ON THE PERIODICITY OF INVENTORIES

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Abstract

This article studies inventories and monetary policy by estimating VAR models. The complex roots detected in our estimation generate cycles of around 55 to 70 months, which are quite close to actual business cycle lengths. This implies that production and inventories follow damped oscillations (stable sine curves), implying that a boom is the seed of the following recession, and vice versa. Interestingly, the peaks and troughs of policy interest rate precedes those of production in the U.S. (i.e., forward-looking monetary policy), but not in Japan. The central banks in both countries react sharply to demand shocks, but not to supply shocks, because booms after positive demand shocks last longer as firms replenish reduced inventories, while booms after positive supply shocks are short-lived as the initial accumulation of inventories suppresses production in subsequent periods.

Keywords: Inventories, Inventory cycle, Business cycle, Monetary policy, Damped oscillations, Phase shift, Spectrum

JEL classification codes: E32, E58, C32

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1 Introduction

Understanding inventories helps the understanding of business cycles. This article is motivated especially by so-called inventory cycles (see Figures 1 and 2), which are phase diagrams of year-on-year percentage changes in production/shipment (on the y-axis) and inventories (on the x-axis). These clockwise movements are stable in past and present data, and they are especially useful for short-run economic forecasts.

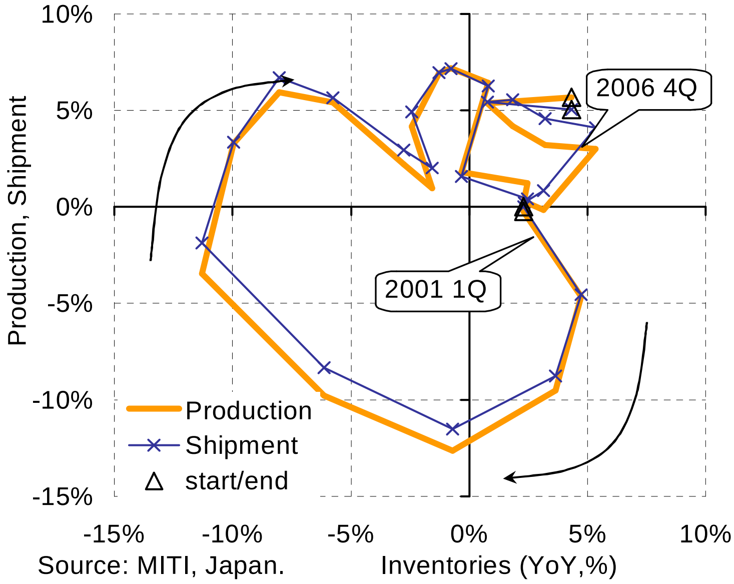

Figure 1: Inventory cycle in Japan. Source: MITI, Japan.

This article has two objectives. The first objective of this article is to shed light on some earlier economic thinking, especially on Kitchin cycles. By the early 20th century, Kitchin (1923), Juglar (1860), Kuznets (1930), and Kondrachieff (1935) found cycles of roughly of 3.4, 10, 20, and 50 years, respectively.[1] Later, Schumpeter (1939) excavated

[1] A summary of the major early thoughts is as follows.

Name	Period (yrs)	Main Driving Force
Kitchin Cycle	3.4	Inventories
Juglar Cycle	10	Investment
Kuznets Cycle	20	Construction
Kondratieff Cycle	50	Technological Revolution

There are three remarks. First, most longer cycle lengths are integer multiples of shorter ones. This implies that observed cycles are not completely distinguishable from one another. For example, three Kitchin cycles could be misidentified as one Juglar cycle.

Second, the main driving forces in the table are provided by later analyses. For example, the data used by Kitchin are bank clearings, commodity prices, and interest rates, whereas Kondratieff uses wholesale prices, interest rates and wages, foreign trade and the production of some metals. Hence,

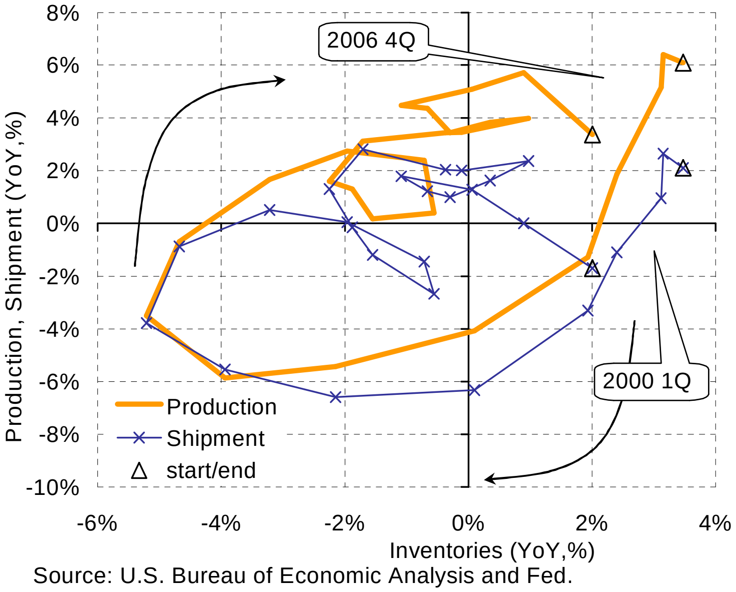

Figure 2: Inventory cycle in the U.S. Source: U.S. Bureau of Economic Analysis and Fed.

and sorted out their findings (excluding Kuznets (1930)), and Burns and Mitchell (1946) conducted a comprehensive study of business cycles. Of particular importance, it seems that most of these earlier studies implicitly presupposed damped oscillations, or perhaps limit cycles.[2] Among them, the cycles detected in this article seem to correspond to those found by Kitchin, because the length of the Kitchin cycle is closest to our estimates, and interestingly researchers today consider inventories to be the driving force behind the Kitchin cycle.[3]

Note that it seems that most modern views take a different stance from the earlier thinking. For example, Prescott (1986) argues that the term *business cycle* is inaccurate. Instead, he proposes the concept of *business cycle phenomena,* because "some systems of low-order linear stochastic difference equations with a nonoscillatory deterministic part, and therefore no cycle, display key business cycle features" (Prescott

Kitchin himself supposedly did not recognise his finding as an inventory cycle.

Third, all of these cycles are empirical findings with little theoretical background, and their empirical techniques may not be defensible by modern standards. Indeed, Harvey (1993, pp.195-196) demonstrates that the moving average that Kuznets uses generates spurious cycles. Hence, it should be understood that the existence of these cycles has not yet been confirmed econometrically.

[2] See Hassler, Lundvik, Persson, and Soderlind (1992) for a related discussion.

[3] See Knetsch (2004), for example.

(1986), p.10). Essentially, Prescott's business cycle phenomena are exponential decays: successive deviations of variables from their steady states and their returning processes.

On the contrary, the most important message of this article is that cycles in this article are damped oscillations (stable sine curves), rather than successive exponential decays. In our view, business cycles are endogenously generated; a boom is the seed of the following recession, and a recession is the seed of the following boom. A casual conjecture on Figures 1 and 2 tells us a more specific story. As the stockout avoidance and inventories-as-sales-facilities models suggest,[4] the target level of inventories is increasing in demand. Hence, when demand is strong (i.e., in a boom), firms accumulate inventories above the normal level to capture a good sales opportunity. This accumulation of inventories itself augments economic activities, because firms use more labour input and intermediate goods. Importantly, however, such a high level of inventories is justified only by demand stronger than the normal level and is the source of the recession that follows. Once firms start cutting their production to adjust their inventories, it not only reduces labour income and the use of intermediate goods, but also decreases the target level of inventories through weaker demand; hence, firms keep reducing their inventories, even when the amount of inventories reaches its normal level. However, such a low level of inventories is desirable only with demand weaker than usual, which, in turn, is the source of the boom that follows. This process repeats itself.

The second objective is to investigate the *dynamics* between inventories and monetary policy. There has been much empirical research on inventories and monetary policy. In the U.S., for example, Gertler and Gilchrist (1994) find that, after a tightening monetary policy shock, small firms decumulate their inventories, while large firms accumulate them, suggesting a difference in creditworthiness between small and large firms. During tight monetary policy periods, large firms can finance their inventories, while small firms cannot (see also Barnanke and Gertler (1995)). Kashyap, Lamont, and Stein (1994) also report essentially identical results by using firm level data. For Japan, several studies such as Yoshikawa et al (1993) emphasise the importance of the inventory channel as

[4] See Kahn (1987 and 1992) for the former and Bils and Kahn (2000) for the latter.

a transmission mechanism of monetary policy. A tight monetary policy first negatively affects inventory investments, which then affects real economic activity, because a low level of inventories as working capital may not be sufficient to lubricate trades (see also Teruyama's survey (2001)). In sum, much evidence shows the importance of the inventory channel. Built on this existing literature, however, this article aims to take a further step; i.e., we focus on the dynamics of monetary policy.

This paper conducts two sets of vector autoregression (VAR) estimations: three- and six-variable VAR using Japanese and U.S. data. Each estimation uses three types of data sets: level data, HP-filtered seasonally adjusted data (HP-s.a.), and year-on-year change (YoY) data. The purpose of the three-variable VAR is to test the existence of inventory cycles, while the six-variable VAR, which additionally includes policy interest rate and price indicators, investigates the implications for monetary policy.

For both countries, the three-variable VARs find one conjugate pair of complex roots and its significance. Importantly, the implied cycle lengths are close to the actual average of post-war business cycles; e.g., the implied cycle lengths for Japan are 55 to 63 months, which are close to the average length of the post-war business cycles (50 months).

In addition, we find that the peaks and troughs of inventories lag behind those of production/shipment[5] by 12 to 14 months. Each detected lag is quite close to 1/4 of the estimated business cycle length; in the parlance of difference equations, the phase shift (time lag) between production/shipment and inventories is around $\pi/2$ (orthogonal).[6] The orthogonal phase shift implies that the locus of the phase diagram in the (inventories, production/shipment) plane must have a clockwise movement with a nearly circular trajectory, which is consistent with Figures 1 and 2.[7] Also, this means that the contemporaneous covariance between production/shipment and inventories is almost zero (i.e., orthogonal), although they are dynamically closely related; contemporaneous variances and covariances alone cannot capture dynamic interactions.

Monetary policy is the main interest in the six-variable estimations. The most impor-

[5]Production and shipment move together very closely, and hence they are interchangeable in most discussions.

[6]One cycle is 2π in terms of argument

[7]See Appendix A.2.1 for Figure 2.

tant observation is that monetary policy reacts sharply to a demand shock (a shock in the shipment equation), but not to a supply shock (a shock in the production equation). This is perhaps because the boom after a positive demand shock tends to last longer than that after a supply shock.[8] This is consistent with the stockout avoidance model, in which the target level of inventories is an increasing function of demand. According to this model, a positive demand shock reduces inventories and, as a result, production continues to rise to replenish inventories. On the other hand, a supply shock increases inventories, and subsequently firms cut their production to adjust their inventories.

Interestingly, the phase shift between the overnight call rate and production is around 2 months in the Japanese data. Considering the fact that statistics are released 1 to 3 months after the period to which they refer, monetary policy of the Bank of Japan (BoJ) is quite timely. However, the lag for the U.S. Fed is around -4 months! The negative lag means that the Fed's monetary policy is preemptive/forward-looking.

The main technical challenge of this article is the treatment of non-stationarity. Rather than addressing the issue directly, this article shows in two ways that the estimation bias is not very strong. First, by using Monte Carlo experiments, we find that the real unit root affects the estimated period length and phase shifts only negligibly. Second, to check for robustness, VARs are estimated by using two additional data series (HP-s.a. and YoY data, as mentioned above). In these two stationary data sets, we obtain results quantitatively quite similar to those of level data.

The plan of this paper is as follows. The next section reviews theories on how to compute the cycle length and phase shifts from VAR estimates. The results of the three- and six-variable VARs with Japanese data are discussed in Sections 3 and 4, respectively. The estimation results with the U.S. data are discussed in Appendix, because the quality of the U.S. data set (and, as a result, its estimation performance) is not as good as the Japanese one. Though the three-variable VAR is something of a subset of the six-variable VAR, the former has its own worth; it allows for Monte Carlo experiments, and the estimation results are more precise and reliable. The final section concludes.

[8] However, this is observed only in the Japanese data, but not in the U.S. data (see Appendix A).

2 Preparations before Estimations

2.1 Conjugate Pair of Complex Roots

This subsection briefly introduces key notations (see Appendix B for computation). We estimate the coefficient matrices of the following VAR.

$$y_t = z_t A + y_{t-1} B_1 + y_{t-2} B_2 + \cdots + y_{t-M} B_M + \xi_t C \tag{1}$$

where A, B and C are real coefficient matrices, and z_t, y_t and ξ_t are the row vectors of exogenous variables (time trend, seasonal dummies, etc.), endogenous variables and *iid* shocks, respectively.

It is known that any complex roots, if they exist, must appear in pairs − any complex root $z = a + bi$ has its conjugate $z^H = a - bi$, where $i = \sqrt{-1}$. It is also known that if there are complex roots, the solution of an endogenous variable includes a term such as

$$\alpha_{kj} \rho_{kj}^t \sin\left(\theta_{kj} t + \beta_{kj} \right)$$

where t is time and α_{kj}, β_{kj}, ρ_{kj} and θ_{kj} are parameters that are functions of elements in VAR coefficient matrices B_m and the variance-covariance matrix of the error term. The subscript kj implies that the term is in the solution of the k-th variable and is related to the j-th eigenvalue (and its conjugate).

The economic meanings of these parameters are as follows. α_{kj} is a kind of size parameter. $\rho_{kj} = \rho_j = \sqrt{a_j^2 + b_j^2}$ is the absolute value of the complex roots.[9] $\theta_{kj} = \theta_j = \arctan(b_j/a_j)$ is the frequency of the sine function, and hence the length of one period is $2\pi/\theta_j$. β_{kj} is the phase, which shows the "initial state" of the k-th variable right after a shock.[10] $(\beta_{kj} - \beta_{lj})/\theta_{kj}$ is the phase shift (in time) between the k-th and l-th variables. If it is x months, then it means that the peaks and bottoms of the k-th variables precede those of the l-th variable by x months. It can be shown that the phase

[9] For example, if there is a ρ_j whose absolute value is unity, then the term represents a unit root while all ρ_j must be less than 1 in absolute terms to have a stable system.

[10] See footnotes 21 and 22 to understand the intuition of the "initial state."

shift (in argument), $s_{kl,j} = \beta_{kj} - \beta_{lj}$, is a function only of the elements in matrices B_m, although β_{kj} alone depends on past and present shocks as well.

2.2 Phase Shifts

Phase shifts have important implications in dynamic relationships among variables, because, intuitively, they indicate time lags among variables. This subsection briefly reviews (a) the limitation of contemporaneous covariances and (b) the empirical implication for inventories.

2.2.1 Limitation of Contemporaneous Covariances

A phase plane exhibits a spiral only if there is at least one pair of conjugate complex roots, and the shape and direction of spiral depend on phase shifts (Figure 3). It is clear that, even when two variables have a close dynamic relationship with each other, their contemporaneous covariance is close to zero if their phase shift is near $\pm\pi/2$.

Of course, the entire story is not so simple. If the true data generating process (DGP) is very noisy, the effect of endogenous dynamic relationships, governed by matrices B_m, may be swamped by the initial effects of shocks. In such cases, contemporaneous covariances are determined mainly by matrix C in (1). Nonetheless, the limitation of contemporaneous second moments can be very serious. Indeed, we find that inventories and production have a close dynamic relationship, but their contemporaneous covariance is close to zero (see Section 3.4).

2.2.2 Implication for Inventory Cycles

To have phase diagrams such as inventory cycles (Figures 1 and 2), the value of the phase shift between production/shipment and inventories must be around $\pi/2$. This value is predicted through the following two observations. First, *the phase shift must be positive*, because the direction of inventory cycles is clockwise. Second, *the phase shift should be around either $+\pi/2$ or $-\pi/2$*, because the contemporaneous correlation between inventories and production/sales is close to zero in the data.

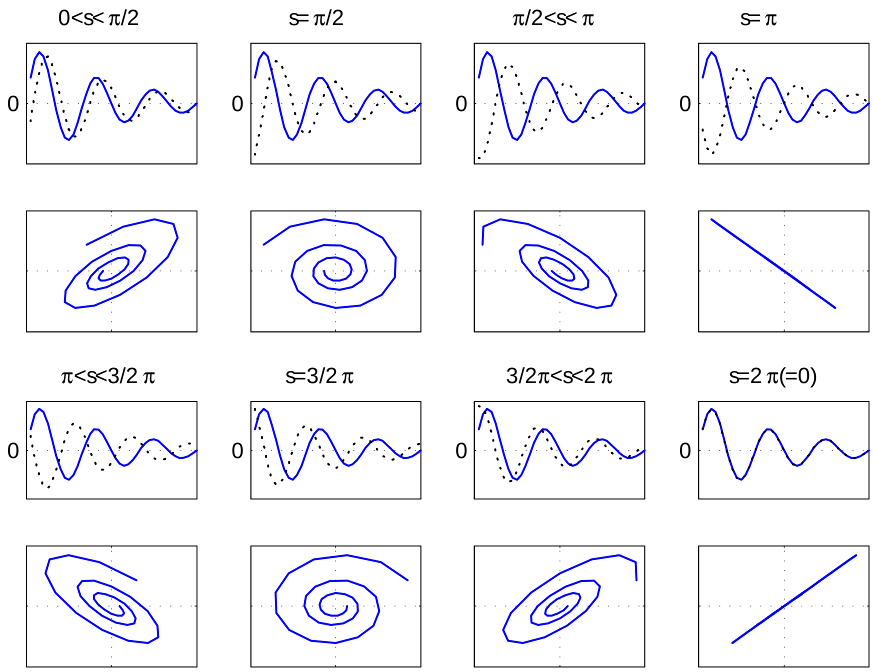

Figure 3: Impulse response functions and phase diagrams. s shows a phase shift. The solid and dotted lines in the IRFs correspond to the variables on the y- and x-axes in the phase diagrams, respectively.

2.3 Non-Stationarity

One of the challenges of this article is the use of level data, which almost inevitably causes the non-stationarity problem. This article indirectly tackles this problem in two ways: Monte Carlo simulations and the comparison with filtered data sets.[11]

First, Monte Carlo experiments reveal that there is one *real* unit root under the assumption that the true DGP has no time trend. However, they strongly reject the hypothesis that the absolute value (norm) of business cycle complex roots is +1 under most maintained hypotheses. Moreover, the same Monte Carlo experiments show that the effects of the unit root on the estimated cycle length and phase shifts are quantitatively negligible (see Section 3.2.4 for numerical results).

Second, to check the robustness of the estimated results, this paper also implements two additional VARs: estimations based on (i) HP-s.a. and (ii) YoY data. Presumably,

[11] In addition, as preliminary tests, Johansen's (1991) trace tests indicate that there exists at least one cointegration vector at the 1% level. For these trace tests, two preliminary estimations are conducted: one includes constant and seasonal dummies, and the other additionally includes the linear time trend. These tests are conducted by using PcGive, an econometric software; however, the trace test with a fifth-order polynomial time trend is not conducted.

the level data set is subject to the non-stationarity problem, while filtered data are subject to an artificial endogeneity problem. It is also well known that HP-filtered data may generate spurious cycles (see Cogley and Nason (1995)). See also Nelson and Kang (1981) for spurious cycles due to improper detrending. Rather than directly tackling these problems separately, this article compares these three specifications to evaluate how seriously the estimates are biased. As shown below, these three estimates show results very similar to each other, supporting the view that the estimated business cycles are not strongly biased.

2.3.1 Sketch of Monte Carlo Experiments

This subsection sketches the Monte Carlo Experiments conducted in this article. Assume that the true data generating process follows a VAR(1) process to keep exposition simple.

$$y_t = y_{t-1}B + \xi_t C$$

where ξ_t is assumed to be *iid*. Matrix B is first estimated by OLS. If there are no multiple roots, $\hat{B}$ can be decomposed by eigenvalue matrix $\hat{\Lambda}$ and eigenvector matrix $\hat{V}$.

$$\hat{B} = \hat{V}\hat{\Lambda}\hat{V}^{-1}, \qquad \hat{\Lambda} = \begin{bmatrix} \lambda_1 & & 0 \\ & \ddots & \\ 0 & & \lambda_K \end{bmatrix}$$

where K is the number of roots (number of endogenous variables times VAR order).

The idea of our Monte Carlo experiments in this article is as follows. For example, if the first eigenvalue is suspected to be a unit root, then the true data generating process (DGP) is assumed to be generated by $\check{B}$ such that

$$\check{B} = \hat{V}\check{\Lambda}\hat{V}^{-1}, \qquad \check{\Lambda} = \begin{bmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & \lambda_K \end{bmatrix}$$

Keeping $\hat{V}$ unchanged, the $\check{B}$ is constructed based on $\check{\Lambda}$. Then, by generating artificial innovations $\{\check{\xi}_t^j\}_{j=0}^N$,[12] $\check{B}$ and C matrices yield artificial data sets $\{\check{y}_t^j\}_{j=1}^N$, where N is the number of trials in Monte Carlo experiments. Estimates such as period length are computed for each $\check{y}_t^j$, and their distributions are obtained by stacking such estimates for $j = 1, \cdots, N$. Though true V and Λ are unknown, presumably $\hat{V}$ and $\hat{\Lambda}$ do not vary far from them given tight estimations.

2.4 Theories for Identification

In VAR estimations, although it affects *only* IRFs but not other results such as phase shifts and spectra, we need an identification assumption. To determine matrix C in (1), this article exploits two implications of the stockout avoidance model for real variables.[13] First, shocks in the inventory equation do not affect current production or sales. The law of motion of inventories (accounting identity) says that unsold goods at the end of period U_t are simply the result of today's production Y_t and sales S_t and not the cause.

$$U_t = U_{t-1} + Y_t - S_t \tag{2}$$

Second, the stockout constraint (3) implies that inventories play no role, if production can respond to the sales shocks; if firms could observe today's demand shock, firms would produce the minimum amount of products which exactly meets their demands and there would be no inventories. Note that firms hold inventories to protect themselves from demand uncertainties.

$$S_t = \min\{Y_t + U_t, D_t\} \tag{3}$$

For other variables in six-variable VARs, following Christiano, Eichenbaum and Evans (1999),[14] we assume that the O/N call rate can respond to any of the current

[12] A row vector $\check{\xi}_t^j$ is generated by resampling $\hat{\xi}_t = \left(y_t - y_{t-1}\hat{B}\right) C_{hol}^{-}$, where $\hat{B}$ is estimated by the simple OLS and C_{hol}^{-} is the inverse of the upper triangular matrix C_{hol} such that $\frac{1}{T}\sum_t \xi_t^T \xi_t = C_{hol}^T C_{hol}$. Generating $\check{\xi}_t^j$ by the standard normal distribution does not change the results quantitatively; as long as its variance is unchanged, the distribution of $\hat{\xi}_t$ has only negligible effects.

[13] However, similar discussions in this subsection also hold under the theories of buffer stock inventories and production smoothing (cost shock model).

[14] See Sims (1986), Leeper, Sims, and Zha (1996), Leeper, Sims, and Zha (2003) and Kim (1999),

shocks. We also assume that neither CPI nor material prices can respond to the current shocks to the three real variables. Because material price index is regarded as a leading indicator of CPI, it can respond to contemporaneous CPI shocks.

In this relation, there are two additional comments. First, in the stockout avoidance model, it is shown that the target level of inventories is an increasing function of demand.[15] This is the main reason why we use level data. The author personally believes the reason for tight estimations in this article is the use of level data; most existing inventory studies use inventory investment (time difference of the level of inventories). Second, Shibayama (2007) shows that, with a stockout constraint and a production chain, a rational dynamic stochastic general equilibrium model can generate such cycles at least potentially; this article, as an empirical counterpart, aims to find empirical evidence of damped oscillations.

3 Three-Variable VAR

This section describes the results of the three-variable VAR, in which production (output), shipment (sales) and inventories as well as the exogenous seasonal dummy variables and time trend are regressed. The three-variable VARs allow us to establish valid Monte Carlo simulations. Contrarily, in the six-variable VARs, there exist several pairs of complex roots similar to each other. Such roots are mixed with each other in some Monte Carlo experiments, which prevents us from tracking the behaviour of one specific pair of complex roots throughout the simulations.

3.1 Description of Details

3.1.1 Original Data

This section analyses the data of industrial production in Japan.[16] The data estimated in three-variable VAR are production (output), shipment (sales) and inventories. All

among others, for the opposing view.

[15] The same is also true in Bils and Kahn (2000).

[16] The data are available on the website of the Ministry of Economy Trade and Industry of Japan. http://www.meti.go.jp/english/statistics/index.html

of them are of "mining and manufacturing" (i.e. all sectors) from January 1978 to December 2006. All variables are the average of physical units of goods weighted by value-added in the baseline year. The data quality is thought to be extremely high, given the ministry's strong authority over Japanese manufacturers.

3.1.2 Data Format

(I) Benchmark Estimation (with Level Data) The benchmark estimation uses non-seasonally adjusted level data. It also includes seasonal dummies and a 5th-order polynomial time trend. The former and latter are included to eliminate seasonality and trend, respectively.

Polynomial Time Trend: The benchmark estimation includes the 5th-order polynomial of time. This time trend well mimics the HP-filter with smoothing parameter $\lambda_M = 130,000$.[17] Given the HP-filter's popularity, the HP-filtered series (the original series minus the HP-trend) is preferable in detecting cycles *recognised by practitioners*. While the HP-filter artificially causes an endogeneity problem (and spurious cycles), the exogenous 5th-order polynomial itself does not bias OLS estimates, and it eliminates almost the same trend as the HP-filter does. In our estimation, however, it is important to note that the estimated cycle length is very sensitive to the specification of the time trend (see Section 3.2.3).

Seasonal Dummy: In addition, the VAR estimation also includes the seasonal dummies. However, the fixed seasonal dummies cannot completely eliminate seasonality. Visually examining the plots of the fitted and actual data, it seems that seasonal fluctuation is growing over time.

(II) Estimation with HP-Filtered Seasonally Adjusted Data This estimation uses HP-s.a. data with $\lambda_M = 130,000$, which, by construction, are stationary. However,

[17] Numerical experiments, demonstrate that the smoothing parameter for monthly data, which is equivalent to $\lambda_Q = 1600$ for quarterly data, is slightly less than $\lambda_M = 130,000$. The rule of thumb $\lambda_M = 14,400$ generates a too well-fitted HP-trend series (i.e., not smooth enough). See Ravn and Uhlig (2002) for analytical discussions.

both the HP-filter and seasonal adjustment are essentially moving averages of past and future values, which implies that the residuals can be correlated to the regressors.

(III) Estimation with YoY Data This estimation uses YoY change data. If the original data are $I(1)$, then YoY data are stationary. The main problem with YoY data is that they could magnify the effect of noise.

3.1.3 Order Selection Criterion

For the level data (not seasonally adjusted), some information criteria suggest very long VAR orders (maximum time lag of endogenous variables), perhaps because the fixed seasonal dummy cannot perfectly eliminate the seasonality. Judging from the AIC and SIC of HP-s.a. and YoY estimations, it seems that the best VAR order is somewhere between 2 to 4. Hence, the VAR order in this article is always 3 to facilitate comparisons. Fortunately, the quantitative effect of changing the VAR order is not substantial for any of the following results (see below). Most estimates are quantitatively robust against changes in the VAR order.

3.1.4 Bootstrapping

The bootstrapping method is used to compute the standard deviations of estimates and confidence intervals. In addition, the standard deviations of period length and phase shifts are computed, as long as a cycle exists for all the trials in the bootstrapping.

3.2 Roots of Coefficient Matrix

3.2.1 Implied Cycles

The estimated conjugate pair $0.95 \pm 0.11i$ is the evidence that the endogenous variables follow a sine curve (see Table 1). These complex roots imply a cycle with 56.1 months long (s.d. = 2.6 months), which is near the post-war average in Japan (50.3 months).[18]

[18] In Japan, a governmental committee determines the business cycle dates. http://www.esri.cao.go.jp/en/stat/di/041112rdates.html

It is possible to compute the standard deviation of the cycle length because no trials in the bootstrapping experiments lack these complex roots.

The other three cycles are 2.7 to 4.4 months in length. One possibility is that they are evidence that the inventories work as buffers at very high frequencies (see Section 3.3). However, they may simply capture high-frequency noise and seasonality that cannot be perfectly eliminated by dummy variables.[19] In any event, it is difficult to establish their statistical significance, because they are often mixed with each other in the bootstrapping, and are therefore almost impossible to distinguish.

The estimated period length does not change considerably in the other two data sets: 55.6 months (s.d. = 5.4 months) in the HP-s.a. data, and 57.6 months (s.d. = 6.7 months) in the YoY data.

Table 1: Estimated business cycle roots (three-variable VARs with Japanese data).

Panel I: Level						
Roots	0.95±0.11i	0.6887	-0.35±0.45i	-0.28±0.28i	0.07±0.45i	
Norm	0.9541	0.6887	0.5723	0.4014	0.4555	
Angle	±0.0357	0	±0.7081	±0.7503	±0.4502	
Cycle length	56.05	+inf	2.82	2.67	4.44	
Panel II: HP-s.a.						
Roots	0.96±0.11i	0.64807	-0.34±0.29i	-0.10±0.38i	-0.1429	0.047281
Norm	0.9704	0.64807	0.4493	0.3944	0.1429	0.047281
Angle	±0.0359	0	±0.7783	±0.5816	0	0
Cycle length	55.64	+inf	2.57	3.44	+inf	+inf
Panel III: YoY						
Roots	0.96±0.11i	0.82106	-0.31±0.44i	-0.23±0.23i	0.2686	-0.24244
Norm	0.9651	0.8211	0.5423	0.3228	0.2686	0.2424
Angle	±0.0347	0	±0.6949	±0.7526	0	0
Cycle length	57.63	+inf	2.88	2.66	+inf	+inf

Notes: The norm of a root is its absolute value. The angle of a root is the angle between its real and imaginary parts, which is equivalent to the frequency of the sine curve that is generated by the root. See section 2.1 for other terminologies.

3.2.2 Phase Shifts

With respect to the business cycle roots detected in the level data, the peaks and troughs of inventories lag behind those of production and shipment by 12.4 and 12.1 months, respectively. As expected, the phase shift between production/shipment and inventories is close to 1/4 of the period length. There is almost no time lag between production and shipment.

[19] Just having complex roots itself is not very interesting at all. It is important to have complex roots that correspond to the business cycle.

Table 2: Phase Shifts in Business Cycle

(Cycle length)		Sales	Inventories
Level	(56.1)	-0.3146 mths	12.416 mths
HP-s.a.	(55.6)	0.2110 mths	13.527 mths
YoY	(57.6)	-0.4733 mths	14.209 mths

Note: Time-lags from production.

3.2.3 Effect of Time Trend

In most specifications of the time trend, the VAR estimation detects one significant pair of business cycle complex roots. However, the estimated cycle length crucially depends on the choice of time trend, while the effect of the VAR order is not very strong. For example, the VAR with a linear time trend shows that the length of one business cycle is 168.5 months (see Table 3). This means that the estimated cycle length with the level data is not robust against the specification changes of time trend, while the phase shift between production/shipment and inventories is almost always close to 1/4 of the business cycle's length. In addition, the specification of the time trend affects the norm of the largest *real* root (see the next subsection).

Table 3: Length of Estimated Business Cycle

Time Poly. Order:	1	2	3	4	5	6	8	10
VAR(2)	206.6	109.4	98.35	75.78	59.88	60.14	58.32	53.35
VAR(3)	168.5	102.5	90.44	69.67	56.05	56.79	55.01	50.28
VAR(4)	153.4	100.3	94.71	72.78	57.11	57.59	55.25	51.00

Note: Estimation based on the level data.

3.2.4 Effect of Unit Root

In the level data estimation, we cannot rule out the possibility that the real root (0.6887) in the level data is a unit root. Certainly, 0.6887 appears to be far from +1, but the norm of this root is strongly affected by the time trend; as the order of the time trend polynomial decreases, the norm moves towards +1. At limit, the hypothesis that there is one real unit root is not rejected under the maintained hypothesis that there is no time trend in the true DGP.

However, these Monte Carlo experiments show that the existence of the real unit root only slightly affects the cycle length and phase shifts. Figures 4 and 5 show the selected distributions under the maintained hypotheses, that there is one *real* unit root

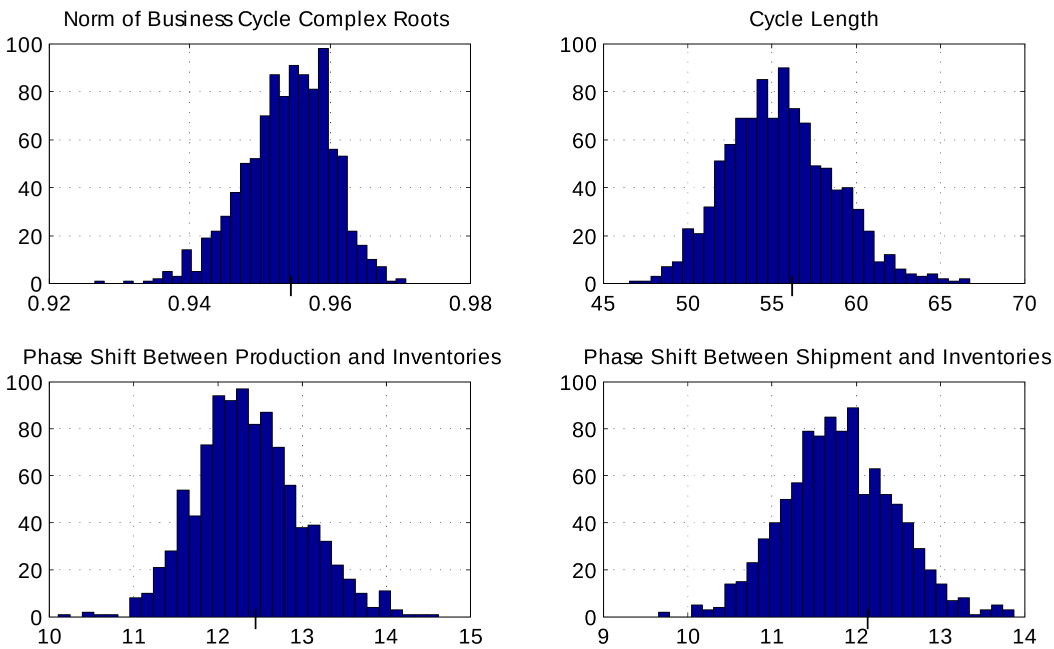

Figure 4: Distributions generated by 1,000 trials. H_M: There is one real unit root. Ticks on the x-axis show the true value in $H_{M.}$

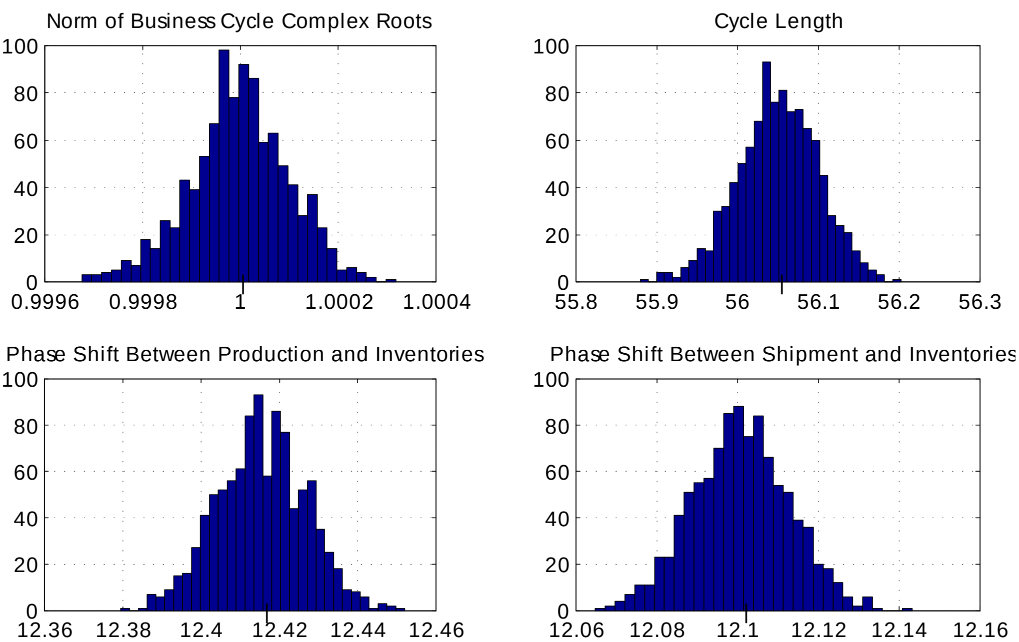

Figure 5: Distributions generated by 1,000 trials. H_M: There is one pair of complex unit roots. Ticks on the x-axis show the true value in $H_{M.}$

and that there is *one pair of unit complex roots*, respectively. Both experiments assume that the true DGP has the 5th-order polynomial time trend. These results show that the estimates are very precise and the distributions are skewed only slightly. For example, the upper-right panel of Figure 4 shows that the distribution of the cycle length centres on 55 months, which is very close to the true value in the DGP (56.1 months, as denoted by "|" on the x-axis). Also, the top-left panel in Figure 5 suggests that the absolute value (norm) of the estimated business cycle complex roots (0.9541) is far enough from +1. Even though the true DGP is assumed to have no time trend, the same exercise still suggests that the business cycle complex roots are not unit roots.

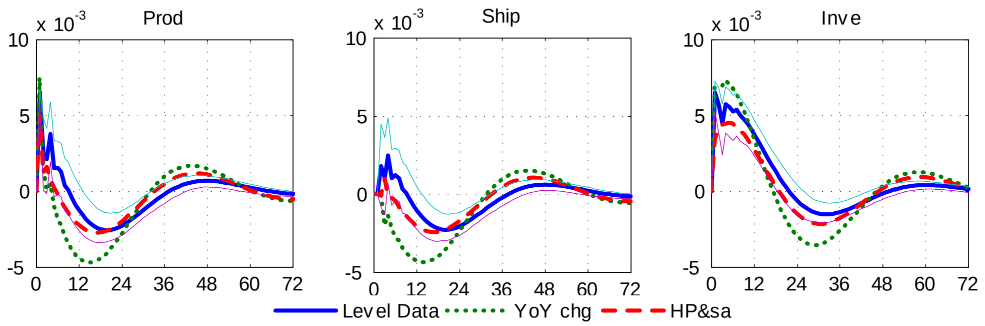

Figure 6: IRFs due to a positive shock in the production equation. Narrow lines show the 95% confidence intervals of level data estimations based on the bootstrapping method.

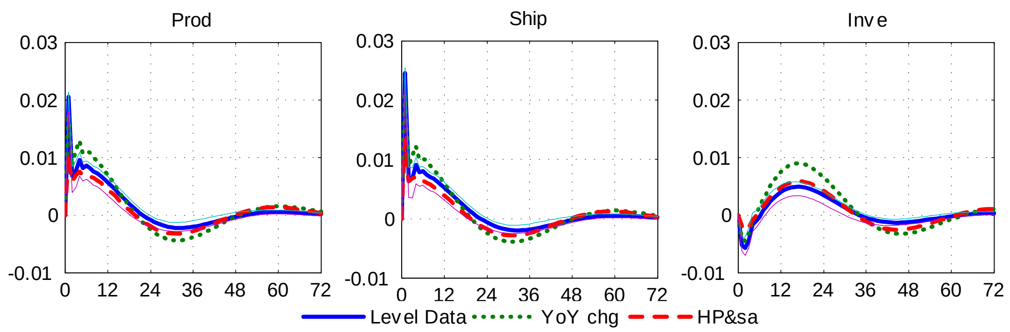

Figure 7: IRFs due to a positive shock in the shipment equation. Narrow lines show the 95% confidence intervals of level data estimations based on the bootstrapping method.

3.3 Impulse Response Functions

Clearly, all of the impulse response functions show the shape of sine curve fluctuations. Visually reviewing the distance between two peaks in each IRF, we can see that the length of one cycle is roughly 56 months, almost same length implied by the business cycle complex roots.

Technology Shock: Figure 6 shows impulse responses to a production shock, which can be regarded as a technology or supply shock. After a positive shock, both production and shipment increase. Inventories increase due to the law of motion of inventories (2). Sales do not increase as much as production does; hence, for production shocks, output is more volatile than sales. This corresponds with the theory of cost shock models.[20]

However, more importantly, production returns to zero roughly 9 months after the shock. The effects of a positive production shock disappear quickly. This is because a positive production shock induces an increase in inventories[21] − but, because having excess inventories is costly for firms, they want to reduce such excess inventories by cutting production.

Demand Shock: On the other hand, Figure 7 shows that after a positive sales shock, which can be regarded as a demand shock, production stays above zero for more than 20 months. Right after a positive demand shock, inventories decrease due to the law of motion of inventories (2).[22] However, such a level of inventories is too low, and firms want to increase their production in order to recover their inventories. Also, note that the initial impacts of a demand shock are much larger than those of a supply shock (compare the units of the y-axes).

Indeed, we can draw more implications. In the theoretical literature, the stockout

[20] Cost shock models in the inventory literature emphasise the effect of production cost. The idea is that because the source of shock lies on the production side, production is more volatile than sales. In addition, inventory investment increases when production increases due to a low cost shock (procyclical inventory investment).

[21] In phase diagrams such as Figures 1 and 2, starting from the origin, a positive supply shock is plotted as a jump to the northeast of the origin.

[22] In phase diagrams such as Figures 1 and 2, starting from the origin, a positive demand shock is plotted as a jump to the northwest of the origin.

avoidance model suggests that the target level of inventories is an increasing function of sales. Thus, after a positive demand shock, firms want not only to replenish their reduced inventories, but also to raise the level of inventories so that it meets the new, higher level of sales. Actually, the subsequent increase in production is slightly larger than that of sales (otherwise, inventories would decrease). As a result, even though the source of the shock is on the demand side, output is more volatile than sales. In the sense that demand shocks are magnified by inventories, inventories are regarded as *destabilising factors in business cycle frequencies.*

In contrast, while inventories drop sharply right after a positive demand shock, more than half of the initial effect of the shock on production and shipment disappears within one period. This shows that inventories work as buffers in a very short time period. In this sense, production smoothing theory is still very much alive at very high frequencies.[23]

These findings can be summarised as follows. Inventories are destabilising factors at business cycle frequencies but are stabilising factors at very high frequencies. This view is in line with Wen (2002).

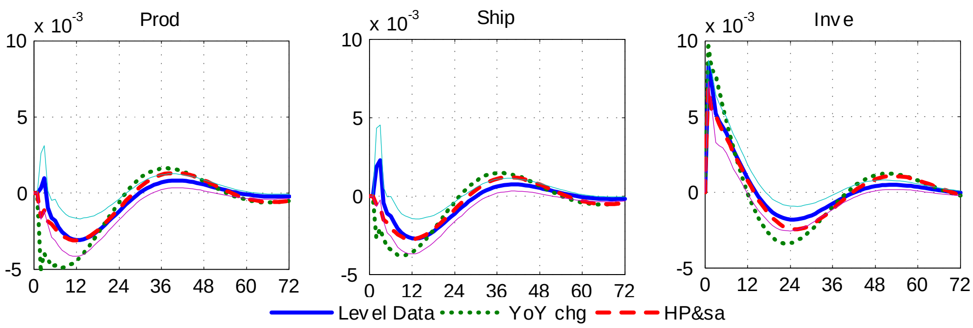

Figure 8: IRFs due to a positive shock in the inventory equation. Narrow lines show the 95% confidence intervals of level data estimations based on the bootstrapping method.

Inventory Shock: After a positive shock to the inventory equation, both sales and production decline (Figure 8). In a sense, a shortage of inventories is akin to an increase in demand, and vice versa, because firms have an incentive to replenish (or cut) them

[23] Originally, inventory literature started with the production smoothing theory, which says that firms have an incentive to smooth the time-path of production due to a convex cost function; firms hold inventories to protect themselves from unexpected demand shocks.

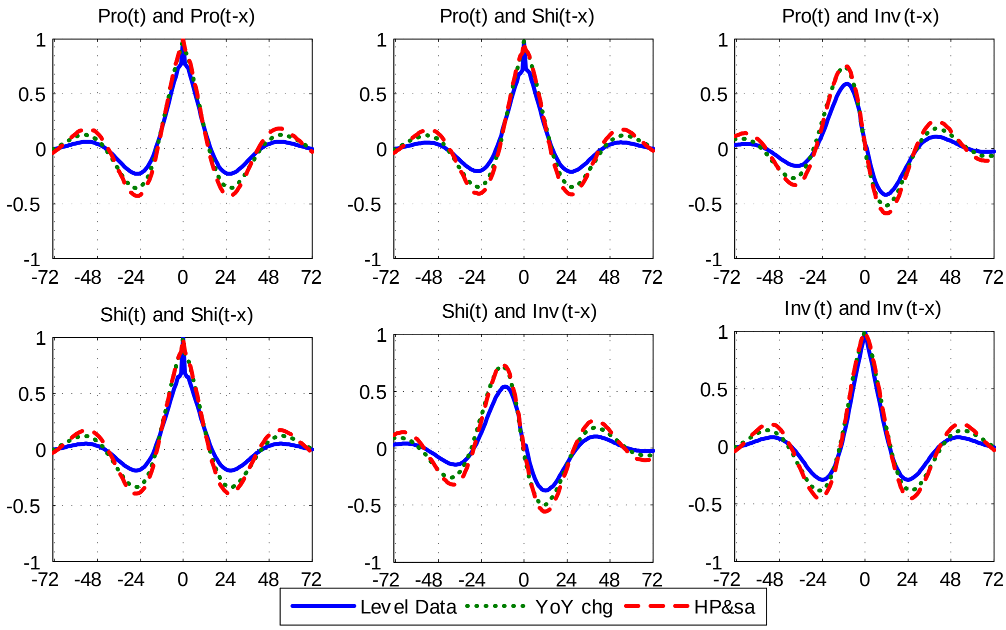

Figure 9: Cross correlations.

to their normal level.

3.4 Cross Correlations and Spectral Densities

The cross correlations and spectra are computed from the estimated coefficients in equation (1).[24] Note that, with non-stationary processes, neither is well defined; thus, we should focus on the cross correlations and spectra in the HP-s.a. and YoY data sets. Nonetheless, the results in the benchmark data quite markedly resemble those based on the two stationary data sets. Both cross correlations and spectral densities show that (a) there is a cycle with business cycle frequency, and (b) the contemporaneous correlation fails to capture the dynamic relationship among variables.

3.4.1 Cross Correlations

The cross correlations (Figure 9) show several observations worth mentioning. First, the cross correlation between production/shipment and inventories reaches its peak and bottom when the time lag is around ±12 months, which is consistent with the estimated

[24] See Chapter 10 of Hamilton (1994); however, the phase shifts are computed in this chapter in a different way (See Appendix B).

phase shift. Second, the contemporaneous correlation between production/shipment and inventories is close to zero; thus, the contemporaneous correlation alone cannot capture their dynamic relationship. Third, the autocorrelations reach their bottom around ±25 months, implying that the dominant cycle is around 50 months in length ($= 25 \times 2$), which is not very different from the finding in Section 3.2.1 (see also Appendix A.4). Fourth, the spikes in autocorrelations of production and shipment at 0 month imply a very high frequency component that affects both production and shipment. This is indirect evidence of buffer inventory models (see also Figure 7).

3.4.2 Spectral Densities

The spectral densities[25] (Figure 10) show several observations worth mentioning. First, all the cospectra and quadrature spectra reach their peaks or bottoms at around 56 months, which again implies that the cyclical component with a period length of around 56 months, is most influential. Second, the cospectrum between production/shipment and inventories is almost zero for all period lengths, which implies that the contemporaneous covariance cannot capture their dynamic relationship in any frequency. However, the existence of a dynamic relationship between production/shipment and inventories is evident in their quadrature spectra. Finally, the quadrature spectrum between production and shipment is almost zero for all period lengths, which means that there is almost no time lag between them.

[25] It may be worth reviewing the two spectral densities for multiple variables.

First, a cospectrum has the same meaning as a spectrum with one variable. For the components of cross covariances reflected in contemporaneous covariance, a cospectrum attributes such components to each frequency. For example, if the absolute value of a cospectrum density reaches its peak at frequency f, it implies that the cycle with frequency f makes the largest contribution to the contemporaneous covariance. The integral of cospectral densities over the whole frequency domain $0 \leq f \leq 2\pi$ is equal to the contemporaneous covariance.

Second, a quadrature spectrum essentially represents anything other than the corresponding cospectrum. For the components of cross covariances *not* reflected in contemporaneous covariance, a quadrature spectrum attributes such components to each frequency. For example, if the absolute value of a quadrature spectrum density reaches its peak at frequency f, it implies that the cycle with frequency f makes the largest contribution to the cross covariance with a time lag of $\pi/2f$ periods (1/4 of the period length $2\pi/f$). Remember that if two variables follow a sine curve, and the phase shift between them is 1/4 of the period length, then the contemporaneous correlation of these two variables is zero, even though both follow essentially the same process. In other words, a quadrature spectrum represents the relationship that is *not* reflected in contemporaneous covariance due to phase shift. The integral of quadrature spectral densities over $0 \leq f \leq 2\pi$ is equal to zero.

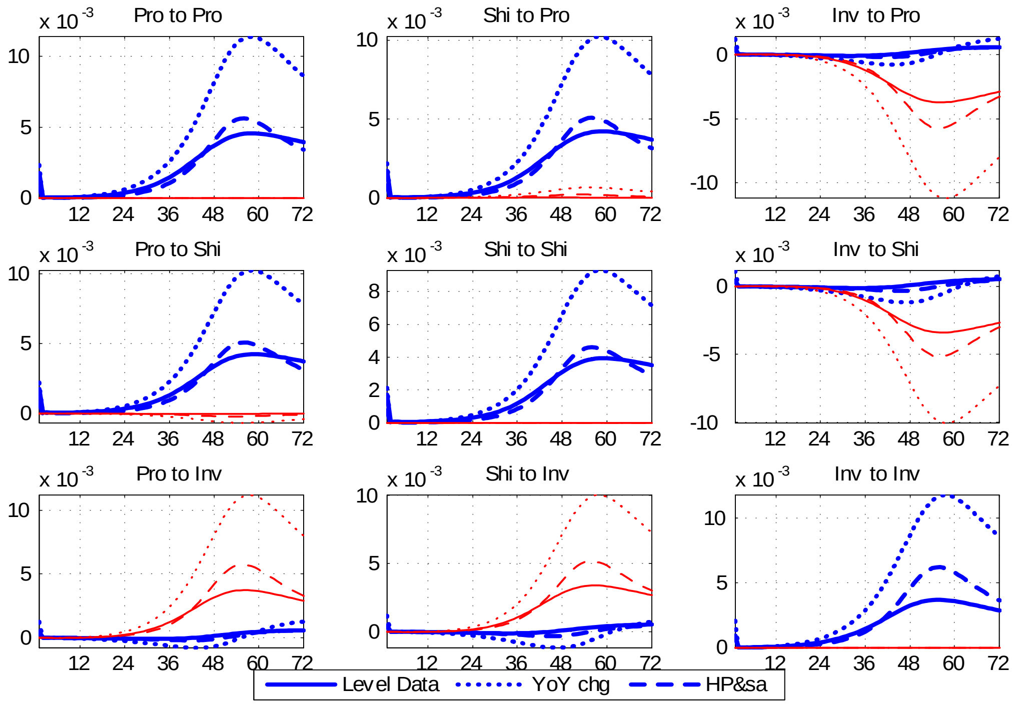

Figure 10: Co- and quadrature spectra. Bold lines show cospectra and narrow lines show quadrature spectra.

4 Six-Variable VAR

This section describes the results of the six-variable VAR estimations, to investigate the interaction between monetary policy and inventories.

4.1 Description of Details

Original Data Though the BoJ's direct policy instrument is the *un*collateralised O/N call rate (and excess reserves under the zero-interest rate policy), its data length is short. Hence, the collateralised O/N call rate, which exhibits movements quite similar to those of the uncollateralised O/N call rate, is adopted in this analysis.[26] For the Consumer Price Index (CPI), the general (overall) index excluding fresh foods and imputed rents is used,[27] while the material price index in the Corporate Goods Price Index (CGPI) is included as a leading inflation indicator.[28] To avoid zero-interest rate periods, the

[26]See "How to Download Long-Term Time-Series Data Files" on http://www.boj.or.jp/en/theme/research/stat/market/short_mk/tanki_rate/index.htm
[27]See http://www.stat.go.jp/english/data/cpi/index.htm
[28]See "Index by Stage of Demand and Use" on

estimation period is from January 1978 to December 1998.

In the HP-s.a. data set, following convention, the O/N call rate and CGPI are not seasonally adjusted. In the YoY data set, the YoY change in the O/N rate is used, although it is presumably stationary.

4.2 Roots of Coefficient Matrix

Each estimation finds two or three pairs of complex roots that correspond to the business cycle. Selected point estimates of the roots are shown in Table 4. Roots omitted from the table are complex roots with very high frequencies (i.e., shorter than 6 months).

Table 4: Estimated business cycle roots (six-variable VARs with Japanese data).

Panel I: Level						
Roots	0.96±0.10i	0.82±0.12i	0.9139	0.6127	0.5026	-0.3693
Norm	0.9640	0.8287	0.9139	0.6127	0.5026	0.3693
Angle	±0.0316	±0.0461	0	0	0	0
Cycle length	63.2	43.4	+inf	+inf	+inf	+inf
Panel II: HP-s.a.						
Roots	0.97±0.11i	0.85±0.09i	0.80±0.01i	0.40±0.25i		
Norm	0.9755	0.8545	0.8010	0.4735		
Angle	±0.0352	±0.0339	±0.0036	±0.1746		
Cycle length	56.8	59.0	552.8	11.5		
Panel III: Year-on-Year						
Roots	0.96±0.11i	0.92±0.10i	0.9741	0.7934	0.50964	-0.3746
Norm	0.9655	0.9270	0.9741	0.7934	0.50964	0.3746
Angle	±0.0376	±0.0361	0	0	0	0
Cycle length	53.2	55.4	+inf	+inf	+inf	+inf

Note: See Table 1 for notes.

The roots in the second column, at first glance, may seem to indicate one identical cycle, but the point estimates of the phase shifts differ considerably among the three data sets. On the other hand, the phase shifts of the largest norm roots are consistent among the three data sets (except for CPI in the YoY estimation), and are compatible with those in the three-variable estimations. In addition, none of the other roots is robust against a change in the VAR order. Overall, it is concluded that there exists one business cycle pair of complex roots (perhaps the same cycle as in the three-variable estimations) in the six-variable estimations. This conclusion is also supported by the cross correlation and spectrum analysis below.

http://www.boj.or.jp/en/theme/research/stat/pi/cgpi/index.htm#04

Compared to the three-variable estimations, the cycle length now becomes longer in the level data estimation (63.2 months), while it becomes shorter in the YoY data estimation (53.2 months).

Table 5: Estimated phase shifts (six-variable VARs with Japanese data).

unit: months	(Cycle length)	Shipment	Inventories	O/N call	CPI	Com. Price
Level data	(63.2)	0.2828	11.655	3.2501	-13.619	-6.8463
	(43.4)	2.4008	6.4567	4.7944	9.3550	-3.3411
HP-s.a.	(56.8)	-0.0434	12.660	4.4041	-13.348	-2.4408
	(59.0)	7.1328	8.5986	7.2003	11.619	6.9404
YoY	(53.2)	-0.0095	12.824	1.9726	12.267	-4.1758
	(55.4)	0.0013	11.386	-8.7080	0.5821	11.306

Note: Time-lags from production.

4.2.1 Phase Shifts

The O/N call rate lags behind production by 3.3 months in the level data, suggesting that the BoJ reacts to real variables fairly quickly.[29] However, it is not forward-looking; perhaps good monetary policy would anticipate the cyclical patterns of economic variables, given the long time lag before the effects of monetary policy are realised (shown below). Indeed, it seems that the Fed's monetary policy anticipates such cyclical patterns (see Appendix).

4.3 Impulse Response Functions

One caveat of the six-variable analysis is the price puzzle.[30] In other respects, however, the estimation results are consistent with theoretical predictions.

Supply vs. Demand Shocks: Monetary policy is tightened after a positive shipment (demand) shock (Figure 12), while its response to a positive production (supply) shock is ambiguous (Figure 11). Indeed, following a positive supply shock, although the response is not estimated tightly, the point estimates of all three IRFs show that the BoJ loosens its monetary policy. Considering the behaviours of other IRFs, this is because (i) a boom

[29] It is important to note that phase shifts do not indicate the speed of responses to *shocks*. Instead, for example, we can interpret the phase shift between the O/N call rate and an endogenous variable as a speed of the BoJ's response to the *cyclical component* of that endogenous variable.

[30] See Sugihara et al (2000), Teruyama (2001) and Yoshikawa et al (1993). Almost all versions in these studies show temporal price increases after a tight monetary policy shock in Japanese data.

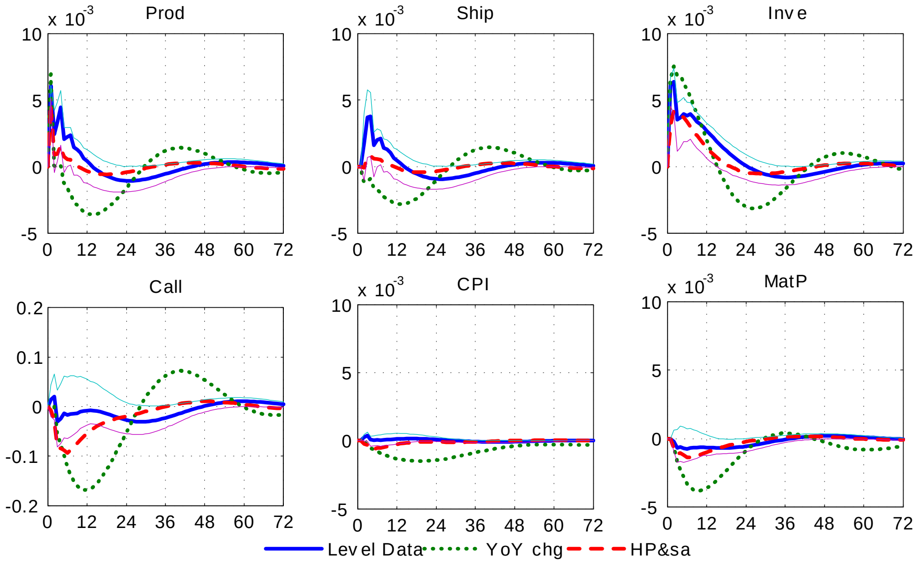

Figure 11: IRFs due to a positive shock in the production equation. Narrow lines show the 95% confidence intervals of level data estimations based on the bootstrapping method.

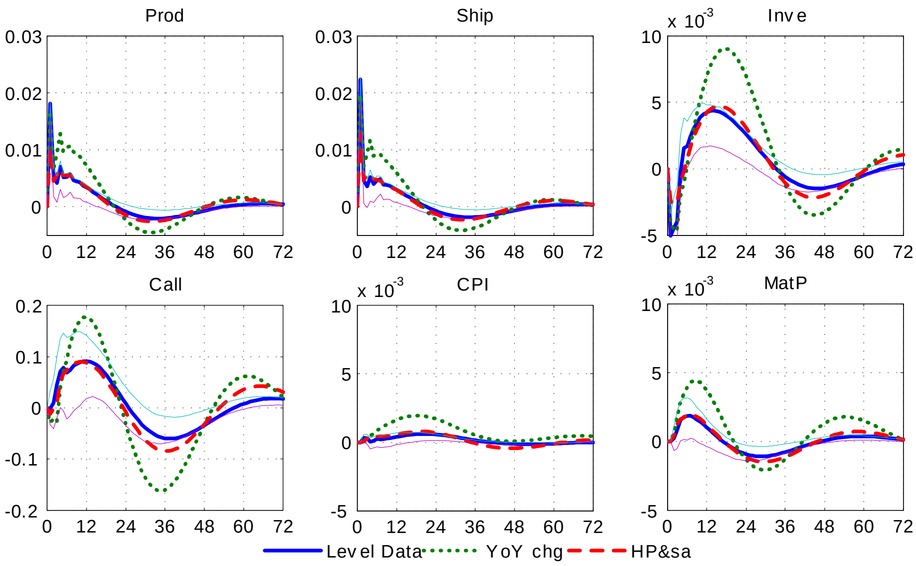

Figure 12: IRFs due to a positive shock in the shipment equation. Narrow lines show the 95% confidence intervals of level data estimations based on the bootstrapping method.

lasts longer after a positive demand shock than after a positive supply shock, and (ii) the leading inflation indicator and CPI increase after a positive demand shock but not after a positive supply shock. Hence, it is important to discriminate between demand and supply shocks, in order to analyse monetary policy.

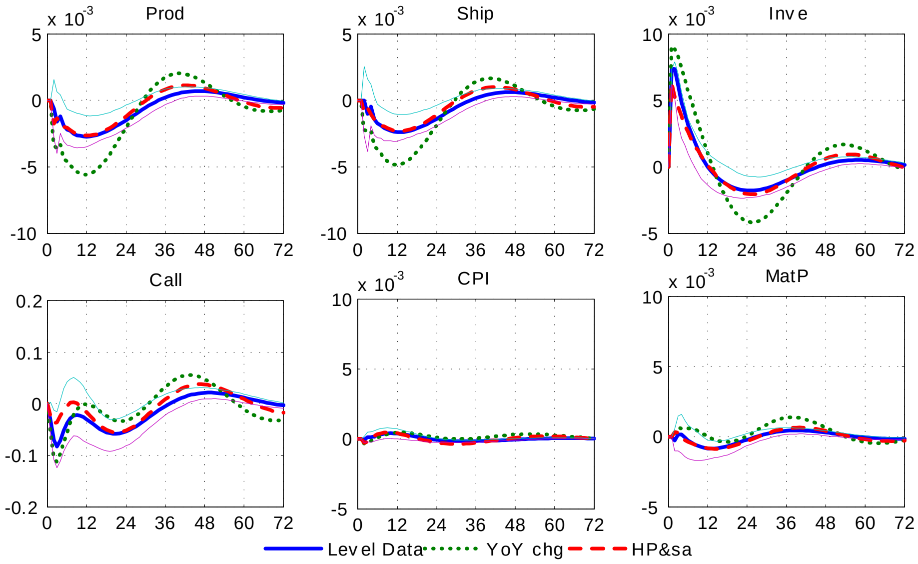

Figure 13: IRFs due to a positive shock in the inventory equation. Narrow lines show the 95% confidence intervals of level data estimations based on the bootstrapping method.

Inventory Shock: A positive deviation of inventories from the steady state is akin to a negative demand shock (Figure 13). As a result, the O/N call rate declines after a positive inventory shock.

Price Shocks: The O/N call rate increases after a positive material price shock, but decreases after a positive CPI shock. These patterns seem to reflect the features of the BoJ's monetary policy.

On one hand, after a positive CPI shock, both the O/N call rate and production decline, possibly because the major CPI shocks tend to arise from increases in public prices and energy prices in Japan.[31] In other words, large CPI shocks are often regarded as exogenous negative shocks; indeed, production and shipment decline after a positive CPI shock.

[31] The effects of the changes in VAT rate on CPI are adjusted in our data.

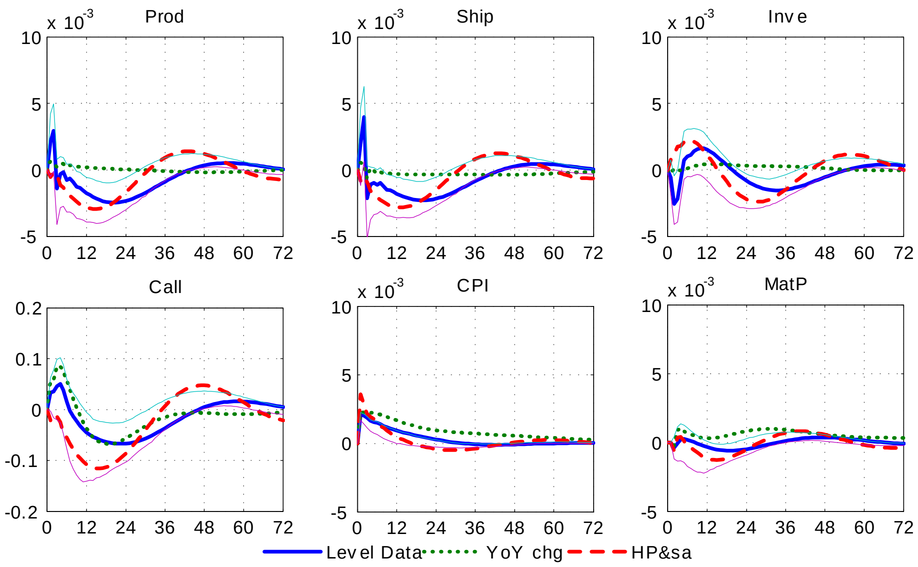

Figure 14: IRFs due to a positive shock in the CPI equation. Narrow lines show the 95% confidence intervals of level data estimations based on the bootstrapping method.

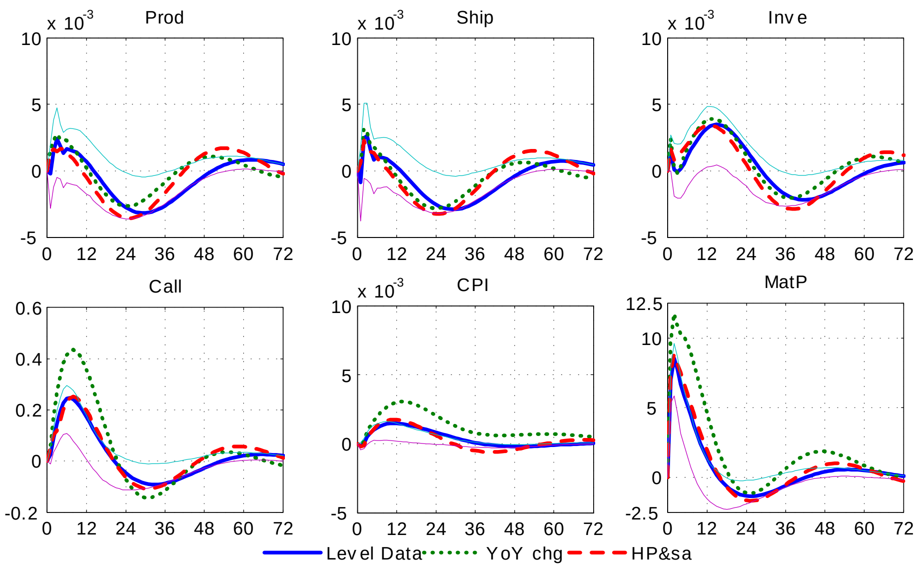

Figure 15: IRFs due to a positive shock in the leading inflation indicator equation. Narrow lines show the 95% confidence intervals of level data estimations based on the bootstrapping method.

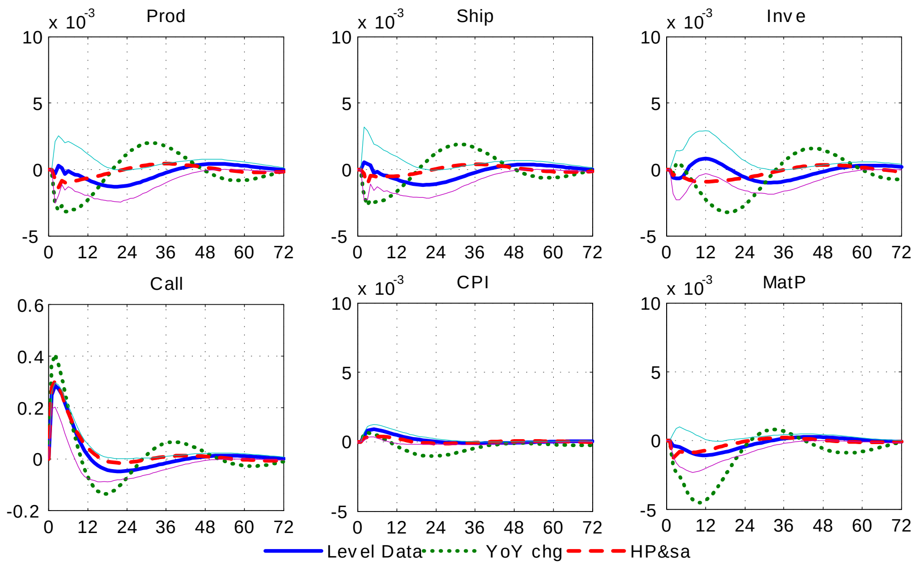

Figure 16: IRFs due to a positive shock in the O/N call rate equation. Narrow lines show the 95% confidence intervals of level data estimations based on the bootstrapping method.

On the other hand, the BoJ tends to focus on leading inflation indicators, while CPI is often considered as a lagging indicator. Moreover, the BoJ traditionally has been concerned with the exchange rate. Because exports are the growth engine of the Japanese economy (though this situation is changing), a strong yen, which reduces the exporters' profit margins and competitiveness, has been considered something that the central bank has to defeat. Hence, the BoJ's reaction to the leading inflation indicator may represent its reaction to the exchange rate; a strong yen implies low import prices (especially on raw materials), and is followed by an expansionary monetary policy.

Call Rate Shock: The effects of O/N call rate shocks (monetary policy shocks) on production, shipment, and inventories are unclear and mixed. In the level data estimation, production and shipment decline several periods after a positive call rate shock, although they decline right after the shock in the HP-s.a. and YoY data. Existing studies find a long time lag before the effects of monetary policy materialise.[32]

Bils and Kahn (2000) find that the inventory investment is positively correlated to the interest rate; this is considered a puzzle because a high interest rate gives rise to a

[32] See Bernanke and Gertler (1995) and Christiano, Eichenbaum, and Evans (1999) among others.

high inventory carry cost. There is one natural way to address this puzzle; if demand decreases sharply while production cannot adjust quickly, firms are "forced" to accumulate inventories due to the law of motion of inventories (2). However, VAR estimations show no substantial differences between the IRFs of production and shipment.

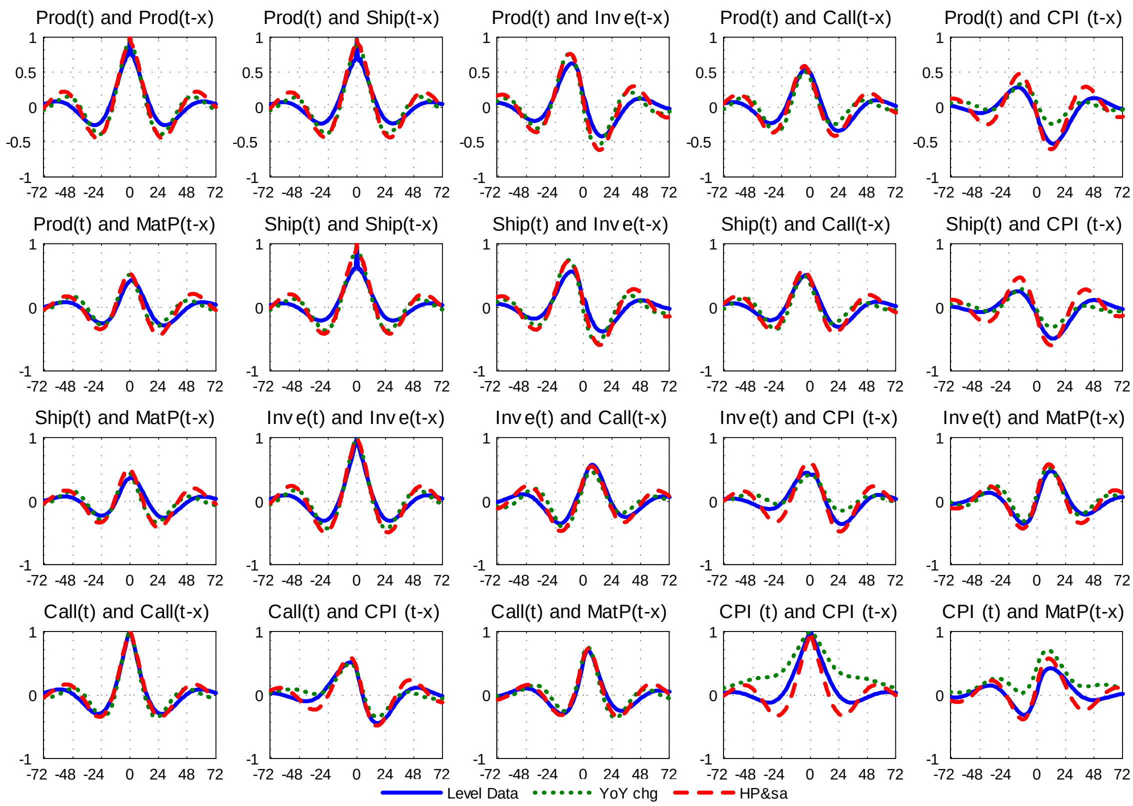

Figure 17: Cross correlations.

4.4 Spectral Analysis and Cross Correlations

Cross correlations and spectra confirm the findings discussed above. First, the cross correlation between the O/N rate and production/shipment (and material price) reaches the peak with a 2 to 4 months lag. This is consistent with the estimated phase shifts between them. Second, as quadrature spectra suggest, there are dynamic relationships between the O/N rate and other variables that are not reflected in the contemporaneous correlations. Finally, most of the co- and quadrature spectra reach their peaks or bottoms

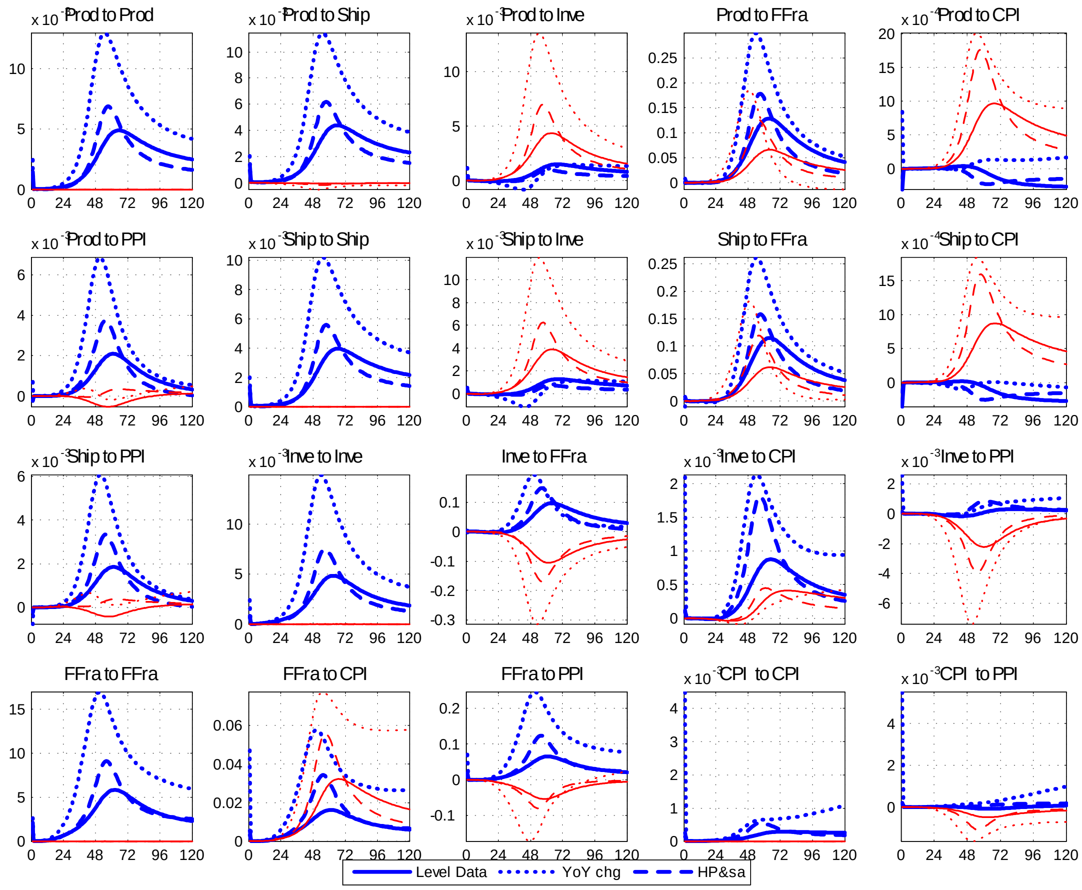

Figure 18: Spectrum densities. Bold lines show cospectra and narrow lines show quadrature spectra.

at around 53 to 63 months, implying that the cycle with a period length of 53 to 63 months is the most important cyclical factor.

5 Conclusion

To study inventory cycles (see Figures 1 and 2), a number of VARs (equation (1)) are estimated in this article. The key feature of our estimations is that we focus on the level of inventories, rather than inventory investment, because it is the level of inventories that plays a key role in the theoretical literature on inventories. To check the robustness of our estimation results, we use three data formats: level data with polynomial time trend, HP-filtered seasonally adjusted data and year-on-year change data. There may be weaknesses with any of the data formats. For example, the level data set causes the problem of non-stationarity and the estimated cycle length is quite sensitive to the specification of the time trend. Also, there is a possibility of spurious cycles from employing HP-filtered data. Our strategy is, however, rather than tackling these econometric issues directly and individually, to compare these three different data choices in order to check how severely estimations are distorted. Indeed, these data specification generate results quite similar to each other, which suggests that the econometric problems in each data format do not distort the estimation results very severely. Paying due heed to these issues, we can summarise our finding as follows.

First, in terms of periodicities, our estimations find a pair of complex roots, and these roots generate cycles of around 55 to 70 months, which are quite close to actual business cycle lengths. This implies that production and inventories follow damped oscillations (stable sine curves), implying that a boom sows the seed of the following recession, and vice versa. Also, the estimated phase shift (time lag) between inventories and production is close to 1/4 of the cycle length, which has two implications. Firstly, inventories have information for near-future economic conditions; for example, in Japan, if inventories have bottomed out, then production will peak around 16 to 20 months later. Secondly, examining only contemporaneous variance-covariance matrices may fail to capture economic dynamics; indeed, the contemporaneous covariance between production and inventories is close to zero in spite of their close dynamic relationship. The monetary policy is another well-known example; its effects on prices and real variables materialise after a long time lag.

Second, related to the theoretical literature on inventories, we find the evidence that supports the following three leading inventory theories: buffer stock inventories, cost shock mechanism and stockout constraint model. In addition, we find that (a) Inventories stabilise an economy, working as buffers against demand shocks *at very high frequencies*, while they amplify demand shocks *at business cycle frequencies*, and (b) demand shocks are much more important than supply shocks. For the latter, this is not only because the initial impact of demand shocks seems to be larger than that of supply shocks, but also because booms after positive demand shocks last longer than those after positive supply shocks. The impulse response functions show that inventories increase after positive supply shocks, which induces firms to cut their production to adjust their inventory levels, while they decrease after positive demand shocks, which encourages firms to produce more. Note that, according to the stockout constraint model, with a positive demand shock, the desired level of inventories becomes higher, and thus firms not only replenish inventories reduced by a positive demand shock, but also accumulate their inventories further to catch up with a higher demand level.[33]

Third, we find that monetary policy is forward-looking in the U.S., but not in Japan. Specifically, the phase shift between production and policy interest rate shows that the peaks and bottoms of Fed funds rate *precede* those of production by 4 to 8 months in the U.S., while overnight call rate *lags behind* production by 2 to 4 months in Japan. Note that the phase shift is only related to the *cyclical components* (or predictable components) of economic activity. On the other hand, the central banks in both countries respond to *news* (innovations on shocks) reasonably quickly. In addition, both central banks sharply react to demand shocks, but not to supply shocks, because, as mentioned before, booms after positive demand shocks tend to last longer than those after positive supply shocks.

Finally, we would like to emphasise the importance of inventories in business cycle research. This is not only because inventories may generate cycles, but also because inventories may help disentangle demand and supply shocks. The results reported in this article hence suggest that, in understanding monetary policy, it may be essential to analyse inventories explicitly.

[33] Note that the results in this paragraph critically depend on the identification assumption (see Section 2.4). Also, note that the difference in the length of booms between supply and demand shocks is observed only in Japanese data, but not in the U.S. data (see Appendix A.3).

Appendix

A Six-Variable VAR with U.S. Data

This section describes the estimation results of the six-variable VARs with the U.S. data. The main problems with the U.S. data are (a) the pool of surveyed firms and survey methods are perhaps different between production and shipment/inventories because they are provided by different institutions, (b) the quality of real inventory data is not very good, and (c) data of real inventory before seasonal adjustment is not available. Compared to Japanese data, the estimations with the U.S. data are less precise. In addition, a couple of IRFs are not consistent among the (i) level, (ii) HP-s.a. and (iii) YoY data sets.

Nonetheless, we find that (1) one pair of complex roots exists, and the implied cycle length is fairly close to the post-WWII average, (2) inventories lag behind production/shipment by 1/5 to 1/6 of the business cycle length, and (3) the Fed reacts to supply shocks less sharply than to demand shocks. However, unlike the estimations for Japan, the last finding is not very clear. In addition, the lifespans of booms due to a positive demand and supply shocks are almost the same in the U.S. estimation, and the behaviours of inventories are not very different in response to those two types of shocks.

A.1 Description of Details

Original Data All data are monthly data from January 1978 to December 1998. Although more data are available for the United States, the same period used in the Japanese estimations is used here for the sake of comparison (expanding the data period makes the estimation more precise, but only slightly). Although production data are compiled by the Board of Governors of the Federal Reserve System,[34] real shipment and inventory data are estimated by the U.S. Bureau of Economic Analysis.[35] The latter

[34] U.S. production data are available at http://www.federalreserve.gov/releases/G17/

[35] Shipment and inventory data in nominal terms are available from the U.S. Census Bureau: http://www.census.gov/indicator/www/m3/hist/naicshist.htm

are, as building blocks, compiled to estimate U.S. national income (GDP), and "their quality is significantly less than that of the higher level aggregates," according to the Bureau. Shipment and inventories are of "manufacturing" (not including trading sectors) for comparison. As a monetary policy indicator, the effective monthly Fed funds rate (FF rate) is used.[36] Inflation is measured by the Consumer Price Index for All Urban Consumers (CPI-U) excluding food and energy, while PPI (raw materials) is used as a leading inflation indicator.[37]

Data Formats Again, there are three data sets: (i) level, (ii) HP-s.a. and (iii) YoY data. All estimations are based on equation (1) with order 3. The estimation with the level data uses the 5th-order time trend without seasonal dummies because only seasonally adjusted real shipment and inventories are available. For all three data sets, CPI-U is seasonally adjusted for simplicity, while FF rate and PPI (raw materials) are not, because the latter two are not considered to have seasonality.

Unit Root For the three-variable VAR with the level data, Monte Carlo experiments again suggest that there exists one real (not complex) unit root in the U.S. data set (the results are omitted). The results based on the stationary data sets (HP-s.a. and YoY data) are relatively similar to those based on the level data, though such similarities are not as strong as in the Japanese estimations.

A.2 Roots of Coefficient Matrix

Selected point estimates of the roots are shown in Table 6. Roots omitted from the table are complex roots with very high frequencies (shorter than 8 months) and some short real roots.

For the estimations of real shipment and inventories, see Herman, Donahoe, and Hinrichs (1976). For data, see the website of the Bureau of Economic Analysis:
http://www.bea.gov/national/nipaweb/nipa_underlying/SelectTable.asp

[36] See the Fed's website: http://www.federalreserve.gov/Releases/H15/data.htm

[37] Both are available at http://www.bls.gov/home.htm

Table 6: Estimated business cycle roots (six-variable VARs with U.S. data).

Panel I: Level

Roots	0.93±0.09i	0.88±0.06i	0.73±0.08i	0.43±0.37i	0.8051	-0.5162
Norm	0.9374	0.8870	0.7307	0.5622	0.8051	0.5162
Angle	±0.0291	±0.0219	±0.0365	0.2261	0	0
Cycle length	68.52	91.15	54.83	8.85	+inf	+inf

Panel II: HP-s.a.

Roots	0.95±0.11i	0.77±0.03i	0.74±0.11i	0.43±0.30i	0.9511	-0.4983
Norm	0.9443	0.7667	0.7454	0.5221	0.9511	0.4983
Angle	±0.0363	±0.0109	±0.0455	±0.1970	0	0
Cycle length	55.12	182.80	43.93	10.15	+inf	+inf

Panel III: YoY

Roots	0.93±0.13i	0.9084	0.8128	0.7677	0.7323	0.52±0.39i	0.9855	-0.5763
Norm	0.9376	0.9084	0.8128	0.7677	0.7323	0.6445	0.9855	0.5763
Angle	±0.0428	0	0	0	0	±0.2047	0	0
Cycle length	46.73	+inf	+inf	+inf	+inf	9.77	+inf	+inf

Note: See Table 1 for notes.

There are many conjugate pairs of complex roots that correspond to long cycles, but only the first pair in each panel seems to be robust against a change in the VAR order. For this cycle, phase shifts are consistent among all three data sets. In addition, cross correlations and spectra also show that the dominant cycle is 47 to 69 months in length, which is close to the post-war average (67 months).[38]

A.2.1 Phase Shifts

The phase shift between production and inventories is 1/5 to 1/6 of the cycle length, implying that the trajectory of the inventory cycle is a shrinking ellipse with a major (longer) axis running from the northeast to the southwest around the origin (Figure 2).

The FF rate precedes production by 4 to 8 months. It seems that the Fed's monetary policy is forward-looking/preemptive; it anticipates the cyclical patterns of economic variables.

Table 7: Estimated phase shifts (six-variable VARs with U.S. data).

unit: months	(Cycle length)	Shipment	Inventories	FF rate	CPI-U	Com. Price
Level data	(68.5)	1.9612	12.748	-8.0767	-0.6753	1.7430
	(91.2)	2.9152	19.939	16.323	-16.551	22.389
	(54.8)	-4.5560	1.5431	0.6742	-0.3231	-3.0252
HP-s.a.	(55.1)	2.1547	10.436	-4.0955	1.3363	2.7509
	(182.8)	0.5981	5.2254	39.217	5.5583	18.872
	(43.9)	-3.3573	2.9764	-0.2556	-1.0580	-4.0754
YoY	(46.7)	1.9730	7.4541	-3.6056	-2.9659	0.7352

[38] See NBER's "U.S. Business Cycle Expansions and Contractions" at http://nber.nber.org/cycles/cyclesmain.html

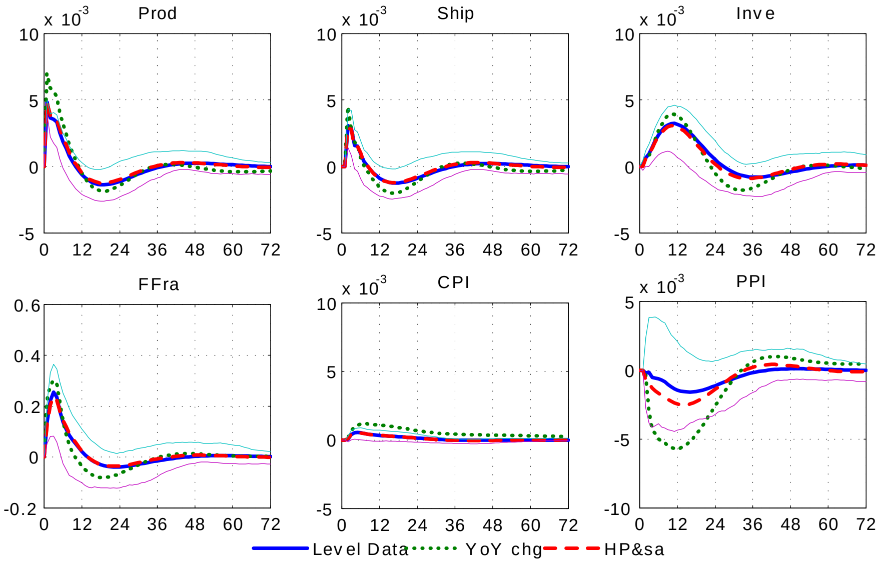

Figure 19: IRFs due to a positive shock in the production equation. Narrow lines show the 95% confidence intervals of level data estimations based on the bootstrapping method.

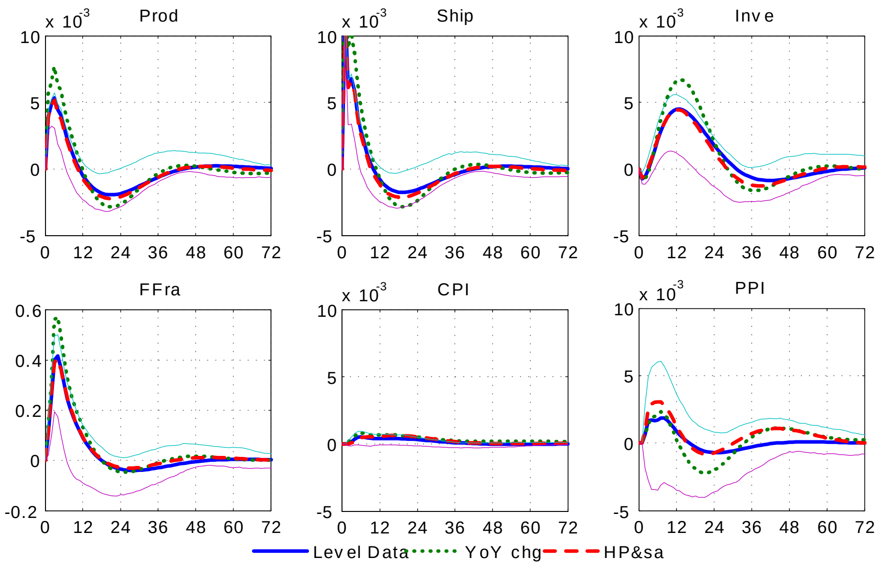

Figure 20: IRFs due to a positive shock in the shipment equation. Narrow lines show the 95% confidence intervals of level data estimations based on the bootstrapping method.

A.3 Impulse Response Functions

As with the estimation for Japan, there exists a somewhat perverse price puzzle. In addition, the estimated IRFs have a wide confidence interval (especially for the FF rate and prices).

Supply vs. Demand Shocks: Monetary policy is tightened after both positive demand and supply shocks (Figures 20 and 19). However, the Fed raises the FF rate much more sharply in response to a demand shock than a supply shock, because the leading inflation indicator increases after a demand shock but decreases after a supply shock. In addition, the initial effect of a demand shock is stronger than that of a supply shock.

Unlike Japanese estimations, the lifespans of booms do not differ between demand and supply shocks. The author's conjecture is that this is because of differences between the surveyed firms in production and shipment/inventories statistics. For example, if a firm's figures are included in production statistics but not in shipment statistics, then the demand shock that hits that firm increases production but not shipment.

Price Shocks: The IRFs to shocks to CPI and PPI raw materials are similar to each other, but the latter, a leading inflation indicator, has stronger effects than the former. It seems that the central banks react to leading inflation indicators but not to CPI both in Japan and in the United States.

Fed Funds Rate Shock: Again, the price puzzle arises; after a positive FF rate shock, CPI rises (Figure 24). Though the confidence interval is very wide, inventories also increase after a positive FF rate shock. This could be because firms cannot cut their production quickly enough to counterbalance the decline in demand, but this is difficult to verify because data are collected from different pools of sampled firms.

A.4 Spectral Analysis and Cross Correlations

Like the Japanese data, the U.S. data also show the S-shape cross correlations between inventories and other variables, which shows the existence of time lags between them.

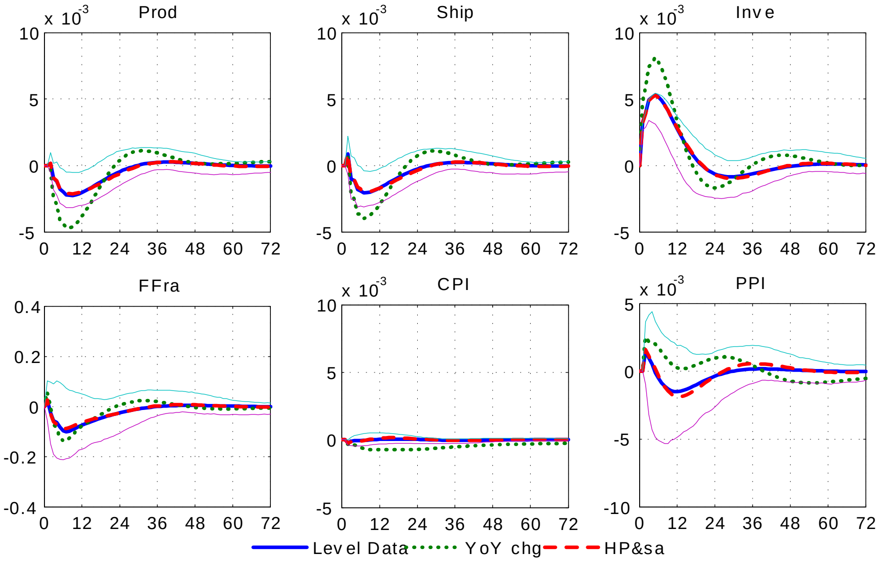

Figure 21: IRFs due to a positive shock in the inventory equation. Narrow lines show the 95% confidence intervals of level data estimations based on the bootstrapping method.

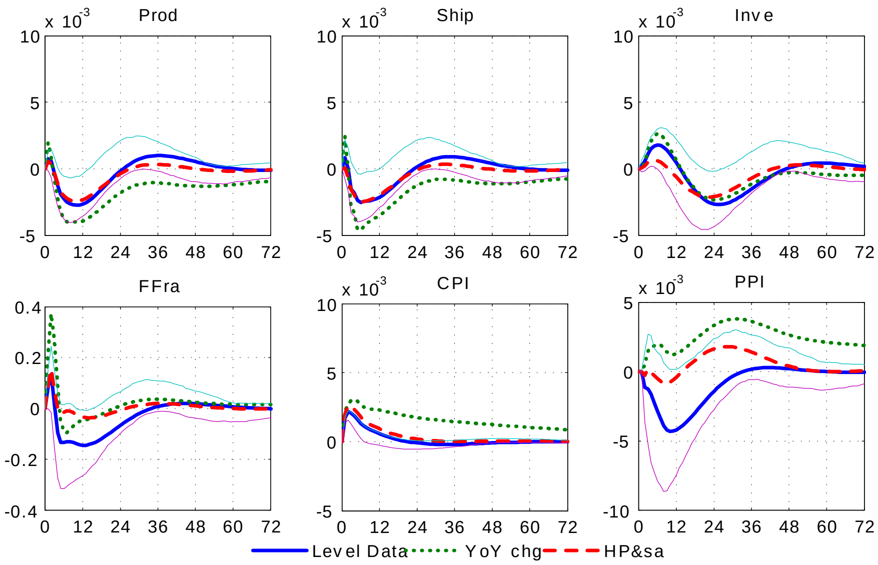

Figure 22: IRFs due to a positive shock in the CPI equation. Narrow lines show the 95% confidence intervals of level data estimations based on the bootstrapping method.

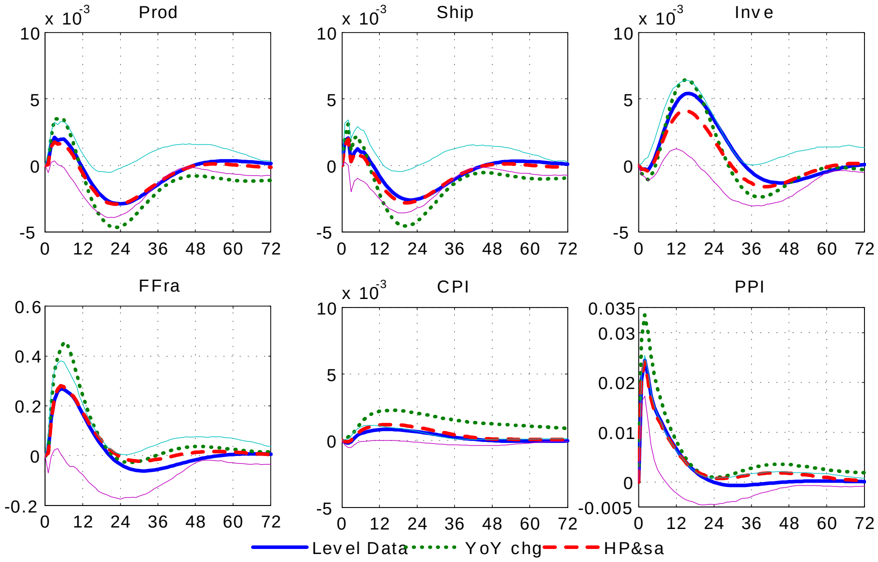

Figure 23: IRFs due to a positive shock in the leading inflation indicator equation. Narrow lines show the 95% confidence intervals of level data estimations based on the bootstrapping method.

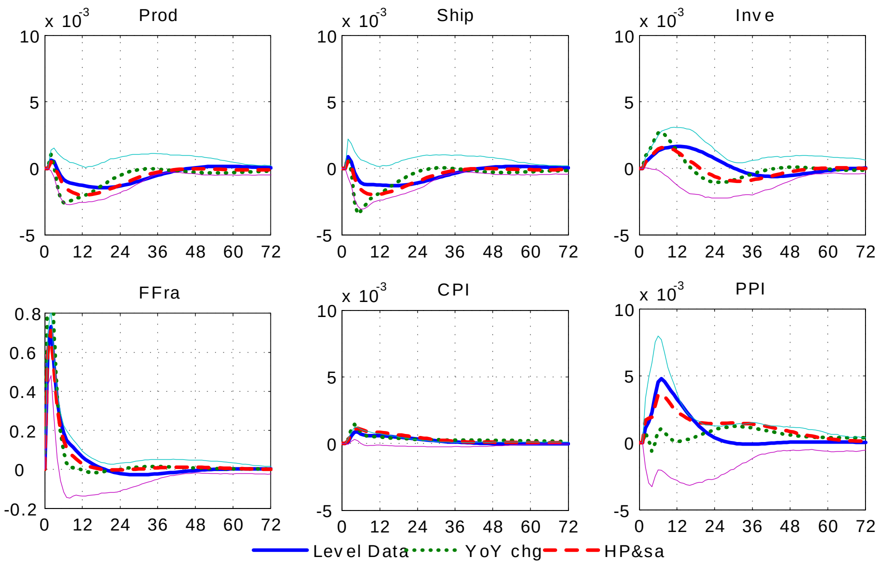

Figure 24: IRFs due to a positive shock in the FF rate equation. Narrow lines show the 95% confidence intervals of level data estimations based on the bootstrapping method.

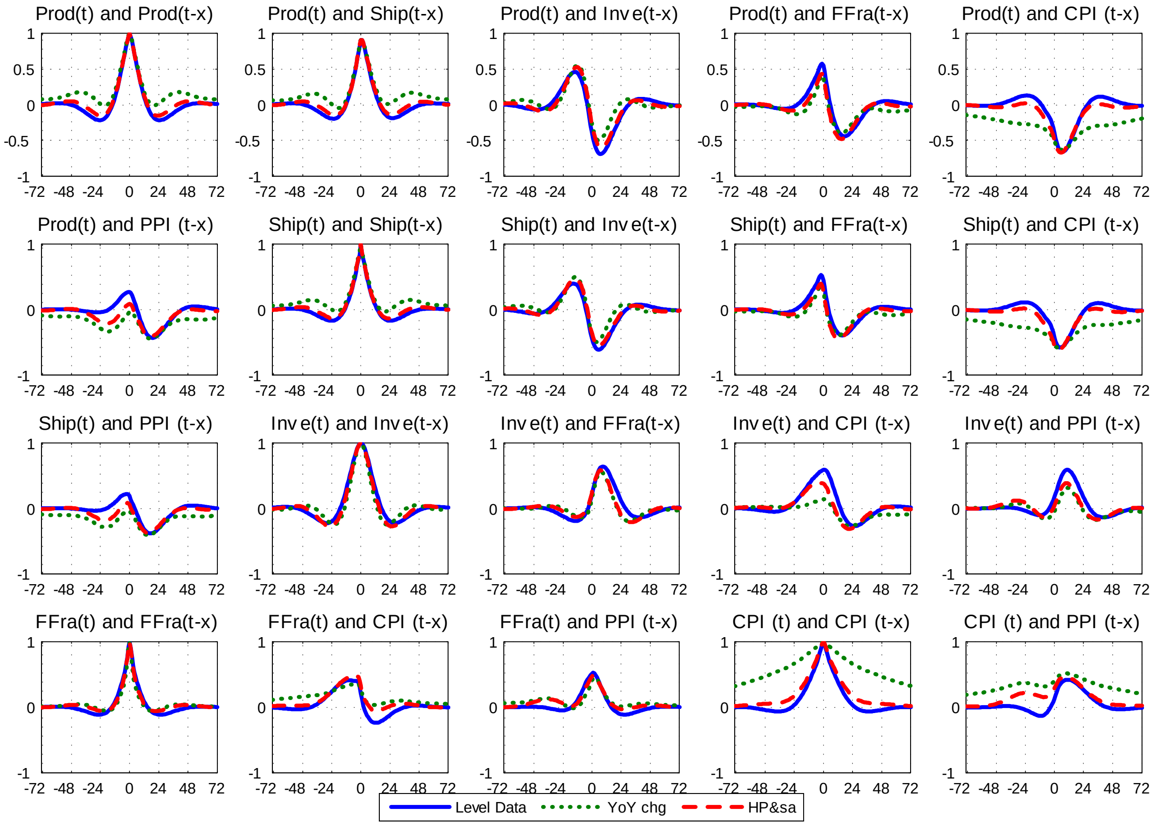

Figure 25: Cross correlations.

The correlations between production/shipment and the FF rate peak around 0 to -2 months, showing that the Fed reacts to these variables with a short time lag, which may seem to be inconsistent with the finding in the phase shift between them (see Appendix A.2.1). However, this is because of very high frequency components; by definition, the Fed cannot react to *iid* shocks in advance. Remember that the phase shift between production and the FF rate shows the Fed's reaction to the *cyclical component* of, but not to *shocks* to, production, but the cross correlation between them reflects the Fed's reaction to both the cyclical component and shocks. On the other hand, the correlations between production/shipment and the FF rate reach their bottom at around 15 to 20 months, which shows that it takes more than one year for the effect of monetary policy to fully materialise.

The spectra show that the quadrature spectrum plays a major role mainly with inventories (Figure 26). Most of the spectra of CPI and PPI raw materials with other variables have a sharp spike at 0 months (making it difficult to distinguish them from the y-axis), which means that their behaviour is dominated by shocks, with weak cyclical linkages with other variables. Also note that most of the spectra have their peak or bottom at around 60 months, which means that the cyclical component with a 60 months long is a key driving factor in the business cycle. The quadrature spectra of the FF rate with other variables have their peak or bottom at business cycle frequencies, showing that contemporaneous covariances are not sufficient to evaluate the Fed's monetary policy.

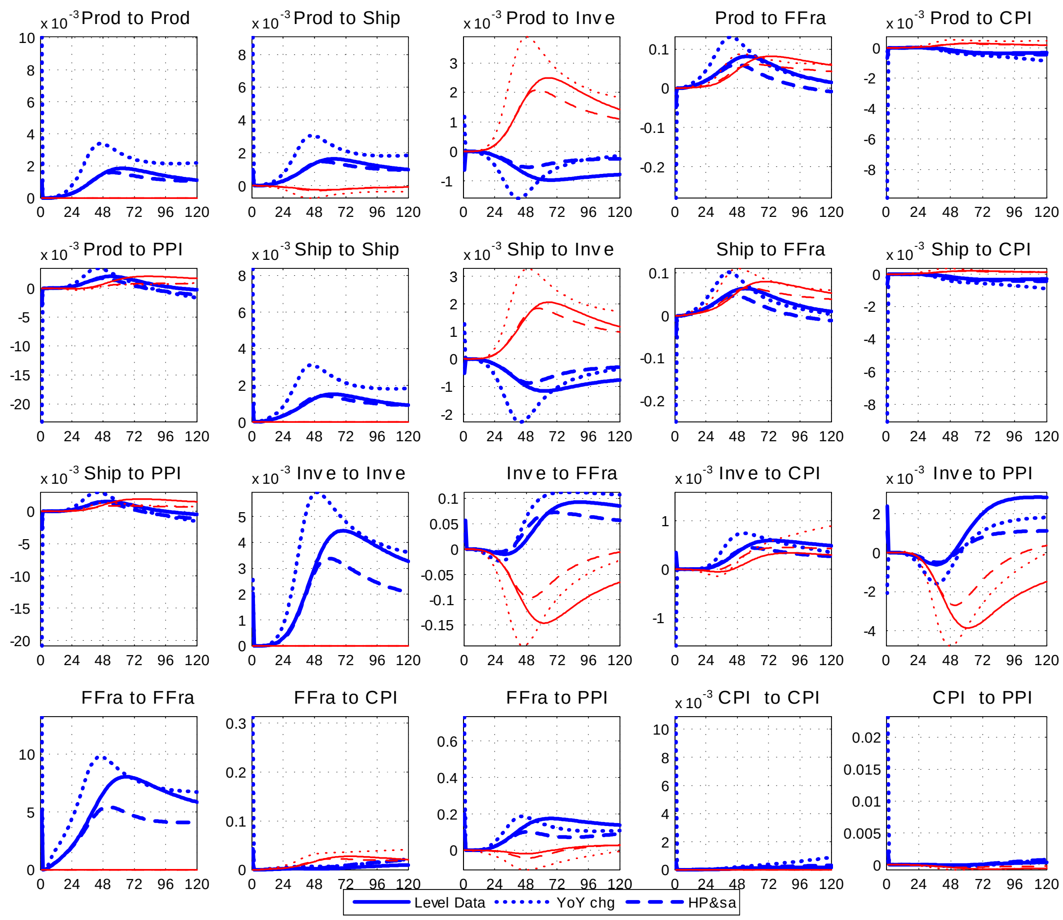

Figure 26: Spectrum densities. Bold lines show cospectra and narrow lines show quadrature spectra.

B Computation of Phase Shifts

This section omits commonly used techniques, but briefly describes how to compute phase shifts in a given system of difference equations. It may be useful for some readers since the author personally experienced some difficulty in finding references for the computation of phase shifts.

B.1 Computational Summary

Suppose that we have obtained a VAR(M) estimation without exogenous variables (see equation (1)). Then, it can be rewritten in the form of VAR(1) by redefining the vector of endogenous variables.

$$
\begin{aligned}
Y_t &= Y_{t-1}\mathbf{B} + \xi_t \mathbf{C} && (4)\\
\mathbf{B} &\equiv \begin{bmatrix} B_1 & \cdots & B_{M-1} & B_M \\ I & & 0 & 0 \\ & \ddots & & \vdots \\ 0 & & I & 0 \end{bmatrix}, \quad
\begin{matrix} \mathbf{C} \equiv \begin{bmatrix} C & 0 & \cdots & 0 \end{bmatrix}, \\ \\ Y_t \equiv \begin{bmatrix} y_t & \cdots & y_{t-M+1} \end{bmatrix} \end{matrix}
\end{aligned}
$$

where y_t and ξ_t are row vectors of endogenous and exogenous variables, respectively. ξ_t is assumed to be *iid* over time and equations.

Let Λ and V be the matrices of eigenvalues and eigenvectors of $\mathbf{B}$ in (4), respectively.

$$
\Lambda = \begin{bmatrix} \lambda_1 & & 0 \\ & \ddots & \\ 0 & & \lambda_n \end{bmatrix}, \quad V = \begin{bmatrix} V_1 & \cdots & V_n \end{bmatrix} \tag{5}
$$

where n is the number of roots ($M \times \#$ of endogenous variables) and V_j is the eigenvector that corresponds to the j-th eigenvalue. Then,

- Frequencies (θ_j): $\Theta = diag\begin{bmatrix} \theta_1 & \cdots & \theta_n \end{bmatrix} = \arctan\left(\Im[\Lambda]./\Re[\Lambda]\right)$
- Cycle lengths ($2\pi/\theta_j$): $2\pi./\Theta = diag\begin{bmatrix} 2\pi/\theta_1 & \cdots & 2\pi/\theta_n \end{bmatrix}$
- Phase (β_{lj}): $\Phi = \arctan\left(\Im[V]./\Re[V]\right) + \textit{nuisance term}$
- Phase shifts between k and l: $\Phi_{k.} - \Phi_{l.} = \begin{bmatrix} \Phi_{k1} - \Phi_{l1} & \cdots & \Phi_{kn} - \Phi_{ln} \end{bmatrix}$

In terms of notations, "./Θ" signifies the element-by-element multiplication of Θ^{-1} from right, $\Phi_{k.}$ is the k-th row of Φ and $\Re[V]$ and $\Im[V]$ mean the real and imaginary parts of V, respectively. Φ_{lj} is the phase of the l-th endogenous variable with respect to the cycle corresponding to the j-th eigenvalue.

There are a few comments. (i) If the r-th eigenvalue is real, then frequency θ_r is positive infinity and phase shifts between any variables are zero. (ii) The unit of $\Phi_{k.} - \Phi_{l.}$ is radian. To convert the unit from radian to time, it should be divided by a proper frequency, as in the main text. (iii) In actual computation, it is necessary to take care the fact that any $\hat{\beta}_{lj} \equiv \beta_{lj} \pmod{2\pi}$ are equivalent to β_{lj}.

B.2 Derivation

If λ_j and λ_i are conjugate each other (denote conjugate by upper bar: $\lambda_i = \bar{\lambda}_j$), then V_j and V_i are also conjugate each other ($V_i = \bar{V}_j$). This is evident because $\bar{\lambda}_j$ and $\bar{V}_j$ must satisfy the definition of the eigenvalue-eigenvector if λ_j and V_j satisfy it.

$$(\mathbf{B} - \lambda_j I) V_j = 0 \Leftrightarrow \overline{(\mathbf{B} - \lambda_j I) V_j} = 0 \Leftrightarrow \left(\mathbf{B} - \bar{\lambda}_j I\right) \bar{V}_j = 0$$

Note that $\bar{\mathbf{B}} = \mathbf{B}$ and $\bar{I} = I$ since the identity matrix and $\mathbf{B}$ are both real. Denote such λ_j and V_j as follows.

$$\begin{aligned}
\lambda_j &= a_j + b_j i = \rho_j (\cos\theta_j + i \sin\theta_j) \\
\bar{\lambda}_j &= a_j - b_j i = \rho_j (\cos\theta_j - i \sin\theta_j) \\
V_j &= R_j + M_j i \\
\bar{V}_j &= R_j - M_j i \\
R_j &= \begin{bmatrix} R_{1j} \\ \vdots \\ R_{nj} \end{bmatrix}, \; M_j = \begin{bmatrix} M_{1j} \\ \vdots \\ M_{nj} \end{bmatrix}
\end{aligned}$$

where $\rho_j = \sqrt{a_j^2 + b_j^2}$ and $\theta_j = \arctan b_j/a_j$. It is obvious that both $\lambda_j^t V_j$ and $\bar{\lambda}_j^t \bar{V}_j$ are elementary solutions of the difference equations (4). By De Moivre's formula,

$$\begin{aligned}
\lambda_j^t &= \left(\rho_j \left(\cos\theta_j + i \sin\theta_j\right)\right)^t = \rho_j^t \left(\cos\theta_j t + i \sin\theta_j t\right) \\
\bar{\lambda}_j^t &= \left(\rho_j \left(\cos\theta_j - i \sin\theta_j\right)\right)^t = \rho_j^t \left(\cos\theta_j t - i \sin\theta_j t\right)
\end{aligned}$$

However, we prefer the elementary solutions that do not have imaginary root i. Because any linear combination of these solutions can be also elementary solutions,

$$\begin{aligned}
\eta_i^{\Re} &= \frac{1}{2}\left(\lambda_i^t V_i + \bar{\lambda}_i^t \bar{V}_i\right) = \rho_i^t \left(R_i \cos\theta_i t - M_i \sin\theta_i t\right) \\
\eta_i^{\Im} &= \frac{1}{2i}\left(\lambda_i^t V_i - \bar{\lambda}_i^t \bar{V}_i\right) = \rho_i^t \left(M_i \cos\theta_i t + R_i \sin\theta_i t\right)
\end{aligned}$$

By the formula of linear combination of trigonometric functions (synthesis formula),

$$\begin{aligned}
R_j \cos\theta_j t - M_j \sin\theta_j t &= \psi_j \cdot \sin\left(\theta_j t + \hat{\beta}_j\right) \\
R_j \cos\theta_j t + M_j \sin\theta_j t &= \psi_j \cdot \sin\left(\theta_j t + \tilde{\beta}_j\right) = \psi_j \cdot \cos\left(\theta_j t + \hat{\beta}_j\right)
\end{aligned}$$

where $\cdot$ signifies element-by-element multiplication, and

$$\hat{\beta}_j = \begin{bmatrix} \hat{\beta}_{1j} \\ \vdots \\ \hat{\beta}_{nj} \end{bmatrix} = \arctan\left(\frac{M_j}{R_j}\right), \quad \psi_j = \begin{bmatrix} \psi_{1j} \\ \vdots \\ \psi_{nj} \end{bmatrix} = \begin{bmatrix} \sqrt{R_{1j}^2 + M_{1j}^2} \\ \vdots \\ \sqrt{R_{nj}^2 + M_{nj}^2} \end{bmatrix}$$

Interestingly, there is a kind of duality between eigenvalues and eigenvectors. Therefore, the two real elementary solutions are written as

$$\begin{aligned}
\eta_j^{\Re} &= \psi_j \cdot \rho_i^t \sin\left(\theta_j t + \hat{\beta}_j\right) \\
\eta_j^{\Im} &= \psi_j \cdot \rho_i^t \cos\left(\theta_j t + \hat{\beta}_j\right)
\end{aligned}$$

The solution of a linear difference equation is a linear combination of elementary solutions.

$$y_t = \cdots + \omega_j \psi_j \cdot \rho_i^t \sin\left(\theta_j t + \hat{\beta}_j\right) + \omega_{j'} \psi_j \cdot \rho_i^t \cos\left(\theta_j t + \hat{\beta}_j\right) + \cdots$$

Weights $\{\omega_\tau\}_{\tau=1}^n$ are determined by the initial condition (past and present innovations in our case) of a given problem. By using the synthesis formula again, it is shown that the phase of the l-th variable with respect to the j-th eigenvalue β_{lj} must satisfy

$$\begin{aligned}
& \alpha_{lj} \rho_i^t \sin\left(\theta_j t + \beta_{lj}\right) \\
= \; & \omega_j \psi_{lj} \rho_i^t \sin\left(\theta_j t + \hat{\beta}_{lj}\right) + \omega_{j'} \psi_{lj} \rho_i^t \cos\left(\theta_j t + \hat{\beta}_{lj}\right) \\
= \; & \left(\psi_{lj} \sqrt{\omega_j^2 + \omega_{j'}^2}\right) \rho_i^t \sin\left(\theta_j t + \hat{\beta}_{lj} + \bar{\beta}_{lj}\right)
\end{aligned}$$

where $\bar{\beta}_{lj} = \bar{\beta}_j = \arctan\left(\omega_j / \omega_{j'}\right)$ is common to all l.

Hence,

$$\begin{aligned}
\alpha_{lj} &= \psi_{lj} \sqrt{\omega_j^2 + \omega_{j'}^2} \\
\beta_{lj} &= \hat{\beta}_{lj} + \bar{\beta}_j
\end{aligned}$$

It is clear that the phase shift between the k-th and l-th variables is independent from the initial value (past and present innovations in our case) because $\bar{\beta}_j$ is cancelled out.

$$\beta_{kj} - \beta_{lj} = \hat{\beta}_{kj} - \hat{\beta}_{lj}$$

Remember that $\bar{\beta}_j$ is dependent on ω_τ but $\hat{\beta}_{lj}$ is not.

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