Stable partial agglomeration in a New Economic Geography model with urban frictions.

Sylvain Barde

Department of Economics

University of Kent

Abstract

This paper extends the Puga (1999) model by introducing urban frictions. It assumes that the agglomeration of manufacturing in a city imposes a cost on the inhabitants of the agglomerated region. Furthermore, an implicit function methodology is developed to provide a numerical stability function that does not require prior analytical work. Simulations reveal that these numerical stability conditions are consistent with the original Puga (1999) analytical predictions.

The central finding is that the extension significantly alters the agglomeration properties of the original Puga framework. In particular, partial agglomeration becomes a stable long run outcome in both with and without migration. Furthermore, the level of sensitivity of the agglomeration to the friction cost market parameters is shown to be different in the both cases. This outlines the need to evaluate the imperfectness of migration when modifying the urban geography as a policy implication

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Address for Correspondence: Department of Economics, University of Kent, Canterbury, Kent, CT2 7NP. Telephone: 01 227 82 76 39 Email: sb71@kent.ac.uk

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1. Introduction

According to recent work by Robert-Nicoud (2004) and Ottaviano and Robert-Nicoud (2006), partial agglomeration cannot be stable in a New Economic Geography framework. However, this disagrees with the urban mismatch literature, in which several studies point out that partial agglomeration is possible if the labour markets and the urban land markets are allowed solve together[1]. This is especially true in Smith and Zenou (1995), which show, in a two-city case, that it affects the migration between cities, and can help sustain partial agglomeration. Breukner and Zenou (1999) confirm this by showing that adding a land market to a Harris-Todaro (1970) framework, the extra frictions from urbanisation provides a force which limits migration. The purpose of this paper is therefore to investigate whether this is also the case in a NEG framework.

This paper shows that stable partial agglomeration can be obtained by integrating the costs of urbanisation into the Puga (1999) model. The choice of this model rests on the fact that due to its analytical results, its integration of vertical linkages and its capacity to cope with both migration and non-migration it is a cornerstone of the NEG literature. Including even a simple frictional urbanisation cost that solves simultaneously with the labour market means that the cost of urbanisation will increase as agglomeration grows, introducing an extra dispersion force, which makes partial agglomeration a much more likely outcome. This study shows that this

[1] These papers are mainly: Smith and Zenou (1995), Breukner and Zenou (1999), Wasmer and Zenou (2000) Breukner, Thisse and Zenou (2002) and Zenou (1999).

has several important implications. One is that this modifies the Robert-Nicoud (2004) and Ottaviano and Robert-Nicoud (2006) results mentioned above. Another implication is that this can potentially create problems for researchers who attempt to use the original Puga stability conditions in empirical studies, as we shall show that the agglomeration properties of the model are sensitive both to urban friction and the freeness of migration. The cases of Head and Mayer (2004) and Brakman *et al* (2006) are illustrations of this and this study shows how their results could be modified by the inclusion of a land market, and its effects on the stability conditions.

The remained of the paper is organised as follows: section 2 modifies the original Puga (1999) framework to include urban frictions. Section 3 presents the numerical method of evaluating the stability of the system that will be used throughout the paper. The simulations of the migration and non-migrations versions of the extended model are then presented in sections 4 and 5. Section 6 discusses the consequences of the extension and section 7 concludes.

2. The Puga model with urban frictions

The spatial mismatch literature mentioned above suggests the possibility of stable partial agglomeration when the labour market and some form of urban friction are allowed to solve simultaneously. This section therefore introduces such a friction, even if it is in a simplified manner, in the Puga (1999) framework. First of all, the agglomeration of the manufacturing sector in the Puga model is assumed to proxy for urbanisation. We then assume the existence of an urbanisation cost, which is a positive function of the size of the manufacturing sector in a region. The burden of this regional urbanisation cost is assumed to be equally shared between the workers in a region. This creates a simple setting, in which agglomeration of manufacturing in one region is matched by a relatively high urban friction costs in the agglomerated region, and by low urban friction costs in the region devoid of manufacturing activity.

In order to clarify exactly how the extension modifies the Puga (1999) model, it is important to briefly reiterate the original framework. The full model, as specified in appendix of Puga (1999) consists of the following set of equations (1.1)-(1.4).

$$\sigma\pi = \begin{array}{c}\hat{q}^{\mu(1-\sigma)}\hat{w}^{(1-\mu)(1-\sigma)} \times \\ \left(T\hat{q}^{(\sigma-1)}\left(\gamma\left(\hat{w}L + \hat{K}r(w)\right) + \left(\gamma - \mu(\sigma-1)\right)\hat{n}\pi + \mu\hat{n}\hat{q}^{\mu}w^{(1-\mu)}\right) - \hat{q}^{\sigma\mu}w^{\sigma(1-\mu)}\right)\end{array} \tag{1.1}$$

$$q^{1-\sigma} - T\hat{q}^{\mu(1-\sigma)}\hat{w}^{(1-\mu)(1-\sigma)}n = 0 \tag{1.2}$$

$$L - (1-\mu)\hat{n}\left(\hat{q}^{\mu}w^{-\mu} + (\sigma-1)\hat{\pi}w^{-1}\right) + \hat{K}r_w(w) = 0 \tag{1.3}$$

$$q^{-\gamma}w = \frac{1}{M}11^T q^{-\gamma}w \tag{1.4}$$

The system variables are profits π, the number of firms n, the industrial price index q, wages w, and in the case of migration, labour L[2]. This system has one more endogenous variable than equation, and solving it requires fixing either firms n or profits π. Fixing the number of firms n allows to solve for profits, and models the short run equilibria. Setting profits equal to zero allows to solve for the equilibrium number of firms in the long run. For a full explanation of the derivations of these equations, the reader is referred to Puga (1999).

The assumption that is made is that all manufacturing firms and workers locate in a city. In a given region, the urban frictions resulting from the existence of a city therefore depend on the size of the manufacturing sector in that region. This depends

[2] In the case where migration is not allowed, the variable L becomes exogenous, and the equation (1.4), which models the equalisation of real wages through migration, is dropped.

first of all on n_i, the number of manufacturing sector firms, located in region *i*. Secondly, this includes the number of manufacturing workers located in that region, given by the first part of equation (1.3) and equal to:

$$(1-\mu)n_i\left(q_i^{\mu}w_i^{-\mu}+(\sigma-1)\pi_i w_i^{-1}\right)$$

Given this, the size of the city or urban area *C* in region *i* can be measured by the number of manufacturing sector agents present in the region:

$$C_i=(1-\mu)n_i\left(q_i^{\mu}w_i^{-\mu}+(\sigma-1)\pi_i w_i^{-1}\right)+n_i \tag{2}$$

The per capita cost of urban friction c_i is a positive function of city size:

$$c_i=\chi(C_i)^{\varphi} \tag{3}$$

The positive parameter φ is included to allow for the fact that the effect of city size on the demand for locations may be non-linear and χ is a calibration parameter which quantifies how much an increase in city size increases the per-capita costs of agglomeration. Replacing city size C_i, the urbanisation friction cost per unit of labour can be written as:

$$c_i=\chi\left((1-\mu)n_i\left(q_i^{\mu}w_i^{-\mu}+(\sigma-1)\pi_i w_i^{-1}\right)+n_i\right)^{\varphi} \tag{4}$$

Because the friction cost c_i is imposed on all of the population of region *i*, it only enters the model via the disposable income of consumers. It is important to explain at this point why the urban friction cost c_i is paid by workers of all sectors, both manufacturing and agricultural. The assumption that the costs of urbanisation are equally spread between agricultural and manufacturing worker seems initially counter-intuitive. It can be satisfactorily explained, however, by existence of economy-wide agglomeration costs, as well as the need to conserve the intersectoral properties of the Puga (1999) model.

Even though the framework presented above is simplified, it is not unreasonable to assume that there exists some costs to agglomeration that are imposed on all workers in society. A first example would be negative externalities to agglomeration, which depend positively on the size of the urban area or the manufacturing sector and which impose a cost on all of society in a region. Pollution is a good example of such an externality. Another urbanisation cost that can be spread out over the entire population is the provision of urban infrastructure to the manufacturing sector as agglomeration occurs. The cost of providing this infrastructure is usually covered by the state, and paid for by taxes. If developing urban infrastructure is a priority for the state, then one could imagine that the agricultural sector would also contribute taxes towards providing the infrastructure. Importantly, regardless of how these costs are shared over the population, the mechanism that is captured by this extension is that if manufacturing starts to agglomerate in one region, then the social cost imposed by this agglomeration will be higher in that region than in the non-agglomerated region, reflecting the costs of urbanisation.

Another central reason why this assumption is made is that it preserves one of the properties of the intersectoral adjustment in the Puga (1999) model. In the original model, all workers in a region *i* earn the same wage at equilibrium, whether they supply their work to the agricultural or manufacturing sector. Because they are faced with the same prices, their real wage is also the same. This means that at equilibrium, the marginal worker is indifferent to working in either sector and there is no intersectoral adjustment. This equalisation of real wages between sectors at equilibrium makes sense from an economic point of view, and it is important that this property be conserved in the extension. What this fundamentally means is that in the long run, the cost to the manufacturing worker of living in a city is equal to the cost to the agricultural worker of not living in the city. Because both earn the same wage this means that at equilibrium, the marginal worker is still indifferent between moving location and changing sectors, at the same time.

As explained above, it is assumed that the urban frictions directly reduce the wages of workers, in other words, they are taken from the wages as a lump sum before the workers start consuming. The consumer utility function is the same as in Puga (1999), and there is now a difference between wages *w* and the disposable income of workers *w-c*. Solving the utility maximisation problem gives the aggregate consumer expenditure on manufactures, where *γ* is the share of expenditure on manufactures:

$$\gamma\left(w_i - c_i\right)L_i \tag{5}$$

This new consumer expenditure then enters the demand function alongside other manufacturing and agricultural expenditures in the same fashion as in Puga (1999).

Furthermore, as was the case in the original model, in the long run the real wages in both regions will equalise when migration between the regions is allowed. Including the effect of urban rent on real wages and the assumption that all sectors pay the rent, the long run equality of the real wages between regions can now be expressed as:

$$q_i^{-\gamma}\left(w_i - c_i\right) = q_j^{-\gamma}\left(w_j - c_j\right) \tag{6}$$

The simple way in which the rent *c* feeds back into the model is a property of the assumption that both manufacturing and agricultural workers pay the rent. In Puga (1999), they have the same utility function by assumption and the same wage by construction. This allows the consumption of all workers in a region to be determined simply as $\gamma \times wL$. . Because the frictional urbanisation costs apply in some way to agricultural an manufacturing workers, one can write the aggregate consumption of labour $\gamma \times \left(w - c\right)L$, as shown in equation (5), and maintain the analytical simplicity of Puga (1999). This is an important aspect, as the aim of the chapter is to see how the frictions modify agglomeration in the Puga model. It is therefore important to keep the extended model comparable to the original framework.

A more refined model would separate the urban and rural costs linked to the expanding urban area, as well as allow for a nominal wage differential between the urban manufacturing and agricultural worker. Real wages would still adjust at equilibrium, but such a model would require that the two sources of worker consumption in the expenditure equation have to be separated, as the disposable income of agricultural and manufacturing workers become different. The effects of the frictions on agglomeration shown later in this chapter would still exist, but the model would be less tractable.

Given the existing Puga model and the urban friction modifications shown above, the extended system can be described by equations (7.1)-(7.5) below:

$$\sigma\pi = \hat{q}^{\mu(1-\sigma)}\hat{w}^{(1-\mu)(1-\sigma)} \times \left(T\hat{q}^{(\sigma-1)}\left(\gamma\left(\left(\hat{w}-\hat{c}\right)L+\hat{K}r\left(w\right)\right)+\left(\gamma-\mu\left(\sigma-1\right)\right)\hat{n}\pi+\mu\hat{n}\hat{q}^{\mu}w^{(1-\mu)}\right)-\hat{q}^{\sigma\mu}w^{\sigma(1-\mu)}\right) \tag{7.1}$$

$$q^{1-\sigma}-T\hat{q}^{\mu(1-\sigma)}\hat{w}^{(1-\mu)(1-\sigma)}n=0 \tag{7.2}$$

$$L-\left(1-\mu\right)\hat{n}\left(\hat{q}^{\mu}w^{-\mu}+\left(\sigma-1\right)\hat{\pi}w^{-1}\right)+\hat{K}r_{w}\left(w\right)=0 \tag{7.3}$$

$$c=\chi\left(\left(1-\mu\right)\hat{n}\left(\hat{q}^{\mu}w^{-\mu}+\left(\sigma-1\right)\hat{\pi}w^{-1}\right)+n\right)^{\varphi} \tag{7.4}$$

$$q^{-\gamma}\left(w-c\right)=\frac{1}{M}11^{T}q^{-\gamma}\left(w-c\right) \tag{7.5}$$

This extended model still nests the original Puga (1999) model. If the friction share parameter χ is set to zero, equation (7.4) reduces to $c=0$, the extra variable c disappears from the model, and the model reverts to the original setting (1.1)-(1.4). This also means that this model still solves as explained above. Any short run equilibrium can be computed by setting the firm mass exogenously and calculating profits. The long run equilibria are solved for by setting profits exogenously to zero and solving for the firm mass.

3. An implicit function method for determining stability

Before moving on to simulations of the extended system (7.1)-(7.5), this section explains the numerical methodology developed to asses the effect of the extension on the stability of the symmetric equilibrium. This methodology is based on the fact that most of the numerical solvers used for simulating problems such as the system (1.1)-(1.4) require that it be re-arranged in the form shown in (8.1), by subtracting the right hand side from both sides of each equation. The solver algorithms then typically search for the set of variables that make the right hand side equal to zero. It is possible to take advantage of this to directly calculate, as a part of the simulation, a numerical stability function for the symmetric equilibrium. This calculation rests on the implicit function theorem, extended to a simultaneous system of equations.

$$\begin{cases} F_1\left(\pi, q, w, L, n\right) = 0 \\ F_2\left(\pi, q, w, L, n\right) = 0 \\ F_3\left(\pi, q, w, L, n\right) = 0 \\ F_4\left(\pi, q, w, L, n\right) = 0 \end{cases} \tag{8.1}$$

$$\begin{cases} \pi = f_1\left(n\right) \\ q = f_2\left(n\right) \\ w = f_3\left(n\right) \\ L = f_4\left(n\right) \end{cases} \tag{8.2}$$

The system $F_1 - F_4$ describes a set of implicit functions $f_1 - f_4$ as described in (8.2) if the equations $F_1 - F_4$ are continuous and twice differentiable, and if, for a set of points $\{\pi_*, q_*, w_*, L_*, n_*\}$ which are a solution to (8.1), the following holds:

$$|J| = \begin{vmatrix} \frac{\partial F_1}{\partial \pi} & \frac{\partial F_1}{\partial q} & \frac{\partial F_1}{\partial w} & \frac{\partial F_1}{\partial L} \\ \frac{\partial F_2}{\partial \pi} & \frac{\partial F_2}{\partial q} & \frac{\partial F_2}{\partial w} & \frac{\partial F_2}{\partial L} \\ \frac{\partial F_3}{\partial \pi} & \frac{\partial F_3}{\partial q} & \frac{\partial F_3}{\partial w} & \frac{\partial F_3}{\partial L} \\ \frac{\partial F_4}{\partial \pi} & \frac{\partial F_4}{\partial q} & \frac{\partial F_4}{\partial w} & \frac{\partial F_4}{\partial L} \end{vmatrix} \neq 0 \tag{9}$$

If the condition holds, in other words the matrix of partial derivatives of the system is non-singular, then by the implicit function theorem, the following is true in the region around the solution point $\{\pi_*, q_*, w_*, L_*, n_*\}$:

$$\begin{pmatrix} \frac{\partial F_1}{\partial \pi} & \frac{\partial F_1}{\partial q} & \frac{\partial F_1}{\partial w} & \frac{\partial F_1}{\partial L} \\ \frac{\partial F_2}{\partial \pi} & \frac{\partial F_2}{\partial q} & \frac{\partial F_2}{\partial w} & \frac{\partial F_2}{\partial L} \\ \frac{\partial F_3}{\partial \pi} & \frac{\partial F_3}{\partial q} & \frac{\partial F_3}{\partial w} & \frac{\partial F_3}{\partial L} \\ \frac{\partial F_4}{\partial \pi} & \frac{\partial F_4}{\partial q} & \frac{\partial F_4}{\partial w} & \frac{\partial F_4}{\partial L} \end{pmatrix} \begin{pmatrix} \frac{\partial \pi}{\partial n} \\ \frac{\partial q}{\partial n} \\ \frac{\partial w}{\partial n} \\ \frac{\partial L}{\partial n} \end{pmatrix} = \begin{pmatrix} -\frac{\partial F_1}{\partial n} \\ -\frac{\partial F_2}{\partial n} \\ -\frac{\partial F_3}{\partial n} \\ -\frac{\partial F_4}{\partial n} \end{pmatrix} \tag{10}$$

The determinant of the large matrix is simply the Jacobian $|J|$ of the original system expressed in (9). By using Cramer's rule on (10) one can see that:

$$\frac{\partial \pi}{\partial n} = \frac{\begin{vmatrix} -\frac{\partial F_1}{\partial n} & \frac{\partial F_1}{\partial q} & \frac{\partial F_1}{\partial w} & \frac{\partial F_1}{\partial L} \\ -\frac{\partial F_2}{\partial n} & \frac{\partial F_2}{\partial q} & \frac{\partial F_2}{\partial w} & \frac{\partial F_2}{\partial L} \\ -\frac{\partial F_3}{\partial n} & \frac{\partial F_3}{\partial q} & \frac{\partial F_3}{\partial w} & \frac{\partial F_3}{\partial L} \\ -\frac{\partial F_4}{\partial n} & \frac{\partial F_4}{\partial q} & \frac{\partial F_4}{\partial w} & \frac{\partial F_4}{\partial L} \end{vmatrix}}{|J|} \tag{11}$$

Equation (11) shows that two pieces of information are needed to calculate this result, which are the Jacobian $|J|$ and the $\partial F/\partial n$ terms present in (10). This is where numerical methods prove useful, as many non-linear solvers provide direct estimates of the matrix J at the solution point. Furthermore, the way the equations are formatted, shown in Equation (8.1), the right hand side of $F_1 - F_4$ is equal to zero for the solution set $\{\pi_*, q_*, w_*, L_*, n_*\}$. Given a disturbance ε close to the computational tolerance of the solver[3], the following approximation can therefore be made:

$$\frac{\partial F_i}{\partial n} \simeq \frac{F_i\left(\pi_*, q_*, w_*, L_*, \left(n_* + \varepsilon\right)\right)}{\varepsilon}$$

Once the solver calculates the solution set $\{\pi_*, q_*, w_*, L_*, n_*\}$, it is therefore possible to obtain numerical estimates for both $\partial F/\partial n$ and $|J|$ which allow us to directly determine the stability of the equilibrium $\partial \pi/\partial n$ using (11), without any extra analytical work. This can be done without actually having to work out explicitly the partial derivatives or the implicit functions that describe the equilibrium.

Because Puga (1999) provides analytical solutions for the stability breakpoints of the symmetric equilibrium, it is possible to test the validity of this implicit function methodology. For the migration version, the analytical prediction made in Puga (1999) is that the symmetric equilibrium is stable if the following condition is met:

$$\tau > \left(1 + \frac{2(2\sigma - 1)\left(\gamma + \mu(1-\gamma)\right)}{(1-\mu)\left((1-\gamma)\left[\sigma(1-\gamma)(1-\mu) - 1\right] - \gamma^2\eta\right)}\right)^{\frac{1}{\sigma-1}} \tag{12}$$

Figure 1 shows a plot of (12), as well as the implicit function stability measure derived from a simulation of (1.1)-(1.4). Both use the parameter values from Puga (1999)[4].

[3] This is usually left to the discretion of the researcher, but typical values are around 10e-6 to 10e-9

[4] These are the values in Figures 1-3 of Puga (1999):
$\gamma = 0.1$, $\theta = 0.55$, $\mu = 0.2$, $\sigma = 4$ and $\eta = 11$

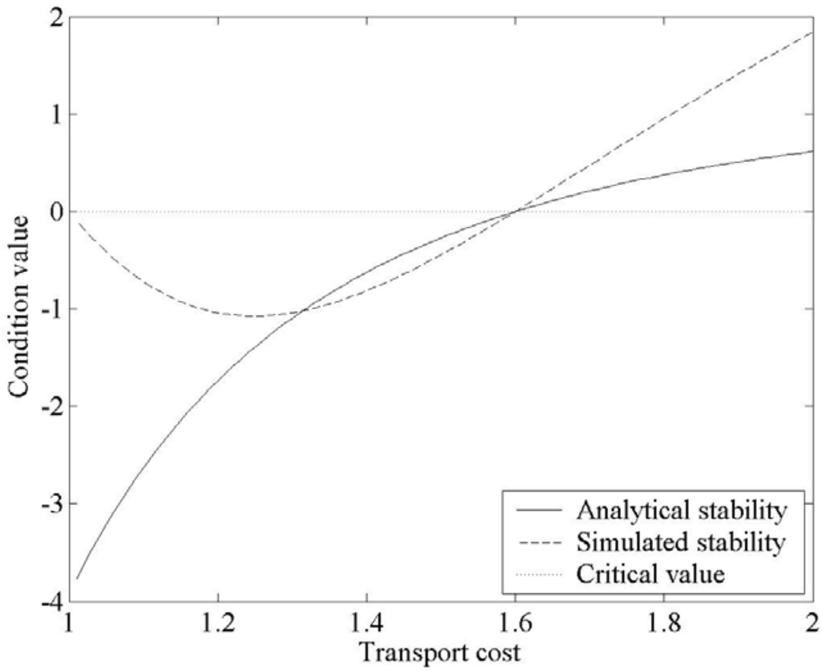

Figure 1: Analytical and Simulated Stability Conditions
Puga (1999) migration case

The important result is that although the functional form is clearly different, both functions change sign at the same point, at $\tau = 1.6$. The difference between the two functional forms probably stems from the fact that the Puga (1999) analytical condition is obtained through a simplification, and therefore only includes the parts of the analytical jacobian that determine the change in sign.

In Puga (1999) the analytical stability condition in the absence of migration is given by equation (13), which is quadratic in $\tau^{1-\sigma}$. A positive value of the function indicates a long-run stability of the symmetric equilibrium and a negative value, in between the roots of the quadratic, means the symmetric equilibrium is unstable, and the only long run outcome is agglomeration.

$$\begin{aligned}&(\sigma(1+\mu)-1)\left[(1+\mu)(1+\eta)+(1-\mu)\gamma\right]\left(\tau^{1-\sigma}\right)^2\\&-2\left[\left(\sigma\left(1+\mu^2\right)-1\right)(1+\eta)-\sigma(1-\mu)\left(2(\sigma-1)-\gamma\mu\right)\right]\tau^{1-\sigma}\\&+(1-\mu)\left(\sigma(1-\mu)-1\right)(\eta+1-\gamma)>0\end{aligned} \tag{13}$$

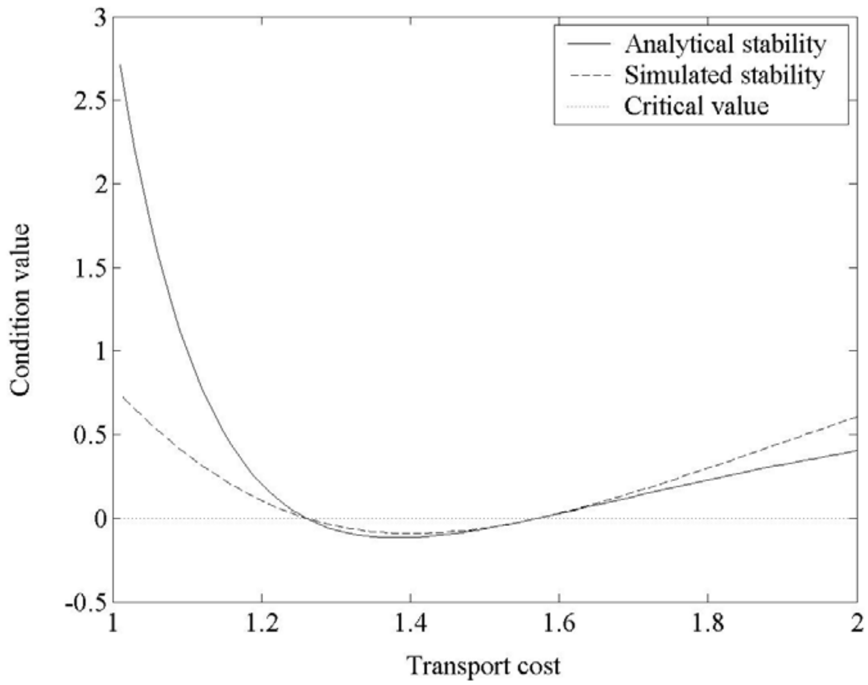

Figure 2: Analytical and Simulated Stability Conditions
Puga (1999) non-migration case

Figure 2 shows a plot of (13) and the numerical stability condition obtained through the implicit function methodology[5]. As for the migration case, the analytical and numerical breakpoints correspond exactly. Again, the functional forms are slightly different, but this can be explained by the fact that as for (12), condition (13) is obtained though a simplification and only contains the parts of the analytical jacobian that relate to the change in signs. The important result, however, is that as for the migration case, the two functions share the same roots and their signs change in the same way.

Figures 1 and 2 confirm that even though the jacobian $|J|$ and the shocks $\partial F/\partial n$ are not exact values but numerical approximations given by the solver, the implicit function methodology can predict the breakpoints of the system, without any prior analytical work. Although this is of little interest for the Puga (1999) model as the analytical stability conditions already exist, it can prove useful in situations where the analytical calculations are not tractable. This could be situations where the presence of

[5] The parameter for figure 2 are the values from Figure 4 of Puga (1999): $\gamma = 0.1$, $\theta = 0.55$, $\mu = 0.4$, $\sigma = 4$ and $\eta = 11$

more regions, more sectors, or more agglomeration channels makes the analytical derivation of a stability function more complicated.

4. Stability of the extended Puga model with migration

The migration version of the extended model, in other words the full system (7.1)-(7.5), is simulated using the same parameters as for Figure 1, so that the effect of the extension on the equilibrium can be evaluated. For the purpose of this section, the parameters on the extension are set at 0.05 for the friction share χ and at 0.5 for the non-linearity parameter φ. In this simulation, therefore, the urban friction is a relatively small effect, but it will nevertheless be shown to be important. In order to provide an element of sensitivity analysis, simulations were carried out with χ and φ parameters around the values used. The results of these simulations are presented in Figures 5 and 6, in appendix. Figure 3 shows the long run equilibrium path of the share of manufacturing for the system as a function of transport cost. Two important observations can be made from the analysis of this figure.

First of all, Figure 3 shows that at low levels of transport cost agglomeration ceases to be a stable outcome, and manufacturing disperses itself over regions again. This is not the case in the original model, where there is only one bifurcation away from the symmetric equilibrium, around $\tau = 1.6$, and it is the only stable outcome for lower levels of the transport cost.

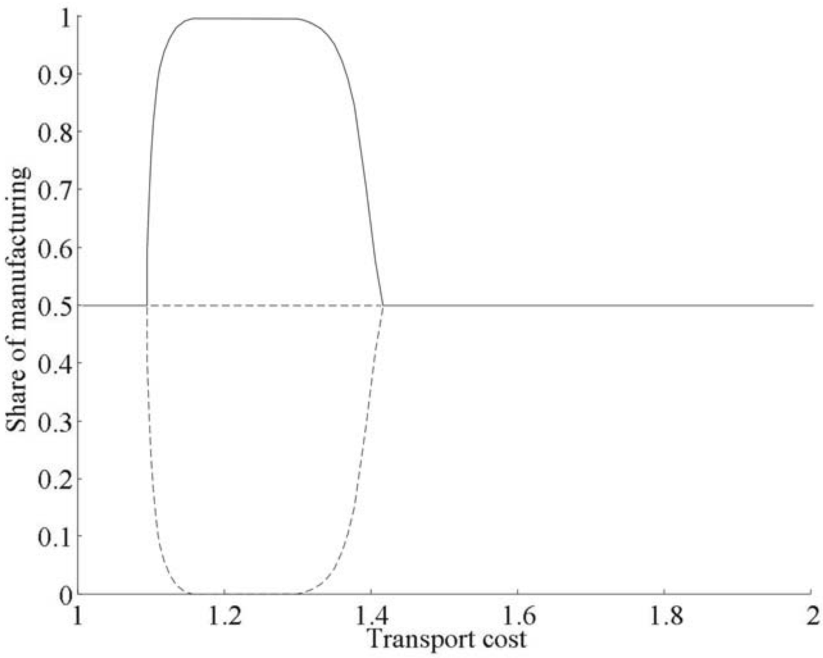

Figure 3: Path of Spatial Equilibrium w.r.t Transport Cost
Extended model with migration

The economic rationale for this is shown in equation (5). With urban frictions, disposable income is reduced, which reduces consumer demand for all goods, and slightly reduces the profitability of locating in a city. With migration, total agglomeration implies a large city and high per capita friction costs, as all the firms are located there, as well as most of the labour. As transport cost drop and manufacturing output increases, so do nominal wages, attracting even more labour and pushing the friction costs up again.

As transport costs keep falling, a critical level is passed where it becomes profitable for a firm to switch regions. The cost of moving is having to pay the transport cost twice, once while importing the intermediate inputs from the city and the second when exporting the output back there, where most of the demand is still located. As transport costs fall, so does this cost, for obvious reasons. The benefit of moving is the increase in consumer expenditure following the reduction in city size and frictions when the firm decides to move. As transport costs fall, this benefit increases. The agglomerated city size increases as transport costs fall, and the positive effect on expenditure of a firm relocating will be bigger the lower the transport cost. In that respect, the migration case becomes much similar to the non migration case, in that the agglomerated equilibrium will disperse when transport costs fall below a critical level.

The second observation is that the partially agglomerated portion of the equilibrium path, between the $share = 0.5$ and $share = 0$ or 1 lines, is now stable in the long run, which was not the case in the original Puga (1999) model. This can be inferred from Figure 3, as the partial agglomeration path curves are located in between the breakpoints rather than on either side, as was the case for the tomahawk bifurcations in Puga (1999). Even though for this set of parameters total agglomeration in one of the two regions is the dominant form of agglomeration, the paths leading to it are also stable in the long run. This is probably the more important of the two changes present in this part of the model, as it shows that the inclusion of urban frictions can indeed lead to situations where the economy agglomerates only partially in a region.

The reason for this is again linked to equation (5). In the original Puga (1999), at the symmetric equilibrium, all variables are equal in both regions. Assuming the level of transport costs allows for agglomeration to occur, if one firm moves from region 2 to region 1, it will slightly increase the demand for labour in region 1, pushing wages up. This in turn creates a small migration of workers from region 2 to region 1, attracted by the slightly higher wages. Region 1 now has a bigger wage bill than region 2, which translates into higher demand for goods. This triggers the migration of more firms from region 2 to region 1, wanting to locate close to this demand, pushing up wages again and creating migration. This bang-bang effect continues until all the firms have located in region 1.

With urban frictions reducing disposable income, this bang-bang property is inhibited. The first firm moves from region 2 to region 1, wages increase and a few workers move to follow the firm. With the extension, this now increase the per capita friction cost, reducing the proportion of the wage bill spent on goods. So, on the one hand, the wage bill is larger, but on the other hand less of it is spent on goods. If the two balance out, effective demand doesn't increase and no more firms will want to move. The partial equilibrium has become stable, and it takes a further drop in transport cost to increase agglomeration.

The sensitivity analysis in appendix, more particularly Figure 5, shows that for some higher values of the χ and φ parameters, partial agglomeration is the only form of agglomeration possible, even with migration. Total agglomeration remains the dominant type of agglomeration in most cases, but it is possible to show situations where this does not happen. Furthermore, Figure 5 shows that the portions of the long-run equilibrium paths for which there is partial agglomeration are stable. The relevance of this is that it shows that the stability of the partially agglomerated equilibrium is a property of the extended model, and not the accidental by-product of the parameters chosen.

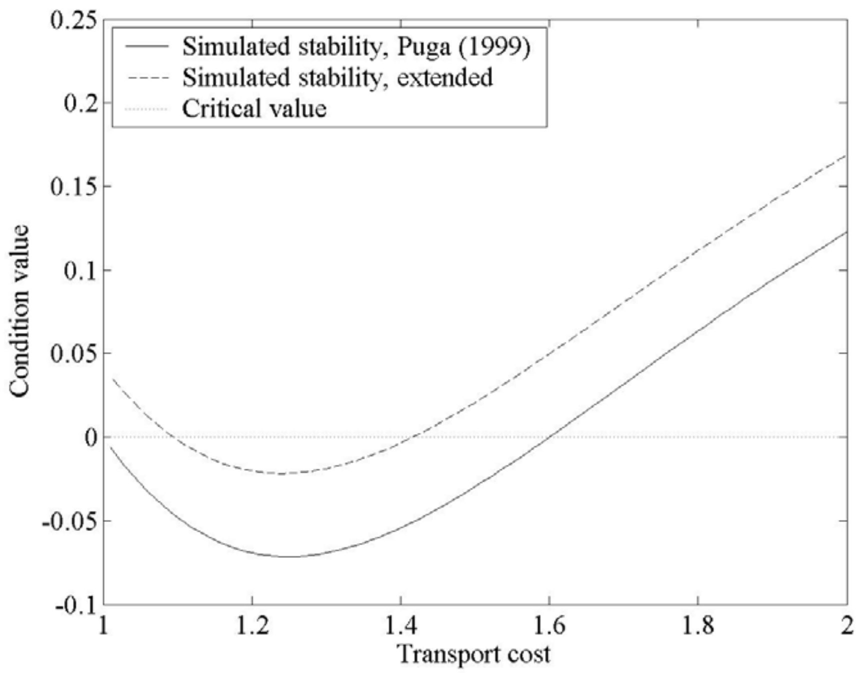

Figure 4: Comparison of Simulated Stability Conditions
Original and extended migration cases

A comparison of the symmetric equilibrium stability condition obtained using the implicit function method with the stability condition for the original Puga model is shown in Figure 4. It reveals that the presence of an urban friction reduces the range of transport costs over which agglomeration occurs. This is also true of all the sensitivity simulations shown in Figure 6, in appendix. This is where the implicit function methodology developed previously proves useful. It facilitates the analysis of stability, as the inclusion of the per capita friction cost variable c and its interaction with wages w and the number of firms n makes the determination of analytical stability functions much more complicated.

Including urban frictions, even in an abstract manner, therefore seems to reduce the range over which any form of agglomeration is possible, partial or total. This can be linked to the reduction in consumer disposable income resulting from the existence of an urbanisation friction, and the resulting reduction in consumer demand. In particular, as shown above, this is what explains the observed instability of agglomeration for low transport costs.

Importantly, an analysis of Figures 4 and 6 shows that the occurrence of agglomeration in the simulations is sensitive to the friction share parameter χ. In other words, the presence of rents is strong dissaglomeration force and agglomeration only occurs for low values of χ. With the original Puga model parameters, any value of χ significantly above the maximum one tested (0.06) leads to the complete absence of agglomeration. Furthermore, the non-linear parameter φ is also less than one, and any value significantly above the ones tested here again leads to the complete stability of the symmetric equilibrium. What this shows is that in the presence of migration, the urban friction has a large leverage on the presence and type of agglomeration, and small variations in parameters have large influences on the agglomeration outcome for the manufacturing sector.

Compared to the migration version of the Puga (1999) model, agglomeration is less likely, due to the extra dissaglomeration force introduced by the urban land market. When agglomeration does, occur, however, it can be both partial and stable in the long run. This is an important result, as it confirms that including the costs of agglomeration, an improvement in itself, can produce a richer range of agglomeration outcomes.

5. Stability of the extended Puga model in the absence of migration

Similar to the structure of the original Puga (1999) model, the non-migration version of the extended model consists of the reduced system (7.1)-(7.4), where the equation which equalises real wages over regions, (7.5), is dropped. Again, the main model parameters for this version are the same as the ones used in the non-migration version of Puga (1999), in Figure 2. For the friction parameters, the friction share parameter χ is set at 0.5 and the non-linearity is left at 0.5. This means that for an equally sized city, the per capita cost of agglomeration is higher than in the previous simulation, and increases in city size will have a higher impact on friction costs. As for the previous section, a sensitivity analysis was carried out on the friction cost parameters. The main results of these simulations are shown in Figures 9 and 10 in appendix.

Figure 7 shows the long run equilibrium path of the share of manufacturing that results from the simulation.

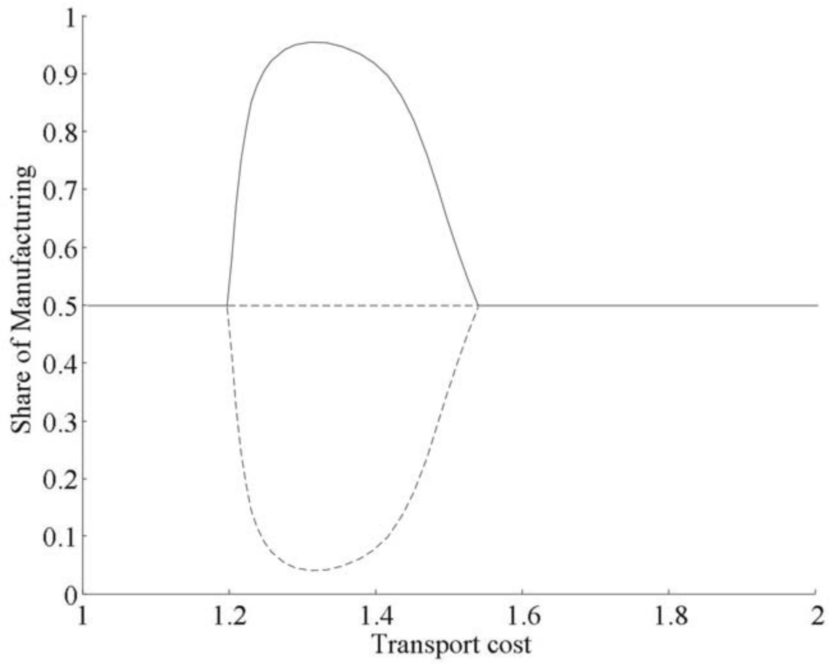

Figure 7: Path of Spatial Equilibrium w.r.t Transport Cost
Extended model with migration

Comparing the result of this simulation with the original version of the non-migration Puga (1999) model, the effect of the extension is initially less surprising than in the migration case. Indeed, there is at least no major change in the bifurcations. Overall, in both the original and extended models, the symmetric equilibrium stays stable for low and high values of the transport costs, and agglomeration only occurs for an intermediate range.

The difference with the original Puga model is that there is now no total agglomeration, and a partially agglomerated equilibrium is the only alternative to the symmetric equilibrium. This is also visible in Figure 9 in appendix. The economic reasons behind this are similar to the migration case. The purchasing power of workers is lower than in the original model, which makes locating in a city less profitable for firms. The difference, in this version, is that because there is no migration, workers are not able to follow firms as they change regions, making total agglomeration much more difficult. In a situation where a large proportion of manufacturing is located in region 1, the wages manufacturing firms have to pay will be high, because without migration labour is scarce. At the same time, the urban area is large, because most of the firms are located there, so the per capita urban friction costs are high, and a significant part of the wages paid are not converted into consumer demand. Manufacturing costs are therefore high, due to high wages, and the presence of frictions means that consumer demand is lower than in the original model. Furthermore, due to the absence of migration a large portion of the demand is still located in region 2, and a firm wanting to ship goods there is subject to transport costs.

At some point during the agglomeration process, which depends on the friction cost parameters, it will become profitable for a firm to locate in region 2, where labour inputs are more plentiful and cheaper, and cater for the demand located there. At this point, the city located in region two has only a small share of manufacturing firms. Urbanisation costs are therefore lower than in region 1, and more of the region 2 wage bill gets converted into demand. Total agglomeration in region 1 is therefore extremely difficult in this case, as the alternative of locating in a region that has cheaper labour inputs and a relatively high demand will become profitable before total agglomeration occurs.

The possibility of a stable partially agglomerated path without migration in the Puga model is not new, as it was shown in figures 6 and 7 of Puga (1999) it is possible to obtain a path similar to Figure 7 above. The relevance of the extension in this case is that the partial agglomeration is achieved with the same structural parameters as in the original Puga (1999)[6]. The partial agglomeration result of Puga (1999) relies on an extreme value of the θ parameter, which is set at 0.94. θ being the elasticity of agricultural output with respect to labour, this implies an incredibly labour-intensive agricultural sector, with an elasticity of agricultural output with respect to land of only

[6] These are σ, μ, γ, and θ.

0.06. This means that although partial agglomeration can be obtained in the non-migration version of Puga (1999), it is not a general property of the model. This is not the case here, where the structural parameters are unchanged compared to the original version of the Puga model and where the partially agglomerated equilibrium is a widespread result that stems from urban frictions.

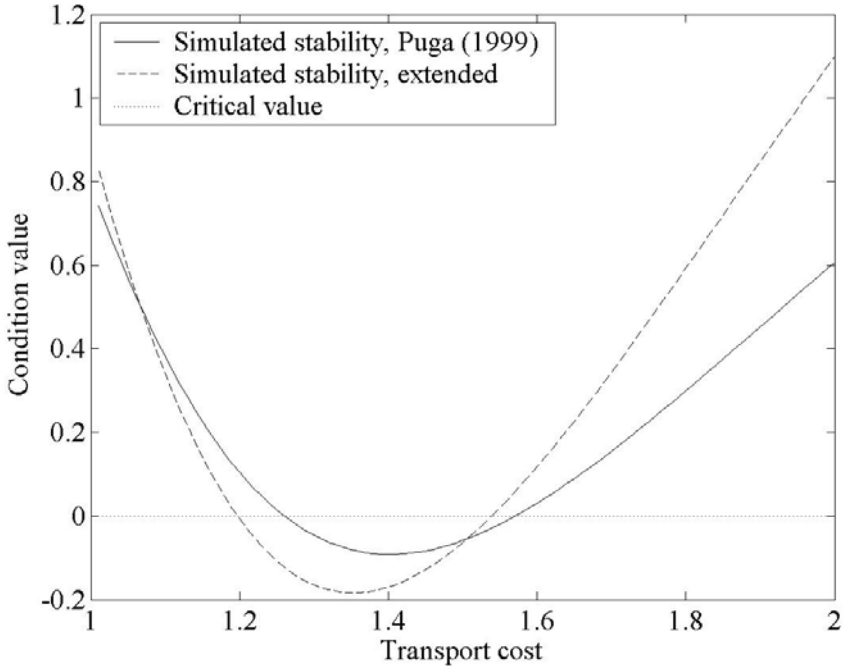

Figure 8: Comparison of Simulated Stability Conditions
Original and extended non-migration cases

Another important aspect of the non-migration version of this extension is its effect on the range of transport costs over which the symmetric equilibrium is stable. Figure 8 shows a comparison of the original and extended stability functions calculated using the implicit function method. When compared to the migration case in Figure 4, what stands out is that in the non-migration case, the effect of the extension on the range of agglomeration is more complex and ambiguous. The presence of urban frictions doesn't simply push the stability function up, as was the case for migration. A quick look at the sensitivity analysis in Figure 10 in appendix confirms that this is also the case on a wider range of friction costs parameters.

For the higher range of transport costs, the effect of the extension on the stability function is the same as in the migration case. Because consumer demand is constrained to be spread over regions and transport costs between regions are still high, the presence of frictions decreases disposable income and reduces the profitability of locating in a city. The counter-intuitive aspect is that although rents reduce consumer spending, agglomeration is sustainable below the original Puga (1999) critical point. What explains this sustained agglomeration is that in the absence of migration and for low values of the transport cost, partial agglomeration is just as profitable as dispersion. The presence of frictions just makes it stable in the long run.

The profitability argument stems from the absence of migration. At the symmetric equilibrium, just to the left of the original lower breakpoint, the spreading of manufacturing firms across regions means that wages are low. As a result, consumer demand is lower than in the presence of a city, where the high labour demand and absence of migrant labour create high wages. This is what underpins the profitability of partial agglomeration. Because labour supply is fixed by the absence of migration, wages will strongly increase in response to agglomeration. Therefore, it is profitable for some firms to agglomerate in one region, increase the city size and boost consumer demand with higher wages. The presence of some agglomerated firms also reduces the cost of procuring intermediate inputs. Furthermore, because of the relatively low level of transport costs, it is possible to ship the output produced in this small city to the other region, where a lot of demand is still located. This will be the case until the transport cost drops below a certain point, at which point the dispersion force of being able to costlessly ship intermediate inputs from and outputs to any region will become too strong to make partial agglomeration profitable.

Importantly, this effect is also visible in the original Puga model, and does not depend on the presence of a cost of urbanisation. In the original non-migration version, for transport costs just below the lower breakpoint, there exists a partially-agglomerated long run equilibrium, even though it is unstable in the long run. However, because the urban frictions inhibit the bang-bang transitions of the model, in the extended case this partial agglomeration equilibrium becomes stable.

In terms of sensitivity, an analysis of Figures 9 and 10 in appendix indicates that this version of the model is less sensitive to the friction share χ than the migration version. Increasing χ affects the range and scope of agglomeration, but to a lesser extent than the migration version. The parameter is also set an order of magnitude higher than in the migration version. In the absence of migration of workers, the city size will never increase as much as in the migration case, so one would expect to have to increase χ more to obtain similar effects.

6. Comparison with the Puga (1999) results and discussion

Section 2 shows that the extended model developed nests the original Puga framework. What this means is that it is possible to use exactly the same structural parameters as in Puga (1999) and modify the urban friction independently to evaluate the effect of the extension. The previous section show the main result is that partial agglomeration becomes can be a stable long run equilibrium. This is an important point for NEG, in view of recent analytical results in Robert-Nicoud (2004) and Ottaviano and Robert-Nicoud (2006), which suggest that partial agglomeration is always unstable in Dixit-Stiglitz models. Their result is challenged by fact that the inclusion of urban frictions can create stable partial agglomeration. This is a point that needs to be explored further. In particular, analytical work needs to be carried out, along the lines of the studies mentioned above, to explore the general properties of NEG models with specific urban frictions

In addition, the figures in appendix show that small changes in urban costs can have large effects on the location of the agglomeration path. In particular, the sensitivity of the agglomeration to the friction cost parameters is different in the two versions of the model. This underlines the need to evaluate and understand the degree of imperfectness in migration when targeting urban costs. The same policy can have a different effect on agglomeration depending on how free migration. As mentioned in Combes *et al* (2005), this is an aspect of the field that has received little attention.

Another central finding is the complex influence that urban frictions have on the stability function in the absence of migration. This affects studies that test the analytical Puga (1999) breakpoints. A first example is the sectoral test of the Puga breakpoints carried out in Head and Mayer (2004). Using data from two pairs of countries USA-Canada and France-Germany, this study calculates the range of transport costs over which 21 different industrial sectors should be agglomerated. When comparing the range of transport costs with the actual situation in those sectors, the finding is that nearly all the industrial sectors should be disaggregated. Head and Mayer point out that cautious interpretation is needed as the ranges are quite sensitive to the parameters chosen.

Another case is the Brakman *et al* (2006) test of agglomeration in the EU. This study attempts to tackle the often thorny issue of testing new economic geography models. Using EU data, an equilibrium wage equation derived from the Puga model is estimated. This provides estimates for the elasticity σ and the transport cost τ. The Puga (1999) specification is then used with these estimates to determine the level of agglomeration in the EU using. As is the case in Head and Mayer (2004), Brakman *et al* recognise that the assumptions behind the Puga model are unrealistic, and a lot of corner-cutting has to be done in order to get reasonable estimates of the parameters, particularly as the analytical Puga breakpoints in are only valid for a two-region world. The central result of the study is that agglomeration forces seem to be too weak to explain the spatial structure of the EU. Importantly, they claim that agglomeration forces that arise from Puga the puga model are smaller than those generated in Krugman (1991).

Using the extended model and the different stability functions obtained, there is little doubt that the predictions agglomeration in Head and Mayer (2004) and Brakman *et al* (2006) would be different. The relative weakness of agglomeration they find in the Puga model compared to other NEG models would be affected, as this paper shows that with an urbanisation cost the breakpoints can be further apart. This is not to say that the problems pointed out in these two studies would necessarily be solved by this extended version of Puga (1999), particularly as far as the number of regions or sectors is concerned. However, what is clear is that urban frictions of the kind presented here affect both the range and scope of agglomeration and not including them in an empirical study will probably bias the results.

7. Conclusion

The central finding of this paper is that it is possible to obtain stable partial agglomeration within a vertically linked Dixit-Stiglitz model by accounting for the extra social costs of urbanisation. This is true for both versions of the Puga framework, when migration between regions is allowed, or not. Furthermore, simulations using a wider range of friction cost parameters have shown that whilst the presence of partial agglomeration is widespread, it is also sensitive to the parameters chosen. This is especially true for the migration case, where very small changes in the parameters have large effects in terms of agglomeration.

Importantly, what drives the partial agglomeration result in both cases is the reduction in the disposable income due to urban friction costs. In the original Puga model, all of the wages are spent on goods. The extension introduces a friction that reduces the disposable income of workers and reducing demand as a result, making it slightly less profitable for firms to agglomerate. In the simple framework assumed, this happens as a result of an urbanisation cost that is paid for by the entire population. The examples mentioned are pollution or the cost of urban infrastructure. However, even with more complex urbanisation frictions and a more realistic spread of costs it would be possible to obtain the results shown in this study. What this means is that stable partial agglomeration can result directly from a reduction in demand linked to the existence and size of a city. Any mechanism which reduces the disposable income of workers based on the size of the urban area would provide a similar effect. This has been shown to have important theoretical consequences on existing new economic geography theory.

Last of all, the implicit function methodology developed in this study shows its relevance, allowing the comparison of stability functions between the versions without requiring prior analytical derivations. In the migration case, the presence of frictions creates a dispersion force which reduces range of transport costs over which agglomeration occurs. The non-migration case, however, is more complex, and comparison of the original and extended stability shows that depending on the land market parameters the range can be narrower, or wider than in the initial Puga (1999) specification.

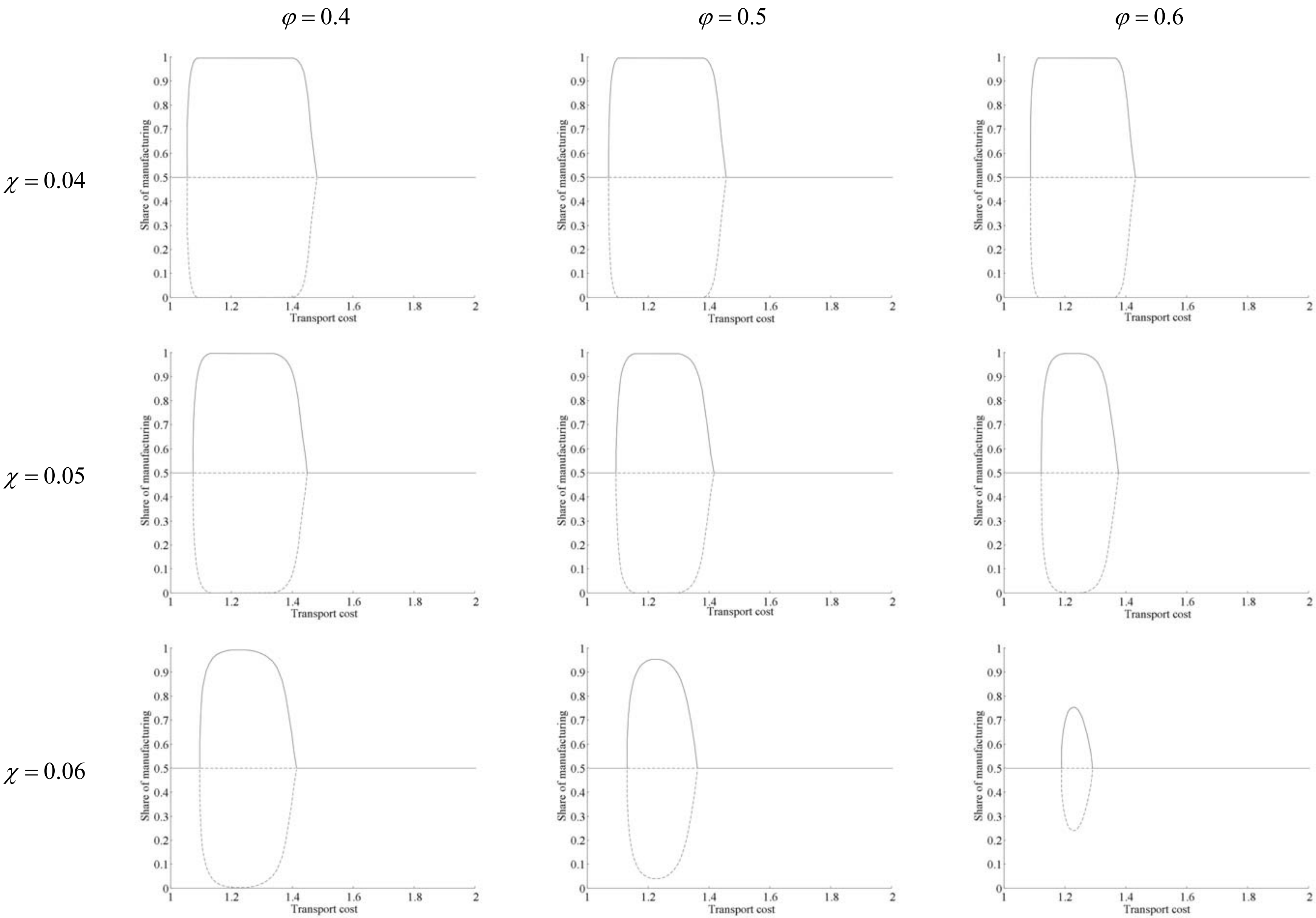

Figure 5: **Sensitivity of the migration equilibrium to friction cost parameters, equilibrium path**

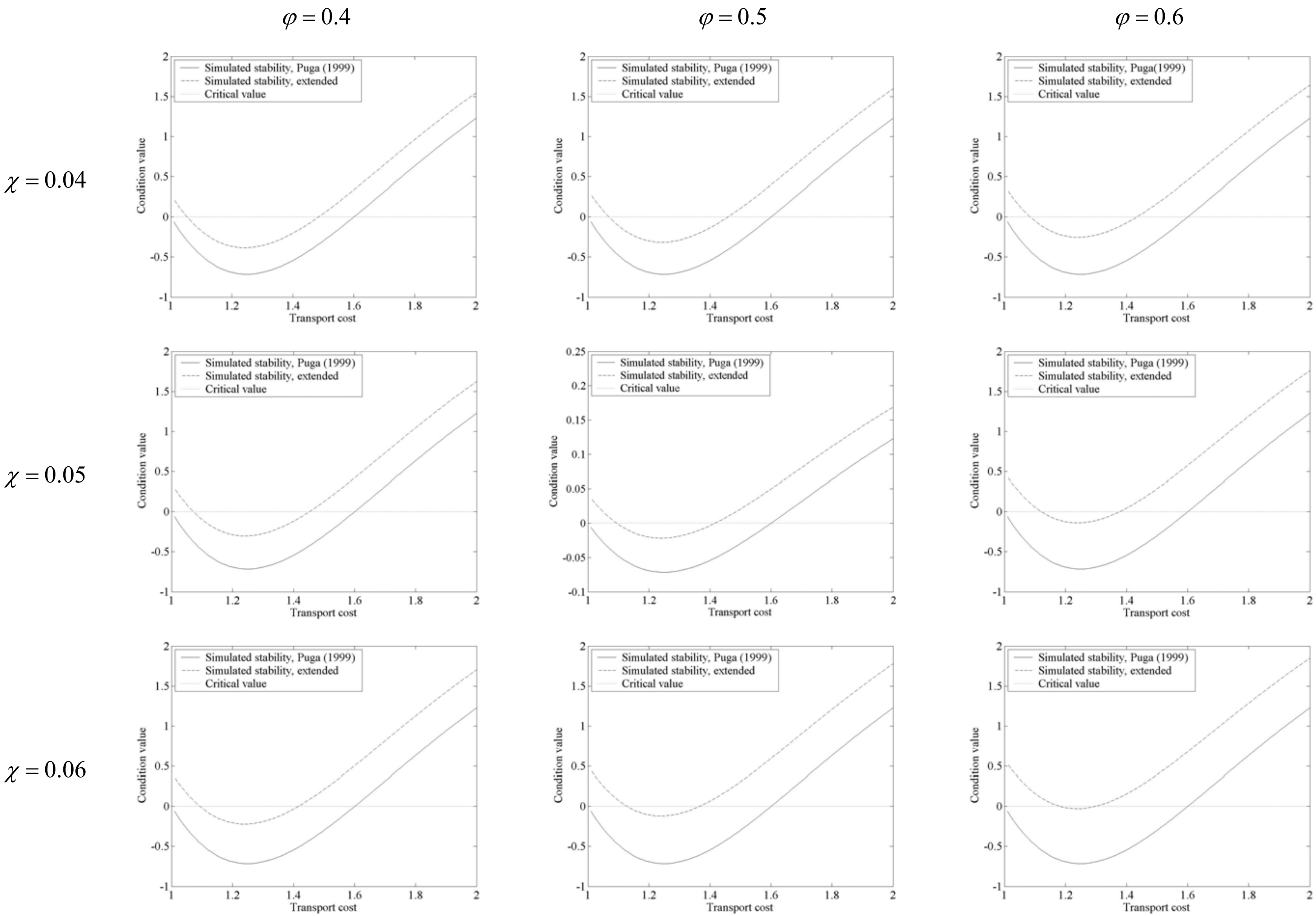

Figure 6: **Sensitivity of the migration equilibrium to friction cost parameters, stability condition**

$\varphi = 0.4$ $\varphi = 0.5$ $\varphi = 0.6$

$\chi = 0.4$

$\chi = 0.5$

$\chi = 0.6$

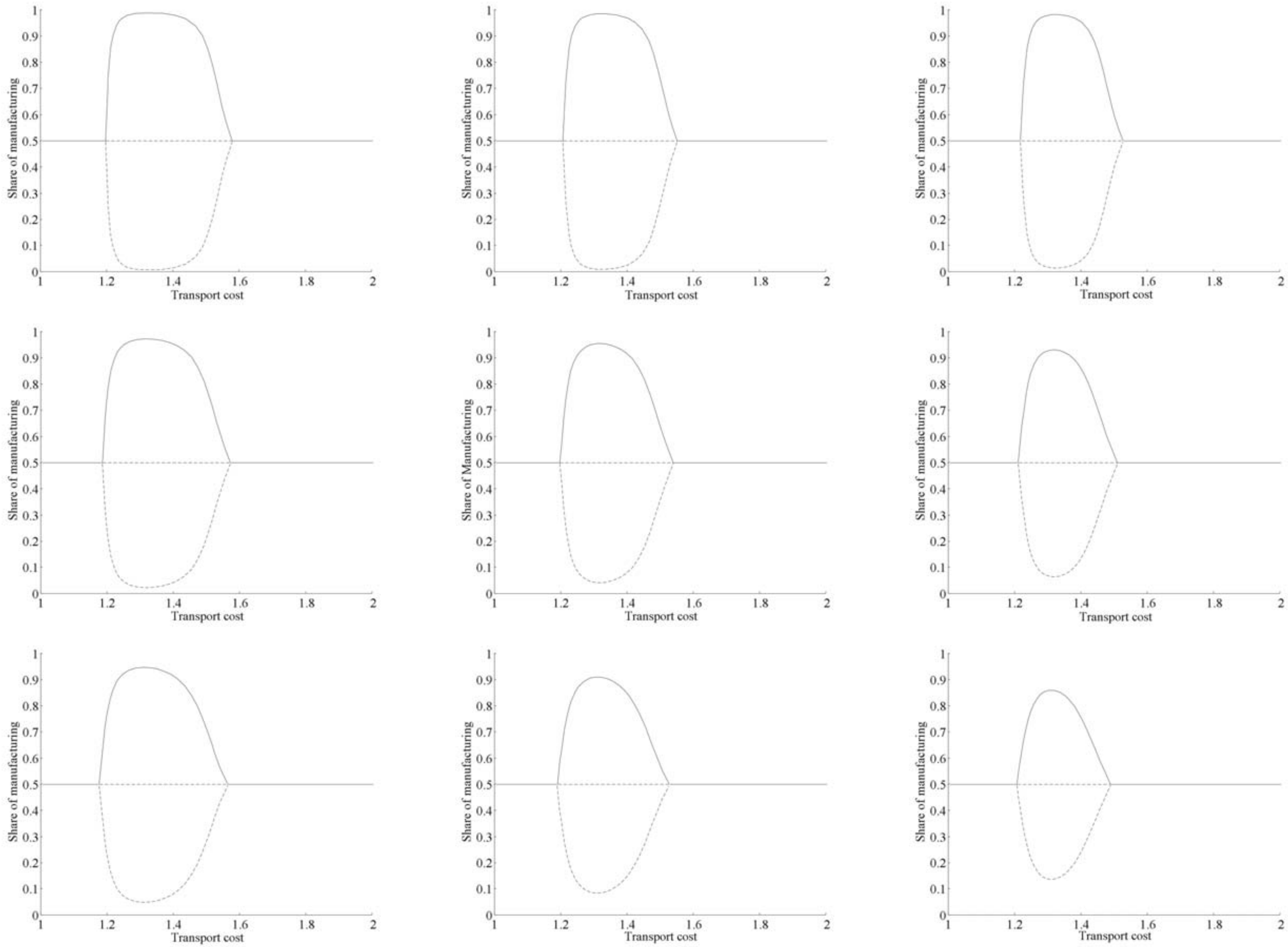

Figure 9: Sensitivity of the non-migration equilibrium to friction cost parameters, equilibrium path

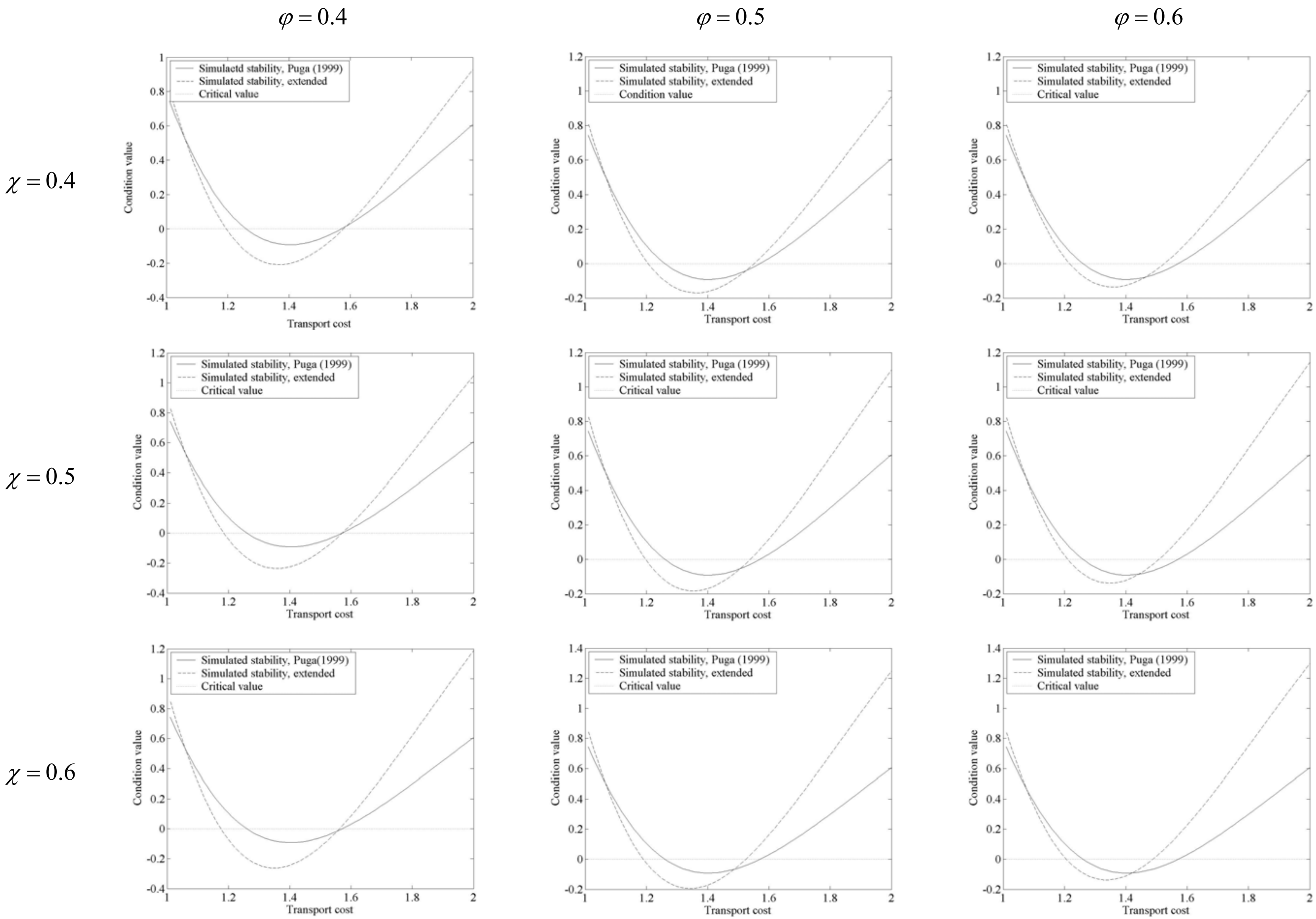

Figure 10: Sensitivity of the non-migration equilibrium to friction cost parameters, stability condition

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