. On the other hand, fiscal policy affects the overall level of current demand, scaling output up or down, and is ultimately less effective at stabilizing private sector welfare losses following a shock. The implication of our analysis for monetary policy is that (i) more attention should be paid to commitment devices for the fiscal policy maker to avoid aggressive acts of stabilization and that (ii) because of the possibility of deleterious impact arising from an exuberant fiscal policy maker, a monetary policy maker should be somewhat concerned with output rather than simply inflation stabilization. The recent call for independent fiscal councils by the OECD (2010) and other policymaking institutes thus seems to be well judged.

We also clarify the role of monetary policy under the fiscal theory of the price level (FTPL), where the price level must adjust to alter the value of nominal debt and ensure that it equals a given stream of fiscal surpluses. Here inflation control is no longer the concern of the monetary authority with stabilization left to the fiscal policy. If we find this outcome unattractive and wish to maintain MTPL then, as already argued, we need to call for ongoing fiscal commitments to stabilize the level of debt in expectation, which may imply the announcement of plans consistent with this objective. Overall, it turns out that in order to maintain a simple interest rate strategy to stabilize the economy there are a number of key restrictions that need to be placed on the expected operation of fiscal policy. On the other hand if the MTPL cannot be maintained, for example if interest rates have hit the zero lower bound, then we are able to ask ourselves what kind of fiscal policy rule is required to ensure determinacy. It turns out that unstable debt levels and aggressive fiscal policy may be required to stabilise an economy, in which the interest rate is constrained, and this will complicate the transition from MTPL to FTPL and back to MTPL again, or what has been termed the exit strategy in policy circles.

Apart from analyzing Ricardian (which allows the MTPL) and non-Ricardian

fiscal policy (which leads to the FTPL), we also find it useful to partition fiscal policy into two separate zones: moderate and aggressive. The former (latter) is characterized by the weight on output and inflation in the fiscal rule being below (above) the reciprocal of the share of government expenditure in output.<sup>5</sup> Consider first Ricardian fiscal policy.<sup>6</sup> It turns out that the standard (Taylor Rule) case for monetary stabilization is a special case, where fiscal policy is necessarily moderate. But under an aggressive (but Ricardian) fiscal policy we find that the Taylor Principle is increasingly insufficient to guarantee a unique REE and some additional targeting of output by monetary policy is required. Under non-Ricardian fiscal policy, we find that the further characterization of fiscal policy as moderate or aggressive places additional restrictions on the feasible sequences of interest rates; if fiscal policy is non-Ricardian and aggressive then the inflation stabilization role of an interest rate rule will be irrelevant to the determination of a unique REE. This result might be of some use in considering the appropriate response of fiscal policy under a lower zero bound, which might be to deliver highly aggressive fiscal policy.

In Section 2 we provide a short overview of some of the tensions that arise between monetary and fiscal policy via the public sector present value budget constraint. Section 3 develops the model we use to characterize operational monetary and fiscal policy. Section 4 derives conditions for this model that ensure the existence of a unique REE in each of four regions where fiscal policy is either moderate or aggressive and Ricardian or non-Ricardian. Section 5 concludes with

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<sup>5</sup>For output the threshold results from whether fiscal policy acts to overcompensate for booms or recessions by overstimulating demand to turn boom into recession and vice versa. For inflation, the result derives from the New Keynesian Phillips curve derived in Section 3.2 and stated in B4, which expresses inflation as the current value of expected output gaps. If these output gaps, via fiscal policy are also a function of inflation, then it turns out that the appropriate discount rate to apply on future output gaps falls and hence inflation becomes more sensitive to output gaps, requiring a more aggressive interest rate rule. See Section 4.1.1, 4.1.2 and 4.3 for further details.

<sup>6</sup>Ricardian fiscal policy is defined under Proposition 4.1 and involves a fiscal commitment to stabilise the nominal value of public liabilities. In the absence of such a commitment non-Ricardian fiscal policy occurs.

some observations on policy. The Appendix explains the fiscal set-up, the detailed derivations of the model of section 3.

## 2. The Public Sector Present Value Budget Constraint

Traditionally economists have argued that the public sector faces a present-value budget constraint (PVBC) similar to that of private agents. Given a quantity (real value) of public liabilities, the government *must* plan for its expected stream of discounted net surpluses to be just sufficient to meet these liabilities. In other words, the government's PVBC must hold identically for any feasible equilibrium sequence of the economy's other variables, notably the price-level and the interest rate. In this section we set out the restriction implied by the PVBC for the interest rate. This is a key illustration of the results we derive in Sections 3 and 4 for a fully-fledged dynamic model. We show that a more (less) aggressive fiscal policy reduces (increases) the upper bound on the feasible sequence of interest rates. In later sections we show that increasingly aggressive fiscal policy, either towards output or inflation, correspondingly increases the need for offsetting monetary responses.

We can take the analysis further and, indeed, a recent literature due to Leeper (1991), Sims (1994), Woodford (1997, 2001) and Cochrane (2001), relaxes the requirement that the PVBC is an identity in all states of nature. Nevertheless, the PVBC continues to be a relationship that is satisfied in equilibrium. A defining characteristic of this fiscal theory of the price-level is a presumption that fiscal authorities do not typically coordinate their 'actions' – specifically their temporal (contingency) sequences for tax rates and government expenditure. We also characterize the mutual restrictions on monetary and fiscal policy in this case.

### 2.1. Some budgetary arithmetic

We outline the budgetary implications of the public sector PVBC in terms of the interaction of monetary and fiscal policy. Consider a deterministic economy, in

which wealth takes one of two forms: money, which earns no interest, and one-period nominal, riskless bonds, which do earn interest. The period public sector flow budget constraint is given by:<sup>7</sup>

$$\frac{B_t}{(1 + i_t)} = B_{t-1} + P_t(G_t - T_t) - (M_t - M_{t-1}), \quad (2.1)$$

where  $B_t$  is the nominal quantity of debt maturing in  $t + 1$ ,  $i_t$  is the nominal interest rate between period  $t$  and  $t + 1$ ,  $P_t$  is the aggregate price level,  $(G_t - T_t)$  is the real primary deficit in period  $t$ , and  $(M_t - M_{t-1})$  is seigniorage raised in period  $t$ . A central assumption is that the monetary-fiscal sequences avoid Ponzi schemes, such that the sequence of nominal debt stocks have a zero value in the limit,

$$\lim_{T \rightarrow \infty} B_{t+T} \left( \prod_{j=0}^T (1 + i_{t+j}) \right)^{-1} = 0. \quad (2.2)$$

This condition ensures that the PVBC is satisfied and, given the level of outstanding liabilities at the start of any time period, the ensuing temporal sequence of net surpluses plus seigniorage is able to meet those liabilities. Let  $T_t$  denote the period  $t$  tax yield. We will analyze fiscal rules (or regimes) of the following form

$$T_t = \lambda_t G_t - \frac{(M_t - M_{t-1})}{P_t} + \gamma \frac{B_{t-1}}{P_t}. \quad (2.3)$$

Fiscal policy is characterized by the sequence  $\{(\lambda_{t+s}, \gamma_{t+s})\}_{s=0}^T$ , that is by choices on the size of deficit,  $(1 - \lambda)G$ , and on the extent to which outstanding debt is retired,  $\gamma$ . Let  $0 < \gamma < 1$ , which corresponds to the portion of outstanding debt carried over from the previous period. For simplicity, let us further assume that seigniorage revenue is rebated lump sum to the private sector. The fiscal regime is now indexed simply by restrictions on the sequence  $\{\lambda_{t+s}\}_{s=0}^T$  and this is the key to understanding the implications of (2.2) for our class of fiscal policy rules. First, given  $\gamma$ , the fiscal authority, looking forward from any time  $t$ , will always do enough to repay the outstanding debt in existence at the start of time

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<sup>7</sup>Annex A shows the steady-state implication of this constraint.

$t$ . Thus fiscal solvency hinges on the present value of future surpluses and deficits in  $t+1, t+2, \dots$ . So, we need to clarify the implications of (2.2) for that sequence. It turns out that as time  $T \rightarrow \infty$  the fundamental requirement for fiscal solvency on any monetary-fiscal program is that:

$$\sum_{s=0}^T \left[ \left\{ \prod_{j=0}^{s-1} (1 + i_{t+j}) \right\}^{-1} (1 - \gamma)^{T-s} (1 - \lambda_{t+s}) P_{t+s} G_{t+s} \right] \rightarrow 0. \quad (2.4)$$

In other words, the discounted sum of net government liabilities must tend to zero.

Rewriting our solvency condition in real terms, and assuming that the net deficit is constant,

$$(1 - \lambda) \bar{G} \sum_{s=0}^T \left[ \left\{ \prod_{j=0}^{s-1} \frac{(1 + \pi_{t+1+j})}{(1 + i_{t+j})} \right\} (1 - \gamma)^{T-s} \right]. \quad (2.5)$$

We require, for solvency, that the expression in square braces tend to zero in the limit. Expression (2.5) in turn can usefully be re-written as

$$(1 - \lambda)(1 - \gamma)^T \bar{G} \sum_{s=0}^T \left[ \left\{ \prod_{j=0}^{s-1} \frac{(1 + \pi_{t+1+j})}{(1 + i_{t+j})} \right\} \left( \frac{1}{1 - \gamma} \right)^s \right]. \quad (2.6)$$

A sufficient condition for this expression to reach zero in the limit is simply that the term in square braces is convergent, as opposed to having a zero limiting value. It can then be shown that this will be the case as long as the following requirement is met infinitely often:

$$i_s - \pi_{s+1} < \gamma \quad \forall s \geq T. \quad (2.7)$$

This expression has a very obvious interpretation in that it requires that the fiscal authority must eventually repay a sufficient portion of the debt each period so that the discounted sum of net public liabilities tends to zero.<sup>8</sup> Alternatively, we

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<sup>8</sup>Actually this expression is an approximation, since we ignore the cross term:

may think of it as saying that the debt retirement schedule places an upper bound on the feasible real interest rate sequence. That interpretation will be convenient in what follows.

### 3. The Model

#### 3.1. The Representative Agent

In this section we set out the key structure of the model. The equations characterizing equilibrium choices are derived in detail in the appendix. The utility function,  $V_0$ , for a representative agent,  $j$ , is given by,

$$V_0 = \sum_{t=0}^{\infty} \left\{ \left( \frac{1}{1+\delta} \right)^t E_0 U \left( C_t^j, \frac{M_t^j}{P_t}, L_t^j \right) \right\}. \quad (3.1)$$

Here  $\delta$  is the subjective discount rate and  $E_0$  is the expectation operator at time 0. The utility function is assumed to be concave and separable in its arguments,  $C$ , consumption,  $M/P$ , real money balances, where  $M$  is nominal money balances and  $P$  is the aggregate price-level,  $L$  is leisure, which is equal to  $1 - N$ , where available time is normalized to unity and  $N$  is labor input. In practice we shall assume log-separability, that is,  $U(\cdot) \equiv \left[ \log C_t^j + \log \left( \frac{M_t^j}{P_t} \right) + \log(1 - N_t^j) \right]$ . The representative agent maximizes expected utility subject to a sequence of per period flow constraints:

$$M_{t-1}^j + B_{t-1}^j + W_t N_t^j + \Pi_t - T_t \geq P_t C_t^j + M_t^j + \frac{B_t^j}{1+i_t}, \quad (3.2)$$

where,  $P_t C_t^j = \int_0^1 p_t(z) c_t^j(z) dz$ , and where (3.2) holds for all  $t \geq 0$ , and in each state of nature,  $M_{-1}^j$  and  $B_{-1}^j$  given. Let  $z$  index goods in the economy. Then, we have that  $c_t^j(z)$  denotes the representative agent's consumption of good ( $z$ ),

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$[(p_{t+1}/p_t) - 1] \times \gamma$ . The expression (2.7) is closely mimicked by the term we derive from a dynamic macro model in Section 4, equation 4.2 and equations 4.12, 4.13 and 4.14 for the selection of Ricardian or non-Ricardian fiscal policy, respectively. But in the latter case, note that because the debt retirement rate is too low then monetary policy is further restricted.

which costs  $p_t(z)$ . In addition,  $W_t$  denotes the nominal wage,  $\Pi_t$  denotes profits remitted from firms where each agent receives essentially a pooled dividend,  $T_t$  denotes lump sum taxes,  $B_t$  denotes the nominal stock of bonds held over at the end of period  $t - 1$  and  $i_t$  is the economy-wide period nominal interest rate. The evolution of financial wealth,  $F$ , is given by

$$F_t^j = M_{t-1}^j + B_{t-1}^j, \quad \forall t \geq 0. \quad (3.3)$$

This sequence of equations together with the transversality condition,  $\lim_{T \rightarrow \infty} E_0 \prod_{j=0}^{T-1} (1 + i_{t+j})^{-1} W_{t+T}^j \rightarrow 0$  help ensure that the agent's optimization problem is well behaved. Consumption is defined over the Dixit-Stiglitz aggregator function, where  $\theta$  is the elasticity of demand for good ( $z$ ),

$$C_t^j \equiv \left[ \int_0^1 c_t^j(z)^{\frac{\theta-1}{\theta}} dz \right]^{\frac{\theta}{\theta-1}}, \quad (3.4)$$

with the aggregate price-level defined accordingly as:

$$P_t \equiv \left[ \int_0^1 p_t(z)^{1-\theta} dz \right]^{\frac{1}{1-\theta}}. \quad (3.5)$$

The derivation of the agent's optimal supply and demand decisions are standard and are relegated to Annex B.

### 3.2. The Representative Firm

We assume that there are a large number of infinitely-lived monopolistically competitive firms, who use only labor in the production of their differentiated good. Each period these firms receive a symmetric productivity shock. So for firm  $i$  the production function at time  $t$  is given by

$$Y_{i,t} = A_t N_{i,t}^\sigma, \quad (3.6)$$

where  $0 < \sigma < 1$ , and  $A_t$  is the productivity innovation. We further assume that firms face Calvo-type restrictions in setting prices but that firms meet demand whether or not they have been able to change prices that period. The optimal price is given by

$$\tilde{p}_t = \frac{\theta}{(\theta - 1)} \frac{\sum_{k=0}^{\infty} (\alpha\beta)^k E_t(\mu_{t+k} P_{t+k}^\theta Y_{t+k}^d \Lambda_{t+k})}{\sum_{k=0}^{\infty} (\alpha\beta)^k E_t(\mu_{t+k} P_{t+k}^{\theta-1} Y_{t+k}^d)}. \quad (3.7)$$

$\beta$  is the rate of time preference,  $\alpha$  denotes the probability of the firm having to charge the same price next period as it did the period before,  $\mu_t$  is the marginal utility of consumption,  $\Lambda_t$  is real marginal cost and  $Y_t^d$  is an index of aggregate demand. Equations (3.5) and (3.7) jointly imply the New Keynesian Phillips curve. We make standard assumptions on firms' involvement in the labour market. Again, Annex B contains further details.

### 3.3. Monetary and Fiscal Policy

We turn now to consider our class of feedback policy rules. As is well understood this sticky price system implies demand determined output which may differ from its flexible price steady-state (see Woodford, 2003). We shall therefore consider the setting of the per period nominal interest rate and the primary surplus in order to stabilize this system, that is policy rules of the form:

$$i_t = \phi^m(Y_t - Y_t^*, \pi_t - \pi_t^*), \quad (3.8)$$

and

$$s_t = \phi^f(Y_t - Y_t^*, \pi_t - \pi_t^*, \gamma B_{t-1}), \quad (3.9)$$

where  $i_t$  is the nominal interest rate set in period  $t$ ,  $Y_t - Y_t^*$  is output gap,  $\pi_t - \pi_t^*$  is the difference between inflation and its target rate in period  $t$ ,  $\gamma$  is the rate at which the stock of nominal debt,  $B_{t-1}$ , is retired and  $s_t$  is the per period level of primary surplus. Monetary policy is modelled as the control over the short-term (one-period) nominal interest rate. In the presence of (some) sticky prices this implies some leverage over interest sensitive endogenous variables. We shall

assume that the fiscal authority, as is standard, sets taxes in response to the level of contemporaneous government expenditure (which we assume is exogenous), that seigniorage is returned lump-sum to the private sector, and crucially that taxes respond to the level of debt outstanding at the start of the period. It is the response of taxes to a given expenditure stream and the evolution of endogenous variables that sets the per period fiscal surplus.

Fiscal policy has a number of channels through which it acts. The first is the direct channel by affecting aggregate demand, that is period output, in the economy. When the fiscal policy authority runs a procyclical surplus over the business cycle, output will stay closer its flex-price level.<sup>9</sup> The second channel operates by interacting with the monetary policy maker, who is acting to stabilize inflation. Because expected inflation has output as an argument and output is being somewhat tempered by fiscal policy, the interest rate is better able to stabilize inflation because the fiscal policy maker is also stabilizing output. Taylor (1999b) has recently demonstrated that a relationship such as (3.9) can be used to model the course of the surplus in US data.

## 4. Stability and Determinacy

In this section we analyze the local stability conditions to ensure a unique REE for the model derived in Section 3. We have linearized that model and appendix B sets out these linear relations in full for: consumption (B1); money demand (B2); the evolution of wealth (B3); New Keynesian Phillips curve (B4); output (B5); interest rate rule (B6); fiscal rule (B7); and public sector budget constraint (B8). Our analysis leads to a model with 8 endogenous variables and Annex B shows how we reduce the dimensions of this system to three. This set of three stochastic

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<sup>9</sup>We shall see that the stabilizing properties of this first channel are restricted because if the fiscal policy maker attempts to stabilize output or inflation over some threshold there will more work for monetary policy to do to stabilize this economy. Hence, we partition the fiscal policy response into moderate and aggressive. Note unlike Leeper (1991) aggressive (active in Leeper's terminology) fiscal policy here is not about the stability of debt but about the fiscal policy maker's propensity to stabilize the economy.

difference equations can then be written in compact form as:

$$E_t z_{t+1} = A z_t + G x_t \quad \forall t \geq 0, \quad (4.1)$$

where the vector of endogenous variables,  $z_t$ , is given by:

$$z_t = \begin{bmatrix} \hat{b}_{t-1} \\ \hat{c}_t \\ \hat{\pi}_t \end{bmatrix}.$$

$A$  is a  $3 \times 3$  matrix and  $G$  a  $3 \times 1$  matrix of coefficients and  $x_t$  represents the bounded forcing process acting on bonds, consumption and inflation, which is the transpose of  $[\hat{f}_t, \hat{h}_t, \hat{x}_t]$ . The square matrix  $A$  will have 3 eigenvalues corresponding to each endogenous variable. Standard analysis tells us that the existence of a unique rational expectations solution for the vector,  $z_t$ , depends upon the number of eigenvalues of the square matrix,  $A$ , that lie outside the unit circle (see Blanchard and Kahn, 1980), corresponding to the number of non-predetermined variables. In this case a locally unique equilibrium depends on there being two eigenvalues that lie outside the unit circle. These roots in the familiar Ricardian case are associated with inflation (forward-looking due to the Calvo specification) and consumption, as agents consume out of present-value income. However, these familiar root conditions rely on a restriction in the government's budget constraint that flows from the robustness of the debt repayment schedule. We enlarge on this point below.

Because of the block triangular structure, the analysis of the roots of the model is relatively straightforward. We note that the matrix  $A$  can be partitioned into pre-determined and non-predetermined variables, with matrix  $B$  corresponding to the pre-determined variable and the  $2 \times 2$  matrix  $D$  corresponding to the non-predetermined variables. Note that money simply does not matter for this equilibrium as it is pinned down by the consumption and inflation.<sup>10</sup> What actually matters is technical, in the sense of what provides the unstable root.

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<sup>10</sup>See Chadha et al (2008) on this point.

See Annex B for further details.

$$A = \begin{bmatrix} B & C \\ 0 & D \end{bmatrix}$$

#### 4.1. The Ricardian case

We can locate the MTPL or Ricardian case by analyzing the eigenvalue of matrix  $B$ , which corresponds to the debt accumulation equation, and is simply given by  $(1 - \gamma)\beta^{-1}$ . Then we have that:

$$(1 - \gamma)\beta^{-1} < 1,$$

$$\gamma > (1 - \beta). \tag{4.2}$$

The similarity of this expression with (2.7) is clear. In the event that this root does not lie in that space, then a stable root has to be recovered from matrix  $D$  and we analyze this non-Ricardian (FTPL) case below.

**Proposition 4.1.** *A Ricardian regime requires that the rate at which government debt is retired,  $\gamma$ , is no less than one minus the rate of time preference.*

There is a clear intuition for the Ricardian case in that there is a commitment to retire debt at a rate  $\gamma$  more than the rate it is expanding,  $\delta/(1 + \delta) = (1 - \beta)$ . Should this condition not be met then monetary policy will be directly constrained as we make clear below under an FTPL. We shall consider the implications for monetary policy when a Ricardian regime is in place, under MTPL, for Regions 1 and 2 and when Proposition 4.1 is violated when considering Regions 3 and 4, that is a non-Ricardian regime.

First, we concentrate on establishing the determinacy conditions for the remaining  $2 \times 2$  matrix describing the evolution of the two non-predetermined variables of  $z_t$ , that is  $\hat{c}_t$  and  $\hat{\pi}_t$ . We show that such a matrix will admit a number of distinct parameter constellations under which both eigenvalues will lie outside the unit circle. We will discover that the first case corresponds to ‘moderate’ fiscal policy and the second to ‘aggressive’ fiscal policy. Recall that aggressive fiscal policy is simply indexed by the extent to which the surplus responds to endogenous state variables i.e.,  $\phi_y^f$  and  $\phi_\pi^f$ .

#### 4.1.1. Region 1: ‘Moderate’ fiscal policy under MTPL

Once we have partitioned matrix  $A$  we can analyze the equation of motion for the decoupled equations:

$$E_t \bar{z}_{t+1} = D \bar{z}_t + G \bar{x}_t \quad \forall t \geq 0,$$

where  $\bar{z}'_t = [\hat{c}_t \ \hat{\pi}_t]$ ,  $G$  is a parameter matrix and  $\bar{x}_t$  is the vector of forcing variables acting on consumption and inflation, respectively. In Annex B we show that matrix  $D$  is given by:

$$D = \begin{pmatrix} 1 + \frac{\phi_y^m \phi_3}{1 - \phi_y^f (1 - \phi_3)} + \frac{\beta^{-1} \kappa \phi_3}{1 - \phi_y^f (1 - \phi_3)} & \frac{\phi_y^m \phi_\pi^f}{1 - \phi_y^f (1 - \phi_3)} + \phi_\pi^m - \beta^{-1} + \frac{\beta^{-1} \kappa \phi_\pi^f}{1 - \phi_y^f (1 - \phi_3)} \\ -\frac{\beta^{-1} \kappa \phi_3}{1 - \phi_y^f (1 - \phi_3)} & \beta^{-1} - \frac{\beta^{-1} \kappa \phi_\pi^f}{1 - \phi_y^f (1 - \phi_3)} \end{pmatrix}.$$

Assuming a Ricardian regime, we establish the threshold values for  $\phi_y^f$  and  $\phi_\pi^f$  such that the fiscal policy can be thought of as moderate and not acting with sufficient force to alter standard prescriptions on the operation of interest rate rules. Our first condition is standard and requires that:

$$\frac{(1 - \beta)}{\kappa} \phi_y^m + \phi_\pi^m > 1. \quad (4.3)$$

In the absence of fiscal policy, ( $\phi_y^f = \phi_\pi^f = 0, c = y$ ), equation (4.37) is both necessary and sufficient for a unique REE; this is the case analyzed in Woodford

(2003, Chapter 4) and it corresponds to the Taylor Principle.<sup>11</sup> Let us make these conditions more general by considering what restrictions we must place on  $\phi_y^f, \phi_\pi^f \neq 0$  such that a unique REE is attained.

Note that for  $\phi_y^f < \left(1 - \frac{c}{y}\right)^{-1}$ , i.e., our moderate fiscal policy,  $Tr(D)$  will in general be positive unless  $\phi_\pi^f$  breaches some bound. Hence, the first condition for moderate fiscal policy is simply that fiscal policy does not try to push output too strongly back to equilibrium. Note therefore that the determinacy conditions under moderate fiscal policy will also imply some restriction on  $\phi_\pi^f$ , as well as those in the monetary reaction function. We therefore run through the relevant conditions for Case 2i (see Annex B):

$$\phi_y^m + \kappa\phi_\pi^m > \frac{(\beta - 1) + \left(1 - \frac{c}{y}\right) \left((1 - \beta)\phi_y^f + \kappa\phi_\pi^f\right)}{c/y}. \quad (4.4)$$

Solving for  $\phi_\pi^f$ , we obtain:

$$\frac{\frac{c}{y}(\phi_y^m + \kappa\phi_\pi^m) + (1 - \beta) - \left(1 - \frac{c}{y}\right)(1 - \beta)\phi_y^f}{\kappa\left(1 - \frac{c}{y}\right)} > \phi_\pi^f. \quad (4.5)$$

The next condition requires:

$$\frac{\frac{c}{y}\phi_y^m + \beta^{-1}\kappa\left(\frac{c}{y} - \left(1 - \frac{c}{y}\right)\phi_\pi^f\right)}{\left(1 - \phi_y^f\left(1 - \frac{c}{y}\right)\right)} > -\delta. \quad (4.6)$$

And solving for  $\phi_\pi^f$ , we obtain:

$$\frac{\delta\left(1 - \phi_y^f\left(1 - \frac{c}{y}\right)\right) + \frac{c}{y}(\phi_y^m + \beta^{-1}\kappa)}{\beta^{-1}\kappa\left(1 - \frac{c}{y}\right)} > \phi_\pi^f. \quad (4.7)$$

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<sup>11</sup>It is clear that it is not only the weight on inflation in the interest rate rule that counts, but the sum of the weights (i.e., the weight on inflation plus the weight on output).

Conditions (4.5) and (4.7) indicate there exists an upper bound on fiscal policy action towards inflation, which we interpret, in conjunction with the limit on  $\phi_y^f$ , as the limit to fiscal moderation. It is straightforward to see that (4.5) and (4.7) collapse to an identical expression when restriction (4.3) holds with equality. So the attainment of condition (4.7) as an inequality will relax the restrictiveness of condition (4.5), where  $\phi_\pi^m$  is an argument, and so when condition (4.7) is satisfied so will condition (4.5).<sup>12</sup>

Therefore only if fiscal policy does not observe the constraints derived here will condition (4.3) be insufficient to determine a unique REE. We discuss the case of  $\phi_y^f > \left(1 - \frac{\epsilon}{y}\right)^{-1}$  in Section 4.1.2 but discuss briefly here why an inflation stabilizing fiscal rule may require some further stabilization of inflation and output via the monetary rule. Mechanically this is because when inflation is also targeted by the fiscal rule, the current period inflation rate becomes more sensitive to a given stream of expected inflation and so will require more aggressive stabilization by the interest rate rule. We can see this point quite easily by noting that when the fiscal surplus targets inflation, output becomes a function of inflation. When we then substitute out for inflation we then find that inflation is more sensitive to a given stream of expected output gaps i.e. the Phillips curve becomes more vertical.

$$\pi_t = E_t \sum_{i=0}^{\infty} \beta^i \frac{1}{1 - f(\phi_\pi^f)} \kappa y_{t+i} \quad (4.8)$$

In effect, even though inflation continues to be pinned down by the expansion of output, when the fiscal policy maker also uses output to stabilize inflation there are reduced output effects from any given stream of demand shocks meaning that inflation may respond instead. Note that inflation may become more volatile unless further more aggressive interest rate policy is adopted. Moderate and

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<sup>12</sup>Note that that the optimal weights derived for inflation and output stabilization in Chadha and Nolan (2003) conform to stable region outlined by Proposition 4.2.

Ricardian fiscal policy is thus a necessary pre-condition for the Taylor Principle to effect a determinate equilibrium.

**Proposition 4.2.** *Under moderate fiscal policy, defined by explicit limits on  $\phi_y^f$  and  $\phi_\pi^f$ , a greater than equiproportional interest rate response to inflation is sufficient to ensure a unique REE.*

#### 4.1.2. Region 2: ‘Aggressive’ fiscal policy under MTPL

It is possible for the fiscal rule to locate an alternate stable region where matrix  $D$  delivers two eigenvalues greater than  $|1|$ . When fiscal policy remains Ricardian but  $\phi_y^f$  and  $\phi_\pi^f$  are above the limits outlined in section 4.1.2, in order to ensure a unique REE, the monetary authority itself will have to become more aggressive. This aggression is specifically induced by the chosen fiscal parameters but interest rate policy can still determine a unique REE under MTPL. Following the conditions for a local unique rational expectations equilibrium, following Case 2ii in Annex B we first find that,

$$\phi_y^m + \kappa\phi_\pi^m > \frac{-(\beta + 1) + \left(1 - \frac{c}{y}\right) \left((1 + \beta)\phi_y^f + \kappa\phi_\pi^f\right)}{c/y}. \quad (4.9)$$

We also find that:

$$1 - \left(1 - \frac{c}{y}\right) \left(\phi_y^f + \kappa\phi_\pi^f\right) + \frac{c}{y} \left(\phi_y^m + \kappa\phi_\pi^m\right) - 2\beta F - \delta\beta F - \beta\frac{c}{y}\phi_y^m - \kappa\left(\frac{c}{y} - \left(1 - \frac{c}{y}\right)\phi_\pi^f\right) > -\beta F,$$

And this expression turns out to be equivalent to Condition (4.3), so we move to the next condition;

$$(1 + \beta)\phi_y^m + \kappa\phi_\pi^m > \frac{-(2(1 + \beta)) + (2(1 + \beta))\phi_y^f\left(1 - \frac{c}{y}\right) - \kappa\frac{c}{y} + 2\kappa\phi_\pi^f\left(1 - \frac{c}{y}\right)}{c/y}. \quad (4.10)$$

In other words, we see that monetary responses are rising in fiscal responses. We can combine (4.9) and (4.10) to yield the key requirement for determinacy in this system:

$$(2 + \beta) \phi_y^m + 2\kappa\phi_\pi^m > \frac{-(3(1 + \beta)) + (3(1 + \beta)) \phi_y^f \left(1 - \frac{c}{y}\right) - \kappa \frac{c}{y} + 3\kappa\phi_\pi^f \left(1 - \frac{c}{y}\right)}{c/y}. \quad (4.11)$$

We note that condition (4.11) shows that aggressive fiscal policy, in the spirit of Section 2, *per se* raises the lower bound for the sequence of interest rates. We also find that the extent to which the interest rate rule can stabilize the economy by reacting to output is enhanced. Note that  $\phi_y^m$  in this region is premultiplied by a number near three rather than near zero (*i.e.*  $1 - \beta$ ), as in Region 1. Recall that fiscal policy acts through aggregate demand and aggression towards output, as indexed by  $\phi_y^f$  and  $\phi_\pi^f$  leads, in general, to the need for a substantially more aggressive interest rate rule than that suggested by (4.3) alone. Unabated aggressive fiscal policy will set out to drive output when it is above (below) its flexible price level back below (above) its flexible price level and hence will require more aggressive monetary policy, particularly towards output. But note rather than ceding control over the determinacy of a unique REE, monetary policy can act with some strength, particularly towards output, which has an increasing fraction set by (aggressive) fiscal policy. Unless such fiscal policy can be ruled out by commitment there is a strong case for monetary policy to act strongly towards output as well as inflation in order to determine a unique REE.

**Proposition 4.3.** *Under aggressive fiscal policy, just greater than an equiproportional response to inflation is generally insufficient to ensure a unique REE, as more aggressive output stabilizing policy is generally required.*

## 4.2. The Non-Ricardian case

Recall from our discussion of Proposition 4.1 that should matrix  $B$  not yield a (backward) stable root, then we have to recover such a root from matrix  $D$ . In this case, the eigenvalue of matrix  $B$  will be given by:

$$\gamma < \delta\beta. \quad (4.12)$$

Hence, for our purposes, policy elasticities need to generate ‘saddlepath’ behavior from matrix  $D$ ; where we require one root to lie outside the unit circle, and one root to lie inside the unit circle. Again we are required to distinguish between ‘moderate’ and ‘aggressive’ fiscal responses. We note that in this case, the government’s PVBC can only hold if the interest rate sequence does not react much to inflation, and so the stream of interest rates no longer meets condition (4.3). We will find, as well as this upper bound on the feasible interest rate sequence, the FTPL also places restrictions on the lower bound for interest rates.<sup>13</sup>

### 4.2.1. Region 3: ‘Moderate’ fiscal policy under FTPL

We employ Case 3 from Annex B to uncover the conditions under which the  $D$  matrix will yield a saddlepath. It turns out that in this case necessary and sufficient conditions on monetary policy to ensure determinacy in the non-Ricardian regime are:

$$-1 - \frac{1 + \beta}{\kappa} \phi_y^m - \frac{2(1 + \beta)}{\kappa} [1 - (1 - c/y)] \phi_y^f < \phi_\pi^m < 1 - \frac{1 - \beta}{\kappa} \phi_y^m. \quad (4.13)$$

The point here is that the interest rate rule ends up being severely restricted: it is *necessary* for monetary policy to breach Condition (4.3) and move no less than

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<sup>13</sup>In this sense the MTPL places lower bound restrictions on the interest rate sequence and the FTPL places both lower and upper bound restrictions on the sequence.

equiproportionally with inflation.<sup>14</sup> In this region, the fiscal policy maker, who has not committed to stabilize the debt stock requires inflation to stabilize the real value of public liabilities. In this case, the monetary policy maker must not act to stabilize inflation i.e. by raising real rates, because that will act to raise the current value of government liabilities (see equation B8 in Annex B) because the current value of future surpluses would be reduced by such action. Under moderate fiscal policy, the left hand side of the conditions are not particularly restrictive as we note that for  $\phi_y^m, \phi_y^f \geq 0$  they are bounded above at -1. As output stabilization by the interest rate rule has limited efficacy in this set-up, (since the rate of transformation from output to inflation is relatively low i.e.  $(1 - \beta)/\kappa$ ), it is still possible for the interest rate rule to stabilize output to some great extent, even under the FTPL.

**Proposition 4.4.** *Under non-Ricardian but moderate fiscal policy, monetary policy cannot act sufficiently to stabilize inflation and the Taylor Principle must be violated.*

#### 4.2.2. Region 4: ‘Aggressive’ fiscal policy under FTPL

Similarly, using Case 3 of Annex B, under aggressive fiscal policy, we can write the necessary and sufficient conditions to ensure determinacy in the non-Ricardian regime. But note here that the left hand side of the conditions markedly reduce the set of options for the monetary policy maker.

$$\frac{2 \left[ \left(1 - \frac{c}{y}\right) \kappa \phi_\pi^f - (1 + \beta) \left(1 - \phi_y^f \left(1 - \frac{c}{y}\right)\right) \right]}{(c/y)\kappa} - \frac{(1 + \beta) \phi_y^m}{\kappa} - 1 < \phi_\pi^m < 1 - \frac{1 - \beta}{\kappa} \phi_y^m. \quad (4.14)$$

We find a similar story under aggressive fiscal policy to that under moderate. There is an upper bound constraint on the interest rate sequence, once we

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<sup>14</sup>A version of this result was originally exposted in Woodford (1996).

introduce the FTPL, and some role for output stabilization by the interest rate rule remains. But for sufficiently aggressive fiscal stabilization of inflation, we find that the extent to which the interest rate stabilizes inflation becomes irrelevant. In this region the bounds on interest rate stabilization are likely to matter from below and from above. In the next subsection we will illustrate these findings in a little more detail.

**Proposition 4.5.** *Under non-Ricardian and aggressive fiscal policy, monetary policy operates within a significantly constrained parameter sub-space and can be treated as being required to be fixed.*

### 4.3. Illustration of Key Results

In this section, we illustrate some the key results outlined in Sections 4.1 and 4.2 with some simple plots of determinate zones under various monetary and fiscal strategies. Figure 1 plots, for standard parameter values (see Table 1), the zone of determinacy (denoted by REE) in  $\phi_y^m - \phi_\pi^f$  space for (4.4). The central (thick) line shows, for  $\phi_\pi^m = 1$  (loosely speaking when the Taylor Principle is just being met), the rate at which the monetary authority must increase the weight on output should the fiscal rule stabilize inflation to a significant degree. Unless ruled out by commitment to fiscal moderation, more aggressive inflation stabilization by the interest rate rule would act to mitigate the impact of inflation stabilization by the fiscal rule. The dotted line shows, for example, that when  $\phi_\pi^m = 2$  the fiscal authority must place a weight of over 3 on inflation before output stabilization by the interest rate rule is strictly necessary.

Figure 2 illustrates the zone of determinacy for Condition (4.7) where the interest rate rule is constrained to employ output stabilization alone. Note the central (thick) line corresponds very closely to the central line for Figure 1. This is because the two conditions collapse to an identical one when  $\phi_\pi^m - \frac{(1-\beta)}{\kappa} \phi_y^m = 1$ . We illustrate the limited implications for the interest rate rule that result from altering

the extent of (moderate) fiscal stabilization as the shifts in the curves partitioning stability are relatively small. This is because there is little intertemporal shifting of demand brought about by fiscal surpluses stabilizing output in this set-up. However, for significantly large inflation weights in the fiscal rule, output stabilization by the interest rate rule would become a requirement since inflation has a clear intertemporal price, as the rate at which expected inflation is traded for current output.

For Figure 3, we maintain the partition for Ricardian fiscal policy, such that the MTPL obtains, but illustrate the implications of allowing the fiscal rule to place a large weight on output stabilization. In each case we maintain an equiproportional response on the interest rate rule to an inflation shock,  $\phi_\pi^m = 1$ . We therefore show the extent to which output must be targeted by the interest rate rule as ‘aggressive’ fiscal policy places an increasingly higher weight on output. Note also that greater inflation stabilization by the fiscal rule also impacts directly on the extent of required output stabilization by the interest rate rule.

Figure 4 illustrates the FTPL, in which fiscal policy becomes non-Ricardian. In this case, the two equations for output and inflation must deliver a saddlepoint. As a result, monetary policy is constrained to the *converse* of the Taylor Principle, see (4.12) i.e., real rates should *not* rise in the face of an inflation shock. This might be interpreted as a what might happen under a zero bound, when policy cannot fall to offset falling inflation. The upper line in Figure 4 illustrates the upper bound constraint on  $\phi_\pi^m$  and the lower line illustrates that a large set of negative inflation weights on the interest rate rule are possible and that the implicit constraints on output in the monetary rule are somewhat attenuated.

Figure 5 illustrates the FTPL under the further supposition that fiscal policy is aggressive. Note that, in this case, the extent to which fiscal policy stabilizes inflation acts to constrain further the feasible choices for inflation stabilization by the interest rate rule. In fact we show that for sufficiently large weights on inflation in the fiscal rule, i.e.,  $\phi_\pi^f > 3$ , it is not inflation stabilization that matters for the attainment of a unique REE but the weight the interest rate places on

output.

These Figures illustrate the main results of this paper: that stabilization policy pursued jointly by monetary and fiscal policy may well require some additional weight on output in the monetary rule. Under moderate fiscal policy that additional weight may not be a requirement. But there are conditions when that additional weight appears to be more substantial. Specifically, we show that if fiscal policy responds to any great extent to inflation or output, monetary policy needs to respond robustly towards output to be sure of bringing about a unique REE. Finally, we have shown that under the FTPL interest rates cannot stabilize inflation but that output stabilization remains possible and under aggressive fiscal policy may be a requirement of a unique REE. Without well understood restrictions on fiscal policy, interest rates may need to respond more forcefully to output.

## 5. Conclusions

In this paper we analyse a simple micro-founded model in which both monetary and fiscal policy may play a role in stabilization. By closing fiscal policy down we note that this model reverts to the set of inflation and output equations employed with such widescale effect in the literature (see Woodford, 2003). This model turns out to be a special case where fiscal policy is ‘moderate’, non-inflation stabilizing and Ricardian. This is a key finding, which implies that a policy maker looking to implement standard policy prescriptions involving the Taylor Principle must ensure fiscal policy stabilizes debt and is not too aggressively stabilizing: simply announcing an inflation target is not going to be enough.

We studied the conditions for ensuring a unique REE for this model when fiscal policy can be Ricardian or non-Ricardian, and is also either moderate or aggressive. Monetary policy invoking the ‘Taylor Principle’ is sufficient to obtain stability under non-aggressive fiscal policy with the important proviso that fiscal policy does not act against inflation to any great extent. But we also show that

the Taylor Principle is palpably insufficient to ensure stability when fiscal policy is aggressive and strictly ruled out when non-Ricardian.

In effect there are two possible policy conclusions for the Ricardian (monetary and) fiscal policy maker to make. Option 1: In order to maintain a Taylor Principle, commit fiscal policy to be bounded by what we term non-aggressive policy i.e. where the weight on output in the fiscal rule is constrained and where the fiscal policy maker does not act to stabilize inflation. We can note such restrictions being placed on cyclical deficits in the EMU under the Pact for Stability and Growth or in attempts to pass balanced budget amendments in the USA. Option 2: In the absence, or infeasibility, of such a fiscal commitment, the monetary authority will have to consider augmenting the Taylor Principle with an output commitment or accept a higher lower bound constraint on the weight on inflation in the monetary rule than implied by the Taylor Principle. We note that this restraint on fiscal policy is typically implemented or suggested by the IMF under Article IV consultations.

Finally it should not escape our attention that if the Taylor Principle cannot be observed, perhaps because of the zero bound constraint or some such other impediment to monetary policy, then non-Ricardian fiscal policy may well be required for the economy to obtain stability. The growth of public debt under the current crisis thus seems to be relevant to note and any tendency to switch regime, from MTPL to FTPL and then back again, will make statements about any exit strategy pivotal.

## Annex A:

### A1 The Budget Constraint

The period constraint is,

$$\frac{B_t}{(1+i_t)} = B_{t-1} + P_t(G_t - T_t) - (M_t - M_{t-1}). \quad (\text{A.1})$$

We analyze fiscal rules of the form,

$$T_t = \chi_t G_t - \left( \frac{M_t - M_{t-1}}{P_t} \right) + \gamma \frac{B_{t-1}}{P_t}. \quad (\text{A.2})$$

Let  $S_t \equiv (1 - \chi_t)G_t$  denote the per period net surplus. If we substitute A.2 into A.1, deflate, we find in steady state that.

$$b = (1 + \delta)(1 - \gamma)b + (1 - \delta)S. \quad (\text{A.3})$$

## Annex B:

### B1 The Solution

Using the model set out in Section 3 of the main paper and derived in detail below, we confine ourselves to consider equilibria in which the vector of endogenous variables, given the policy rules in place, remains close to the value it would take in a stationary deterministic equilibrium.

#### The Model

#### The Representative Agent Optimality conditions

Let  $\{\mu_t\}_{t=0}^{\infty}$  be a (state-dependent) temporal sequence of Lagrange multipliers associated with the flow budget constraint of the agent. At each date and in each state the following equations, (1.1)–(1.4) are amongst requirements for an interior optimum;

$$\frac{1}{P_t C_t} = \mu_t; \quad (5.1)$$

$$\frac{1}{M_t} + \left( \frac{1}{1 + \delta} \right) E_t \mu_{t+1} = \mu_t; \quad (5.2)$$

$$\frac{1}{1 - N_t} = \mu_t W_t; \quad (5.3)$$

$$\mu_t = (1 + i_t) \left( \frac{1}{1 + \delta} \right) E_t \mu_{t+1}. \quad (5.4)$$

Using equations (1.1) and (1.4) in (1.2) uncovers the money demand relation:

$$\frac{M_t}{P_t} = C_t \left( \frac{1 + i_t}{i_t} \right). \quad (5.5)$$

(1.1) and (1.4) imply the consumption Euler equation:

$$E_t P_{t+1} C_{t+1} = \frac{1 + i_t}{1 + \delta} P_t C_t. \quad (5.6)$$

The labor supply function results from (1.1) and (1.3):

$$N_t = 1 - C_t \left( \frac{W_t}{P_t} \right)^{-1}. \quad (5.7)$$

### The Representative Firm

It is well known that in our framework the demand for the  $i^{\text{th}}$  product is given by

$$c_t(i) = \left( \frac{p_t(i)}{P_t} \right)^{-\theta} Y_t^d, \quad (5.8)$$

where  $Y_t^d$  denotes aggregate demand, the sum of private expenditure and government expenditure. In the model we develop there will be an effect directly from government expenditure. A cost-minimizing firm required to meet current demand will hire labor according to the following optimality condition,

$$w_t = \Lambda_t (\partial Y_{i,t} / \partial N_{i,t}), \quad (5.9)$$

where  $\Lambda_t$  measures real marginal cost. Total per-period profits then are given by

$$\Pi_t(i) = p_t(i) \left( \frac{p_t(i)}{P_t} \right)^{-\theta} Y_t^d - \Lambda_t \left( \frac{p_t(i)}{P_t} \right)^{-\theta} Y_t^d. \quad (5.10)$$

As regards price setting behavior we follow Calvo (1983) and many subsequent analysts and assume that firms which set prices in period  $t$  face a probability,  $\alpha$  ( $0 \leq \alpha < 1$ ) of having to live with the same decision next period. More generally, we assume that a firm which sets its price this period faces the probability  $\alpha^k$  of having to charge the same price in  $k$ -periods time. The firm now has to choose its optimal price.<sup>15</sup> The optimal price,  $\tilde{p}_t$ , is therefore given by:

$$\tilde{p}_t = \frac{\theta \sum_{k=0}^{\infty} (\alpha\beta)^k E_t(\mu_{t+k} P_{t+k}^\theta Y_{t+k}^d \Lambda_{t+k})}{(\theta - 1) \sum_{k=0}^{\infty} (\alpha\beta)^k E_t(\mu_{t+k} P_{t+k}^{\theta-1} Y_{t+k}^d)}. \quad (5.11)$$

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<sup>15</sup>The details of this problem are well understood, and we leave them to an appendix, available upon request.

Here  $\mu_{t+k}$  is marginal utility where the main impact on optimal prices is given by the stream of current and expected real marginal costs.

The evolution of the aggregate price-level is given by a weighted average of this period's optimal price and last period aggregate price:

$$P_t = [(1 - \alpha)\tilde{p}_t^{1-\theta} + \alpha P_{t-1}^{1-\theta}]^{1/(1-\theta)}. \quad (5.12)$$

B2 State vector,  $z_t$

These linear approximations to the model's key equations are straightforward to derive and are given below:

$$\hat{c}_t = E_t \hat{c}_{t+1} - \hat{i}_t + E_t \hat{\pi}_{t+1}; \quad (\text{B1 Consumption Euler Equation})$$

$$\hat{m}_t = \hat{c}_t - (1/\delta)\hat{i}_t; \quad (\text{B2 Money Demand Equation})$$

$$\hat{w}_{t+1} = (b/w)\hat{b}_t + (1 - b/w)\hat{m}_t; \quad (\text{B3 Evolution of Wealth})$$

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \kappa \hat{y}_t; \quad (\text{B4 New Keynesian Phillips Curve})$$

$$\hat{y}_t = (c/y)\hat{c}_t + (1 - c/y)\hat{s}_t; \quad (\text{B5 Output})$$

$$\hat{i}_t = \phi_y^m \hat{y}_t + \phi_\pi^m \hat{\pi}_t; \quad (\text{B6 Interest Rate Rule})$$

$$\hat{s}_t = \phi_y^f \hat{y}_t + \phi_\pi^f \hat{\pi}_t; \quad (\text{B7 Fiscal Rule})$$

$$\hat{b}_t = (1 - \gamma)\beta^{-1}\hat{b}_{t-1} + [(1 - \gamma)\beta^{-1} - 1]\hat{s}_t + \hat{i}_t - E_t \hat{\pi}_{t+1}. \quad (\text{B8 Debt Accumulation})$$

In a series of simple steps we substitute out the following endogenous state variables:  $\hat{i}_t$ ,  $\hat{m}_t$ ,  $\hat{w}_{t+1}$ ,  $\hat{y}_t$ ,  $\hat{s}_t$ . Note first that we can substitute output as it is simply a weighted average of consumption and the choices on the fiscal rule, B7.

$$\hat{y}_t = \frac{\frac{\varepsilon}{y}\hat{c}_t + (1 - \frac{\varepsilon}{y})\phi_\pi^f}{1 - (1 - \frac{\varepsilon}{y})\phi_y^f} \equiv F(\Phi). \quad (\text{B5}')$$

We normalize the natural rate to zero. Substitution of the interest rate rule, A6, and the New Keynesian Phillips curve, B4, solved for  $\hat{\pi}_{t+1}$ , allows us to re-write the consumption Euler equation in terms of consumption and inflation alone. In the representative agent set-up here government debt is part of financial wealth and so is uniquely pre-determined for the next period as a weighted average of debt and money holdings, it does not need to enter the state vector. Similarly and finally, we note that the quantity of money in this model is simply pinned down by period consumption and interest rate choices we do not need to specify money as a separate variable within the state vector.

### B3 Jacobian Matrix

There is a well developed theory for the existence of a unique rational expectations solution for a system of linear difference equations. The resulting system of difference equations can be written in compact form as:

$$E_t \mathbf{z}_{t+1} = A \mathbf{z}_t \quad \forall t \geq 0, \quad (\text{B9})$$

where the vector of state variables,  $\mathbf{z}_t \in \mathfrak{R}^n$  and is given by:

$$\mathbf{z}_t = \begin{bmatrix} \hat{b}_{t-1} \\ \hat{c}_t \\ \hat{\pi}_t \end{bmatrix},$$

where  $n = 3$  and the  $n \times n$  matrix  $A$  has constant coefficients and is given by:

$$A = \begin{bmatrix} (1 - \gamma) \beta^{-1} & \frac{\frac{\epsilon}{y}}{1 - \phi_y^f (1 - \frac{\epsilon}{y})} & (\{(1 - \gamma) \beta^{-1}\} \phi_\pi^f + \phi_\pi^m - \beta^{-1}) \\ 0 & (\{(1 - \gamma) \beta^{-1}\} \phi_y^f + \phi_y^m + \kappa \beta^{-1}) & \frac{\phi_\pi^f (1 - \frac{\epsilon}{y})}{1 - \phi_y^f (1 - \frac{\epsilon}{y})} \\ 0 & 1 + \frac{\frac{\epsilon}{y} \phi_y^m}{1 - \phi_y^f (1 - \frac{\epsilon}{y})} & (\{(1 - \gamma) \beta^{-1}\} \phi_y^f + \phi_y^m + \kappa \beta^{-1}) \\ 0 & \frac{\beta^{-1} \frac{\epsilon}{y} \kappa}{1 - \phi_y^f (1 - \frac{\epsilon}{y})} & \frac{\phi_y^m \phi_\pi^f}{1 - \phi_y^f (1 - \frac{\epsilon}{y})} + \phi_\pi^m - \beta^{-1} \\ 0 & -\frac{\beta^{-1} \frac{\epsilon}{y} \kappa}{1 - \phi_y^f (1 - \frac{\epsilon}{y})} & \frac{\beta^{-1} \kappa \phi_\pi^f}{1 - \phi_y^f (1 - \frac{\epsilon}{y})} \\ & & \beta^{-1} - \frac{\beta^{-1} \kappa \phi_\pi^f}{1 - \phi_y^f (1 - \frac{\epsilon}{y})} \end{bmatrix}$$

## B4 Eigenvalues

The solution of this system is  $\mathbf{z}_t = A^t \mathbf{z}_0$  and hence the dynamic behaviour of the system depends upon the stability, or otherwise, of the square matrix,  $A$ . The stability of which will in turn depend upon the values of roots of the characteristic polynomial, specifically whether their moduli is either less than or equal to or greater than unity. The roots of  $A$  are given by the solutions to the characteristic polynomial, where,  $\lambda$  represent the roots and  $I$  is the identity matrix:

$$\det(A - \lambda I) = 0 \quad (\text{B10})$$

The square matrix  $A$  will have  $n$  eigenvalues corresponding to each state variable. One way to arrive at the eigenvalues is through use of the Jordan canonical decomposition which requires that there also exists a nonsingular  $n \times n$  matrix,  $B$ , such that :

$$A = B^{-1} \Lambda B$$

where  $\Lambda$  represents a diagonal matrix in which each entry  $\Lambda_{ii}$  contains one of  $n$  distinct eigenvalues of  $A$ ,  $\lambda_n$ , ordered by increasing absolute value. This matrix  $\Lambda$  can thus further be decomposed into the following form:

$$\mathbf{\Lambda} = \begin{bmatrix} \mathbf{\Lambda}_1 & 0 \\ 0 & \mathbf{\Lambda}_2 \end{bmatrix}.$$

Such that the  $m$  eigenvalues of  $\mathbf{\Lambda}_1$  lie inside the unit circle and the  $n - m$  eigenvalues of  $\mathbf{\Lambda}_2$  lie outside the unit circle. Standard analysis tells us that the existence of a unique bounded rational expectations solution for the vector,  $z_t$ , depends upon the number of eigenvalues of the square matrix,  $A$ , that lie inside the unit circle (see Blanchard and Kahn, 1980),  $\lambda_m$ , corresponding exactly to the number of non-predetermined state variables,  $m$ , which will, of course, imply that the number of eigenvalues on or outside the unit circle,  $\lambda_{n-m}$ . For our model,  $n - m = 2$ .

But because of the block triangular structure of the matrix,  $A$  is decomposable or reducible such that we note that the matrix  $A$  can be partitioned into pre-determined and non-predetermined variables, with  $2 \times 2$  matrix  $D$  corresponding to the Jacobian for the non-predetermined variables, and  $B$  corresponding to the Jacobian for the pre-determined variable(s).

$$A = \begin{bmatrix} B & C \\ 0 & D \end{bmatrix}$$

The canonical decomposition can then be undertaken on matrices  $B$  and  $D$  separately from the eigenvalues can be obtained. First, we consider the case of  $B$  which will deliver a stable eigenvalue if its diagonal element is less than one in which case we will require the matrix  $D$  to deliver two unstable eigenvalues in order for the sequence  $\{\mathbf{z}\}_{t=0}^{\infty}$  to be bounded to a unique rational expectations equilibrium. Secondly, we consider the case in which  $B$  does not deliver a stable eigenvalue and then in which case the matrix  $D$  will be required to deliver one stable and one unstable eigenvalue. It is thus the choice on the diagonal of the  $B$  matrix that determines how monetary and fiscal policy should operate in order to ensure a unique rational expectations equilibrium.

Let us concentrate on the matrix  $D$  for which we outline the conditions under which the roots,  $\lambda_1$  and  $\lambda_2$  lie inside or outside the unit circle. First, we factor the characteristic polynomial and examine whether the roots fall outside the unit circle,  $p(1)$ .

$$\det(A - \lambda I) = p(\lambda) = (\lambda - \lambda_1)(\lambda - \lambda_2) \quad (\text{B11})$$

$$p(1) = (1 - \lambda_1)(1 - \lambda_2) = 1 - (\lambda_1 + \lambda_2) + \lambda_1\lambda_2 \quad (\text{B12})$$

Clearly in the case where both roots lie inside or outside the unit circle the  $p(1) > 0$  but in the case of positive roots either side of the unit circle A9 will be negative.

*Case 1* -  $\lambda_{1,2} < |1|$ ,  $p(1) > 0$ .

For  $\lambda_{1,2} < |1|$ , then  $\lambda_1\lambda_2 < 1$  and  $\lambda_1 + \lambda_2 < 2$ .

*Case 2* -  $\lambda_{1,2} > |1|$ ,  $p(1) > 0$ .

For  $\lambda_{1,2} > |1|$ , then  $\lambda_1\lambda_2 > |1|$  and  $\lambda_1 + \lambda_2 > |2|$ .

*Case 3* -  $\lambda_1 < |1|$  and  $\lambda_2 > |1|$ ,  $p(1) < 0$ .

For  $\lambda_1 + \lambda_2 > 0$ ,  $\lambda_1\lambda_2 - (\lambda_1 + \lambda_2) < -1$  and  $\lambda_1\lambda_2 + (\lambda_1 + \lambda_2) > -1$ ;

For  $\lambda_1 + \lambda_2 < 0$ ,  $\lambda_1\lambda_2 - (\lambda_1 + \lambda_2) > -1$  and  $\lambda_1\lambda_2 + (\lambda_1 + \lambda_2) < -1$ .

Clearly for the MTPL, Case 2 applies and for the FTPL Case 3 applies. We then must examine the parameters of the  $D$  matrix and determine whether subject to the eigenvalue from the  $B$  matrix whether one or two unstable eigenvalues are required to ensure the satisfaction of a unique rational expectations equilibrium for the state vector  $\mathbf{z}_t$ .

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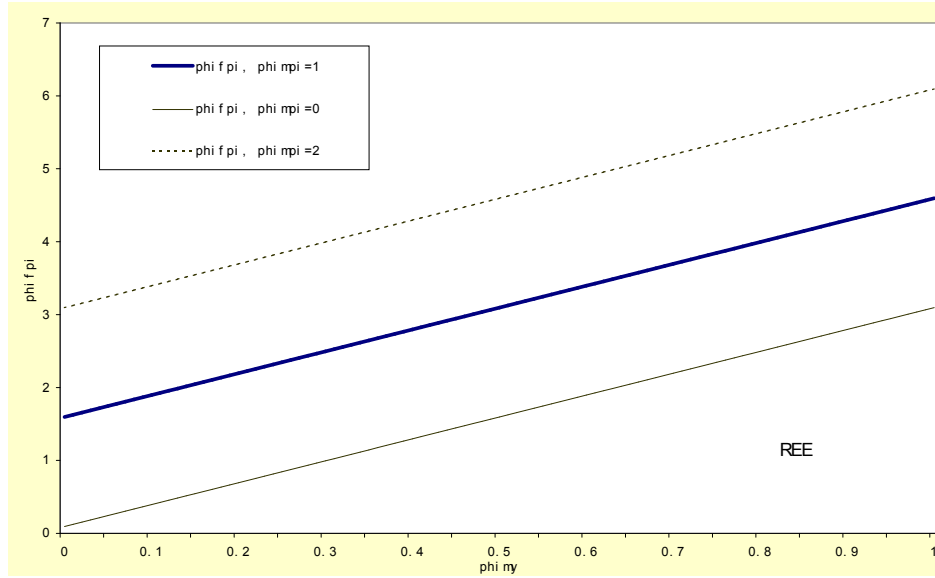


Figure 5.1: Boundary Conditions for 4.4

**Table 1: Table of Parameters**

Symbol	Value	Description
$r$	0.05	Real interest rate
$\beta$	0.95	Subjective discount factor
$\delta$	0.053	Subjective discount rate
$\gamma$	0.06	Rate of debt retirement
$\frac{c}{y}$	0.6	Steady-state consumption-output ratio
$\frac{m}{w}$	0.1	Steady-state money-wealth ratio
$\kappa$	0.5	Phillips curve slope
$\frac{w}{c}$	0.7	Steady-state wealth-consumption ratio

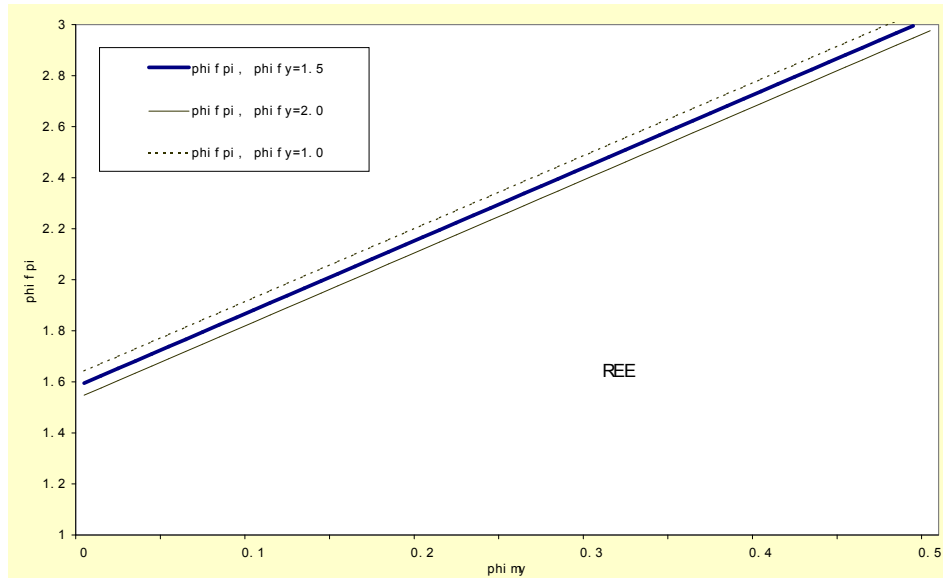


Figure 5.2: Boundary Condition for 4.7

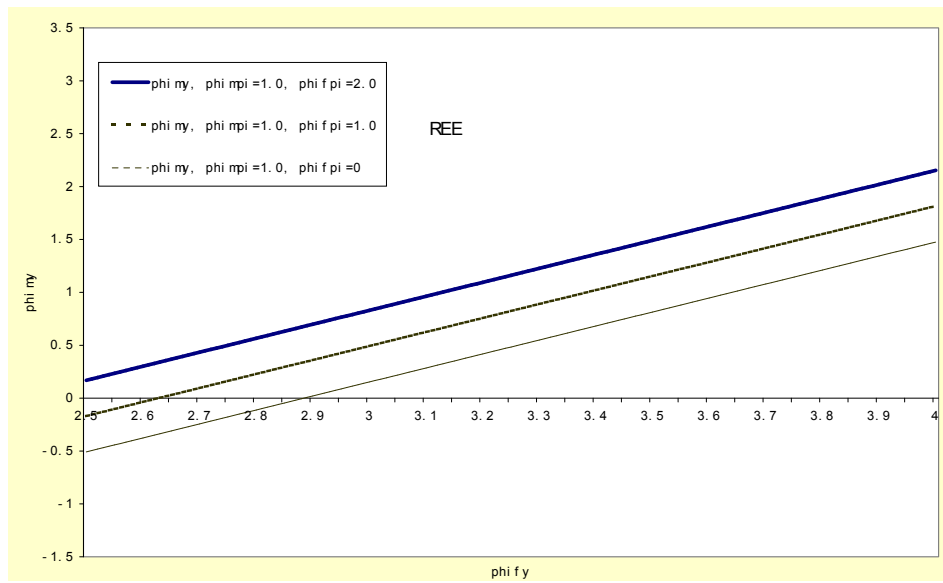


Figure 5.3: Boundary Conditions for 4.11

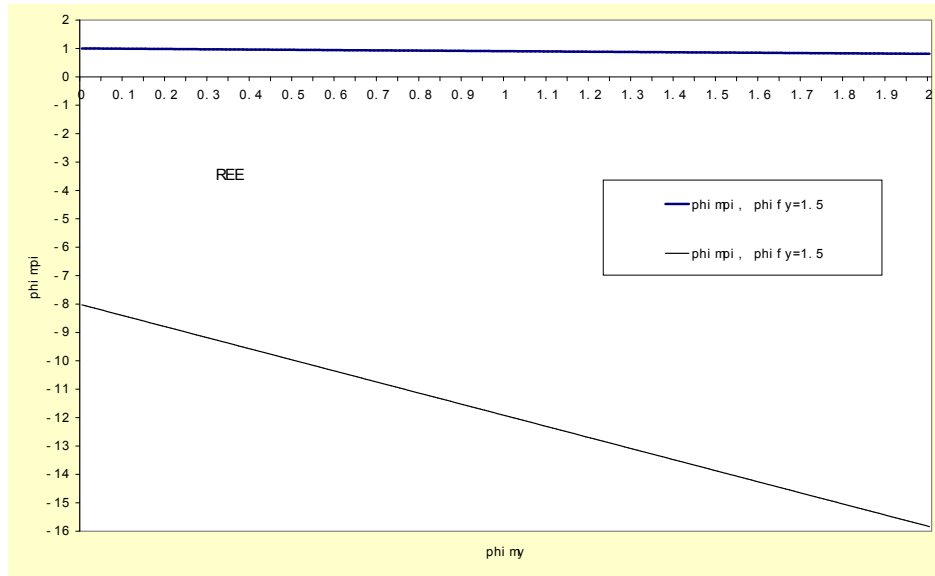


Figure 5.4: Boundary Condition for 4.13

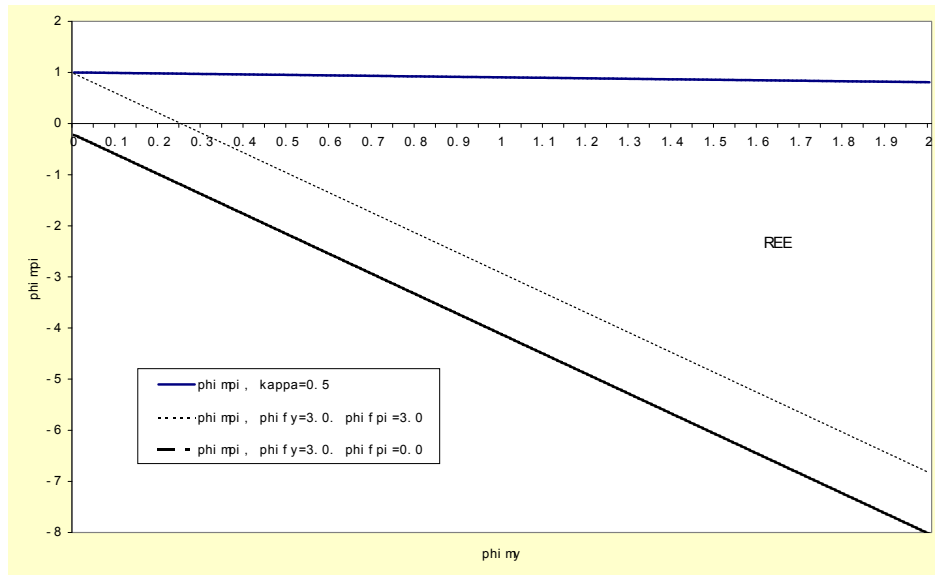


Figure 5.5: Boundary Condition for 4.14