Monetary Policy Analysis: an undergraduate toolkit[*]

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Abstract

We develop simple diagrams that can be used by undergraduates to understand interest rate setting by policy-makers. We combine an inflation target, Fisher equation, policy reaction function and short and long run aggregate supply analysis to give a depiction of the policy problem. We illustrate the appropriate response by the policy maker to each of a positive shock to demand, a negative supply shock and dislodged inflation expectations. We also illustrate the problems of a zero bound for policy rates within this framework and consider the role of an interest rate rule in offsetting money market perturbations. Some key readings are introduced.

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1   Introduction

The nuts and bolts of setting monetary policy are often hard to get across to students. There are a number of key hurdles to overcome.

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First, the conceptual idea of how setting interest rates may (or may not) act to stabilise an economy comprising many households, firms, financial institutions and a significant government sector. Secondly, there are a host of institutional details to convey such as the framework for monetary policy, the relationship between the Finance Ministry and the Central Bank and what might be the ultimate objectives of stabilisation policy. Thirdly, the theory of monetary policy is itself really developing into a branch of 'robust' control theory and so is subject to severe technical barriers at the frontier.[1] And finally, there is the aspect of the real data: how do we convey the idea that the observed economy is not some clearly identifiable mass but a construct based upon a myriad of observations or surveys announced on a daily basis? The mixture of institutional detail, high theory, data and, at times, low politics makes monetary policy courses a daunting mix for instructor and student alike.

We tend to start monetary policy courses with an analogy related to one of driving cars, steering ships or taking a shower! In which, the policy maker is cast as the driver, pilot or bather in question. But the user has severe information problems, he (or she) cannot know with a high degree of certainty where he might currently be compared to where he would like to be. He also does not quite know how the machine will react when he asks it to help him get to where he would like to be. Finally, it may also be some time before he realises that he is or is not where he thinks he would like to be and so he may frequently under or even overshoot his final destination. Should your head be reeling, you would now be pleased to know that I have chosen to side-step almost entirely these kinds of control issues in this chapter.

What will concern us mostly in this chapter is the rather prosaic set of issues to do with where should interest rates go if the economy has a demand-induced boom, a supply induced contraction or indeed if inflation expectations become dislodged.[2] These questions will be considered within the context of a simple two-quadrant and then four-

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[1] As well as the hurdle of deriving aggregate relationships from first principles (so-called micro-foundatations), the resulting equations need to be understood and manipulated to examine issues such a determinacy, learnability and various solutions for the policy rule can be examined according to various loss functions for the monetary policy maker. For example, rather than the well known quadratic loss function which seeks to minimise the deviations of a variable from its target, a policy maker may seek to minimise the losses from the worst possible (probable) outcome and act with a so-called min-max loss function. Appendix A.3 illustrates the connection between the choice of loss function and the optimal monetary policy rule. See Alan Greenspan (2004) for an introduction to how risk managment issues impact on simple monetary policy decisions.

[2] By which I mean dislodged from their (model-based) connection with the state of the economy.

quadrant diagram that I will develop and use to explore directly two further monetary policy issues: how might the zero-bound for interest rates complicate the monetary policy problem and how might money market shocks complicate monetary policy choices? The level of exposition is appropriate for good undergraduates and I introduce many key readings in modern macroeconomics. In Section 2, I write down a standard New Keynesian model appended with both a supply side, money market clearing condition and a term for the price level as well as inflation. I develop the simple conditions for the determinacy of this system and show that it implies a monetary policy reaction where policy rates rise more than equiproportionally with inflation, the so-called 'Taylor Principle' of active monetary policy. Once the existence of an equilibrium for this economy has been established I return to the policy experiments. For the more technically grounded students the Appendix gives a fuller derivation.

In Section 3, I represent the key relationships diagrammatically for an inflation targeting central bank and consider the appropriate responses to three static problems of a positive demand shock, a negative supply shock and an increase in inflation expectations. In each case, I show that policy (interest) rates will have to rise temporarily to bring inflation back to target. In Section 4, I consider two special cases: what happens to the monetary policy reaction function when nominal rates are bounded at zero and when the money market may directly provide perturbations because the economy may be considered more or less risky over time. In the former case, policy rates are shown to be unable to drive real rates lower as inflation falls and thus there appear to be 'real' limits to the efficacy of interest rates as a stabilisation device under a low and/or falling inflation. In the latter case, I show that a disconnect between interest rates set in the private money markets and policy rates can set-up independent deviations of aggregate demand from potential and so require some offsetting in monetary policy.

With this background exposition in the student's toolkit I conclude that it becomes easier to consider the questions of diagnosis of any given monetary policy problem, institutional development, to track real-time data developments and to consider more complicated games that the policy makers may have to play with their various (ir)rational counterparts. I leave that analysis to more advanced courses.

2   The Basic New Keynesian Model with Money

The point of departure for a simple macroeconomic model suitable for monetary policy analysis has become the New Keynesian (NK)

framework (see McCallum (2001) and King (2002)), which is essentially an aggregate model with dominant supply side dynamics but where sticky prices mean that output may deviate temporarily from its flex-price long run level. The possibility of temporary deviations in output from its flex-price level creates a role for the monetary policy maker. In brief, the basic NK story is that the capacity of output is set by a production function based on usual arguments in land and capital with its accumulation of efficiency shocks (the so-called Solow residuals see, for example, 1987) and short run output is determined by a monopolistically competitive supply side faced with Calvo time dependent price setting.

The NK structure means that the full capacity level of output in this economy lies at a point behind the perfectly competitive frontier, which in principle provides an incentive to push the economy above its full capacity level.[3] Secondly, with prices adjusting only gradually to an optimal mark-up over evolving marginal costs, short run output can deviate from this capacity level. Following any shocks, prices can only be re-set in each period by the fraction of firms who are sent an exogenous (Calvo) signal to re-price - with the fraction given by $1 - \alpha$ in each time period. And so all other firms, α, are faced with having to accept a sub-optimal price for their output for at least one period and the overall price level, which is a linear combination of all firms' prices, is also sub-optimal, which means that there are both distributional and direct output consequences from sticky prices.

Inflation is driven by both the difference between capacity and the short run aggregate level of production chosen by all firms and expected inflation. And so inflation, at least in its temporary deviations from target, is not a monetary phenomenom in this model but really an output gap phenomenom, which is itself controlled by interest rate choices. But nevertheless to this basic model we can also consider appending a simple model of money demand (for which supply by the monetary policy maker is implicitly perfectly elastic), where we assume that households need to hold money balances to meet a given level of planned nominal expenditures. The role of the policy maker is to set interest rates so that output stabilises at the capacity level, that is the so-called output gap is closed, at which point inflation is also stabilised. In the remainder of this section I list and explain the key dynamic equations and examine the policymakers' problem in terms of the determinacy of equilibrium.

The simple New Keynesian model expresses each variables as its log deviation from steady-state. Equation (1) gives aggregate demand, y_t, as

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[3]The implications of this incentive i.e. an inflation bias, will not concern me greatly in this chapter.

a function of this period's expectation, E_t, of demand next period, y_{t+1}, and of the expected real interest rate, where R_t is the policy rate, $E_t\pi_{t+1}$ is the next period expectation of inflation and σ is the intertemporal rate of substitution in output.[4] Equation (2) is the forward-looking New Keynesian Phillips curve that relates current inflation, π_t, to discounted expected next period inflation, where β is the subjective discount factor, and is proportional to the deviation of aggregate demand from supply, where κ is the slope of the Phillips curve.[5] The term κ is related to two deep parameters in the underlying Calvo-Yun model (see Yun, 1996): the probability of firms maintaining a fixed price in the next period, α, and the subjective discount factor, β. In inflation space κ can be shown to be equal to $\frac{(1-\alpha)(1-\alpha\beta)}{\alpha}$ and thus in price space, with the deviation in the price level proportional to inflation (see equation 6), the Phillips curve becomes: $p_t = E_t p_{t+1} + (1-\alpha\beta)(y_t - \hat{y}_t) + \frac{\alpha}{1-\alpha}\epsilon_{A,t}$. Under either formulation inflation or the price level is less responsive to the output gap as $\alpha \to 1$.

Equation (3) says that real balances, $m_t - p_t$, are held in proportion to demand, y_t, and inversely with the opportunity cost of holding non-interest paying money, R_t, with a semi-elasticity, η. Equation (4) is a simple interest rate-based rule that is used to stabilise inflation about its steady state value with the weight on inflation given by ϕ_π. The supply side of the economy, $\tilde{y}_t$, which we interpret as the flex-price, or steady-state, level of output is given by (5). The shocks to this equation account for changes in the short run deviation of flex-price output from its steady-state and can typically be interpreted as productivity, or efficiency, shocks. Finally, the forward-looking Phillips curve, (2), determines the split between current and expected inflation as a function of the current output gap but we can use the current inflation rate to back out the price level: α is the fraction of firms that hold prices fixed and so $(1 - \alpha)$ is the fraction which are given a signal to re-price as a mark-up over marginal costs (see Yun, 1996) thus inflation is simply the ratio of firms that re-price at the new price level, p_t, relative to those that cannot re-price, (6).[6]

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[4] This intertemporal equation also operates as the basic asset pricing equation, or kernel, in a New-Keynesian model.

[5] This compares to various specifications of the Phillips curve through time, relating firstly the inflation rate to the unemployment rate and then the change in inflation to various measures of capacity. The key difference here is that the impact of the output gap is split between current and expected inflation. It is worth reading Bill Phillips (1958) original paper.

[6] Equation (6) is the deviation of inflation and prices from steady state and results from the observation that $P_{t+n} = f\left[\alpha P_{t-1+n}, (1-\alpha)P_{t+n}\right]$ and so if prices are at steady state in the initial period under Calvo pricing.they will move by the ratio of

The system is subject to stochastic shocks, $\epsilon_{A,t}$, $\epsilon_{B,t}$, $\epsilon_{C,t}$, $\epsilon_{D,t}$, which are respectively to demand, mark-up, monetary policy and to aggregate supply.

$$y_t = E_t y_{t+1} - \sigma \left(R_t - E_t \pi_{t+1} \right) + \epsilon_{A,t} \tag{1}$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa \left(y_t - \tilde{y}_t \right) + \epsilon_{B,t} \tag{2}$$

$$m_t - p_t = y_t - \eta R_t \tag{3}$$

$$R_t = \phi_\pi \pi_t + \epsilon_{C,t} \tag{4}$$

$$\tilde{y}_t = \epsilon_{D,t} \tag{5}$$

$$\pi_t = \frac{1 - \alpha}{\alpha} p_t. \tag{6}$$

We can substitute (4) into (1) and into (3), (5) into (2) and solving (6) for p_t into (3) to give us a system of four difference equations that can be written in vector form, if we suppress the stochastic errors, as:

$$E_t \mathbf{x}_{t+1} = \mathbf{\Lambda} \mathbf{x}_t, \tag{7}$$

where the transpose of the vector of state variables $\mathbf{x}_t$ is:

$$\mathbf{x}_t^{'} \equiv \left[\pi_t \, y_t \, m_t \, p_t \right],$$

where $\mathbf{\Lambda}$ is a 4 x 4 matrix of parameters. And so the basic NK model can be boiled down a set of equations linking output and inflation to money and prices via the determination of nominal interest rates.

2.1 Understanding the model dynamics

A question that first concerns macroeconomists when faced with such a model are the 'Blanchard-Kahn local stability conditions',[7] to locate a rational expectations solution to a forward-looking macroeconomic model. In fact much modern macroeconomic theory is concerned with

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those who can re-price to those who cannot.

[7]See Blanchard and Kahn, 1980, which triggered most of the modern work on assessing whether an equilibrium is locally stable.

the conditions under which a given model has a solution, or analogously can be thought to be stable following economic shocks. The existence or not of a unique solution for $\mathbf{x}_t$, given the forcing processes, $\boldsymbol{\epsilon}_t$,[8] and will depend upon matching the number of eigenvalues (or roots) of the matrix $\boldsymbol{\Lambda}$ within the unit circle (less than absolute value of 1) with the number of predetermined state variables.[9]

Predetermined variables are those that we might think of as backward looking and depend upon shocks in previous periods or decisions in previous periods for the attainment of their current levels. On the other hand non-predetermined (also known as forward-looking, or jump variables) depend upon expectational terms for the current value. Note from equations (1) and (2) that both inflation and output are determined with reference to expectations of their own future values and so are non-predetermined variable. This is a key feature of NK macroeconomics, that many of the key variables behave like asset prices rather than traditionally sluggish prices and quantities. This means that the NK economy is somewhat more flexible, faster adjusting, than an examination of the data on a typical economy might suggest.[10] One way to think of the policy problem is that it is necessary to set the coefficients of the policy rule, (4), to ensure local determinacy of the whole system, and this setting is affected by the extent to which key variables are forward-looking.[11]

We can see from inspection of equations (1) to (6) how the structure of this economy responds to shocks. Demand and mark-up shocks, $\epsilon_{A,t}$ and $\epsilon_{B,t}$, immediately impact on output and inflation, respectively and shocks to the policy rate and supply-side, $\epsilon_{C,t}$ and $\epsilon_{D,t}$, also work their way through output and inflation. We can thus see that providing some conditions are met so that inflation and output stabilise after shocks, by which I mean return to their steady-state or target values, then money and prices will also be stabilised. Equation (3) shows that the demand for real balances will be satisfied providing output and the interest rate is stable, which itself is a function of inflation in this simple model. Furthermore equation (6) tells us that a stable path for inflation will also stabilise the price level.

The model is therefore recursive (see the Appendix for a fuller proof) and consequently monetary policy makers have concentrated on

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[8]Which is an analagous 4 x 1 vector for the shocks.

[9]I shall not continue with much matrix algebra in this section but the interested reader is directed to the Appendix for more details.

[10]For example, a rule of thumb for central banks is that the economy responds most actively with a lag of 4-8 quarters to a monetary policy shock but that tends to be considerably longer than that suggested by a typical NK model.

[11]See Woodford (2003) for a comprehensive treatment of this problem.

determining stability by concentrating on the output gap and inflation dynamics. And arguing that the traditional 'bread and butter' of monetary policy, monetary aggregates, impart little or no further information because the observed market clearing levels of money supply are equilibrium outcomes, reflecting stable paths for output and inflation contingent on the policy rule, which therefore have no further information to impart about the state of the economy. The idea that our observations on the economy, that is the time series we have on money, output, inflation and interest rates, are always equilibrium outcomes begs the very difficult question of what models we can use that will simultaneously produce market clearing in all markets and still match the data.

2.2 Inflation-output dynamics

Let us examine the conditions (somewhat loosely) for the determination of monetary stability.[12] First suppress the stochastic terms from equations (1) and (2):

$$y_t = E_t y_{t+1} - \sigma \left(R_t - E_t \pi_{t+1} \right), \tag{8}$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa y_t. \tag{9}$$

Solve (2) for $E_t \pi_{t+1}$ and substitute out the policy rate from (4) to give:

$$y_t = E_t y_{t+1} - \sigma \left(\phi_\pi \pi_t - \beta^{-1} \left(\pi_t - \kappa y_t \right) \right). \tag{10}$$

Now simplify the expression by assuming that $\sigma = 1$ and that $\beta \approx 1$:

$$y_t = E_t y_{t+1} - \phi_\pi \pi_t + \pi_t - \kappa y_t. \tag{11}$$

At steady state the growth rate of output around the trend will be zero and so:

$$E_t y_{t+1} - y_t = \left(\phi_\pi - 1 \right) \pi_t + \kappa y_t = 0, \tag{12}$$

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[12] For a full account see Appendix A.3.

which means that output will be at steady-state providing the
following condition is satisfied:

$$y_t = \frac{(1 - \phi_\pi)}{\kappa}\pi_t, \tag{13}$$

which we note will be positively sloped if $\phi_\pi < 1$ and negatively
sloped if the weight on inflation in the interest rate rule is greater than
one. We can think of these alternate rules for monetary policy as passive
and active, respectively (see, Leeper, 1991). Note that under a passive
rule a positive shock to inflation will imply that output will rise and
hence through the Phillips curve will generate higher inflation in this
and subsequent periods, that is inflation will not be stabilised and will
continue to escalate.[13] But the active rule will imply that higher inflation
will be associated with lower output and this will continue to drive down
future inflation until it is also back to target. In this way the crucial
aspect of this system's determinacy is the adoption of an *active* rule in
the monetary policy maker's reaction function.

To sum up, in this section, I have set out a modern macroeconomic
model. There is an important but largely hidden supply-side based on a
Cobb-Douglas production function and the Solow residual to provide a
measure of productivity growth, which is basically treated as exogenous.
Inflation is set by a Phillips curve and demand responds to the expected
path of real interest rates. The stability of this economy depends on
the monetary policy reaction function, which moves to stabilise inflation
via the output gap. The stability of this system can also be said to be
recursive in that as long as inflation and output are pinned down to a
unique solution path then the money stock and the price level are also
determined in each period. I have also explored a simple exposition of
how the adoption of an active rule will stabilise this economy. In the
following Section, I will create some simple toolkit diagrams, which can
be used to understand the setting of monetary policy.

3 Toolkit Policy Diagrams

I can now represent the key elements of this model within the context
of a simple set of quadrant style diagrams. In which the target inflation

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[13]The argument is the same for a negative inflation shock whereby the active policy
rule will ensure that output is higher in future periods but there is a downward
constraint as nominal interest rates cannot go below zero. I discuss this lacuna in
the section on the zero-bound.

rate is determined by a monetary policy maker using the interest rate as a tool of stabilisation. In this section we will develop this diagram and also consider the appropriate policy response to a positive demand shock, a negative supply shock and the possibility of dislodged (from fundamentals) inflation expectations.

3.1 Basic steady-state equations

The simple model outlined in equations (1) to (6) explained the dynamics of an economy around some steady-state or target level. In this section, we briefly outline those steady-states so that we can depict the economy in a diagrammatic form. At steady-state or target values there will be no expected deviation of output, y_t, from its flex-price level and so $y_t = E_t y_{t+1}$, and inflation will equal expected inflation, which in turn will equal the target level of inflation, $\pi_t = E_t \pi_{t+1} = \pi^T$ and assuming that $\sigma = 1$ and $\beta \approx 1$, we can examine the steady state as follows:

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\begin{aligned}
0 &= E_t y_{t+1} - y_t = \sigma \left(R_t - E_t \pi_{t+1} \right), \\
R &= \pi^T.
\end{aligned}
\tag{14}
$$

$$
\begin{aligned}
0 &= E_t \pi_{t+1} - \pi_t = \kappa \left(y_t - \tilde{y}_t \right), \\
y &= \tilde{y}.
\end{aligned}
\tag{15}
$$

Equations (14) and (15) tell us that at steady state, where there is no expected deviation of inflation or output from its target, π^T, or potential value, $\tilde{y}$, the nominal interest rate will deviate from the long run real rate of interest (the so-called *Wicksellian rate of interest*)[14] by the inflation target and output will be equal to its potential with the output gap at zero. The money market will thus clear as follows:

$$
\begin{aligned}
m - p &= \tilde{y} - \eta \pi^T \\
m &= \tilde{y} + \bar{p} - \eta \pi^T.
\end{aligned}
\tag{16}
$$

[14]The Wicksellian rate of interest is explored in other chapters in this volume but essentially is the real rate of interest consistent with (flex-price) equilibrium fluctuations in output.

Equation (16) tells us that money will be held to finance steady-state demand for steady-state output, $\tilde{y}$, and in direct proportion to the price level minus an 'inflation tax' term, $\eta\pi^T$, because for any positive inflation target there is a steady-state disincentive to hold money balances. This is because the negative of the target inflation rate, $\eta\pi^T$, is equal to return on money holdings, when money balances yield no monetary return. We are now in a position to draw our two-quadrant diagram for interest rate determination and for output determination. For the moment let us put money on one side and concentrate on policy rates, inflation and output.

3.2 Inflation and interest rates

Figure 1 shows the determination of equilibrium in the interest rate-inflation space. There are two lines. The first, labelled, FE, is the Fisher equation and relates nominal interest rates to expected inflation equiproportionally and so has a slope of 1, see equation (13). The FE lines cuts the interest rate axis at the natural, or Wicksellian, rate of interest where nominal interest rates equal real interest rates as inflation is zero at this point. It might be argued that at this point there might be a limited degree of money illusion as at zero inflation equilibrium real and nominal changes are identical and so this is a possible long run solution for a monetary economy. The FE line also cuts the inflation axis when nominal rates are zero and inflation is equal to the negative of the real interest rate, $-\pi = R^{nat}$. At this point money balances gives a return equal to the R^{nat} because inflation is negative, which is the so-called Friedmanite maximum, at which point money holdings would be maximised as they do not suffer an opportunity cost in terms of returns relative to bonds.[15]

The second line, is called IRR and represents the reaction of the monetary policy maker to inflation above or below his or her target. I draw the line as an *active* policy rule. As illustrated in Section 2, equation (13), the slope of this curve is greater than one and means that policy rates rise (fall) by more than any increase (decrease) in inflation implying that real interest rates rise (fall) in order to induce aggregate demand to move back to the full employment level. As drawn there is a unique equilibrium at the inflation target, π^T, which is also equal to the level of inflation expectations, π^e. In this economy what we would therefore expect to see if that, with inflation expectations at target or credibility, interest rates, inflation and output would move in unison around the steady-state: rising and falling together.

In some sense, the equilibrium suggested by Figure 1 is arbitrary as the central bank could easily choose an alternate inflation target and set

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[15]See Friedman (1969) on this suggestion.

interest rates to stabilise inflation around that level. There is a wide-ranging debate in monetary economics about the appropriate level of inflation target and although there appears to be have been an advanced country consensus for a numerical statement of something in the region of 2%, it is not at all clear whether that consensus will persist. There is a conceptual trade-off that involves deciding, on the one hand, upon a level of inflation that is not so high that prices lose their signalling power and introduce a significant degree of uncertainty into the economy, which will lead to economising on monetary balances and also on the divergence of resources to mitigating that uncertainty. And, on the other hand, to bear in mind that inflation should not be set at so low a level that it starts to impact on the ease of relative price adjustment, as some wages and prices and downwardly rigid, or that the zero bound on nominal interest rates may start to become a significant constraint.[16]

3.3   Aggregate dynamics revisited

Figure 2 appends a lower quadrant to the earlier interest rate-inflation space. It shows the aggregate supply curve, which is determined by equation (5), and Phillips curve, which for a fixed level of inflation expectations shown in the top quadrant, slopes upwards with the parameter, κ, see equation (2). I can now assess what happens to the this economy in response to three comparative static shocks: aggregate demand, aggregate supply and inflation expectations.

3.3.1   A positive shock to demand

Points A and B show the initial equilibrium in Figure 2. Now imagine that there has been a shock to output demand from something like an increase in wealth, fiscal expenditure or some relaxation of credit conditions.[17]   Aggregate demand is now in excess of supply at some point C and inflation has increased by $\kappa(C - B)$. With fixed inflation expectations, which is really what is meant by the attainment of credibility, the central bank simply raises interest rates to E, given by the IRR slope and at this point real rates are $(E - D)$ higher than the natural rate, R^{nat}. The increase in real rates bears down on aggregate demand and the demand converges back to point B with interest rates and inflation determined at A. The demand shocks leads to a temporary inflation, boom and increase in policy rates but at the end of the cycle we are back to where we started from in terms of the level of interest

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[16]A good introduction to the debate on optimal inflation can be found at Feldstein, 1979.

[17]See Chadha and Nolan, 2007, for an examination of the interactions between monetary and fiscal policy.

rates and inflation.

3.3.2 A negative aggregate supply shock

Figure 3 helps us understand the correct NK policy response to a negative supply shock. From the initial equilibrium at A and B, a negative supply shock shifts the horizontal line in the lower quadrant upwards and takes with it the locus for aggregate demand which also then goes through the new equilibrium point C. Despite the movements in the AS and AD schedules that lead to the determination of a new steady-state level for supply, the level of demand initially remains at point B, which is clearly in excess of capacity. Excess demand drives inflation up and because inflation can jump in this model, see equation (2), inflation will move to D at the same level of excess demand. At D inflation is above target, policy rates are moved up to E, where again real rates are set in excess of the natural rate. The economy then slides down the locus $D - C$ and policy rates fall from E to A. In this cycle policy rates and inflation are back to where we started from after a temporary escalation in both but output lies at a permanently lower level.

3.3.3 An escalation in inflation expectations

In Figure 4 we add to the two-quadrant diagram a vertical line in the upper quadrant that represents inflation expectations formed today for inflation in the next period, $E_t\pi_{t+1}$. They can also be interpreted as longer run inflation expectations and so reflect the level of monetary policy credibility, which is some inverse function of $\mid E_t\pi_{t+1} - \pi^T \mid$. Initially the economy is at the equilibrium A and B. Now let us suppose that inflation expectations shift to the right because of changes in the monetary constitution such that it is no longer judged that the marginal benefit and the marginal cost of inflation are equalised at the target. In other words, it is perceived that the monetary policy maker derives some benefit from elevated inflation.[18] In this case, inflation expectations will be in excess of actual inflation at D and actual real rates will diverge from expected real rates, which have fallen, and output will start to expand towards E from B as there has been an effective loosening of policy.

There are two possible solutions. First, the increase in inflation expectations are accommodated and a new inflation target is set equal to the elevated level of inflation expectations and the AD slides up the AS to cut it at the higher inflation target. That is the economy

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[18]See Kydland and Prescott, 1977. And for an analysis of dislodged inflation expectations, see Chadha and Corrado (2007).

move to an equilibrium of D and C. At this new inflation target, the economy continues to operate at full capacity and the higher inflation expectations lead to a change in the inflation rate and ultimately to the inflation target. The alternative is more difficult and costly as it requires a significant monetary policy response but also illustrates the importance of having some well understood target for monetary policy. First note that at the new equilibrium CD expected real rates are equal to the natural real rate. And the problem is how to deflate inflation and inflation expectations back to the original target. The central bank could have chosen to treat the initial increase inflation expectations as one of inflation and raised rates along the original IRR to point vertically above D and this would have acted to reduce demand from E to B and inflation back to the original equilibrium, A. If on the other hand now that the economy has settled at CD a shift to the old IRR curve will entail a sharp rise in real rates and the maximum size of the recession from this policy, CGB will occur if inflation expectations are sluggish and there is little credibility, on the other hand, in the event that such a policy quickly restores credibility the economy may jump back quickly from C to B.[19]

4   Two Extensions

Within the context of the framework outlined in the previous section, I can also examine two ongoing monetary policy dilemmas. The first is exactly how the zero-lower bound for monetary policy constrains the scope of interest rate reaction with low inflation or even deflation. The second is how exactly a disconnect between money market interest rates and policy rates leads to complications for monetary policy makers.

4.1   An application to non-linearity

The difference between the FE line and the IRR line represents the deviation of the real policy rate from the natural rate of interest. And so the triangle ABC Figure 5 represents the force acting on the economy via the choice of the level of interest rates when inflation is below target. As interest rates can rise as high as policy makers wish to place them the opportunity for deflationary impetus is reasonably unconstrained. But once nominal rates start to fall they are constrained to remain at or above a lower bound of zero. And so I plot the possible triangle of real rate choices faced by a policy maker who pursues their interest rate paths in a piece-wise linear fashion, with policy rates falling first to

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[19]Sargent (1981) outlines a nice illustration of the benefits of credibility.

zero and then staying there as inflation falls. The x-axis show inflation and the y-axis shows the real interest rate, essentially I simply plot the difference between the FE and IRR curves in inflation-real rate space. Note that at initial equilibrium, A, and at the Friedmanite maximum for money holdings, C, real rates equal the natural or Wicksellian rate.

At the origin, O, nominal rates are zero since both inflation and the real rate is zero. Triangle OBD represents the region over which negative interest rates pertain. The policy maker is able to drive real rates down only to point B, after which real rates will rise, as inflation can fall but nominal rates cannot. But note that real rates along ABC are all below the natural rate and are therefore acting to stimulate the economy. The issue then is whether inflation will fall below $-R^{nat}$ after which it will act to further bear down on demand and whether the increasing real rates over the range BC will be sufficient to stabilise a deflating economy. If not then other monetary policy tools will have to be considered. The Japanese experience since the collapse of the asset price bubble in the early 1990s led to a prolonged debate about how to deal with deflation and led to the suggestion of a number of complementary tools to monetary policy, for example, exchange rate devaluation or the underfunding of government fiscal deficits. The question for policy makers is thus simply does the triangle ABC place sufficient stabilisation policy in the hands of the policy maker when inflation lies in the range $\pi = -R^{nat}$ to π^T? Or should more extreme responses be engendered early in any deflationary episode, so that the slope of the line AB is even more negative i.e. real rates are pushed down very quickly so as to minimise the possibility of a increasing real rates as inflation falls i.e. BC. I leave it to the reader to draw his or her preferred path for rates but it may well not be linear.

4.2   An application to the money market

As the name suggests the two quadrant diagram can be extended with two further quadrants. In Figure 6 we add one quadrant for determining market interest rates with a premium over the policy rate and the second as a clearing condition for the money market based on market rather than policy rates. Let us first suggest that the market interest rate has an external finance premium, efp, over the policy rate and so we draw a EFP line in the top left-hand quadrant, which simply states that the market interest rate, $R^m = R^n + efp$. The magnitude of the efp has been explored in various papers and is likely to vary cyclically over the

business cycle to reflect market risk.[20] Note that in a NK framework the risk premium can be directly linked to the state of the economy and can be thought of as reflecting the marginal costs of loan supply to the private sector and may well be highly correlated with the business cycle such that financial intermediaries may perceive their costs of loan supply to fall in an expansion and rise in a recession, meaning that risk premia are counter-cyclical and act to amplify the business cycle.[21]

In equilibrium the supply of money is set by the full employment level of output (see equation 18) and money demand is decreasing in the level of market interest rates with a slope term reflecting the interest rate elasticity of demand for money, η. At equilibrium, point $ABCD$, inflation is at target, $\pi = \pi^T$, output is at its full employment level, $y = \tilde{y}$, and money demand equals money supply, $M^d = M^s$ at the policy rate and the market interest rate, R^n and R^{efp}.

Now let us suppose that the external finance premia is driven upwards as perceptions of risk in the market economy increase and this reduces the supply of money (or liquidity) at each given market rate. The EFP then jumps up with a new intercept, where $efp' > efp$. In the absence of a fall of velocity induced by higher market interest rates, which would drive the demand curve outwards, the money market will now clear at a higher level of market rates and a lower level of observed nominal money supply, that is EF. But the higher market interest rates and lower money supply will set up a deflationary impetus to the economy as scarce liquidity will drive demand down relative to capacity, G, and inflation will fall. The policy rate is thus cut to H in order to offset the increase in market interest rates, which induces a temporary inflation and ouught to cause both the money supply and demand curves to shift out to a new equilibrium, I, at the higher market interest rate. So when money markets disconnect policy rates, output and inflation may eventually return to their long run level but there has to be a temporary offset of the higher market interest rates by the policy marker.

5   Concluding Remarks

In this chapter, I have outlined a simple macroeconomic model that underpins much of modern macroeconomic analysis. Although not developed here at great length, the main equations (1) to (6) can all be

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[20]See Chadha *et al* (2008) for an examination of the possible links between the money markets and interest rate spreads. Other chapters in this volume also consider this question.

[21]This endogenous interpretation of business cycle generated risk premia is quite different to exogenous view taken by the followers of Minsky.

derived from the first principles of household constrained optimisation. I have shown that the path for output and for inflation are determined by the arguments in the central bank's policy rule. Under such a rule we also show that money and the price level are well determined. One key feature of this model is that output and inflation are forward-looking and respond to the expected path of real interest rates and output respectively. I also show that the monetary policy reaction function does imply a trade-off between output and inflation because increasing (reducing) output has an inflationary (deflationary) implication.

I then transfer the key elements of this model to a series of simple diagrammatic expositions that are suitable for undergraduate study. Specifically, we analyse the equilibrium for interest rates and inflation and the slope of the monetary policy reaction function. We are also able to use the diagram to illustrate the multiplicity of possible equilibria, for example, an inflation target can be set at any point from the Freidmanite minimum upwards, and the relationship from this space to that of inflation-output, which is simultaneously determined. The correct policy response to demand and supply shocks are considered as is that to the possibility that inflation expectations may become dislodged from target and I leave to the student the analysis of what to do if the natural rate of interest changes. Finally we examine some limitations of this tool by considering the limits to the correct policy response as a result of a zero bound constraint on the nominal interest rate and also the possibility that disruption to money markets may cause market rates to disconnect from policy rates.

This chapter takes the intermediate student to the point of understanding more fully many of the issues currently occupying monetary theorists and practioners. That is what are the key equation required to understand more fully the aggregate economic system and how the choice of monetary policy rule plays a crucial role in the system's dynamics. Underpinning much of this work is the observation that it is not possible to understand aggregate dynamics of a monetary economy without reference to monetary policy and the level of credibility it has bestowed upon it. The model structure thus outlined takes the Lucas (1996) critique seriously. Ultimately the student who understands the key role of policy rules, targets and beliefs in determining a monetary equilibrium is better equipped to understand how issues such as learning, uncertainty, robust rules, min-max objectives and so forth play their way out of a basic New Keynesian macroeconomic model.

A   Appendix to Section 2.

A.1   Block triangularity

We note that the 4x4 matrix, $\mathbf{\Lambda}$, can be written in block form, where each block $(\mathbf{A},\mathbf{0},\mathbf{C},\mathbf{D})$ is a 2 x 2 matrix:

$$\mathbf{\Lambda} = \begin{bmatrix} \frac{1}{\beta} & -\frac{\kappa}{\beta} & 0\ 0 \\ \sigma\phi_\pi - \frac{\sigma}{\beta} & \kappa\frac{\sigma}{\beta} + 1 & 0\ 0 \\ \Gamma_1 & \Gamma_2 & 0\ 0 \\ \Gamma_3 & \Gamma_4 & 0\ 0 \end{bmatrix} = \begin{bmatrix} \mathbf{A}\ \mathbf{0} \\ \mathbf{C}\ \mathbf{D} \end{bmatrix},$$

where Γ_i are composite parameters.   The block triangularity, or recursiveness, of matrix $\mathbf{\Lambda}$, with a null matrix in the upper right hand block, means that the eigenvalues of the whole matrix are simply given by the eigenvalues of $\mathbf{A}$, referring to $\begin{bmatrix} \pi_t\ y_t \end{bmatrix}$ and $\mathbf{D}$, referring to $\begin{bmatrix} m_t\ p_t \end{bmatrix}$. Also in this case the determinacy of $\mathbf{\Lambda}$ follows from the determinacy of $\mathbf{A}$ given $\mathbf{D}$ is the identity matrix. In other words by locating a stable path for inflation and output around steady-state or target values then both the money stock and the price level will follow recursively in each period. This is a key result, in that in this model it is the case that controlling the economy at the top level of output and inflation, is sufficient to control other aggregate quantities and prices, in this case the money stock and the price level.

A.2   Determinacy

The determinacy of this system will depend on the stability of $\mathbf{A}$. The dynamics of a first order system depend on the eigenvalues, λ_1 and λ_2, of matrix $\mathbf{A}$ which determines the equation of motion for $\mathbf{x}$ in equation (7). And so for the equation of motion for $\mathbf{A}$ this case, with both inflation and output non-predetermined then determinacy will require matrix A to have two eigenvalues outside the unit circle.

A.2.1 $\lambda_{1,2} > |1|$ i.e. eigenvalues both outside the unit circle

When the roots are both positive, as they will be in this case, the conditions for both eigenvalues to be outside the unit circle are easy to derive. Note first that for all square matrices the eigenvalues, $\lambda_1...\lambda_n$, of the matrix will be related to its trace and determinant in the following way:

$$Det(A) = \lambda_1\lambda_2 \tag{17}$$

$$Trace(A) = \lambda_1 + \lambda_2. \qquad (18)$$

And so now note that for a matrix where both roots are outside the unit circle:

$$Det(A) = \lambda_1 \lambda_2 > 1 \qquad (19)$$

$$Trace(A) = \lambda_1 + \lambda_2 > 2. \qquad (20)$$

Because both roots must be greater than 1, then the following condition must also hold:

$$(\lambda_1 - 1)(\lambda_2 - 1) > 0, \qquad (21)$$

which expands to:

$$\lambda_1 \lambda_2 - (\lambda_1 + \lambda_2) + 1 > 0$$

And thus:

$$Det(A) - Trace(A) > -1$$

$$[\frac{1}{\beta}(\kappa\sigma\phi_\pi + 1)] - [\frac{1}{\beta} + \kappa\frac{\sigma}{\beta} + 1] > -1$$

$$\kappa\frac{\sigma\phi_\pi}{\beta} + \frac{1}{\beta} - \frac{1}{\beta} - \kappa\frac{\sigma}{\beta} - 1 > -1$$

$$\kappa\frac{\sigma\phi_\pi}{\beta} - \kappa\frac{\sigma}{\beta} > 0$$

$$\kappa\frac{\sigma\phi_\pi}{\beta} > \kappa\frac{\sigma}{\beta}$$

$$\phi_\pi > 1. \tag{22}$$

And so providing the nominal interest rate increases by more than any

inflationary shock, the economy can be stabilised around any given inflation target. In other words given an inflationary shock providing the real interest rate increases, inflation can be brought back to target by inducing a reduction in demand and so a closing of any output gap. I have illustrated this form on monetary policy response, which is termed *active*, in Figure 1 of the main text.

A.3   Optimality

Let us append a simple loss function to the trade-off between output and inflation:

$$L^{mp} = \frac{1}{2} \left[\omega_\pi \pi^2 + \omega_y \left(y \right)^2 \right]. \tag{23}$$

This form of loss function for equal weights on $\omega_y = \omega_\pi$ will imply indifference curves that are a series of concentric circles around the point where inflation is at target and output is equal to its flex-price level, the so-called 'bliss-point'. Typically the bliss point used to be thought to lie to the right of the flex-price level of output, thereby bringing about a bias into monetary policy to try and get to a higher indifference curve. The so-called 'inflation bias' stemmed from this perception (Nordhaus, 1995).

We can also use the loss function in (14) alongside the Phillips curve, which we can interpret as setting the rate of exchange between current period inflation and output, to analyse the slope of the monetary policy reaction function in output-inflation space. Let us start from some point where there is a negative output gap, $y < 0$, and evaluate the gain from increasing y,[22] which will be simply given by $-\omega_y y \Delta y$. The resulting loss from increasing output will increase inflation, via the Phillips curve, which will be given by, $\omega_\pi \pi \kappa \Delta y$. Now equating the marginal cost to the marginal benefit any outcome for inflation and output must satisfy the following constraint:

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[22]I am grateful to Walsh (2002) for this simple thought experiment. In this simple example, I have implicitly set $\tilde{y} = 0$.

$$y = -\kappa \frac{\omega_\pi}{\omega_y} \pi. \tag{24}$$

Equation (15) thus shows the slope of the optimal monetary policy reaction function in inflation-output space. The rate of transformation, or slope, is given by the slope of the Phillips curve, κ, and the relative weight on output or inflation in the loss function, (14). Finally note that the form of the loss function will determine the optimal monetary policy reaction function in output-inflation space. For example if we only worry about large deviations of output or inflation from target rather than all deviations, the policy reaction function will be flat over some range and then react aggressively when it is in some danger of being breached - this is a form of min-max reaction.

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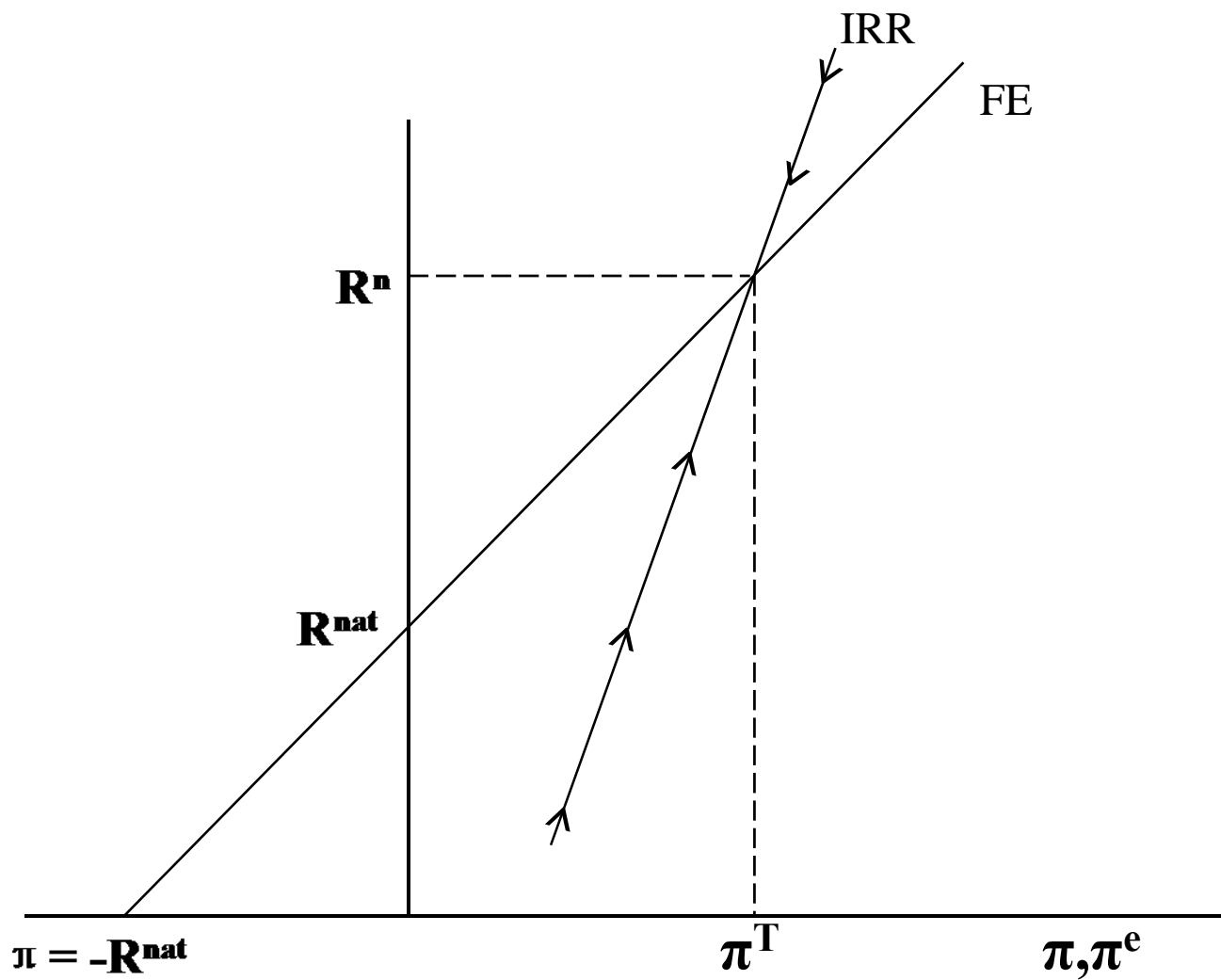
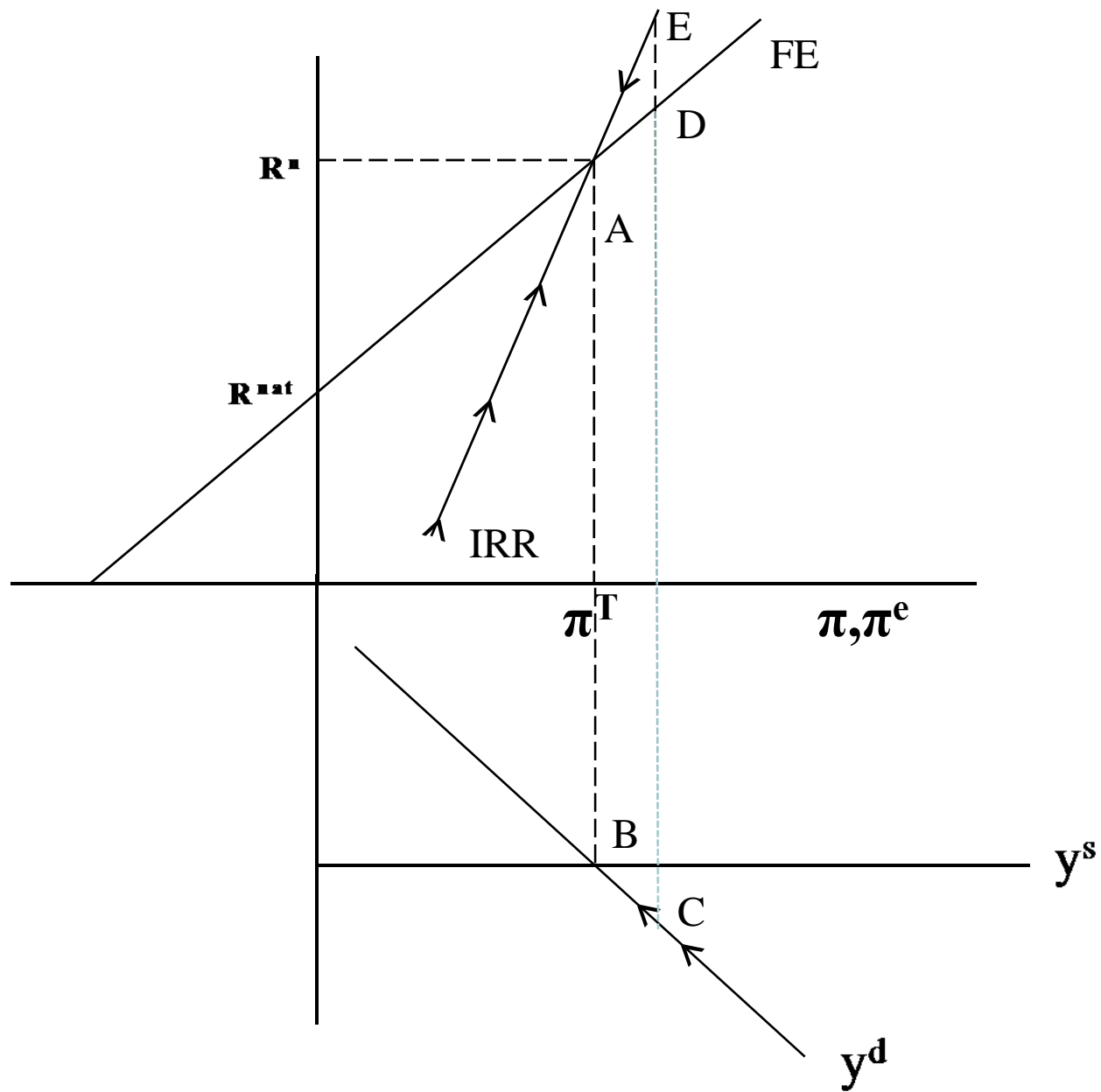**Figure 1: Interest Rates and Inflation Targets**

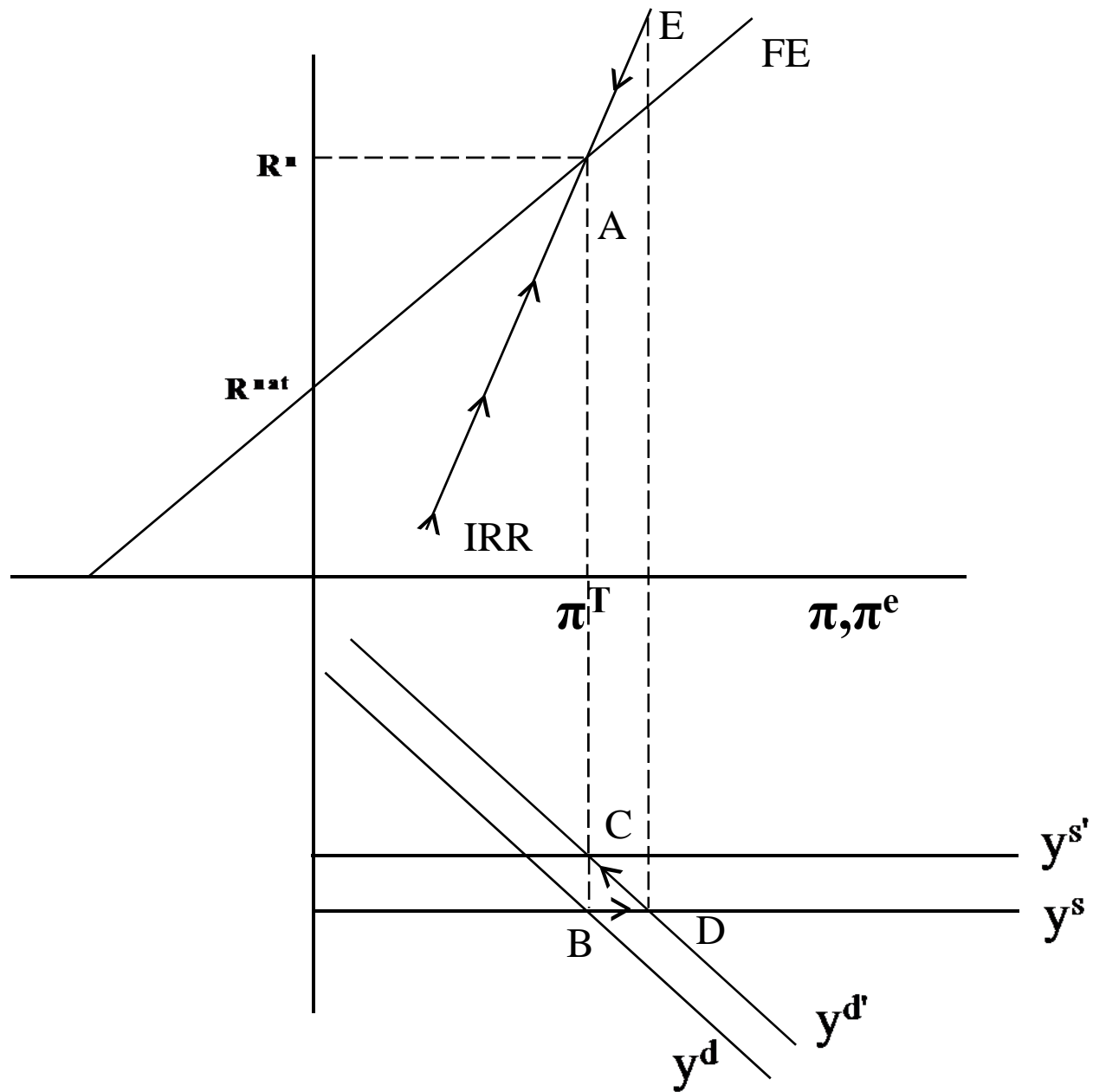**Figure 2: Two Quadrant Diagram – Demand Shock**
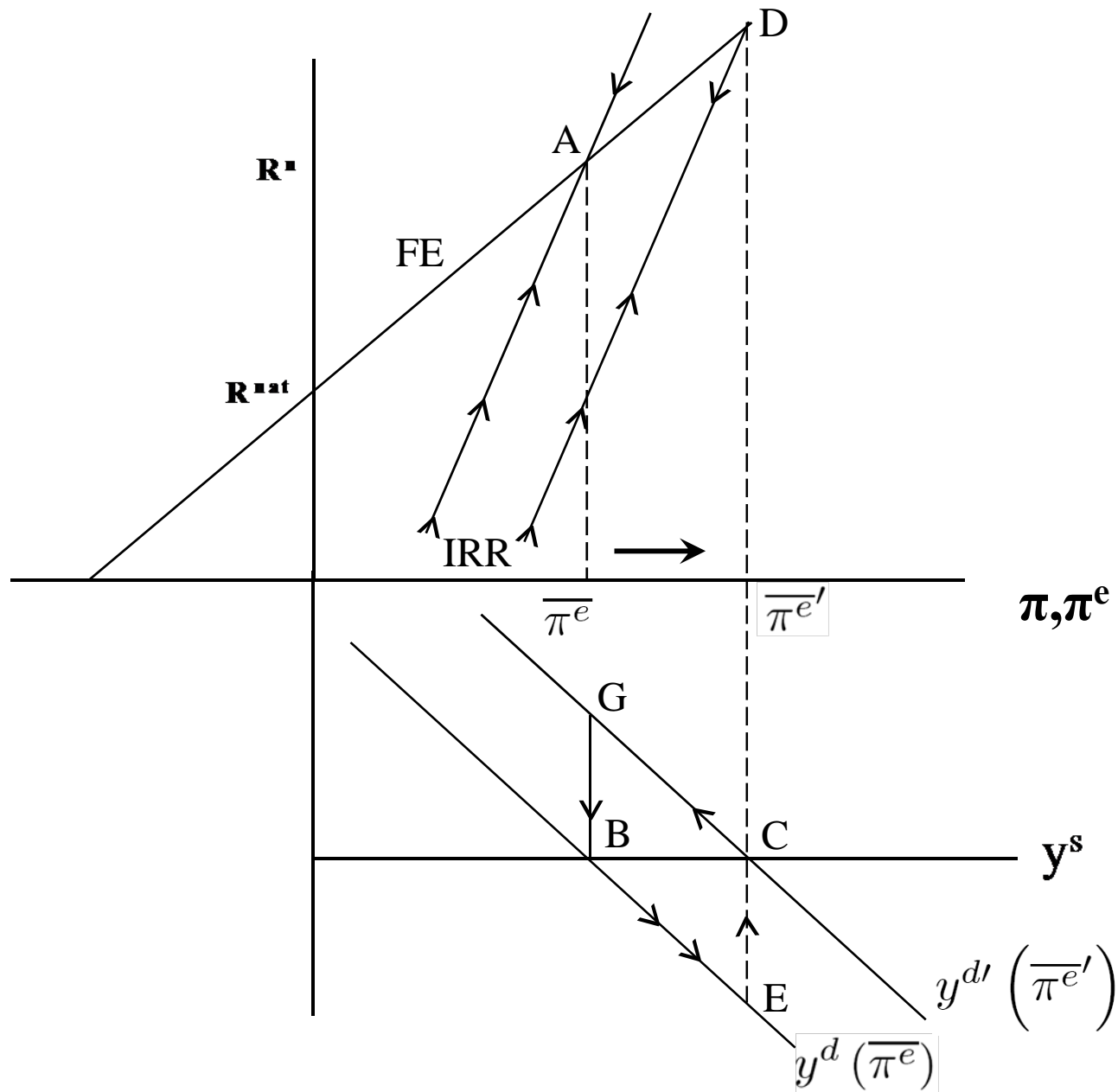
Figure 3: Two Quadrant Diagram – Supply Shock
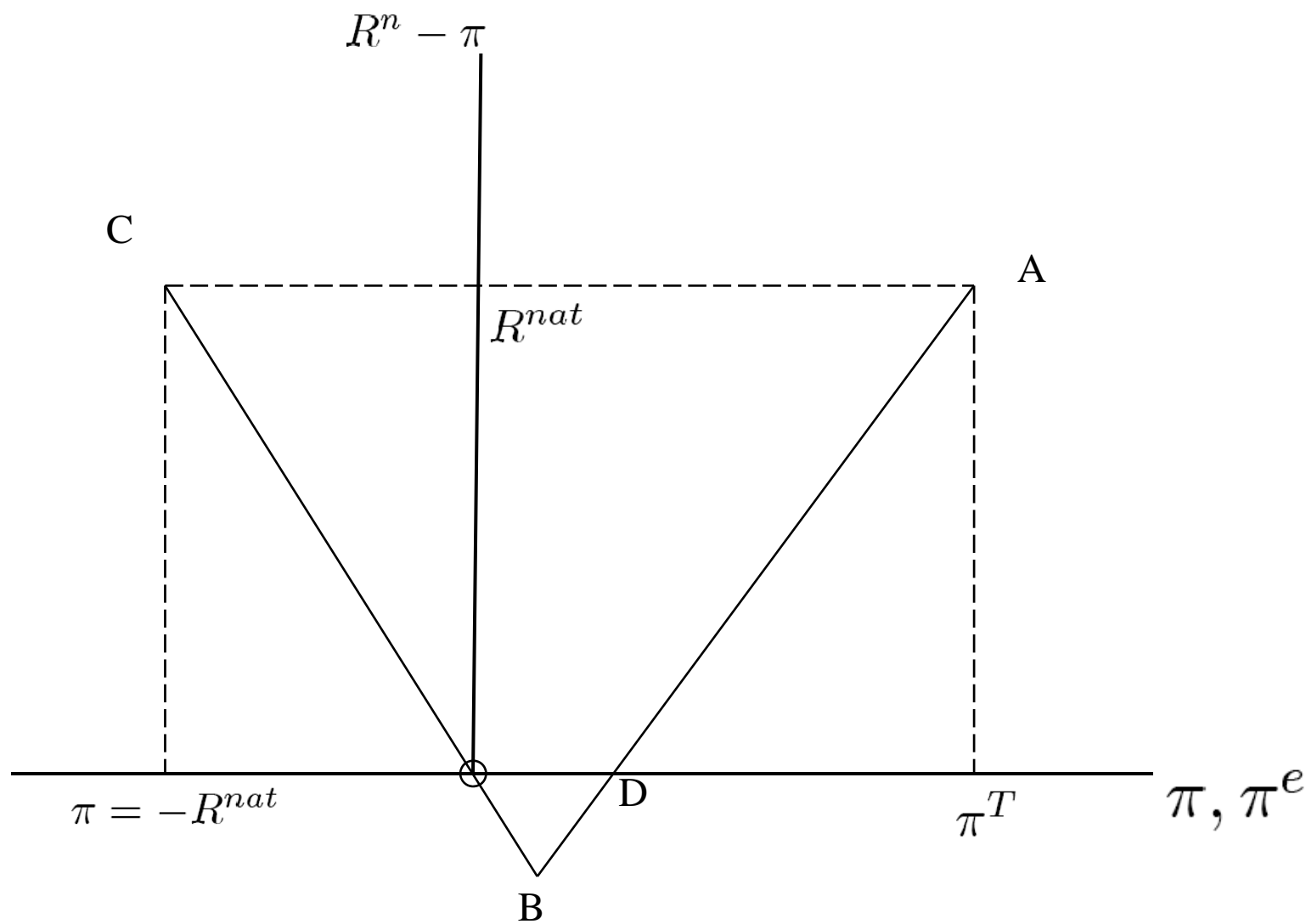
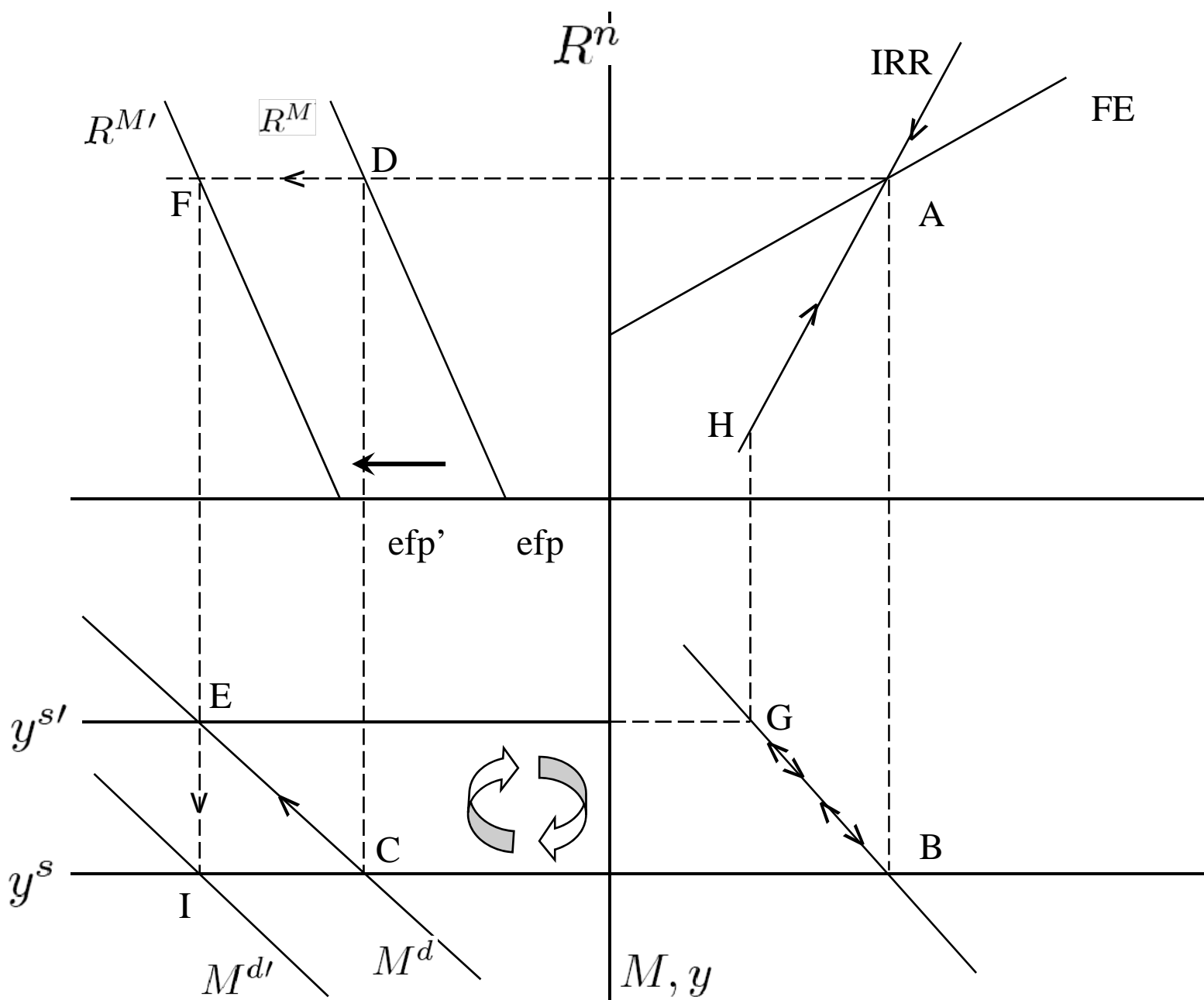**Figure 4: Inflation Expectation Accommodated**

Figure 5: The Zero Bond Problem

Figure 6: The Four Quadrant Diagram