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ESTIMATING WTP WITH UNCERTAINTY CHOICE CONTINGENT VALUATION

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ABSTRACT:

A method for treating Contingent Valuation data obtained from a polychotomous response format designed to accommodate respondent uncertainty is developed. The parameters that determine the probability of indefinite responses are estimated and used to truncate utility distributions within a structural model. The likelihood function for this model is derived, along with the posterior distributions that can be used for estimation within a Bayesian Monte Carlo Markov Chain framework. We use this model to examine two data sets and test a number of model related hypotheses. Our results are consistent with those from the psychology literature relating to uncertain response: a 'probable no' is more likely to suggest a definite no, than a 'probable yes' is likely to suggest a definite yes. We also find that 'don't know' responses are context dependent. Comparing the performance of the methods developed in this paper with the ordered Probit which has been previously used in the literature with this type of data we find our methods outperform the ordered Probit for one of the data sets used.

KEYWORDS: Respondent Uncertainty, Multiple Bound Contingent Valuation, Bayesian MCMC.

JEL CLASSIFICATION: C35, I18 Q5

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Estimating WTP with Uncertainty Choice Contingent Valuation

1 Introduction

There is an increasingly large literature developing to deal with the issue of uncertainty associated with responses in Contingent Valuation (CV) studies. The reason for the development of this literature can be traced back to the recommendation of the NOAA Panel [6] and the inclusion of the "No-Answer" option [11]. How to include and interpret these responses has lead to research on uncertainty in CV responses more generally. To date it has been found that if offered a choice of expressing uncertainty, respondents frequently indicate a degree of uncertainty. The justification for using uncertain responses is that people may be better able to respond to probabilistic intentions rather than absolutes, since they may be inherently uncertain about the nature of their own preferences, or the circumstances surrounding the choices that they are being asked to make. Evans et al. [10] notes that literature since Juster [12] recommend probabilistic responses over definite responses for this reason. They also cite evidence that respondents prefer to reply in terms of 'verbal probabilities' rather than giving numerical probabilities because they are generally poor at either responding to or stating probabilities in numerical terms. Ariely et al. [4] have also argued that economic agents are frequently uncertain about the value they place on goods. They refer to this form of uncertainty as coherent arbitrariness.[1] Thus, there is good reason to assume that we need to taken account of respondent uncertainty in CV design, implementation and analysis.

There are number of ways in which CV studies have been implemented to try and deal with respondent uncertainty (broadly defined). Akter et al. [1] provide a useful overview of this literature. Of specific interest to the present paper is the literature that Akter et al. refer to as the polychotomous choice method (e.g., [9]). This literature can be divided into two parts. First, there are a large number of papers that employ the three option approach. Balcombe and Fraser [7] and many papers cited therein have examined uncertainty and misreporting for the Dichotomous Choice CV three

[1] Understanding the meaning of value is being extensively researched at the interface between economics (eg, [5]), psychology (eg, [15]) and neurobiology (eg, [14]). What these related literatures reveal is that individuals are susceptible to manupluations with respect to value of goods.

option approach. With the three option design, CV data are generated that have allowed respondents to answer yes, no and don't know, where the last category of response captures uncertainty.

The second part of this literature, the part we are interested in, employs a polychotomous response format, termed the Uncertainty Choice (UC) format by Evans et al. [10]. This CV design typically allows respondents to frame their response as yes (Y), probably yes (PY), don't know (DK), probably no (PN), and no (N). This polychotomous response format was initially introduced by Ready et al. [13] who employed six categories. There have since been a number of papers employing the UC approach (e.g., [19], [20], [3], [10], [16], [17], [8]).

A number of statistical methods have been employed in the literature to analyse this type of CV data. Initial efforts involved data recoding (e.g., [13]). An alternative is where some uncertain responses are treated as if they arise within a given threshold (e.g. [18] , [2], [3]). There are also examples of where the uncertain responses are assigned a probability of revealing whether the respondent would derive positive or negative utility from the choice they are offered (e.g. [10]). A useful summary of the range of probabilities for the range of uncertain responses is provided by Boman [8]. In this paper we develop a new way to treat the uncertain responses, differing from those previously employed in this literature.

First, the methods developed in this paper differ from the 'threshold' approaches such as the ordered Probit, that assume that if a bid is sufficiently high or low in order to elicit an indefinite response it is within a threshold. Like Evans et al. [10] we attach a probability to an event that a PY and PN constitutes a bound above or below the stated bid. However, unlike Evans et al. we do not assign a fixed probability that a PY or PN constitutes a bound. Instead we demonstrate that such a probability is estimable (along with the other parameters of the model). Arguably, our approach of attaching a probability to a bound holding is a more direct way of dealing with the uncertain responses. If somebody states that a bound holds with certainty, then it is reasonable to assume that their WTP lies[2] within that bound with probability one. By extension, if they indicate that they are likely, but not certain to respond a certain way, then it seems sensible to attach a probability

[2] This argument is more reasonable where respondents are given the chance to indicate that they are uncertain. In cases that do not allow for uncertain responses then there is less justification for a yes or no constituting a definite bound.

that their indicated response will, in reality hold.

Second, our approach enables us to test whether DK responses can be used informatively, or whether they are better treated as non-informative responses by a group of people that have failed to engage with the questions posed to them. This lack of engagement may perhaps be because they find the survey questions confusing or do not have the time or energy to properly consider them.

Thus, our approach gives us the opportunity to estimate the probabilities and in turn compare them with previous values in the literature. For example, Boman [8] reports that a PY or PN have 0.65% to 0.80% chance of holding. Alternatively, based on a study of the psychology literature, Evans et al. [10] suggest that a PY is around 75% certain to be associated with positive utility whereas a PN is likely to be around 85% certain to be associated with negative utility. We would argue that the value of these probabilities are likely to be context dependent, and should be estimated in each data set rather than set *a priori*.

The parameters that determine the probabilities of truncating the utility distribution (positively or negatively) are not directly 'behavioural' in that they do not describe the behaviour of the respondent, but rather how we interpret and use their responses. Behavioural parameters are those that determine respondents answers, given their preferences, along with the parameters that describe their preferences. Our approach introduces parameters that have a behavioural interpretation. This allows us to estimate the probabilities that a respondent with a positive utility will reply DK or PY etc. As we show in this paper, these structural parameters are needed to identify the non-structural parameters (those that determine the truncation of utility into positive or negative regions). To implement our approach we derive the likelihood for this model which is estimated employing Bayesian methods. Accordingly, we derive the posterior distributions for the parameters within our model and estimate the models using a Bayesian Monte Carlo Markov Chain (MCMC) approach. An appendix is included that contains the derivation of the posterior distributions for the parameters estimated in the models developed.

The paper proceeds by first developing the model in Section 2. Section 3 introduces the prior distributions for the parameters that are used in estimation. In order to illustrate the methods, in Section 4 we employ two data sets. One has previously been employed by Welsh et al. [19] and Evans et al. [10]. The other is an original data set on the WTP of Nigerian Farmers to participate in an irrigation scheme. Within our analysis we test a range of hypotheses that relate to the behavioural model developed. All results are based on the use of the marginal likelihood. Finally, in Section 5 we conclude.

2 The Model

In the following u_i denotes utility; WTP_i is the willingness to pay by the ith individual; $WTP_{\mu,i}$ is the mean willingness to pay for individuals with characteristics x_i; b_i is the bid presented to the individual; and α is a vector of parameters,

$$\begin{aligned} u_i &= WTP_{\mu,i} - b_i + (WTP_i - WTP_{\mu,i}) \\ &= \alpha' x_i - b_i + e_i \\ &\quad \text{where} \\ \alpha' x_i &= WTP_{\mu,i} \\ e_i &= (WTP_i - WTP_{\mu,i}) \end{aligned} \tag{1}$$

It is assumed that e_i is independently and identically normally distributed with mean zero and variance θ.

2.1 Indefinite Responses

We now examine the case where an individual is offered the bids under the 'uncertainty choice' format. In this case an individual is able to indicate whether the response they make is a 'definite yes' (Y), 'probable yes' (PY), 'don't know' (or unsure, DK), 'probable no' (PN) or 'definite no' (N). These responses are assigned five numerical values $L_i = 1, 2\ 3, 4, 5$. (where $1 =$Y$, 2 =$PY$, 3 =$DK$, 4 =$PN, and $5 =$N). From this we need to create a rule whereby the WTP interval for the ith individual is a stochastic function of their responses.

We make the following assumptions:

- 1. A 'Y' implies utility must be positive at the stated bid

2. A ‘N’ implies utility must be negative at the stated bid
3. A ‘PN’ implies utility is negative with a probability at least 0.5 and less than or equal to 1
4. A ‘PY’ implies utility is positive with a probability at least 0.5 and less than or equal to 1
5. A ‘DK’ either (leading to two distinct models): i) implies nothing about the level of utility; ii) implies a 50% chance of utility being positive or negative.

We recognise that our approach is restrictive in that it assumes that Y or N responses indicate positive or negative utility with probability one. In principle, it is possible that using an extension of the methods introduced here, this assumption could be relaxed, but we do not attempt that within this paper. The existing literature suggests that while Y and N may not in reality indicate complete certainty, they are usually considered at least around 95% certain. Our contention is that there is little to be gained by estimating these parameters.

The probability that utility is positive conditionally on a given response, bid and set of parameters Θ will be denoted as:

$$\tau_{k,i} = P\left(u_i > 0 | L_i = k, \Theta, x_i, b_i\right). \tag{2}$$

We further define the cumulative normal probability as Φ_i:

$$\Phi_i = P\left(u_i > 0 | \Theta, x_i, b_i\right) \tag{3}$$

Under our assumptions above, the probabilities $\tau_{k,i}$ in (2) are defined as:

$$\begin{aligned} \tau_{1,i} &= 1 \\ \tau_{2,i} &= \rho_1 + \Phi_i\left(1 - \rho_1\right) \\ \tau_{3,i} &= \omega + \left(1 - 2\omega\right)\Phi_i \\ \tau_{4,i} &= \Phi_i\left(1 - \rho_2\right) \\ \tau_{5,i} &= 0 \end{aligned} \tag{4}$$

Note that in (4) the probability that utility is positive or negative is a function of Φ_i rather than a fixed quantity as assumed in Evans et al. [10]. When generating the latent utility u_i, the greater the predicted utility (based on the bid and other variables), the higher the probability that the utility will be positive should the respondent indicate PY. Conversely, the smaller the predicted utility the larger the probability that the utility will be negative given that the respondent has indicated PN.

It is possible to construct a model in which some of the probabilities were fixed, such that $\tau_{2,i} = \rho_1$ and $\tau_{4,i} = 1 - \rho_2$. However, this is an unsatisfactory approach since if a persons' utility is predicted to be positive on the basis of their bid alone, this should increase the probability that the utility is positive given there indefinite response (PY). For example, if two respondents reply PY, but one has done so at a much lower bid, we are more likely to believe that utility is positive for the individual with the lower bid. Thus, within our model ρ_1 is the probability that utility is truncated positively given the respondent replies PY. The remaining term $\Phi_i (1 - \rho_1)$ reflects the probability that utility will be positive, even though the utility distribution has not been truncated. Treating PNs and PYs symmetrically would require $\rho_1 = \rho_2$.

Under the assumptions stated above the decision to truncate the variables is made according to:

$$\tau_{3,i} = \omega + (1 - 2\omega) \Phi_i \tag{5}$$

With the condition that $\omega \leq \frac{1}{2}$ and noting that for any value of ω that

$$\omega + (1 - 2\omega) \frac{1}{2} = \frac{1}{2} \tag{6}$$

we have the two extremes: $\omega = \frac{1}{2} \Rightarrow \tau_{3,i} = \frac{1}{2}$ and at $\omega = 0 \Rightarrow \tau_{3,i} = \Phi_i$.

The value $\omega = \frac{1}{2}$ is consistent with the view that if a respondent has reported DK, then they have told us there is a 50% chance that their utility is positive or negative (an informative response). The value $\omega = 0$ is consistent with the view that an individual's response gives us no basis for truncating the distribution since their response is non-informative. Accordingly, we assume that their probability of having positive or negative utility is simply what we would predict for the population.

2.2 Behavioral Parameters

Under fixed values of ρ_1, ρ_2 and ω the model is easily estimated. However, our aim in this paper is to estimate the parameters ρ_1 and ρ_2 along with determining whether $\omega = \frac{1}{2}$ or $\omega = 0$ is more consistent with the data. In order to proceed we need to define how these parameters enter the likelihood function. For this purpose the following probabilities are defined:

$$\begin{aligned} P\left(L_i = k | u_i > 0, b_i, x_i, \Theta\right) &= \pi^{+}_{k,i} \\ &\text{and} \\ P\left(L_i = k | u_i < 0, b_i, x_i, \Theta\right) &= \pi^{-}_{k,i} \end{aligned} \tag{7}$$

and the likelihood of a given response is defined as:

$$P\left(L_i = k | b_i, x_i, \Theta\right) = \Gamma_{k,i} \tag{8}$$

It is straight forward to show that, for non-zero values of $\pi^{+}_{k,i}$ and $\pi^{-}_{k,i}$ respectively:

$$\begin{aligned} \Gamma_{k,i} &= \frac{\pi^{+}_{k,i}}{\tau_{k,i}} \Phi_i \\ \Gamma_{k,i} &= \frac{\pi^{-}_{k,i}}{\left(1 - \tau_{k,i}\right)} \left(1 - \Phi_i\right) \end{aligned} \tag{9}$$

and, in general:

$$\Gamma_{k,i} = \pi^{+}_{k,i} \Phi_i + \pi^{-}_{k,i} \left(1 - \Phi_i\right) \tag{10}$$

The conditions in (9) imply:

$$\left(1 - \Phi_i\right) \pi^{-}_{k,i} \tau_{k,i} = \Phi_i \left(1 - \tau_{k,i}\right) \pi^{+}_{k,i} \; . \tag{11}$$

The parameters must also obey the summation conditions:

$$\sum_{k=1}^{5} \pi^{-}_{k,i} = 1 \text{ and } \sum_{k=1}^{5} \pi^{+}_{k,i} = 1 \tag{12}$$

Following from our initial set of assumptions we assume that $\pi^{+}_{5,i} = 0$ and

$\pi^{-}_{1,i} = 0$, thus giving the conditions:

$$\begin{pmatrix} 1-\pi^{+}_{1,i}-\pi^{+}_{3,i} \\ 1-\pi^{-}_{5,i}-\pi^{-}_{3,i} \\ 0 \\ 0 \end{pmatrix} \tag{13}$$
$$= \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ -\Phi_i\left(1-\tau_{2,i}\right) & 0 & \left(1-\Phi_i\right)\tau_{2,i} & 0 \\ 0 & -\Phi_i\left(1-\tau_{4,i}\right) & 0 & \left(1-\Phi_i\right)\tau_{4,i} \end{pmatrix} \begin{pmatrix} \pi^{+}_{2,i} \\ \pi^{+}_{4,i} \\ \pi^{-}_{2,i} \\ \pi^{-}_{4,i} \end{pmatrix}$$

In addition, by employing (11) we obtain the following condition:

$$\left(1-\Phi_i\right)\tau_{3,i}\pi^{-}_{3,i} = \Phi_i\left(1-\tau_{3,i}\right)\pi^{+}_{3,i} \tag{14}$$

This means that overall we have eight unknowns $\left\{\pi^{+}_{k,i}\right\}_{k=1}^{4}$ and $\left\{\pi^{-}_{k,i}\right\}_{k=2}^{5}$ and five restrictions to place on them. Therefore, three further restrictions are required to identify $\left\{\pi^{+}_{k,i}\right\}$ and $\left\{\pi^{-}_{k,i}\right\}$ and consequently the likelihood in equation (10).

2.3 Model Identification

Our strategy for identifying this system is to assume two further sets of assumptions. One relates to the sums of $\pi^{+}_{1,i}+\pi^{+}_{3,i}$ and $\pi^{-}_{5,i}+\pi^{-}_{3,i}$. The second set of assumptions determine $\pi^{+}_{3,i}$ and $\pi^{-}_{3,i}$.We derive these assumptions by employing a linear transformation of our unknown parameters. We recognise that our choice of parameterisation is to some extent arbitrary, as we could have chosen to parameterise the model differently, but equivalently. However, if the posterior distributions of a linear combination of parameters such as $(\psi_2-\psi_1+\psi_3)$ were required, then when using Bayesian MCMC estimation this creates no additional issues, since the posterior distributions for any reparameterisation can be constructed. However, in what follows we only produce the posterior distributions of $\{\psi_i\}$.

The first set of assumptions are that:

$$\begin{aligned} \pi^+_{1,i} &= \psi_1 + (\psi_2 - \psi_1)\,\Phi_i - \pi^+_{3,i} \\ \pi^-_{5,i} &= \psi_1 + (\psi_2 - \psi_1)\,(1-\Phi_i) - \pi^-_{3,i} \\ & \text{where} \\ \frac{\partial \pi^+_{3,i}}{\partial \Phi_i} &\leq 0 \text{ and } \frac{\partial \pi^-_{3,i}}{\partial \Phi_i} \geq 0 \\ 0 &\leq \psi_1 \leq \psi_2 < 1 \end{aligned} \tag{15}$$

where ψ_1 and ψ_2 are intercept and gradient parameters that are to be estimated.

These assumptions state that as the predicted level of utility increases, the probability of respondents **with positive utility** replying Y will increase with Φ_i. Consequently, it also states that respondents **with positive utility** are less likely to respond in an indefinite way (e.g. $\pi^+_{3,i}$, $\pi^+_{4,i}$, and $\pi^+_{2,i}$) as Φ_i rises.

Conversely, these assumptions state that as the predicted level of utility decreases, the probability of respondents **with negative utility** replying N will increase with Φ_i. Consequently, it also states that respondents **with negative utility** are less likely to respond in an indefinite way (e.g. $\pi^-_{3,i}$, $\pi^-_{4,i}$, and $\pi^-_{2,i}$) as Φ_i rises.

The interpretation of the parameters in (15) can most easily be understood by evaluating the functions in (15) at extreme values such as $\Phi_i = 0$ and $\Phi_i = 1$. For example, at $\Phi_i = 0$, $\pi^+_{1,i} = \psi_1 - \pi^+_{3,i}$and $\pi^-_{5,i} = \psi_2 - \pi^-_{3,i}$. At $\Phi_i = 0$, the basis of the bid and the characteristics of the respondent would led us to predict negative utility with probability one. If a respondent has positive utility (in spite of the prediction that $\Phi_i = 0$) this would indicate that the respondent' utility is close to zero. Thus, ψ_1 represents our predicted proportion of respondents that, reply in Y or DK, if they have positive utility, where we predict that utility (though positive) is very small. Coversely, at $\Phi_i = 1$, ψ_2 is the estimated probability that respondents reply Y or DK, when utility is positive, and since $\Phi_i = 1$ we believe that utility is probably very high.

These interpretations remain a little unclear without further describing the determination of $\pi^-_{3,i}$. Furthermore, the assumptions above still require a further condition (that is consistent with $\frac{\partial \pi^+_{3,i}}{\partial \Phi_i} \leq 0$ and $\frac{\partial \pi^-_{3,i}}{\partial \Phi_i} \geq 0$) for the system to be identified. We do this by specifying a function for $\pi^+_{3,i}$ and

$\pi_{3,i}^{-}$ once again observing (14). Two solutions can therefore be described as follows:

$$\begin{aligned} \text{Model (a)}.\pi_{3,i}^{-} &= \pi_{3,i}^{+} = \psi_3 \text{ which is consistent with } \tau_{3,i} = \Phi_i; \\ &\text{or} \\ \text{Model (b) } \pi_{3,i}^{+} &= \psi_3 (1 - \Phi_i) \text{ and } \pi_{3,i}^{-} = \psi_3 \Phi_i \text{ which is consistent with } \tau_{3,i} = \frac{1}{2} \\ \text{where } 0 &< \psi_3 < 1 \end{aligned} \tag{16}$$

From Model (a) if we set $\pi_{3,i}^{-}$ and $\pi_{3,i}^{+}$ equal to a constant (ψ_3) this corresponds to a model in which a given proportion of the respondents simply DK in a non-informative manner. Utility is simply predicted using $x\prime\alpha + e_i$. As we shall see, under this assumption the estimated value of ψ_3 will be very close to the proportion of respondents replying DK.

At the other extreme, Model (b), $\psi_3 (1 - \Phi_i)$ assumes that the DK is highly informative in the sense that a DK response indicates that the respondent is telling us that there is an even likelihood that they have positive or negative utility. Under this condition we know that somebody that has a high expected utility on the basis of their bid $(\Phi_i = 1)$ will have zero probability of replying DK $(\pi_{3,i}^{+} = 0)$, but that somebody with negative utility has a probability of ψ_3 given that $\Phi_i = 1$. In other words, ψ_3 is the probability that somebody who has negative utility, but who we would otherwise be certain to expect to have positive utility (and therefore we would predict has a very small negative utility) will reply DK.

We observe that under Model (b) simply substituting in the definition in (16), into those in (15) gives:

$$\begin{aligned} \pi_{1,i}^{+} &= (\psi_1 - \psi_3) + (\psi_2 - \psi_1 + \psi_3)\,\Phi_i \\ \pi_{5,i}^{-} &= (\psi_1 - \psi_3) + (\psi_2 - \psi_1 + \psi_3)\,(1 - \Phi_i) \end{aligned} \tag{17}$$

Therefore, $(\psi_1 - \psi_3)$ and $(\psi_2 - \psi_1 + \psi_3)$ describe the behaviour of the definite responses, and how they vary with Φ_i where DKs are treated as indicating that there is a equal probability of indicating positive or negative utility.

Returning to (15), in order to interpret the parameters for both models the following conditions are required:

Under Model (a)

$$\begin{aligned} \Phi_i &= 1 \Rightarrow \; \pi^+_{1,i} = \psi_2 - \psi_3 \text{ and } \pi^-_{5,i} = \psi_1 - \psi_3 \\ \Phi_i &= 0 \Rightarrow \pi^+_{1,i} = \psi_1 - \psi_3 \text{ and } \pi^-_{5,i} = \psi_2 - \psi_3 \end{aligned} \tag{18}$$

Under Model (b)

$$\begin{aligned} \Phi_i &= 1 \Rightarrow \; \pi^+_{1,i} = \psi_2 \text{ and } \pi^-_{5,i} = \psi_1 - \psi_3 \\ \Phi_i &= 0 \Rightarrow \pi^+_{1,i} = \psi_1 - \psi_3 \text{ and } \pi^-_{5,i} = \psi_2 \end{aligned} \tag{19}$$

Further conditions are also required to identify this model since survey data will likely contain values of Φ_i very close to zero and one. The probabilities $\left\{\pi^-_{j,i}\right\}$ and $\left\{\pi^+_{j,i}\right\}$ need to lie between zero and one. Therefore, in conjunction with our assumption that $\psi_2 > \psi_1$ we require, for both models:

$$0 < \psi_3 \leq \psi_1 \leq \psi_2 < 1 \tag{20}$$

In model (b) the ψ_2 represents the probability that respondents with positive utility will make a definite response given $\Phi_i = 1$ and the probability that respondents with negative utility will make a definite response given $\Phi_i = 0$. Thus, ψ_2 represents the propensity of respondents with very large absolute utilities to give a definite response.

The interpretation remains the same for model (a) except that it is assumed that there will be a proportion of respondents that always say DK (give a non-informative response). The value of $\psi_1 - \psi_3$ gives the probability that people who have positive or negative utilities can give a definite response, even though we would predict on the basis of their bids that their utilities would be in the opposite direction with probability one (and thus we would expect their actual level of utility to be close to zero). Therefore, $\psi_1 - \psi_3$ represents the propensity of respondents with very small absolute utilities to give a definite response.

Finally, it follows that we can write the likelihood for each of the responses as:

$$\begin{aligned}\Gamma_{1,i} &= \pi_{1,i}^{+}\Phi_i \\ \Gamma_{2,i} &= \pi_{2,i}^{+}\left(\frac{1}{\tau_{2,i}}\right)\Phi_i \\ \Gamma_{3,i} &= \pi_{3,i}^{+}\frac{1}{\tau_{3,i}}\Phi_i \\ \Gamma_{4,i} &= \pi_{4,i}^{-}\frac{(1-\Phi_i)}{1-\tau_{4,i}} \\ \Gamma_{5,i} &= \pi_{5,i}^{-}(1-\Phi_i)\end{aligned} \tag{21}$$

Under the conditions that $\omega = 0$ or $\omega = \frac{1}{2}$ we have two differing solutions:

$$\begin{aligned}\Gamma_{3,i} &= \psi_3 \text{ under } \tau_{3,i} = \Phi_i \ (\omega = 0) \\ &\text{or} \\ \Gamma_{3,i} &= 2\psi_3 (1-\Phi_i)\,\Phi_i \text{ under } \tau_{3,i} = \frac{1}{2}\left(\omega = \frac{1}{2}\right)\end{aligned} \tag{22}$$

2.4 Behavioural Hypotheses

For the models developed in this section all of the parameters have behavioral interpretations. These behavioural parameters provide us with a means to examine a number of hypotheses that relate to how uncertain response data has been treated in the literature previously. Our set of hypotheses are as follows:

- DKs are either fully informative or non-informative - $\omega = 0$ versus $\omega = \frac{1}{2}$
- We can treat symmetrically the treatment of PY and PNs - $\rho_1 = \rho_2$
- A probable response is equivalent to a definite response - $\rho_1 = 1$ and/or $\rho_2 = 1$
- We can treat symmetrically the treatment of PY and PNs but at lower probability (ρ) $\rho_1 = 0.75$ and $\rho_2 = 0.75$

These various hypotheses will be tested using data the Bayesian approach to estimation outlined in the next section.

3 Prior Distributions for Model Parameters

Denote the full parameter set as $\Theta = (\alpha, \theta, \Omega(\omega))$ where $\Omega = \{\psi_i\}, \{\rho_i\}$. Within this structure we examine the case where the latent utilities have been observed:

$$\begin{aligned} u_i - b_i &= \alpha' x_i + e_i \\ & e_i \sim N(0, \theta) \end{aligned} \tag{23}$$

The prior distributions for the parameters α, θ are normal $N(.,.)$ and inverse gamma $IG(.,.)$ respectively:

$$\begin{aligned} f(\alpha) &= N(0, V_0) \\ & \text{and} \\ f(\theta) &= IG(s, v) \end{aligned} \tag{24}$$

For the remaining parameters we adopt a set of uniform priors that obey inequality restrictions such that the parameters lie within the admissible region S (where all the associated probabilities are positive and in the case of $\rho_1$and ρ_2 between 0.5 and 1). For the unrestricted model, that is either Model (a) or Model (b) which is not subject to any of the restrictions on $\rho_1$or ρ_2, the prior is:

$$f(\Omega(\omega)) = \frac{I(\Omega(\omega) \in S)}{C} = \frac{I(\{\psi_i\} \in S_\psi)}{C_\psi} \times 2I\left(\rho_1 \in [\frac{1}{2}, 1]\right) \times 2I\left(\rho_2 \in [\frac{1}{2}, 1]\right) \tag{25}$$

For both models ($\omega = 0$ and $\omega = 1/2$) the conditions that $0 < \psi_3 \leq \psi_1 \leq \psi_2 < 1$ along with $\rho_1 > 0.5$ and $\rho_2 > 0.5$ are sufficient[3] for $\Omega(\omega) \in S$. The

[3]These conditions are sufficient but not necessary for the model to have probabilities bounded between zero and one. Note that π_2^+ and π_4^+ and π_2^- and π_4^- are also required to be bounded between one and zero. We did not establish this condition analytically. However, by simulating from the priors adopted above, we established that these conditions were sufficient (in over 100,000 trials) for π_2^+ , π_4^+, π_2^- and π_4^- to be bounded also.

uniform distribution over the region $S_\psi = \{\{\psi_i\} : 0 < \psi_3 \leq \psi_1 \leq \psi_2 < 1\}$ has an integrating constant of $\frac{1}{6}$. Therefore, we use the value of $C_\psi = \frac{1}{6}$ in (25) as an integrating constant so that the distribution is proper. Although this constant is not required for estimation it is required for the calculation of the marginal likelihoods. Finally, we note that under the hypotheses $\rho_1{=}\rho_1^*$ or $\rho_2{=}\rho_2^*$ or $\rho_2{=}\rho_1 = \rho_2^*$, one or both of the last two terms in the prior above will disappear. The posterior distributions for the model parameters are derived in the appendix.

4 Empirical Section

4.1 The Data

The data employed in this paper is from two different sources. The first was previously employed by Welsh et al. [19] and subsequently by Evans et al. [10]. The data are for a study on the non-use values associated with the Glen Canyon Dam and its impact on the rivarian environment along the Colorado River. A full description of the data can be found in Welsh et al. so we do not repeat this here.

The survey format used was the multiple bound uncertainty format with five levels - Y, PY, DK, PN, and N. The bids offered to respondents were, \$0.5,\$1,\$5,\$10,\$20,\$30,\$40,\$50,\$70,\$100,\$150 and \$200. Therefore, in order to extract data as if it were from the single bound format, we randomly sampled from the replies to bids in a uniform way. That is, one response for each respondent was sampled in a uniform manner by taking their response from one bid, where any bid was equally likely to be chosen. We also experimented with other sampling versions, but they did not have a substantive impact on the results.

The second data set is from a study of farmers in Nigeria, who were asked if they would be willing to participate in an irrigation scheme designed to increase the average yields as well as reduce variability. The Nigerian government, in 2002, implemented an irrigation scheme to increase productivity of small scale farmers in the Northern states of Nigeria. This data set evaluates the impact of improved low cost irrigation technology on the farm households participating in the scheme. The study was conducted in three water scarce villages in Kaduna state, which is situated in the North-west region of Nigeria. Two groups of farmers were asked to respond to CV questions of the uncertainty format type. First, 102 of those already in the scheme (participants) and those (102) not currently in the scheme (non-participants). The CV scenarios differed across the two groups in that there would be an additional cost for non-participants to joining the scheme (of 4,500 naira) since those already participating the scheme had paid for and received equipment that those joining the scheme would subsequently receive. The data was collected using a double bounded methodology, and as with the Glen Canyon Data, we sampled randomly from the two responses obtained from the double bounded responses.

4.2 Results

The main focus of this section will be in examining the relative performance of Models (a) and (b) plus testing the six hypotheses outlined in the preceding section, along with interpreting some of the parameter estimates. Overall twelve nested models were estimated for each data set, along with the ordered Probit. The reason for estimating the ordered Probit relates to its use as a means to estimate this type of data based on an alternative interpretation of the probabilities.

For both data sets, and for all models, MCMC used a burn in of 100,000 draws followed by another 1,000,000 iterations where every 10th value was sampled (leaving 100,000 to be analysed) so as to reduce the dependence of the sequence. Convergence was monitored visually and by using a modified t-tests to check if the first and second halves of the values drawn from the chain had the same mean. All models appeared to converge well.

For the Glen Canyon Dam data we included age, sex and income as explanatory variables. For the Nigerian Data, we did not include descriptors for the respondent as they only had minimal impact on the resulting WTP estimates because of high standard errors for the coefficients on these variables. In both data sets we have employed dummy variables that take account of the different treatments. In the Glen Canyon Dam data there are eight different versions of the survey instrument. In the Nigerian irrigation data we employ a dummy variable to differentiate between participants and non-participants in the scheme.

The logged marginal likelihoods for each of the models, for both data sets, are presented in Table 1. The result in the first row are for the ordered Probit. The first column of numbers for each data set relates to the model where the DKs are treated as informative $\omega = \frac{1}{2}$ and the second where DKs are treated as non-informative $\omega = 0$. The best performing model specification are highlighted by the bold text in Table 1.

For the Glen Canyon Dam data, nearly all the nested models for $\omega = 0$ outperform their counterparts where $\omega = \frac{1}{2}$, since the marginal likelihoods where $\omega = 0$ are larger. In most cases, there is a very big difference in these values. However, it is clear from these results also that the Glen Canyon data prefers the ordered Probit characterisation. None of the models developed in this paper have a marginal likelihood as large as the ordered Probit. Other than the ordered Probit the preferred model is the one where both ρ_2 and ρ_1 are free to vary and $\omega = 0$. The next best performing model is where $\rho_2 = 1$ but ρ_1 is free to vary. This suggests that a PN, is more like a N than a PY is like a Y which is consistent with the previous literature. We also note that the lower values $\rho_2 = \rho_1 = 0.75$ are not supported by this data. In fact, the model where $\rho_2 = \rho_1 = 1$ is preferred to this model.

Next we turn to the Nigerian Data. In this case the majority of the models where $\omega = \frac{1}{2}$ outperform the case where $\omega = 0$. Furthermore, for this data all the models introduced in this paper are preferred to the ordered Probit. Overall the best supported model is the restriction $\rho_2 = 1$. Again, the results are consistent with the previous findings within the literature that a PN is more like a N than a PY is like a Y. We evaluate this claim further below.

{Approximate Position of Table 1}

We now consider the parameter estimates for both versions of the unrestricted model. Our reason for examining the unrestricted model is that it allows to extended our analysis of the hypotheses of interest. Our result for the Glen Canyon data are presented in Table 2, and for the Nigerian Irrigation data in Table 3.

{Approximate Position of Tables 2 and 3}

First, the unrestricted models for the Glen Canyon Dam data the mean WTP for the first treatment group is captured by the intercept (Int) and the coefficients for the dummy variables give the deviation of these WTPs from the first treatment group. In the model specification we also have also included sex, age and income variables. The WTP estimate for the first treatment group are just over \$60. There is no substantial difference in the WTP estimates between models (a) and (b).[4]

In Table 3, the WTP figures are in 1000 NGN. Thus, the WTP for the participant group is around 14600 NGN (equating to a little over \$100 US dollars). The additional WTP figure of around 4784 or 4850 NGN closely reflects the additional costs that a non-participant is prepared to pay over and above those already participating in the scheme. This reflects the fact that participants have already paid for equipment that the non-participants would receive to join the scheme. Thus, the scenario's being presented to the two groups differed in terms of the options that they were being asked to choose from, and the cost of this equipment was around 4,500 NGN.

Next we consider the parameter estimates that deal with uncertainty. As noted in the introduction, the literature has suggested that ρ_2 exceeds ρ_1. Our results are consistent with this literature. In Table 2 for the Glen Canyon Dam data, the preferred model $(\omega = 0)$ gives $\hat{\rho}_2$ =0.871 and $\hat{\rho}_1$ =0.750. For the Nigerian Irrigation data in Table 3, the preferred model $\left(\omega = \frac{1}{2}\right)$ gives $\hat{\rho}_2$ =0.793 and $\hat{\rho}_1$ =0.716. However, these point estimates understate the degree to which the evidence suggests that ρ_2 exceeds ρ_1. Perhaps more informative are the distributions presented in Figures 1 and 2.

{Approximate Position of Figures 1 and 2}

These are for the Glen Canyon Dam data only, as we do not include similar graphs for the Nigerian data for brevity. As can be seen the posterior distributions for ρ_2 are heavily skewed toward unity, whereas the peak of the distribution for ρ_1 are around 0.75. The posterior distributions for the Nigerian Irrigation data does not present quite such a stark contrast between the two distributions. However, similar tendencies are displayed in terms of the skewness of ρ_2 towards one, with ρ_1 having more density at lower values. The posterior distributions of ρ_1 and ρ_2 are, therefore, consistent with the marginal likelihood results that also point to $\rho_2 > \rho_1$.

With regard to the other parameters, for the Glen Canyon Dam data, we can see from Table 2 that for the preferred model $(\omega = 0)$ the value of ψ_3 =0.129 is almost exactly the proportion of respondents replying DK. The

[4] All other models estimated yielded comparable WTP estimates including the ordered Probit.

values $\psi_1 = 0.541$, and $\psi_2 = 0.81$ mean that at $\Phi_i = 0$ a respondent with positive utility has a 54% chance of responding with a Y. However, at $\Phi_i = 1$ this rises to 81%. Around 13% of the remaining 19% are those that choose to reply DK, with the remaining 6% choosing PY. The converse is true for those with negative utility (e.g. there is around a 54% chance of responding definite no if .$\Phi_i = 1$ and the person has negative utility etc.).

With regard to the Nigerian Irrigation data, in Table 3 we can see for the preferred model $\left(\omega = \frac{1}{2}\right)$, the values ψ_1 =0.645, and ψ_2 =0.833 and ψ_3 =0.264 meaning that at $\Phi_i = 0$ a respondent with positive utility has a 64% chance of responding with a Y, with the remainder replying DK. However, at $\Phi_i = 1$ this rises to 83%, with the remainder choosing to respond PY, since within this model, the probability of somebody replying DK is, by definition 0% at .$\Phi_i = 1$. The converse is true for those with negative utility (e.g. there is a 64% chance of responding definite no if .$\Phi_i = 1$ and the person has negative utility etc.).

5 Conclusions

This paper has introduced a new method that allowed for the estimation of the probability that a PY or PN constituted a real bound within the context of a WTP study. Unlike previous studies, the probabilities that an indefinite response would provide a bound was estimated using the data alone, rather than drawing upon estimates from the psychology literature. A structural model was constructed for this purpose and the posterior distributions for the parameters were derived. The model was employed on two data sets which provide mixed evidence about whether the models developed outperform the ordered Probit (the threshold approach).

Overall we found that the informativeness of DKs seems to be context specific; PYs are less like a Y than a PN is like a N. Therefore, we would argue that the treatment of PNs and PYs should be asymmetric. This finding is in line with the psychology literature. We also found that benchmark values of $\rho = 0.75$ and 1 are not generally supported by the data (at least for both parameters simultaneously). The estimates of these parameters seem to be context specific. We also find that our WTP estimates are not that sensitive to the treatment of the uncertain responses. This is in contrast to the findings reported by Evans et al. [10] and Boman [8].

Finally, we note that the approach developed in this paper is restrictive in that it uses a single bounded UC, although it has the potential to be extended into a multiple bounds UC. The single bounded approach is less restrictive than it might first seem, since multiple bounded survey data can be used within a single bounded framework also, though admittedly it does not fully use all the information available. This can be most easily achieved by drawing a random response from the multiple bounded responses. Future work in this area might extend the current approach to fully utilise the multiple bounded data. This would most probably be dealt with by extending the current approach in the context of the multivariate Probit.

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Appendix Posterior Distributions and Estimation

All quantities are as defined in the text. The posteriors for α and θ can then be derived using:

$$\begin{aligned} f(\alpha|\{L_i\},\{u_i\},\theta,\Omega,\{x_i\},\{b_i\}) &\propto f(\{u_i\}|\{L_i\},\alpha,\theta,\Omega,\{x_i\},\{b_i\}) \\ &\quad\times f(\{L_i\}|\alpha,\theta,\Omega,\{x_i\},\{b_i\})\times f(\alpha) \end{aligned} \tag{26}$$

and

$$\begin{aligned} f(\theta|\{L_i\},\{u_i\},\alpha,\Omega,\{x_i\},\{b_i\}) &\propto f(\{u_i\}|\{L_i\},\alpha,\theta,\Omega,\{x_i\},\{b_i\}) \\ &\quad\times f(\{L_i\}|\alpha,\theta,\Omega,\{x_i\},\{b_i\})\times f(\theta) \end{aligned} \tag{27}$$

From the normality assumption:

$$\begin{aligned} f(u_i|L_i=k,\Theta,x_i,b_i) &= f(u_i|u_i>0,L_i=k,\Theta,x_i,b_i)\,\tau_{k,i} \\ &\quad+f(u_i|u_i<0,L_i=k,\Theta,x_i,b_i)\,(1-\tau_{k,i}) \\ &= f_N(u_i|\Theta,x_i,b_i)\left(\frac{(\tau_{k,i})}{\Phi_i}+\frac{(1-\tau_{k,i})}{1-\Phi_i}\right) \end{aligned} \tag{28}$$

Therefore:

$$\begin{aligned} & f(u_i|L_i=k,\Theta,x_i,b_i)\,f(L_i|\alpha,\theta,\Omega,\{x_i\},\{b_i\}) \\ = \; & f_N(u_i|\Theta,x_i,b_i)\times\Gamma_{k,i}\left(\frac{(\tau_{k,i})}{\Phi_i}+\frac{(1-\tau_{k,i})}{1-\Phi_i}\right) \end{aligned} \tag{29}$$

Given [9] this simplifies to

$$\begin{aligned} & f(u_i|L_i=k,\Theta,x_i,b_i)\,f(L_i|\alpha,\theta,\Omega,\{x_i\},\{b_i\}) \\ = \; & f_N(u_i|\Theta,x_i,b_i)\times\left(\pi^+_{k,i}+\pi^-_{k,i}\right) \end{aligned} \tag{30}$$

Consequently, under the assumptions above (where $\pi^+_{L_i}, \pi^-_{L_i}, \Gamma_{L_i}$ and τ_{L_i} are the values of $\pi^+_{k,i}, \pi^-_{k,i}, \Gamma_{k,i}$ and $\tau_{k,i}$ given the observed responses):

$$\begin{aligned} & f(\alpha|\{L_i\},\{u_i\},\theta,\Omega,\{x_i\},\{b_i\}) \\ \propto \; & \prod_{i=1}^{N} f_N(u_i|\Theta,x_i,b_i)\,f(\alpha)\times\left(\pi^+_{L_i}+\pi^-_{L_i}\right) \end{aligned} \tag{31}$$

and

$$\begin{aligned} & f\left(\theta \middle| \left\{L_i\right\}, \left\{u_i\right\}, \alpha, \Omega, \left\{x_i\right\}, \left\{b_i\right\}\right) \\ \propto & \prod_{i=1}^{N} f_N\left(u_i|\Theta, x_i, b_i\right) f\left(\theta\right) \times \left(\pi_{L_i}^{+} + \pi_{L_i}^{-}\right) \end{aligned} \tag{32}$$

Therefore, in this model the fact that both the posteriors for α and θ are multiplied by $\prod_{i=1}^{N}\left(\pi_{L_i}^{+} + \pi_{L_i}^{-}\right)$ means that this quantity must be accounted for when estimating the parameters. In this paper we draw from the proposal distributions:

$$\begin{aligned} & f^*\left(\alpha \middle| \left\{u_i\right\}, \left\{b_i\right\}, \left\{x_i\right\}, \theta\right) = N\left(\hat{\alpha}, V\right) \\ \hat{\alpha} &= \left(V_0^{-1} + \theta^{-1}\sum x_i x_i'\right)^{-1}\left(\theta^{-1}\sum x_i\left(u_i - b_i\right)\right) \\ V &= \left(V_0^{-1} + \theta^{-1}\sum x_i x_i'\right)^{-1} \end{aligned} \tag{33}$$

and

$$f^*\left(\theta \middle| \left\{u_i\right\}, \alpha\right) = IG\left(\frac{s + \sum e_i^2}{2}, \frac{v+n}{2}\right) \tag{34}$$

However, unlike the standard Probit model the proposed draws α^* and θ^* are accepted with probability[5]

$$p = Min\left(\frac{\prod_{i=1}^{N}\left(\pi_{L_i}^{+*} + \pi_{L_i}^{-*}\right)}{\prod_{i=1}^{N}\left(\pi_{L_i}^{+} + \pi_{L_i}^{-}\right)}, 1\right) \tag{35}$$

where $\pi_{L_i}^{+*}$ and $\pi_{L_i}^{-*}$ are evaluated at α^* and θ^*.For the remaining parameters Ω we simply observe that since we can calculate the likelihood function at any point, then:

$$f\left(\Omega|\alpha, \theta\right) \propto \prod_{i=1}^{N} \Gamma_{L_i} f\left(\Omega\right) \tag{36}$$

Therefore, to estimate these parameters we adopted a random walk metropolis-hasting step, such that the proposed parameters Ω^* are accepted with prob-

[5]The normal and gamma ordinates to do not enter this probability calculation since they cancel

ability

$$p = Min\left(\frac{\prod_{i=1}^{N}\Gamma_{L_i}^{*}f\left(\Omega^{*}\right)}{\prod_{i=1}^{N}\Gamma_{L_i}f\left(\Omega\right)}, 1\right) \tag{37}$$

Finally, since the likelihood is calculated as part of the estimation process, it is a relatively quick calculation to use the Gelfand and Dey method for calculating the Marginal Likelihood (ML).

$$ML = \left(\frac{1}{G}\sum_{g=1}^{G}\frac{T\left(\Omega^{g}, \alpha^{g}, \theta^{g}\right)}{f\left(\Omega^{g}\right)f\left(\alpha^{g}\right)f\left(\theta^{g}\right)L_{g}}\right)^{-1} \tag{38}$$

where L_g is the likelihood function evaluated at the posterior draws $\Omega^g, \alpha^g, \theta^g$.

Table 1: Logged Marginal Likelihoods

Glen Canyon Dam			**Nigerian Irrigation Scheme**		
Ordered Probit	-950.6		**Ordered Probit**	-302.9	
Model	**b)**	**a)**	**Model**	**b)**	**a)**
	$\omega = \frac{1}{2}$	$\omega = 0$		$\omega = \frac{1}{2}$	$\omega = 0$
Unrestricted	-963.4	**-959.2**	**Unrestricted**	-296.8	-297.5
$\boldsymbol{\rho}_1 = \boldsymbol{\rho}_2$	-964.2	-959.6	$\boldsymbol{\rho}_1 = \boldsymbol{\rho}_2$	-297.8	-296.9
$\boldsymbol{\rho}_1 = \mathbf{1}$	-961.2	-960.3	$\boldsymbol{\rho}_1 = \mathbf{1}$	-298.9	-298.5
$\boldsymbol{\rho}_2 = \mathbf{1}$	-961.1	-959.5	$\boldsymbol{\rho}_2 = \mathbf{1}$	**-296.3**	-297.8
$\boldsymbol{\rho}_2 = \boldsymbol{\rho}_1 = \mathbf{1}$	-961.3	-959.6	$\boldsymbol{\rho}_2 = \boldsymbol{\rho}_1 = \mathbf{1}$	-296.6	-298.5
$\boldsymbol{\rho}_2 = \boldsymbol{\rho}_1 = \mathbf{0.75}$	-960.4	-960.3	$\boldsymbol{\rho}_2 = \boldsymbol{\rho}_1 = \mathbf{0.75}$	-297.5	-297.6

Table 2: Glen Canyon Dam Results

Unrestricted Model (b) $\omega = \frac{1}{2}$			**Unrestricted Model (a)** $\omega = 0$		
	mean	stdv		mean	stdv
int	62.509	6.52	int	61.16	6.99
d2	5.411	8.39	d2	9.664	9.29
d3	3.935	8.54	d3	3.009	8.98
d4	-23.67	7.87	d4	-22.56	8.39
d5	-15.87	7.66	d5	-16.31	8.01
d6	-0.482	8.63	d6	0.055	9.07
d8	-6.15	8.75	d8	-7.56	9.17
d9	-15.99	8.25	d9	-15.88	8.19
sex	1.48	4.81	sex	4.449	5.18
age	-0.381	0.14	age	-0.382	0.15
inc	0.193	0.061	inc	0.201	0.067
ρ_1	0.732	0.13	ρ_1	0.750	0.13
ρ_2	0.810	0.13	ρ_2	0.871	0.10
ψ_1	0.639	.046	ψ_1	0.541	.054
ψ_2	0.767	.028	ψ_2	0.810	.029
ψ_3	0.362	.035	ψ_3	0.129	.013
θ	1392.5	272	θ	1327.7	335

Table 3: Nigerian Irrigation Model Results

Unrestricted Model (b) $\omega = \frac{1}{2}$

	mean	stdv
int	14.60	0.571
d1	4.850	0.792
ρ_1	0.716	0.137
ρ_2	0.793	0.134
ψ_1	0.645	0.074
ψ_2	0.833	0.047
ψ_3	0.264	0.054
θ	16.73	5.32

Unrestricted Model (a) $\omega = 0$

	mean	stdv
int	14.60	0.596
d1	4.784	0.834
ρ_1	0.716	0.137
ρ_2	0.813	0.129
ψ_1	0.603	0.084
ψ_2	0.850	0.129
ψ_3	0.101	0.21
θ	16.57	6.015

Figure 1: Posterior Distribution of Rho 1

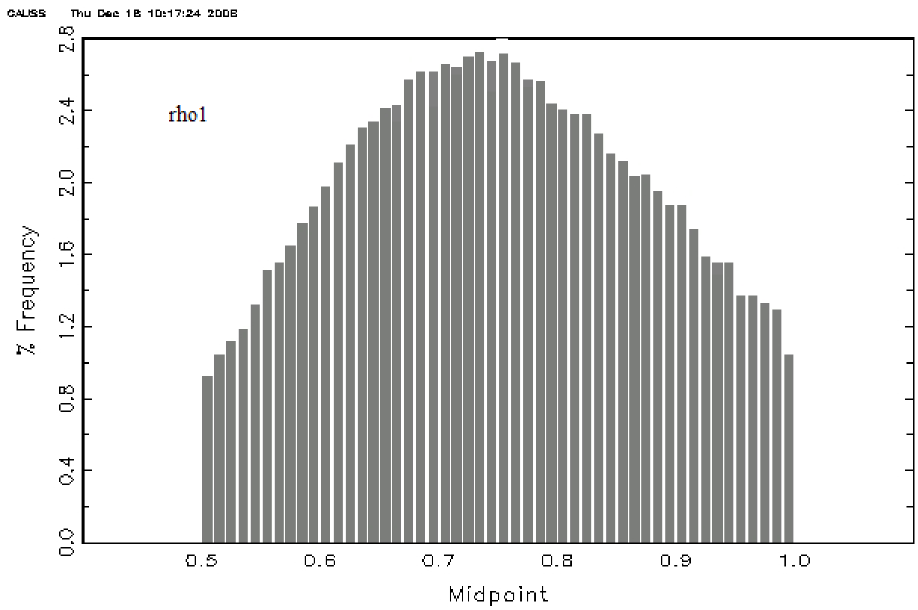

Figure 2: Posterior Distribution of Rho 2