

**Modelling the folk theorem: A spatial Cournot model with explicit  
increasing returns to scale**

Sylvain Barde

*Department of Economics*

*University of Kent*

**Abstract**

This paper attempts to model directly the “folk theorem” of spatial economics, according to which increasing returns to scale are essential for understanding the geographical distributions of activity. The model uses the simple structure of most New Economic Geography papers, with two identical regions, a costlessly traded agricultural sector and a manufacturing sector subject to iceberg costs. This simple setting isolates IRS in manufacturing production function as the only potential agglomerating force. This implies that an unstable symmetric equilibrium means IRS cause agglomeration

The central result is that while a CRS manufacturing sector will always stay at the symmetric equilibrium, the presence of IRS in manufacturing causes the symmetric equilibrium to become unstable and agglomeration becomes the only long run equilibrium for the system

**JEL Classification:** R10, R12, F12

**Keywords:** Agglomeration, increasing returns to scale, imperfect competition.

**Address for Correspondence:** Department of Economics, University of Kent,  
Canterbury, Kent, CT2 7NP. Telephone: 01 227 82 76 39 Email: [sb71@kent.ac.uk](mailto:sb71@kent.ac.uk)

# **Modelling the folk theorem: A spatial Cournot model with explicit increasing returns to scale**

## **1. Introduction**

In a benchmark review of economic agglomeration theory Fujita and Thisse state that “increasing returns to scale are essential for explaining the geographical distributions of economic activities” (1996, p342). Many other reviews, such as Krugman (1998), Ottaviano and Puga (1997), or Brakman et al (2001) similarly point out that returns to scale are a key concept underpinning agglomeration. This has become known in the field as the “folk theorem” of spatial economics. However, to our knowledge, no existing spatial model looks directly at the impact of explicit IRS in manufacturing on agglomeration. Existing New Economic Geography models using the Dixit-Stiglitz (1977) approach do integrate returns to scale, however, they use a production structure with fixed and variable costs to create the returns to scale<sup>1</sup>. This paper investigates the effect of explicit IRS in the manufacturing sector of a two region-two sector model on the stability of the dispersed equilibrium.

Most of the theoretical analyses of agglomeration, especially in NEG, use the two region/two sector setting because it is the simplest one in which the various centripetal and centrifugal forces can be identified. In particular, most of them, such as Puga (1999) focus on deviations from the symmetric equilibrium. The simulation approach used in this paper is therefore similar to the one in Puga (1999). The aim is to link the presence of explicit increasing returns to scale in manufacturing to deviations

---

<sup>1</sup> The first model to use this framework is basic Krugman (1991). Fujita et al (1999) also provides a review of NEG models. For a NEG model with more complex interactions, the reader is referred to Puga (1999).

from the symmetric equilibrium. Because by construction no other agglomeration forces are included in the model, changes in the stability of the symmetric equilibrium can be traced directly to the presence of IRS in the manufacturing sector.

In order to deal with the strategic competition that emerges from including explicit IRS into the production function, a Cournot competition framework is used. Furthermore, there is an existing spatial Cournot literature, such as Andersen and Neven (1991), Gupta *et al* (1997) or Mayer (2000), Combes and Lafourcade (2005) which provide economic basis for agglomeration. Agglomeration in most of these studies rests firm entry in the most profitable locations is the agglomerating force. This study will therefore impose the restriction of a fixed number of firms, so as to be able to assess the agglomerative effect of IRS independently of firm entry.

The remainder of the paper is organised as follows: Section 2 presents the model used in the simulation. Section 3 discusses the behaviour of the model at the symmetric equilibrium and how the presence of IRS in manufacturing modifies the production costs. Section 4 then analyses how the stability of the symmetric equilibrium is affected by the IRS in manufacturing, and section 5 concludes.

## **2. A 2 region 2 sector Cournot model with explicit IRS**

In order to generate a Cournot setting in which IRS is the only agglomerating force, the following assumptions will be made. The two regions will be identical in endowments of land ( $K$ ) and labour ( $L$ ). Distance is normalised to one, as is the TFP for both sectors. Agriculture is freely traded and manufacturing is subject to iceberg transport cost  $e^{\tau}$ . In the first simulation, manufacturing will be subject to CRS, in order

to provide a benchmark, but then IRS will be introduced. In terms of notation, superscripts  $a$  and  $m$  indicate the agriculture and manufacturing sectors respectively.

### *Production and firm behaviour*

The production function used here is similar to the one in Combes and Lafourcade (2001), but introduces increasing returns to scale. In terms of notation, we assume there are two industrial sectors over two regions. For the purpose of the notation,  $r = 1, 2$  and  $h = a, m$ . As in Combes and Lafourcade (2001), a single Cobb-Douglas production function describes both agriculture and mining manufacturing. These two industries differ only in that the elasticities of output with respect to inputs will therefore be industry-specific. The difference with Combes and Lafourcade (2001) is the assumption that for any given industrial sector, there is a fixed number of producers. The spatial Cournot literature shows that free entry in the most profitable locations can cause agglomeration even in constant returns to scale. Therefore, in order to isolate the agglomerative effect of IRS, we assume a fixed number of producers. For purposes of simplification, we choose a single producer per sector and region.

Fujita and Thisse (1996) point out that the presence of IRS in a production function creates non-convexities. For example, a property of Cobb-Douglas functions under CRS is that the share of producer expenditure allocated to an input is equal to the elasticity of output with respect to that input. However, with IRS this sum of expenditures on individual inputs would be greater than the total expenditure of the producer, which makes no sense economically. We show later in this section that most of these problems can be satisfactorily addressed. In particular, an assumption made to correct this problem is that the producers use one set of CRS elasticities and a returns to scale parameter to modify the elasticities. The CRS elasticities, which sum to one, are

used to determine the inputs shares for expenditure. The returns to scale parameter then introduces IRS into the CRS production function.

The production function for the  $h^{\text{th}}$  industry in location  $r$  is a Cobb-Douglas of the following form:

$$y_r^h = A^h (K_r^h)^{\alpha^h} (L_r^h)^{\beta^h} \prod_{i=1}^H (x_r^{i,h})^{\varepsilon^{i,h}} \quad (1)$$

Here  $A^h$  technology,  $K$  is the use of land,  $L$  is labour. Intermediate consumption is introduced through  $x^i$ , which is the use of the  $i^{\text{th}}$  industry's good as an intermediate input, where  $\varepsilon^{i,h}$  is the elasticity of the  $h^{\text{th}}$  industry's production with respect to the input from the  $i^{\text{th}}$  industry. This allows for vertical linkages within the model. Agriculture is assumed to have sole use of land, so  $\alpha^m = 0$ , and does not use intermediate inputs, meaning  $\forall i, \varepsilon^{i,m} = 0$ . Furthermore, as explained above, if  $\Psi^h$  is the industry-specific returns to scale parameter, the following applies:

$$\alpha^h + \beta^h + \sum_{i=1}^H \varepsilon^{i,h} = 1 \quad (\text{CRS elasticities})$$

$$\alpha^h + \beta^h + \sum_{i=1}^H \varepsilon^{i,h} = \left( \alpha^h + \beta^h + \sum_{i=1}^H \varepsilon^{i,h} \right) \Psi^h = \Psi^h \quad (\text{IRS elasticities})$$

As clarified above, the set of CRS elasticities is used for all the input demand determinations, whereas the modified elasticities are the ones used in evaluating the cost-reducing effects of the existence of returns to scale.

The cost minimisation problem is straightforward, and involves minimising (2) subject to (1):

$$C_r^h = cK_r^h + w_r L_r^h + \sum_{i=1}^H p_r^i x_r^{i,h} \quad (2)$$

Where  $c$  is the rental price of capital,  $w$  are the wages and the final summation is the expenditure by the  $h^{\text{th}}$  industry on intermediate inputs from other industries. The cost function obtained through the minimisation is:

$$C_r^h = \chi_r^h (y_r^h)^{\frac{1}{\Psi^h}} \quad (3)$$

Where the  $\chi_r^h$ , the input component of marginal cost, is:

$$\chi_r^h = \left( \frac{(\Psi^h)^{\Psi^h} (c)^{\alpha^h} (w_r)^{\beta^h} \prod_{i=1}^H (p_r^i)^{\varepsilon^{i,h}}}{A^h (\alpha^h)^{\alpha^h} (\beta^h)^{\beta^h} \prod_{i=1}^H (\varepsilon^{i,h})^{\varepsilon^{i,h}}} \right)^{\frac{1}{\Psi^h}} \quad (4)$$

Transport costs are integrated using exponential iceberg costs, as in Samuelson (1952, 1954). The presence of IRS, however, creates a problem for the calculation of transport costs. One can see from the cost function (3) that in CRS, where  $\Psi^h = 1$  average and marginal costs are equal to  $\chi_r^h$ , and applying the multiplicative iceberg costs is straightforward. This is not the case for increasing returns to scale, where  $\Psi^h > 1$ , as the divergence of average and marginal costs means that a choice needs to be made as to which cost the iceberg applies to. We assume that the shipping costs relate to marginal costs of production. It makes economic sense for the increase in the cost of shipping an extra unit of output to relate to the value of that marginal unit. The total cost of producing in  $r$  and shipping to region  $s$  is therefore:

$$C_{r,s}^h = \frac{y_{r,s}^h}{y_r^h} C_r^h + y_{r,s}^h mC_r^h (e^{\tau^h d^{rs}} - 1) \quad (5)$$

Where the marginal cost of production is:

$$mC_r^h = \frac{\chi_r^h (y_r^h)^{\frac{1-\Psi^h}{\Psi^h}}}{\Psi^h} \quad (6)$$

With this assumption, one can see below that the marginal cost of producing an extra unit in  $r$  and shipping it to  $s$  contains two components: the first is the marginal cost

of producing that extra unit, which is the same regardless of the target market. The second part relates to the shipping of the extra unit to the target market  $s$ , which depends only on the marginal cost of production.

$$\frac{dC_{r,s}^h}{dy_{r,s}^h} = mC_r^h + mC_r^h \left( e^{\tau^h d^{rs}} - 1 \right) \quad (7)$$

The total cost of producing in a given region is given by the summation of all cost flows (5) over the  $s$  target regions:

$$\sum_{s=1}^R C_{r,s}^h = C_r^h + \sum_{s=1}^R y_{r,s}^h mC_r^h \left( e^{\tau^h d^{rs}} - 1 \right) \quad (8)$$

The total cost contains two separable components, the cost of producing in a region and the cost of shipping to other regions. An additional assumption made is that the  $mC_r^h$  term in the transport cost component is constant with respect to any output flow. This is necessary to keep equation (8) separable.

$$\forall s, \quad \frac{\partial mC_r^h}{\partial y_{r,s}^h} = 0$$

With the cost function specified, the next step is to derive the demand function for a sector  $h$  in a location  $r$ , in order to be able to close the model.

### *Final consumption and demand*

As is the case for all the vertically linked models, aggregate demand in each location is the sum of final consumption by workers and intermediate consumptions by other producers. The utility function for workers in region  $r$  is Cobb-Douglas, with  $Q_r^h$  the final consumption of the  $h^{th}$  good in region  $r$ .

$$U_r = \prod_{h=1}^H (Q_r^h)^{\mu^h} \text{ with } \sum_h \mu^h = 1 \quad (9)$$

All the income flows, including returns on capital and profits, are spent as final consumption. We assume that entrepreneurs have the same utility function as workers, so that their incomes can be pooled in the labour constraint, pro rata of the location of manufacturing and agricultural firms. In two regions, this gives  $\kappa_r = 0.5$ . The budget constraint is:

$$\sum_{h=1}^H p_r^h Q_r^h = w_r L_r + \kappa_r \sum_{r=1}^R \sum_{h=1}^H (c_r K_r^h + \pi_r^h) \quad (10)$$

Given the exogenously fixed amounts of labour and land, the wage and rents are given by:

$$w_r = \frac{\sum_{h=1}^2 C_r^h \beta^h}{L_r} \quad (11)$$

$$c_r = \frac{C_r^a \alpha^a}{K_r} \quad (12)$$

Solving the utility maximisation problem gives the optimal demand for each good:

$$Q_r^h = \frac{\left( w_r L_r + \kappa_r \sum_{r=1}^R \sum_{h=1}^H (c_r K_r^h + \pi_r^h) \right) \mu^h}{p_r^h} \quad (13)$$

The second source of demand is intermediate consumption from other sectors. As explained above, in order for the model to close, it is important to note that the elasticity used is the CRS version  $\varepsilon^{i,h}$ . The intermediate demand of industry  $i$  inputs in from industry  $h$  in location  $r$  is

$$x_r^{i,h} = \frac{C_r^h \varepsilon^{i,h}}{p_r^i}$$

Combining the two sources of demand yields the aggregate demand for the  $h^{\text{th}}$  industry's good in the  $r^{\text{th}}$  location. Furthermore, in order for the model to close, the output that melts away during transport must also be re-introduced as final

consumption. Because of the relatively small size of the transport cost and in order to minimise the impact on the model's solution, the total transport cost fed back into the regions, pro-rata of the share of total output  $\varpi_r^h$  the region produces. Final demand is therefore:

$$D_r^h = \frac{1}{p_r^h} \left( \left( w_r L_r + \kappa_r \sum_{r=1}^R \sum_{h=1}^H (cK_r^h + \pi_r^h) \right) \mu^h + \sum_{i=1}^H C_r^i \varepsilon^{h,i} + \varpi_r^h \sum_{h=1}^H \sum_{r=1}^R \sum_{s=1}^R y_{r,s}^h m C_r^h (e^{\tau^h d^{rs}} - 1) \right)$$

In order to simplify the notation, this is re-written as:

$$D_r^h = \frac{\Phi_r^h}{p_r^h} \quad (14)$$

Where  $\Phi_r^h$ , the regional expenditure in  $r$  on sector  $h$ , is the term in brackets in the previous equation.

### *Equilibrium and Cournot competition*

The equilibrium condition for the model is that for each industry the sum of all the output flows  $y_{s,r}^h$  from various locations  $s$  and directed to  $r$  is to be equal to demand in  $r$ :

$$D_r^h = \sum_{s=1}^R y_{s,r}^h \quad (15)$$

In order to determine the Cournot solution for all the producers, we require the profit equation for the  $h^{\text{th}}$  industry in location  $r$ . It is important to note that transport costs are taken into account here as a part of production costs. From equations (8) and (13), the total profit for a producer of the  $h^{\text{th}}$  industry located in  $r$  can be written as:

$$\Pi_r^h = \sum_{s=1}^R p_s^h y_{r,s}^h - C_r^h - \sum_{s=1}^R y_{r,s}^h m C_r^h (e^{\tau^h d^{rs}} - 1) \quad (16)$$

Under the Cournot competition framework the following first order condition must apply over all target regions  $s$ :

$$\frac{d\Pi_r^h}{dy_{r,s}^h} = \frac{dp_s^h}{dy_{r,s}^h} y_{r,s}^h + p_s^h - \frac{dC_r^h}{dy_{r,s}^h} - mC_r^h (e^{\tau^h d^{rs}} - 1) = 0$$

As is the case in the Cournot literature and Combes and Lafourcade (2001), the producer assumes that the quantities supplied by other competitors to the same location  $r$  remain constant. The conjectural variation part of the derivative is:

$$\frac{dp_s^h}{dy_{r,s}^h} = \frac{dp_s^h}{dD_s^h} \frac{dD_s^h}{dy_{r,s}^h}$$

This assumption ensures that  $dD_s^h = dy_{r,s}^h$ , and therefore:

$$\frac{dD_s^h}{dy_{r,s}^h} = 1.$$

The conjectural variation then simply becomes:

$$\frac{dp_s^h}{dy_{r,s}^h} = \frac{dp_s^h}{dD_s^h}$$

As a result, a simpler version of the first order condition can be written out:

$$\frac{d\Pi_r^h}{dy_{r,s}^h} = \frac{dp_s^h}{dD_s^h} y_{r,s}^h + p_s^h - mC_r^h e^{\tau^h d^{rs}} = 0 \quad (17)$$

Using this solution, as well as the pricing relation in equilibrium (14) and the equilibrium condition (15), one can find the equilibrium solutions for output flows and price on each market, shown below, in equations (18) and (19).

$$y_{r,s}^h = \frac{(p_s^h - mC_r^h e^{\tau^h d^{rs}}) \Phi_s^h}{(p_s^h)^2} \quad (18)$$

$$p_s^h = \frac{\sum_{r=1}^R mC_r^h e^{\tau^h d^{sr}}}{R-1} \quad (19)$$

Additionally, firms in a given sector  $h$  and location  $r$  will only produce for a regional market  $s$  if the profits they make on that flow are positive. If they are not, the producer drops out of that particular market, and  $y_{r,s}^h = 0$ . The condition can easily be derived from the profit equation (16):

$$p_s^h - mC_r^h \left( e^{\tau^h d^{rs}} - 1 + \Psi^h \right) \geq 0 \quad (20)$$

Equations (3), (4), (6), (11), (12), (14), (16), (18), (19) and (20) above describe the equilibrium of the economy. Equations (3), (4), (6), (11), (12), (14), and (16) enable us to determine the price inputs and the production costs in each location. Using the price and output equations, (18) and (19), as well as the profitability constraint (20) we can determine then the flow of goods from one location to another. Summing those flows yields the equilibrium output per location and sector.

### 3. Model behaviour at symmetric equilibrium

The symmetric equilibrium defined by the system of equations shown in above was simulated using the parameter values shown below in Table 1.

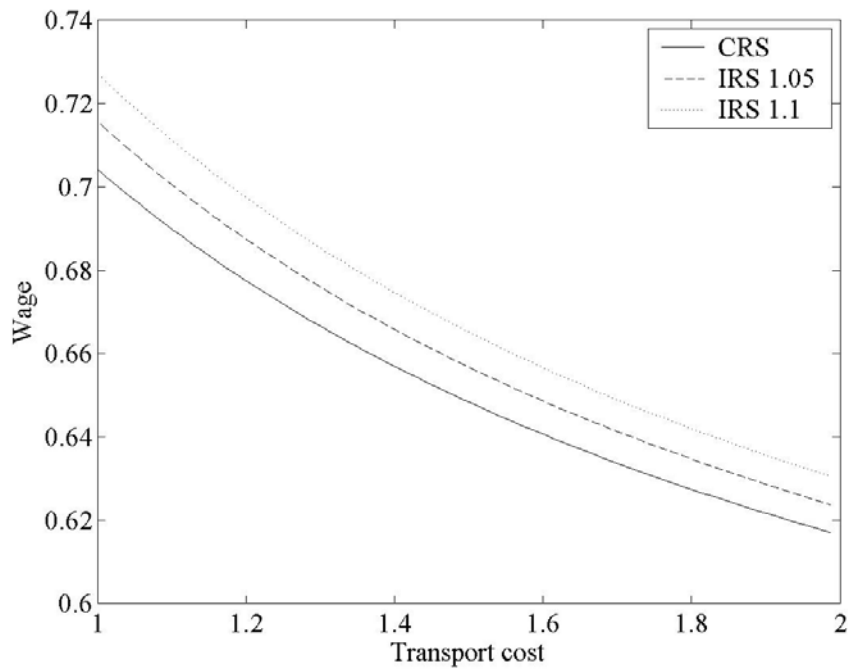
<b>Table 1: Parameter values for the Cournot 2x2 test</b>		
Variable	Symbol	Value
Elasticity of agricultural output w.r.t land	$\alpha$	0.45
Elasticity of agricultural output w.r.t labour	$\beta^a$	0.55
Elasticity of manufacturing output w.r.t labour	$\beta^m$	0.6
Elasticity of manufacturing output w.r.t intermediate input	$\varepsilon$	0.4
Elasticity of utility w.r.t the agricultural good	$\mu^a$	0.5
Elasticity of utility w.r.t the manufacturing good	$\mu^m$	0.5
Labour (per region)	$N$	2
Land (per region)	$K$	6.8406

In order to provide a better understanding of the effect of including IRS in the model, a benchmark simulation was run, with a CRS manufacturing sector. This is then

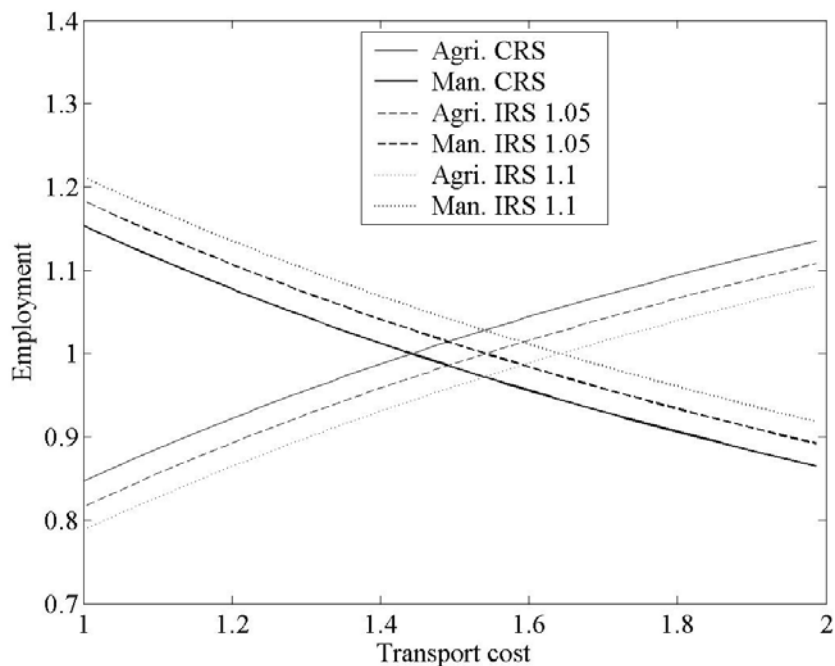
compared to simulations with an manufacturing IRS parameter of 1.05 and 1.1. At this point, it is important to point out a technical issue relating to the integration of IRS in manufacturing. IRS are added to the model by increasing the size of the production elasticities by a factor  $\Psi^m$ . This creates a problem with TFP, as changing the intensities of inputs changes the units in which TFP is measured. Increasing  $\Psi^m$  at a given level of transport cost requires the re-calculation of TFP in the new units. In an applied simulation, where there is data available on inputs and outputs, this recalibration of TFP for a higher level of IRS is just a part of the overall calibration of the model.

However, in the abstract simulations presented there is no “correct” level of outputs and inputs to refer to as a calibration point. Ex ante, all levels of output are equally as valid as a reference point. Furthermore, in order to be able to assess the impact of IRS in the model, we must be able to compare the difference between the CRS and IRS cases for the entire transport cost range, which is not possible if TFP is recalibrated for every level of transport cost. Manufacturing TFP will therefore be assumed to be unitary, for all levels of transport cost and returns to scale. The consequence of this is that the absolute levels of the CRS and IRS curves in the following figures cannot be compared directly, and only the relative slopes are meaningful. Table 2 shows, however, that when correcting for the change in TFP at different levels of transport costs and returns to scale, output increases as expected with the level of returns to scale.

Figure 1 shows that as one would expect, the symmetric wage rate increases as transport costs fall. More open markets encourage more manufacturing trade and higher manufacturing output, resulting on increased competition between both sectors for the fixed labour supply. Furthermore, the presence of IRS in the manufacturing sector does not really impact the labour market. The change in levels is seen in the figure is due to the unitary TFP assumption, and the slopes of the curves are relatively unchanged.



**Figure 1: Symmetric Wage Rate, CRS vs IRS**



**Figure 2: Employment in Agriculture and Manufacturing, CRS vs IRS**

The increase in wage as transport costs fall is explained by a change the structure of employment. Figure 2 shows that as the shipping costs incurred by manufacturing are reduced, labour is transferred from agricultural employment to manufacturing. This is because the expanding manufacturing sector competes with

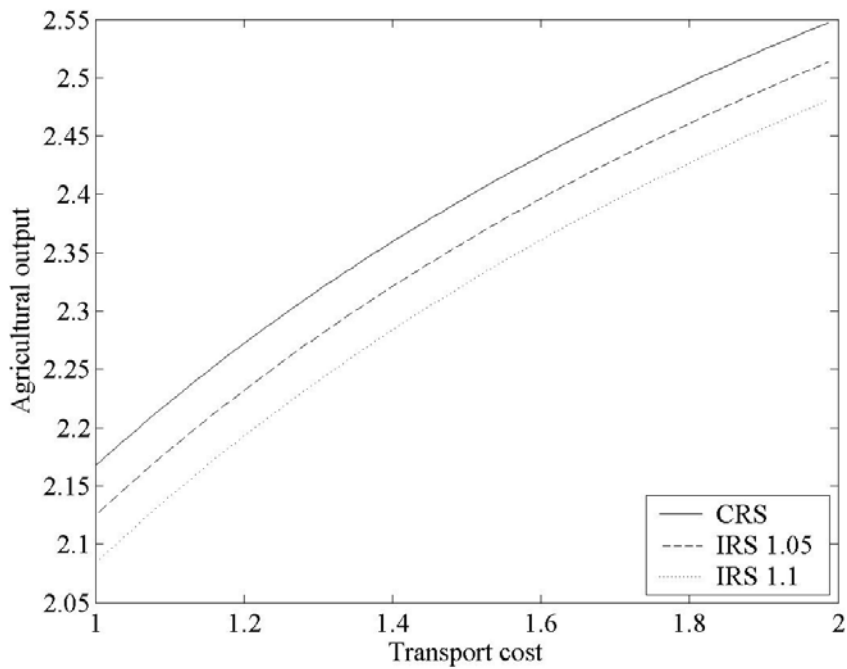
agriculture and draws labour from it. The increase in wages seen in Figure 1 is just the trace of this sectoral reallocation of labour.

However, Figures 1 and 2 show that the presence of IRS does not change significantly the relations between agriculture and the manufacturing sector in the labour market. As for the CRS case, as transport costs go down, manufacturing has a greater demand for labour, and is therefore able to squeeze more labour out of the agricultural sector. The rightward shift of the curves is due the fact that TFP is not adjusted, but the intersectoral adjustment does not appear to happen at an increased rate under IRS. This shows that within this model IRS do not have a major impact on the way the labour market functions.

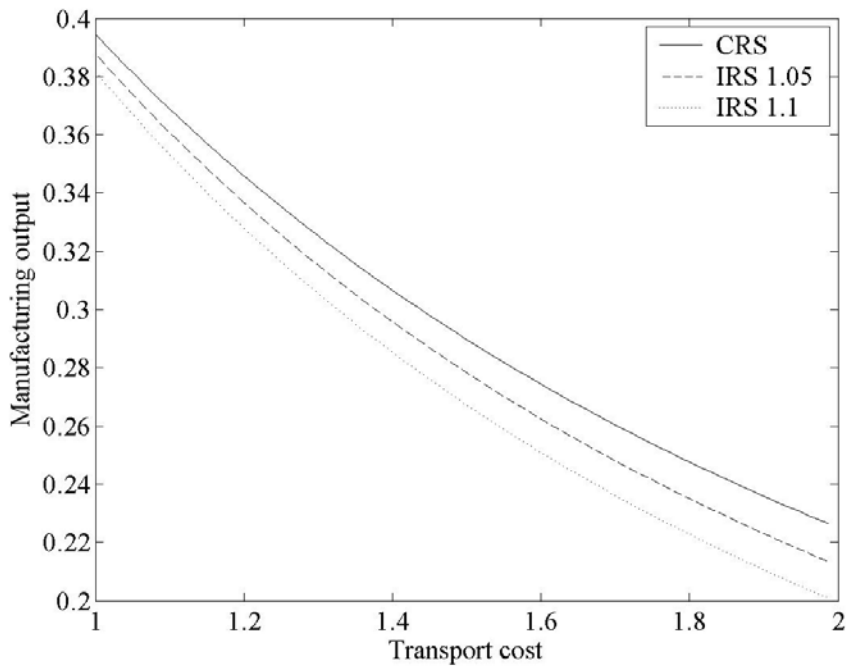
The picture is different if ones look at output. Figures 3 and 4 show the evolution of the output of both sectors as a function of transport costs. As expected, the lower the transport cost, the higher the level of manufacturing output, because of the gradual opening of larger markets for manufacturing goods that result from lower transport costs. The CRS drop in agricultural output with transport costs is a consequence of the increased competition on the labour market and the rise in wages visible in Figure 1. As one can see from figures 3 and 4, at the symmetric equilibrium, the agricultural output is generally higher than the manufacturing output, The reason behind this the assumption that both agricultural and manufacturing TFPs are unitary and the fact that the agricultural production has the exclusive use of land, whereas manufacturing has to compete with final consumers for its intermediate inputs.

As expected, the presence of IRS in the manufacturing sector does not directly affect the production structure of agriculture, which is still assumed to be CRS. The apparent drop in output is due to the upward shift of wages seen in Figure 1, but as explained above, most of the shift in the level wages is itself due to the absence of

recalibration of TFP. The important result is that the slope of the agricultural output curve itself is unchanged.



**Figure 3: Agricultural Output, CRS vs IRS**

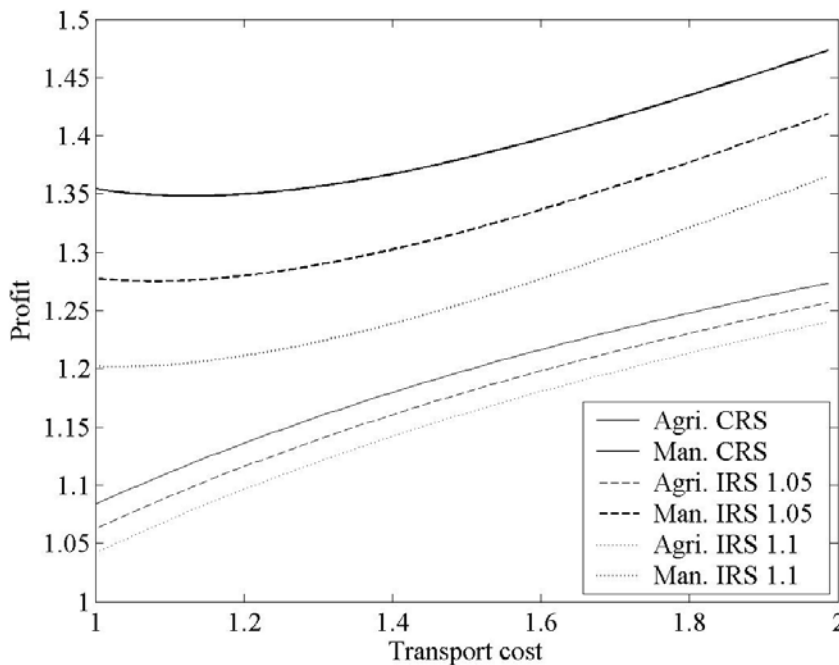


**Figure 4: Manufacturing Output, CRS vs IRS**

Figure 4, shows that IRS do have an effect on the slopes of the curves. For a given reduction in transport costs, the increase in manufacturing output is stronger the higher the level of returns to scale. IRS increases the slope of the output curve through

the cost reduction effect visible in equation 6, and the higher input intensities of the production function. As explained previously, the apparent reduction in output is due to TFP not being adjusted. With an appropriate recalibration of TFP, this will lead to increases in output as transport costs drop. This can be seen in Table 2

Figure 5 shows the profits of both sectors decline as transport costs drop. In agriculture, it is because wages increase as competition for labour with an increasingly larger manufacturing sector leading to the rising labour costs shown in Figures 1 and 2. This is also true for manufacturing, however manufacturing is also faced with the fact that as transport costs drop, the competition between the two producers for both regional markets increases, driving down the price of the manufacturing good. At symmetric equilibrium, manufacturing profits are squeezed from both directions: the price of its output falls, as can be seen in below Figure 7, while the labour costs rise.



**Figure 5: Agricultural and Manufacturing Profits, CRS vs IRS**

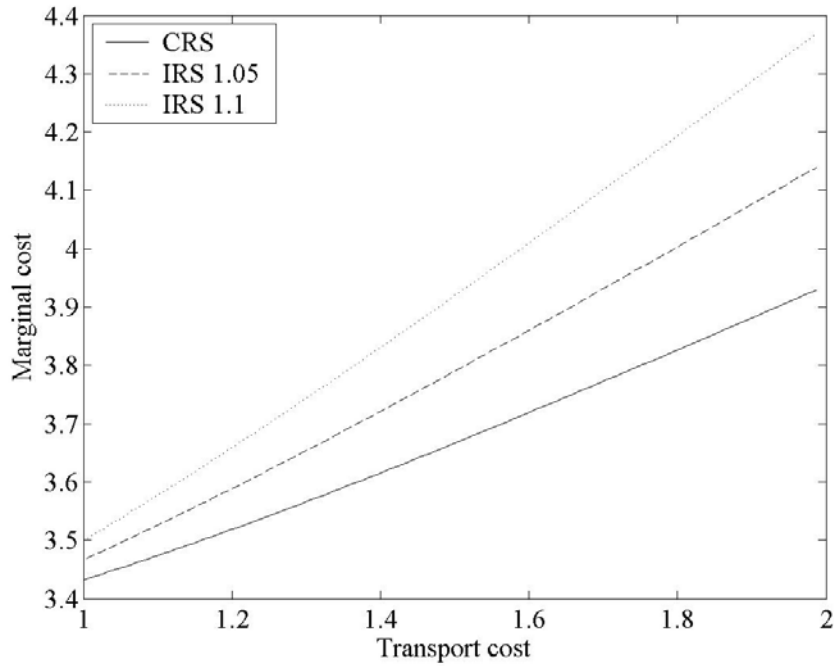
Figure 5 also shows that the effect of IRS on profits is complex. Again allowing for the differences in intercepts, one can see that the equilibrium paths of agricultural

profits are not affected, as was the case for agricultural output. There does however seem to be an effect of IRS on manufacturing profits, as the curve seems to flatten out at low levels of transport cost.

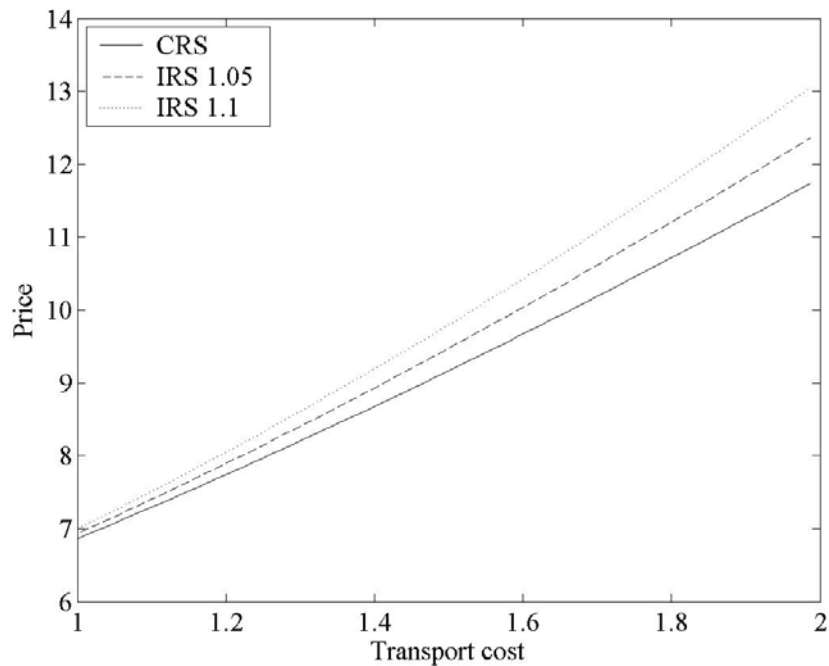
The cost-reducing effect of IRS on the production structure of the manufacturing sector is confirmed by the analysis of the marginal cost of production in Figure 6. Only the manufacturing marginal cost is shown here, the agricultural marginal cost is fixed and equal to 0.5 regardless of transport costs and returns to scale<sup>2</sup>. The fact that the marginal cost of CRS manufacturing drops with transport costs even though labour costs are rising, is due to the input-output structure of the manufacturing sector, As shown in Figure 7, the price of the manufacturing good falls with transport costs, and as one can see from Figure 1, it falls at a faster rate than the rate of increase of wages, therefore causing the marginal cost to fall slightly. The size of this effect therefore depends on the relative shares of labour and intermediate inputs in the production function. The introduction of IRS into manufacturing, however, has a clear effect. In Figure 4 the slope of the output curve becomes steeper, showing that as transport costs fall and manufacturing output increases, the presence of IRS reduces production costs compared to the CRS case. At high levels of transport costs, output is low and as a result, the marginal cost will be relatively high. At low levels of transport costs output is higher, and the cost-reducing effect of IRS increases, causing reductions in marginal cost that are higher than the reductions seen under CRS.

---

<sup>2</sup> This is linked to the fact that the agricultural good is the numeraire, and is explained in more detail in the appendix



**Figure 6: Marginal cost of production in Manufacturing, CRS vs IRS**



**Figure 7: Price of the Manufacturing Good, CRS vs IRS**

Figure 7 indicates that the IRS cost-reduction seen in marginal costs feeds through to price of the manufacturing good. Equation (17) shows that under Cournot competition the manufacturing price in a location is just a weighted average of the marginal costs of the suppliers to that location. It is therefore expected that the reductions in marginal costs should be reflected in the price.

Having examined the symmetric equilibrium of the system under of CRS and IRS, it is important to address the fact that changing the intensities of inputs changes the units in which TFP is measured. As previously explained, this is not a problem for applied work, where data on existing output and inputs provides a reference point for calibration. However, in the simulation carried out here any level of transport cost can provide a reference point. For a given level of transport costs, TFP is calibrated by dividing the CRS output at that level of transport costs by the Cobb-Douglas production function of CRS inputs. Importantly the values of the elasticities used in this Cobb-Douglas have to be in line with the relevant amount of IRS desired<sup>3</sup>. The IRS model can then be simulated with a value of TFP consistent with the level of IRS chosen. The TFP recalibration for selected levels of transport cost  $\tau$  and the resulting outputs are shown below in Table 2.

<b>Recalibrated manufacturing TFP</b>				<b>Recalibrated manufacturing output</b>			
$\tau$	CRS	IRS 1.05	IRS 1.1	$\tau$	CRS	IRS 1.05	IRS 1.1
<b>1</b>	1.000	1.048	1.097	<b>1</b>	0.395	0.420	0.451
<b>1.2</b>	1.000	1.055	1.112	<b>1.2</b>	0.346	0.369	0.397
<b>1.4</b>	1.000	1.061	1.125	<b>1.4</b>	0.307	0.328	0.353
<b>1.6</b>	1.000	1.067	1.138	<b>1.6</b>	0.274	0.294	0.317
<b>1.8</b>	1.000	1.072	1.150	<b>1.8</b>	0.248	0.265	0.287
<b>2</b>	1.000	1.077	1.161	<b>2</b>	0.225	0.242	0.261

Table 2 confirms that for each level of transport costs, higher levels of returns to scale lead to higher manufacturing output, as one would expect from an IRS model.

<sup>3</sup> In other words,

$$A = \frac{y_r^m}{(L_r^m)^{\beta^m} (x_t^m)^{\varepsilon^m}}$$

where  $A$  is TFP,  $y$  is output,  $L$  is labour and  $x$  is intermediate inputs. Simply inserting the relevant IRS values for  $\beta$  and  $\varepsilon$  allows for the calculation of the TFP for that level of IRS.

#### 4. Stability of the symmetric equilibrium

Because we are attempting to check the agglomerative effect of explicit IRS, we assume only one firm per region in this model. We define agglomeration as a situation where a single firm, representing a region, increases the size of its output with respect to other regions. The number of firms therefore does not change but the output of the firm in the agglomerated region goes up relative to other regions. Although the upper bound for the number of producers is the number of regions, their number can be reduced by the profitability constraint. If a producer finds himself in a situation where all output flows are unprofitable, the profitability condition of the model, equation (18), will force him out of production. Total agglomeration is therefore a situation in which only one region produces, and it is unprofitable for other regions to do so. Unfortunately, producers exit the market discretely, and their output drops straight from a positive number to zero. This creates jumps in the price, output and profit equations, and therefore discontinuities in any analytical function that would illustrate agglomeration. It is therefore difficult to provide a complete analytical description of long run equilibria. However, for any equilibrium situation where the number of competitors on a market is set, either at symmetric equilibrium, or at any point between the discontinuities, it is possible to determine analytical conditions that determine whether or not the equilibrium is stable, and if agglomeration is profitable or not. This is not a complete description of all the equilibria, but a set of procedures that allow for the analysis of the local stability of a particular equilibrium. However, because we are only showing the conditions under which the symmetric equilibrium is unstable, this is enough.

This approach rests on the comparison of change in profits of both producers as a result in a change in output of one of the producers, in other words  $d\Pi_r^m/dy_r^m$  and  $d\Pi_s^m/dy_r^m$ . If these first order conditions are negative for both producers, neither of them have an incentive to increase their output. If at least one is positive, then it is possible for at least one of the producers to increase profits by increasing output, and the system moves away from the symmetric equilibrium. The easiest way of doing so analytically is to calculate the change in profits in a region in response to an increase of the home flow  $y_{r,r}^m$ . In this case, the transport cost term is equal to one, and the profit on the flow is:

$$\Pi_{r,r}^m = y_{r,r}^m (p_r^m - mC_r^m \Psi^h) \quad (21)$$

The term in brackets is the profitability of the flow, the amount of profit made on each unit of output sold in the home market. Totally differentiating  $\Pi_{r,r}^m$  with respect to the home output flow yields:

$$d\Pi_{r,r}^m = dy_{r,r}^m (p_r^m - mC_r^m \Psi^h) + (dp_r^m - \Psi^h dmC_r^m) y_{r,r}^m \quad (22)$$

The first term shows the increase in profits due to the extra output at *ex ante* profitability. This is reduced by the second term, which shows the impact of the increase in output on profitability itself. Predictably, the variations in profitability boil down to a combination of price and marginal cost variation. The total differential of the price equation (19) is:

$$dp_r^m = \frac{1}{R-1} dmC_r^m + \frac{e^\tau}{R-1} dmC_s^m$$

This equation relation already reveals the basic mechanism behind agglomeration. For the purpose of simplifying the calculations and underlining the causes of agglomeration, we shall assume that the other producer's output is unchanged.

This is not necessarily the case, and the simulated stability conditions shown below will take this into account and look at the total effect<sup>4</sup>. This assumption just clarifies the analytical insights.

$$dp_r^m = \Omega dmC_r^m \quad \text{with} \quad \Omega = \frac{1}{R-1} \quad (23)$$

$\Omega$ , the ratio between  $dp_r^m$  and  $dmC_r^m$ , is basically the marginal mark-up between prices and marginal cost, and indicates the market power of any individual producer. In the general case, the more producers  $R$  there are on a market, the smaller  $\Omega$  will be. Looking at (22), if the marginal cost of the producer in  $r$  increases, the price in his home market will only increase by a fraction  $\Omega$  of that, and the profitability will fall. If on the other hand the marginal cost falls, price will only fall by a proportion  $\Omega$  of that amount, and profitability increases. Here, with two regions,  $\Omega$  is simply equal to one. All that is left to do is to show under what conditions the marginal cost falls or rises. Totally differentiating the equation (6) yields:

$$dmC_r^m = mC_r^m \left( \frac{1 - \Psi^m}{\Psi^m} \frac{dy_r^m}{y_r^m} + \frac{d\chi_r^m}{\chi_r^m} \right) \quad (24)$$

We can see here that there is already a role played by IRS in causing a fall or increase of marginal cost with respect to output. One can see, that under CRS, where  $\Psi^m = 1$ , the variation in  $dmC_r^m$  collapses down to  $d\chi_r^m$ , and the level of output plays no role. However under IRS, increasing output will reduce the size of the marginal cost, other things equal. One would expect this to be counter-acted by the total differential of the fixed component of marginal cost  $\chi_r^m$  which is a positive function of the cost of

---

<sup>4</sup> The simulated stability conditions in Figures 8 and 9 are obtained using the implicit function methodology developed in Barde (2006). These are based on the numerical Jacobian of the system of equations and therefore contain all the information on partial derivatives.

inputs to production. In order to get a better description of these potential increases, however, we require the total differential of  $d\chi_r^m$ . From equation (4), one gets:

$$d\chi_r^m = \chi_r^m \left( \beta^m \frac{dw_r}{w_r} + \varepsilon^m \frac{dp_r^m}{p_r^m} \right) \quad (25)$$

Replacing (25) in the total differential for manufacturing marginal cost equation (24) gives:

$$dmC_r^m = mC_r^m \left( \frac{1-\Psi^m}{\Psi^m} \frac{dy_r^m}{y_r^m} + \beta^m \frac{dw_r}{w_r} + \varepsilon^m \frac{dp_r^m}{p_r^m} \right) \quad (26)$$

Replacing (23) in (22) and (24) to eliminate  $dp_r^m$ :

$$d\Pi_{r,r}^m = (p_r^m - mC_r^m \Psi^h) dy_{r,r}^m + (\Omega - \Psi^h) y_{r,r}^m dmC_r^m \quad (27)$$

$$dmC_r^m = \frac{mC_r^m}{1 - \varepsilon^m \Omega \frac{mC_r^m}{p_r^m}} \left( \frac{1-\Psi^m}{\Psi^m} \frac{dy_r^m}{y_r^m} + \beta^m \frac{dw_r}{w_r} \right) \quad (28)$$

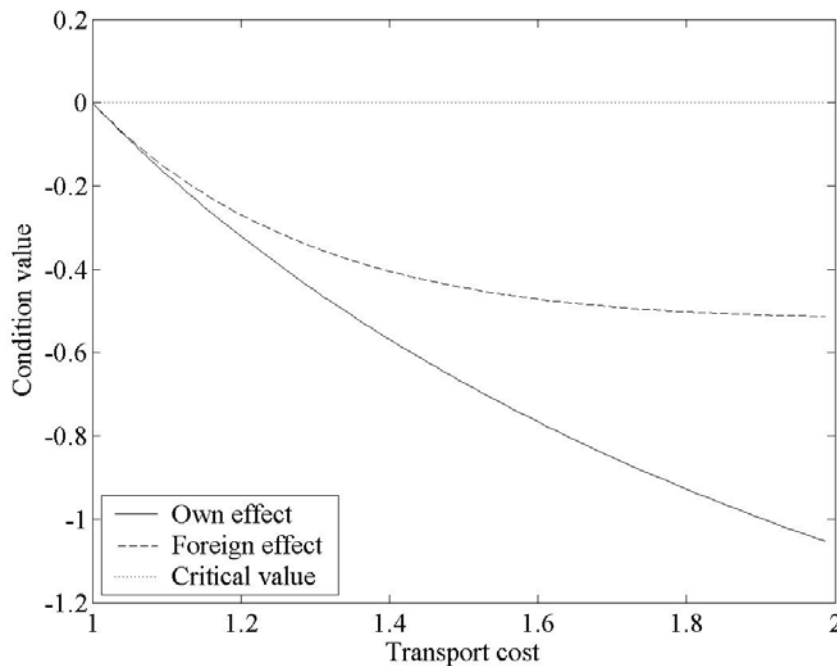
Replacing in (28) in (27) to eliminate  $dmC_r^m$  and assuming that  $dy_r^m / dy_{r,r}^m = 1$  gives the stability condition for the home flow:

$$d\Pi_{r,r}^m = (p_r^m - mC_r^m \Psi^h) dy_r^m + y_{r,r}^m mC_r^m \frac{\Omega - \Psi^h}{1 - \varepsilon^m \Omega \frac{mC_r^m}{p_r^m}} \left( \frac{1-\Psi^m}{\Psi^m} \frac{dy_r^m}{y_r^m} + \beta^m \frac{dw_r}{w_r} \right) \quad (29)$$

The first part of equation (29), the change in profits due to the change in output at *ex ante* profitability is unchanged from (22). Therefore, the important terms of equation (29) are the terms in brackets containing  $dy$  and  $dw$ , and the multiplicative ratio in front of it, which determine the evolution of profitability. Within the brackets, an increase in output  $dy$  has a direct and negative effect under IRS. This corresponds to the cost-reducing effect of IRS on marginal costs above. One would expect  $dw/dy$  to be positive due to the extra pressure on the labour market. For simplicity, this effect is not explicitly included here, but the simulations in section 3 show that the sign is positive. A great deal of the stability analysis therefore rests on the relative strength of these two

effects and whether, for a given increase in output, the IRS-induced reductions in marginal cost brought by the extra output outweigh the higher labour cost incurred.

The ratio in front of the term in brackets is a multiplier which accounts for the negative feedback effect on the profit equation of a rise in prices. The multiplier exists because  $\Omega$ ,  $\varepsilon$  and the marginal cost to price ratio are all individually smaller than one. The numerator  $\Omega - \Psi^h$  is negative, given that the mark up  $\Omega$  is smaller or equal to one, and that the degree of returns to scale  $\Psi^h$  will be equal to or greater than one. This multiplier stems from the vertical linkages in the model<sup>5</sup>. Because of this intermediate consumption, if for any reason the manufacturing price increases, production costs (and marginal costs), will be pushed up, thus pushing prices up even further. This effect also works in reverse with reductions in price. This multiplies any change in profitability due to wage increases or IRS output effects.



**Figure 8: Stability of Symmetric equilibrium, CRS**

<sup>5</sup> Which is why the multiplier term depends on the elasticity of output w.r.t intermediate consumption  $\varepsilon$  and the mark-up  $\Omega$

Equation (29) confirms the importance of IRS in providing incentives to agglomerate. Under constant returns to scale, with  $\Psi^m$  equal to one, there is no direct effect of an increase in output  $dy$  on profitability. The only effect, through the increase in labour costs  $dw/dy$ , is negative. Under CRS, output increases can only lead to reductions in profitability. The first half of equation (29) does show an increase, but it will nearly always be smaller than the drop in profitability, because of the multiplier effect in the profitability term. Figure 8 above shows that without the cost-reducing effects of IRS, deviating from the symmetric equilibrium is detrimental to profits, meaning that the symmetric equilibrium is always stable.

Under IRS, however, profitability does not necessarily drop. Equation (29) shows that the cost-reducing effect of the extra output increases can directly mitigate the increase in wages. The numerical stability conditions visible in Figure 9, in appendix, confirm the analytical indications given above. As the level of increasing returns to scale increases, the home effect, followed by the foreign effect become positive, indicating that it becomes profitable for either the home or the foreign producers, or both, to increase their output, thus moving away from the symmetric equilibrium. In doing so, they will generate a variety of agglomeration patterns which depend on the level of IRS.

In the case where both effects are negative corresponds to a stable symmetric equilibrium, much like the CRS case in Figure 8. If, however the own effect is positive and the foreign effect is negative, as is the case in the first five diagrams, then an increase in home output increases the incentive to produce in the home market and reduces the incentive to produce in the foreign market. This should lead to total agglomeration in the home market, allowing for a discontinuity when the foreign producer is caught by the profitability constraint and exits the market. In the case where

both effects are positive but the own one is larger, both producers have an incentive to increase output and move away from equilibrium.

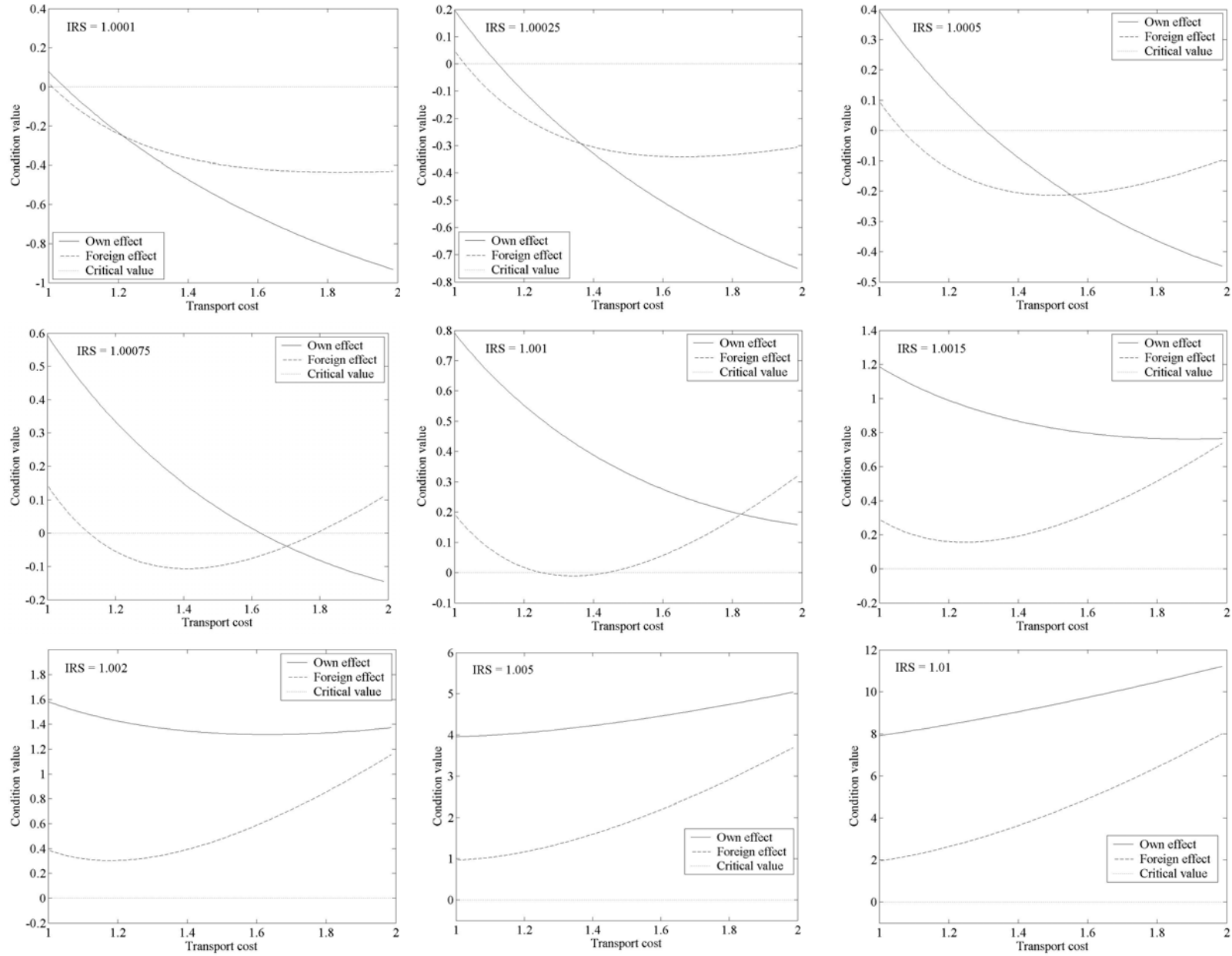
## **5. Conclusion**

This paper set to investigate the folk theorem of spatial economics and investigate how explicit IRS can influence production choices and agglomeration. Although the introduction of IRS in a Cournot model creates analytical problems with the shares of expenditure and allocations of production costs, we show that these can be worked around.

Simulations and analytical investigations of the model show that if the agriculture and manufacturing sectors are both CRS, then the symmetric equilibrium is stable for all transport costs. This corresponds to dispersion of activity across regions, meaning that agglomeration of production in one region is not possible. If, however, IRS are applied to the manufacturing sector then the symmetric equilibrium can become unstable. Within the settings of the model, this means that it becomes profitable for one region to increase its manufacturing output with respect to the other region, thus increasing its share of total output. The importance of this result is the explicit confirmation of the folk theorem specified by Fujita and Thisse (1996).

Because we assume there is a single producer per region, the discontinuities introduced by profitability condition make it difficult to provide an analytical description of the agglomeration process. This is why we instead concentrate on the departure from equilibrium, as under the settings used in the model it is enough to prove that the symmetric equilibrium is unstable. However, this does point out the importance of firm entry in providing the conditions for a full understanding of agglomeration.

The model described is not intended to be a substitute for the Cournot agglomeration models mentioned in the introduction. Rather, it is intended as a complement, which accounts for an extra agglomeration force not linked to firm entry, but to the possible existence IRS in key sectors of the economy. Fixing the number of producers just allows us to underline the existence of this agglomeration force. A direction worthwhile examining would therefore be to relax this assumption of a fixed number of producers and combine both agglomeration forces in a single framework.



Appendix 1

Figure 9 - Stability of Symmetric equilibrium, IRS

## Appendix 2

### Proof of the stability of the agricultural marginal cost

Analytically, one can derive the stability of the agricultural marginal cost from the price equation (17) in section 1. Because the system is at symmetric equilibrium, all the variables are equal over regions. Furthermore, the agricultural good is the numeraire. Therefore,  $p_r^a = p_s^a = p_*^a = 1$  and  $\chi_r^a = \chi_s^a = \chi_*^a$ . Replacing in the price equation for the agricultural sector, with  $R = 2$  gives the price at equilibrium.

$$p_s^a = \frac{\sum_{r=1}^2 \chi_r^a}{R-1} \quad \text{simplifies down to} \quad p_*^a = 2\chi_*^a$$

This explains the value of 0.5 for the marginal cost of the agricultural sector, but does not fully explain why it is constant, i.e. why  $d\chi_*^a/d\tau = 0$ . To show this, one must replace  $p_*^a$  and  $\chi_*^a$  in the output equation (16). Doing so gives

$$y_*^a = \frac{\Phi_*^a}{4\chi_*^a}$$

With an agricultural cost equation equal to:

$$C_*^a = \chi_*^a y_*^a = \frac{\Phi_*^a}{4}$$

The agricultural cost function does not depend on the marginal cost  $\chi_*^a$ , and therefore variations in agricultural costs do not depend on  $\chi_*^a$

## Bibliography

- Anderson S.P, Neven D.J, “Cournot Competition Yields Spatial Agglomeration”,  
*International Economic Review*, Vol. 32, N° 4, p 793-808, 1991.
- d’Aspremont C, Gabszewicz J.J, Thisse J-F, “On Hotelling’s ‘Stability in Competition’”, *Econometrica*, Vol. 47, N° 5, p1145-1150, Sept 1979.
- Barde S, “Stable partial agglomeration in a New Economic Geography model with urban frictions”, *Studies in economic Working Paper*, University of Kent, Working Paper N° ??, 2006.
- Brakman S, Garretsen H, Van Marrewijk C, *An introduction to geographical economics*, Cambridge university press, 2001.
- Christaller W, *Central Places in Southern Germany*, Jena: Fischer, 1933. English version: Prentice-Hall, 1966.
- Combes P-P, “Industrial Agglomeration Under Cournot Competition”, *Annales d’Economie et de Statistique*, vol. 45 1997
- Combes P-P, Duranton G, Overman H.G, “Agglomeration and the Adjustment of the Spatial Economy”, *Papers in Regional Science*, Vol. 84, N° 3, p31-349, 2005.
- Combes P-P, Lafourcade M, “Transport Cost Decline and Regional Inequalities: Evidence from France”, *CEPR Discussion Paper*, N° 2894, 2001.
- Combes P-P, Lafourcade M, “Transport costs: measures, determinants, and regional policy implications for France”, *Journal of Economic Geography*, Vol. 5, p319-349, 2005
- Combes P-P, Overman H.G, “The Spatial Distribution of Economic Activities in the European Union”, *The Handbook of Regional and Urban Economics*, Vol. IV, 2004

- Duranton G, Overman H.G, “Testing for Localisation Using Micro Geographic Data”, *The Review of Economic Studies*, Vol. 72, N° 253, p1077-1106, 2005.
- Ellison G, Glaeser E.L, “Geographic Concentration in US Manufacturing Industries: A Dartboard Approach”, *The Journal of Political Economy*, Vol. 105 N° 5, p 889-927, October 1997.
- Fujita M, Krugman P, Venables A, *The spatial economy; Cities, Regions and International Trade*, MIT press, 1999.
- Fujita M, Thisse J-F, “Economics of Agglomeration”, *Journal of the Japanese and International Economies*, Vol. 10, p339-378, 1996.
- Gupta B, Pal D, “Does Cournot Competition Necessarily yield Spatial Agglomeration?” Working Paper, University of Milwaukee, Jan 1994.
- Gupta B, Pal D, Sarkar J, “Spatial Cournot Competition and Agglomeration in a Model of Location Choice” *Regional Science and Urban Economics*, vol.27, p 261-282, 1997.
- Hanson G.H, “Market Potential, Increasing Returns, and Geographic Concentration”, *NBER Working Paper Series*, N° 6429, February 1998.
- Hanson G.H, “Scale Economies and the Geographic Concentration of Industry”, *NBER Working Paper Series*, N°8013, November 2000.
- Head K, Mayer T, “The Empirics of Agglomeration and Trade”, in *The Handbook of Regional and Urban Economics*, Vol. IV, p2609-2665, 2004
- Hirschman, A.O, *The Strategy of Economic Development*, New Haven, Yale University Press, 1958.
- Hotelling H, “Stability in competition”, *Economic Journal*, vol. 39, March 1929.
- Krugman P, “Space: The final frontier” *Journal of Economic Perspectives*, Vol.12, N°2, Spring 1998.

- Lösch A, *The Economics of Location*, Jena: Fischer, 1940. English version: Yale University Press, 1954.
- Mayer T, “Spatial Cournot Competition and Heterogeneous production Costs Across locations”, *Regional Science and Urban Economics*, vol.30, N°3, May 2000.
- Molho I, “Spatial Search, Migration and Regional Unemployment”, *Economica*, vol. 68, p 269-283, 2001.
- Ottaviano G, Puga D, “Agglomeration in the global economy: A survey of the ‘new economic geography’ ”, *CEP Discussion Paper*, N° 356, 1997.
- Ottaviano G, Tabuchi T, Thisse J-F, “Agglomeration and Trade Revisited”, *International Economic Review*, Vol. 43, N° 2, May 2002.
- Puga D, “The Rise and Fall of Regional Inequalities”, *European Economic Review*, Vol. 43, 1999.
- Samuelson P.A, “The Transfer Problem and Transport Costs: The Terms of Trade When Impediments are Absent”, *The Economic Journal*, Vol.62, N° 246 p278-304, June 1952.
- Samuelson P.A, “The Transfer Problem and Transport Costs: Analysis of Effects of Trade Impediments”, *The Economic Journal*, Vol.64, N° 254 p264-289, June 1954.
- Tabuchi T, “Pricing Policy in Spatial Competition”, *Regional Science and Urban Economics*, Vol. 29 p 617-631, 1999.
- Venables A.J, “Equilibrium Locations of Vertically Linked Industries”, *International Economic Review*, Vol. 37 N° 2, p 341-359, May 1996.
- Von Thünen, *The Isolated State*, Hamburg, Perthes, 1826. English version: Pergamon, 1966.