

Periodically Collapsing Rational Bubbles in Exchange Rates: a Markov-Switching Analysis for a Sample of Industrialised Markets

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Abstract: This paper investigates the presence of periodically collapsing rational bubbles in exchange rates for a sample of industrialised countries. A periodically collapsing rational bubble is defined as an explosive deviation from economic fundamentals with distinct expansion and contraction phases in finite time. By using Markov-switching regime models we were not able to find robust evidence of a bubble driving the exchange rate away from fundamentals. Moreover, the results also revealed significant non-linearities and different regimes. The importance of these findings suggests that linear monetary models may not be appropriate to examine exchange rate movements.

Keywords: Foreign Exchange, Bubbles, Fundamentals, Markov-Switching, Assets.

JEL Classification Codes.: F31, F37, F41.

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1 - Introduction

A rational speculative bubble in the exchange rate is characterised by an explosive path. Explosive behaviour leads the exchange rate to diverge from the equilibrium level defined by monetary fundamentals. The underlying idea about the presence of bubbles concerns the fact that speculators and investors maintain a belief that, despite the currency being overvalued with respect to its fundamentals, it is still profitable to buy additional units of it. Thus, a bubble acquires importance in driving the exchange rate away from the equilibrium determined by its fundamentals because market agents perceive the presence of profit opportunity. In fact, movements of asset prices away from their fundamentals signal the occurrence of self-fulfilling prophecies of market participants, caused by events that are exogenous to the market. Essentially, the presence of bubbles in asset markets implies that market participants are not allocating their savings to the best possible investment. Furthermore, the analysis of rational bubbles, based on rational expectations, has an element of indeterminacy, which usually arises when the current decisions of agents depend both on the current market price and on their expectations of future prices (see Obstfeld and Rogoff, 1997). Given this, an obvious conclusion is that a single hypothesis cannot encompass sequences of prices as only one sequence is the market fundamental price path. The other sequences maybe price bubble sequences (see Blanchard and Fischer, 2001).

In empirical research the traditional literature has followed three different methods of investigating the idea of bubbles driving asset prices away from their economic fundamentals equilibrium. The first one is the variance bound test, also known as volatility test, and it was originally proposed by Shiller (1981) to study the volatility of long-term bonds. A second econometric method is based on the Hausman (1978) test and called the Hausman specification test. The specification test was originally published in 1987 by Kenneth West to investigate the presence of bubbles in the stock market.

The major difficulty of employing either the volatility test or the specification test is related to the fact that both methodologies depend strongly on the chosen model

of exchange rate determination. Hence, the excess volatility observed in some studies may be caused by factors other than the presence of rational bubbles. In particular for the specification test, Flood and Hodrick (1990) argue that the reliability of conclusions based on omitted variables may be weak since the monetary model in general produces misleading results. This may lead bubble tests towards rejection of the null hypothesis of no bubbles.

The presence of rational bubbles driving the exchange rate away from economic fundamentals can also be analysed by the use of unit root and cointegration tests. This methodology consists of examining the possibility of cointegration between the exchange rate and fundamentals (see MacDonald and Taylor, 1993). In essence, the argument is that if asset prices are not more explosive than their determinants, then it may be inferred that rational bubbles are not present as they would generate an explosive component to asset prices. Bubble tests based on unit root tests and cointegration tests have, nevertheless, been potentially misleading in the presence of bubbles collapsing periodically. Evans (1991) was the first to acknowledge this kind of difficulty in examining bubbles using either unit root tests or cointegration tests. According to him the traditional cointegration tests leads to rejection of the null hypothesis of no bubbles more often than if the presence of a particular type of bubble was not taken into account. The central argument described in his paper is that unit root tests or cointegration tests, when faced with periodically collapsing rational bubbles may lead, with a high degree of probability, to a wrong conclusion that bubble components are nonexistent in the stochastic process. Although the arguments developed by Evans have been related to stock price behaviour it can also be extended to exchange rate movements. Sarno and Taylor (2002) state that the concept of periodically collapsing bubbles is important in analysing the behaviour of exchange rates since they collapse almost in finite time.

Sarno and Taylor (1999) investigate the presence of periodically collapsing bubbles in the East Asian stock market. Sarno and Taylor tested for stationarity of the log dividend-price ratio and the *ex-post* rate of return, and then tested cointegration for both series. The underlying idea was that if stationarity or cointegration were found between the series the hypothesis of stock price bubbles could be rejected. The results reached by Sarno and Taylor did not reject the hypothesis of bubbles in the East Asian stock market. Psaradakis *et al.*(2001) used a new procedure for detecting the presence of

periodically collapsing bubbles based on random-coefficient autoregressive models. In fact, they proposed a simpler test procedure which is based essentially on the class of nonstationary varying-coefficient autoregressive models with a stochastic unit root¹. Another test procedure for periodically collapsing bubbles was carried out by Hall *et al.* (1999) which allowed for the possibility of changes in the dynamic behaviour of asset prices across the sample. In effect, the methodology proposed constitutes a generalisation of the augmented Dickey-Fuller (ADF) unit root test applied to the class of Markov-switching regime models (see Hamilton 1988, 1989). An important difficulty is to identify collapsing periods from expanding ones. Hall *et al.* argue that the proposed methodology overcomes some of the econometric problems involved traditional unit root tests (see Evans 1991). They applied the ADF-switching unit root test to investigate the presence of an explosive autoregressive root to the hyperinflation process in Argentina during the 1980s. The results suggested that during that time it was possible to observe a clear explosive regime in the consumer price index associated with the rapid growth in the money supply. The implication was that the 1989 hyperinflation in Argentina was related to adverse fundamentals behaviour, and not to rational bubbles.

This paper embraces the approach based on the unit root and cointegration tests, but uses the Markov-switching regime methodology in order to allow for a more robust analysis of periodically collapsing bubbles. In essence, the contribution of this paper is to investigate the hypothesis of a periodically collapsing bubble underlying the movement of the exchange rate for a set of four industrialised market economy countries (Canada, France, Germany and the United Kingdom). Actually, these countries are usually selected by the literature as a sample to examine the presence of speculative rational bubbles for sample periods of high turbulence in exchange rates. In doing so, the ADF-switching unit root test proposed by Hall *et al.* (1999) is appropriate since this methodology allows consideration of different regimes/states typical of periodically collapsing bubbles in unstable economies. Also, the unit root test approach offers an additional and convenient way of constructing tests of hypotheses of unit roots against alternatives in the sense that the number and the location of change points are unknown. Such pathology is ideally suited to the adoption of the Markov-Switching (MS) methodology.

¹ This class of model was also examined by McCabe and Tremayne (1995), Leybourne *et al.* (1996), Granger and Swanson (1997).

Although the MS unit root test is an advanced tool to detect the presence of rational bubbles in the exchange rate and in its economic fundamentals, it has some limitations. The MS unit root test relies on the researcher's discretion to decide whether explosive behaviour exists at the same time in fundamentals. Deciding whether or not exchange rates depart in an explosive way from fundamentals is not testable with unit root tests. For this reason, this paper uses an additional econometric approach to test for bubbles, based on Markov-switching vector autoregressions (MS-VECM) proposed by Krolzig (1996). The MS-VECM was originally applied to business cycle analysis and allows investigation of the existence of long-run relationships with regime switching. Particularly, such a procedure contains an error correction mechanism which corrects deviations from long-run relationships, and takes into account dynamic adjustments concerning the transition from one regime to another. The importance of using the MS-VECM approach to investigate the presence of bubbles in the stochastic process is that tests applied to time series with uncontrolled for regime switching may lead to misleading conclusion in favour of the presence of bubbles. Hence, this approach allows the analysis of long-run behaviour of time series by eliminating the effect of regime switching on the stochastic process. Artis and Krolzig (2004), Krolzig and Toro (2001) and Krolzig and Sensier (2000) are recently published studies related to the business cycle which make use of MS-VAR modelling. It is still important to emphasise that the employment of the MS unit root test and the MS-VECM approach is new in the empirical literature on the analysis of bubbles in exchange rates.

The paper is structured in five distinct sections. Section 2 outlines the main points of the theoretical model to be employed. Section 3 describes the statistical data, variables used and the research sources. Section 4 presents a theoretical overview of the Markov-switching regime unit root test and the Markov-switching vector autoregression-based cointegration test as well as the empirical results. Finally, Section 5 presents the conclusions.

2 - The Theoretical Model

The model used in this paper follows a standard monetary model of exchange rate determination, which derives the general solution for bubbles under rational expectations. It follows similar versions used by Evans (1991), Taylor and Peel (1998),

Hall *et al.* (1999), Hooker (2000), Psaradakis *et al.* (2001). The model of two countries, domestic and foreign, assumes each country has a money demand equation with the same income elasticity and interest rate semi-elasticity. Thus, it is possible to obtain a single money market equilibrium by combining the money market equilibrium condition in both countries. It leads to the following specification:

$$m_t - p_t = a_1 y_t - a_2 i_t \quad (1)$$

where m_t , p_t , y_t and i_t denotes, respectively, the natural logs of the relative money supply, the relative price level, relative real income and relative interest rate between the domestic and foreign countries. The coefficients a_1 and a_2 correspond to the income elasticity and the interest rate semi-elasticity, respectively.

As the variable i_t in (1) denotes the interest rate differential between the countries and it is assumed that the uncovered interest rate parity (UIRP) condition holds the following expression emerges:

$$i_t = E_t e_{t+1} - e_t \quad (2)$$

where e_t denotes the natural log of the nominal exchange rate at time t and E_t is the mathematical expectation condition on information available at time t . Equation (2) means that an eventual non-zero interest rate differential must be offset by an equivalent expected change in the exchange rate..

The model also assumes that deviations from purchasing power parity (PPP) follow a random walk given by:

$$e_t - p_t = u_t \quad (3)$$

where $u_t = u_{t-1} + \varepsilon_t$ and $\varepsilon_t \sim N(0, \sigma^2)$. Assuming that m_t and y_t are exogenous and substituting (2) and (3) into (1) results in the following representation:

$$e_t = (1-b)f_t + bE_t e_{t+1} + (1-b)u_t \quad (4)$$

$$\text{and } 0 < b \equiv \frac{a_2}{1+a_2} < 1$$

where $f_t = m_t + a_1 y_t$ denotes the market fundamental solution.

From equation (4) a first order expectational difference equation may be derived by repeatedly substituting $E_t e_{t+1}$ for n future time periods. As a result of this, when $j \rightarrow \infty$ and $(1+a_2)^{-1}$ is less than unity, by hypothesis, a non-bubble solution e_t^f emerges as follows:

$$e_t^f = (1-b) \sum_{j=0}^{\infty} b^j E_t f_{t+j} + u_t \quad (5)$$

However, the difference equation (5) can also have an infinite number of solutions if $(1+a_2)^{-1}$ is greater than unity:

$$e_t^f = (1-b) \sum_{j=0}^{\infty} b^j E_t f_{t+j} + B_t = e_t^f + B_t \quad (6)$$

Thus, if $\{B_t\} \neq 0$, and it is assumed to be an implicit process of (6) and defined by (7) as:

$$E_t (B_{t+j}) = B_t \left(\frac{1}{b} \right)^j, \quad \text{for } j = 0, 1, 2, \dots \quad (7)$$

then the solution to (5) is not unique and a potential infinite set of solutions derives from (7).

Therefore, the solution associated with (5) is the market fundamental solution and (6) is a whole set of bubble solutions in which B_t is the exchange rate bubble. The extent of the deviation of the exchange rate from the market equilibrium is a rational bubble captured by the term B_t defined by (7), which drives exchange rate movements away from the market fundamental solution. If such a deviation is perceived by market participants to be significant for speculative purposes, then it will be assigned probabilities and a data generating process (DGP) is formed. The DGP delivers the actual rational aspect to a rational bubble. There will be a particular probability associated with the continuation of the bubble next period against the probability of the bubble bursting. It is worth emphasising that given the bubble's asymmetric probability distribution, then the distribution of the exchange rate innovations will also be asymmetric.

In the real world, this process cannot only be thought of as being characterised by deterministic bubbles. If it is assumed that bubbles do exist, they must be a stochastic

or a periodic process, so that there are periods of expansion as well as periods of contraction or collapse. Evans (1991) embraces this idea, and demonstrates the existence of an important class of periodically collapsing bubbles which is a key focus of this paper. It can be described according to the following representation:

$$B_{t+1} = \begin{cases} (1+r)B_t u_{t+1} & \text{if } B_t \leq \alpha \\ \left[\delta + \pi^{-1}(1+r)\theta_{t+1}(B_t - (1+r)^{-1}\delta) \right] u_{t+1} & \text{if } B_t > \alpha \end{cases} \quad (8)$$

where $(1+r)$ is the discount rate, assumed to be constant, δ and α are real positive parameters such that $0 < \delta < (1+r)\alpha$, $\{u_t\}$ is a sequence of non-negative exogenous i.i.d. random variables with $E_t u_{t+1} = 1$, and θ_{t+1} is an exogenous i.i.d Bernoulli process independent of $\{u_t\}$, such that it takes the value 1 with probability π and 0 with the probability $(1-\pi)$ and $0 < \pi \leq 1$. Note that δ is the mean value of a bubble and α is a positive parameter denoting the magnitude of the bubble, from which a bubble can take on a new dynamic. The idea is that when $B_t > 0$ it implies that $B_s > 0 \quad \forall s > t$. If $B_t \leq \alpha$ the bubble will be growing at mean rate $(1+r)$. On the other hand, when $B_t > \alpha$ the bubble acquires a new expansion dynamic at faster mean rate of $(1+r)\pi^{-1}$ until the bubble collapses with probability $(1-\pi)$. Once the bubble has collapsed it restarts and expands from the mean value of δ .

3 - Data

The data used in this paper are collected from the International Financial Statistic-IFS provided by the International Monetary Fund with end-of-quarter periodicity. They consist of four industrialised market economy countries: Canada, France, Germany and the United Kingdom. The period of analysis extends from 1973:1 to 1998:4, since from 1999:1 onwards a single currency system (the Euro) was introduced.

This study conducts the analysis by using the nominal exchange rate (ER) against the US dollar country by country as the endogenous variable. The regressor

denoting money supply consists of the narrow money supply differential $M1$ ($m1 - m1^* = M1$). The regressor is the real income differential ($gdp - gdp^* = Y$), and real income is represented by GDP in volume.

Finally, the variable differentials are calculated by assuming the United States as the *numeraire* country denoted by an asterisk and in log forms.

4 – Econometric Methodology

In essence, two methodologies are used: the first one is based on unit root tests and the second one takes a cointegration approach. Note that both are based on Markov-switching models.

4.1 - The Markov-Switching Unit Root Test

A number of papers has demonstrated that tests of unit roots and cointegration for economic series may fail in the presence of periodically collapsing bubbles (see Evans 1991). Using Monte Carlo simulations Evans (1991) showed that economic time series with a bubble process may often appear to be stationary in terms of traditional unit root and cointegration tests, even though bubbles are explosive by construction. Taylor and Peel (1998) argue that since bubbles must collapse periodically, standard tests for unit roots and cointegration can generate the misleading conclusion of monotonic mean reversion.

Given these weakness of the familiar augmented Dickey-Fuller (ADF) unit root test in the presence of periodically collapsing bubbles in economic time series, this paper makes use of a more robust test which was explored by Hall *et al.* (1999)² and extends it to a multivariate context. The popular approach to constructing tests of hypotheses to detect the presence of a unit root in time series $\{y_t\}_{t=1}^n$ is based on autoregressive regression models (see Dickey and Fuller, 1981). Nonetheless, the main difficulty of this class of test is that it is only able to detect the presence of a bubble in its expansion phase.

² Note that a similar procedure based on Markov regime-switching regression models was also employed by Van Norden (1996, 1998).

A generalisation to the standard ADF unit root test is to allow for the possibility of the dynamic behaviour of $\{y_t\}$ assuming different characteristics for different periods of the sample. Making use of the class of dynamic Markov-switching models explored in Hamilton (1988, 1989), Hall *et al.* (1999)'s approach consists of modifying the standard ADF unit root test allowing the traditional ADF regression parameters to switch values over different regimes/states. This procedure allows for a dynamic structure consistent with periodically collapsing bubbles. The motivation for this new methodology is the possible existence of two different regimes driving the economic series congruent with the expanding and collapsing phases of the bubble. The generalisation consists of assuming that the parameters governing the ADF regression are time-varying, that is, they change with an unobserved indicator $s_t \in \{0,1\}$, so that the generalised equation can be denoted according to the following representation:

$$\Delta y_t = \mu_0(1-s_t) + \mu_1 s_t + [\phi_0(1-s_t) + \phi_1 s_t] y_{t-1} + \left[\sum_{j=1}^k \psi_{0j}(1-s_t) + \psi_{1j} s_t \right] \Delta y_{t-1} + \sigma_e e_t \quad (9)$$

where $\{e_t\}$ is a sequence of independent and identically distributed (i.i.d) random variables with zero mean and unit variance.

According to Goldfeld and Quandt (1973) and Hamilton (1988, 1989), date t is associated with a regime selected naturally and with a probability which depends upon what regime preceded the process at date $t-1$. The idea is that given a random sequence $\{s_t\}$, characterised as an ergotic Markov chain on the state space $\{0,1\}$, the transition probability associated with each different state space is:

$$\begin{aligned} \Pr(s_t = 1 \mid s_{t-1} = 1) &= p \\ \Pr(s_t = 0 \mid s_{t-1} = 1) &= 1 - p \\ \Pr(s_t = 0 \mid s_{t-1} = 0) &= q \\ \Pr(s_t = 1 \mid s_{t-1} = 0) &= 1 - q \end{aligned} \quad (10)$$

where an additional requirement is that the innovations $\{e_t\}$ in equation (9) must be independent of the state variables \forall_t . By allowing the model's parameters to be

functions of the stochastically chosen regimes which control the process at date t , the equations (9) and (10) constitutes a generalisation of the linear ADF model³.

The MS-switching regime unit root test is applied for the exchange rate and its fundamentals in four industrialised market economy countries: Canada, France, Germany and the United Kingdom. As the fundamentals model of the exchange rate corresponds to the traditional monetary approach, the tests are applied to the exchange rate, money supply differential and output differential⁴. It is also assumed that the time series contain two different regimes: one of them represents the long-run fundamentals solution and the other one represents the non-fundamentals solution. The non-fundamentals solution is supposed to characterise a bubble process. It is important to emphasise that evidence of explosive behaviour found in the exchange rate series combined with similar behaviour in its fundamentals series is suggestive that exchange rate movements is a consequence of market fundamentals movements. On the other hand, if exchange rate behaviour is not followed by similar behaviour in its fundamentals then a rational bubble process may be driving the exchange rate away from the fundamentals solution.

In practice, the MS-unit root test procedure composed of two different regimes consists of testing the null hypothesis of non-stationarity $\phi_0 = 0$ and $\phi_1 = 0$ against the alternative $\phi_0 < 0 (\phi_1 > 0)$ and $\phi_0 > 0 (\phi_1 < 0)$ in equation (9). Note that a negative estimate ϕ_s and a positive estimate ϕ_s statistically significant in the regime $s_t = \{0, 1\}$ imply stationarity and explosive behaviour, respectively. An explosive estimate of ϕ_s for the exchange rate in the regime s_t , not followed by similar estimates in its fundamentals, indicate the possibility of bubbles governing the stochastic process. Table 1 displays the results of estimating (9) for Canada, France, Germany and the United Kingdom. The lag order of the ADF regression is based on the Akaike information criterion⁵.

³ It is important to highlight that these regimes or states changes allow a variety of outcomes to take place.

⁴ Note that the uncovered interest rate parity (UIRP) condition holds.

⁵ Note that all computations are generated by a regime-dependent coefficients and heteroskedasticity MSIAH(s_t)-ARX(p_t) model with two different regimes and p_t lag order for variables in first difference.

Table 1 – Maximum Likelihood Estimates for MS-ADF Regression¹

Country	State Variable ⁶	ϕ_s	
		$s_t = 0$	$s_t = 1$
Canada ²	Exchange Rate [4]	0.1895 (23.884)**	-0.0261 (-1.347)
	Money Supply [4]	0.1621 (15.315)**	-0.0918 (-2.995)**
	Output[4]	-0.1009 (-12.519)**	0.0142 (0.518)
France ²	Exchange Rate [3]	-0.1169 (-3.184)**	-0.0227(-0.723)
	Money Supply [1]	-0.3302 (-19.568)**	-0.0371 (-1.289)
	Output [2]	-0.0914 (-3.755)**	0.0513 (4.238)**
Germany ²	Exchange Rate [5]	-0.1101(-3.378)**	-0.0296 (-0.825)
	Money Supply [1]	0.0119 (0.3704)	-0.0898(-2.067)
	Output [4]	-0.053 (-2.585)*	0.015 (0.075)
UK ²	Exchange Rate [3]	-0.1889 (-4.519)**	-0.1235 (-2.539)*
	Money Supply [4]	-0.1595 (-4.217)**	-0.0052 (-0.139)
	Output [1]	-0.0773(-3.795)**	0.1004 (3.77)**

¹ Figures in square bracket are the lag order in the ADF regression and those in parentheses are *t*-values.

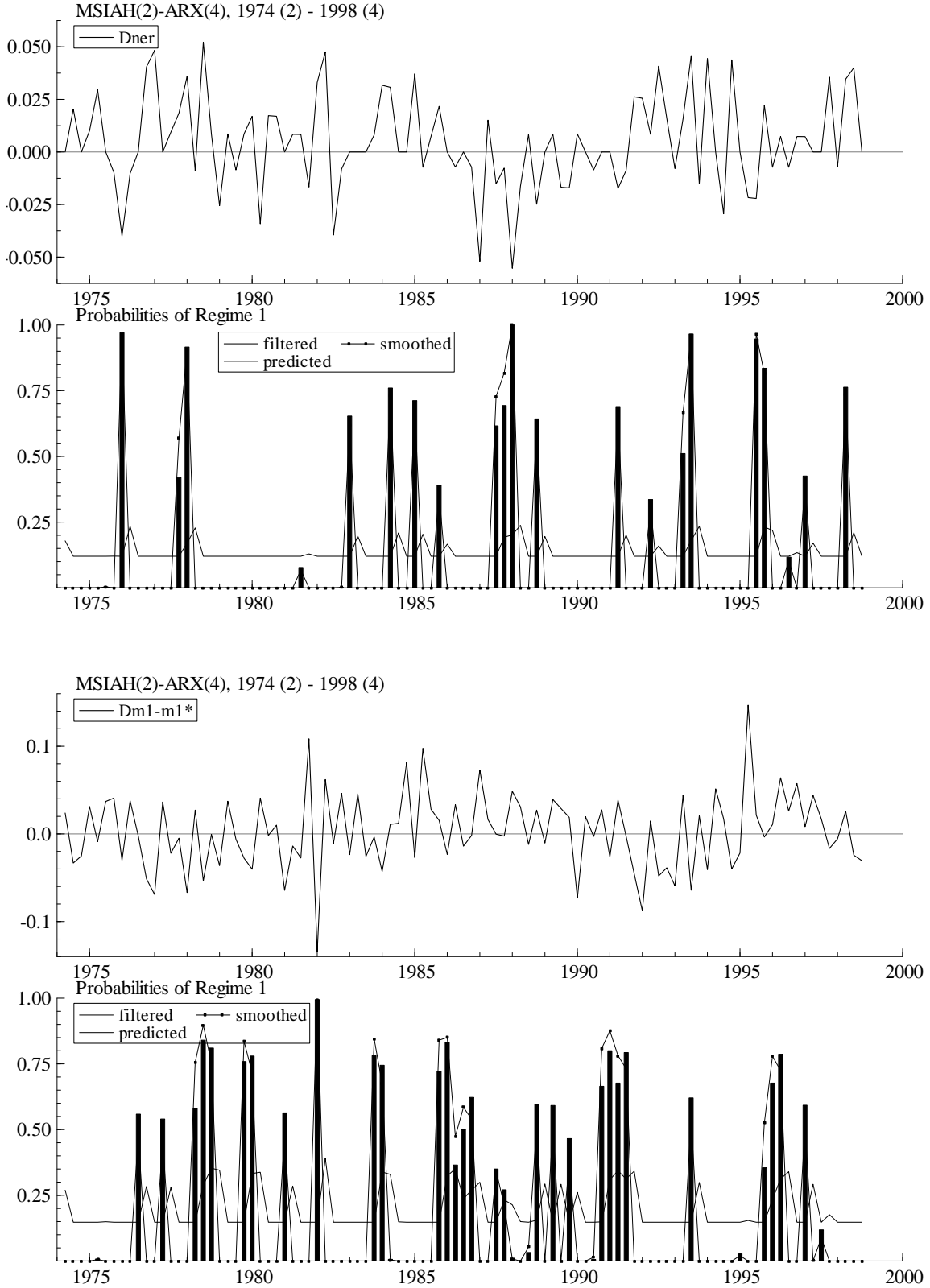
² Critical Values: * = (5% level) , ** = (1% level).

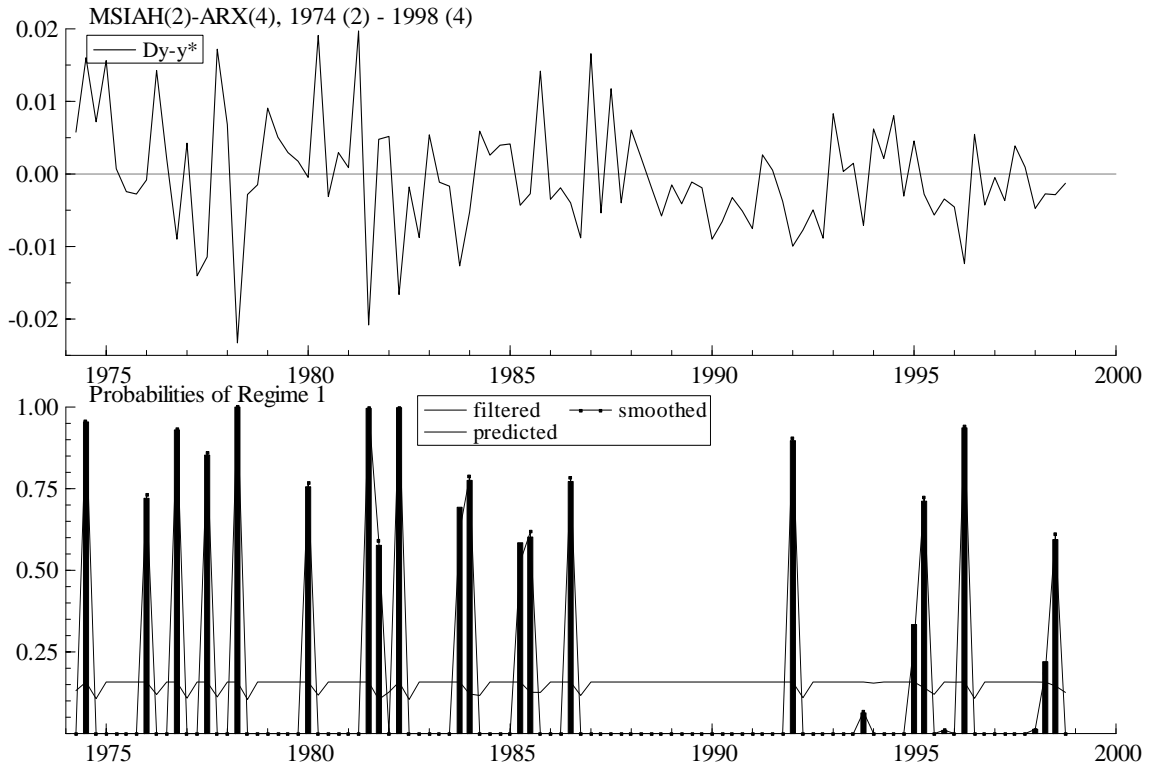
The results in Table 1 for Canada show a statistically significant positive estimate of ϕ_0 ($s_t = 0$) and a non-significant negative estimate of ϕ_1 ($s_t = 1$) for the exchange rate. Note, nevertheless, that the computation for the money supply also demonstrates a significantly explosive result of ϕ_0 in regime $s_t = 0$. Thus, the positive estimate in regime $s_t = 0$ for both variables rejects the hypothesis of bubbles driving the exchange rate. As the exchange rate and the money supply demonstrated both explosive behaviours in $s_t = 0$, the hypothesis of the presence of bubbles is rejected in favour of a fundamental solution. This conclusion for Canada is partially reinforced by analysing the probabilities considering the sample as a whole in regimes $s_t = 0$ and $s_t = 1$ (see Figure 1-Canada). The probability of the exchange rate movements remaining in regime $s_t = 0$ associated to the probability of the money supply remaining in the regime $s_t = 0$ rejects the presence of bubbles. Nonetheless, in some quarters before and after 1980 and 1990 the exchange rate demonstrates an explosive behaviour as in regime $s_t = 0$ associated to non-explosive fundamentals evidenced in regime $s_t = 1$. The graphical

⁶ Note that all variables are expressed in logarithmic form.

analysis for these periods may reveal some evidence of bubbles and requires further investigations.

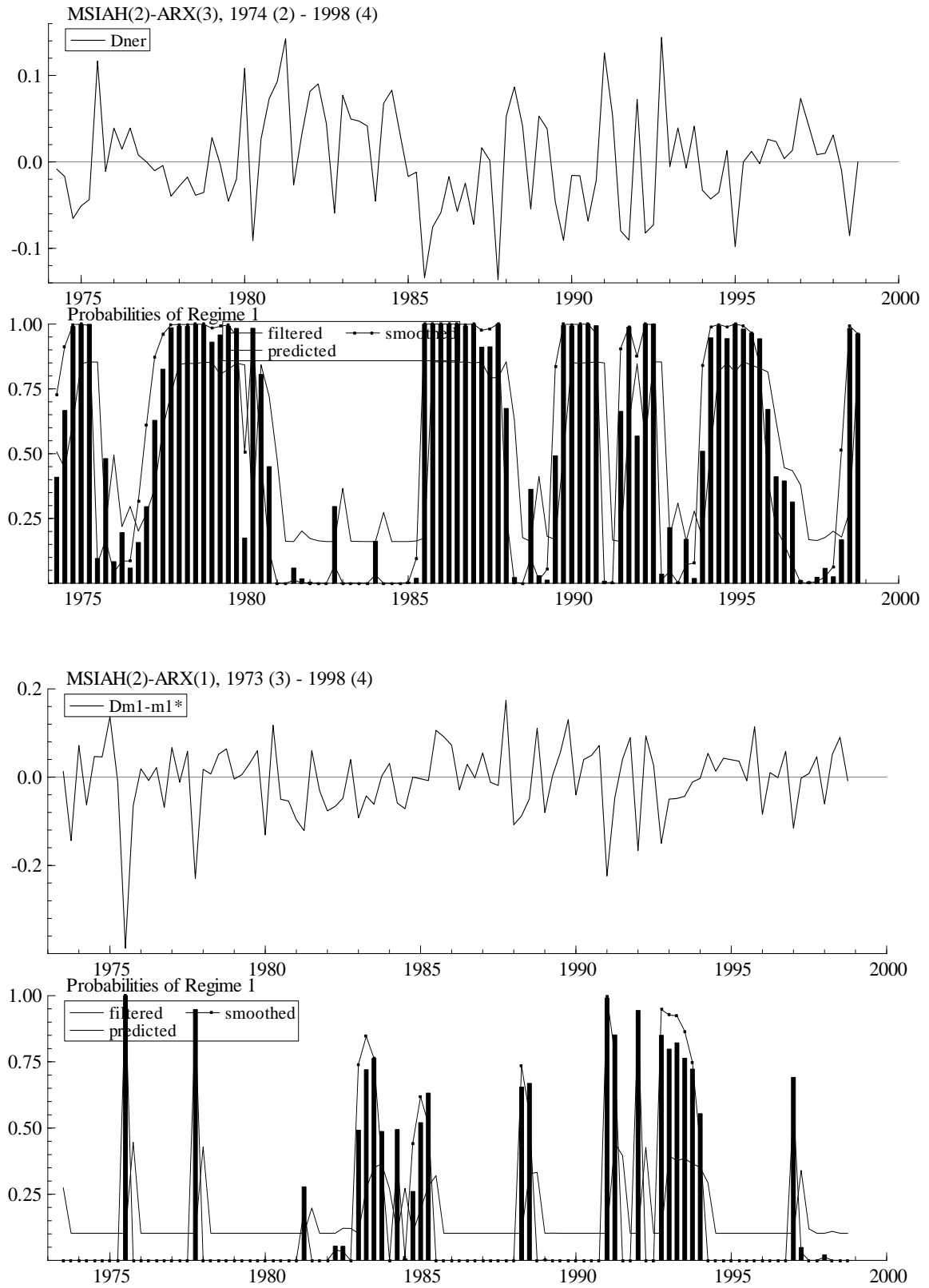
Figure 1-Canada

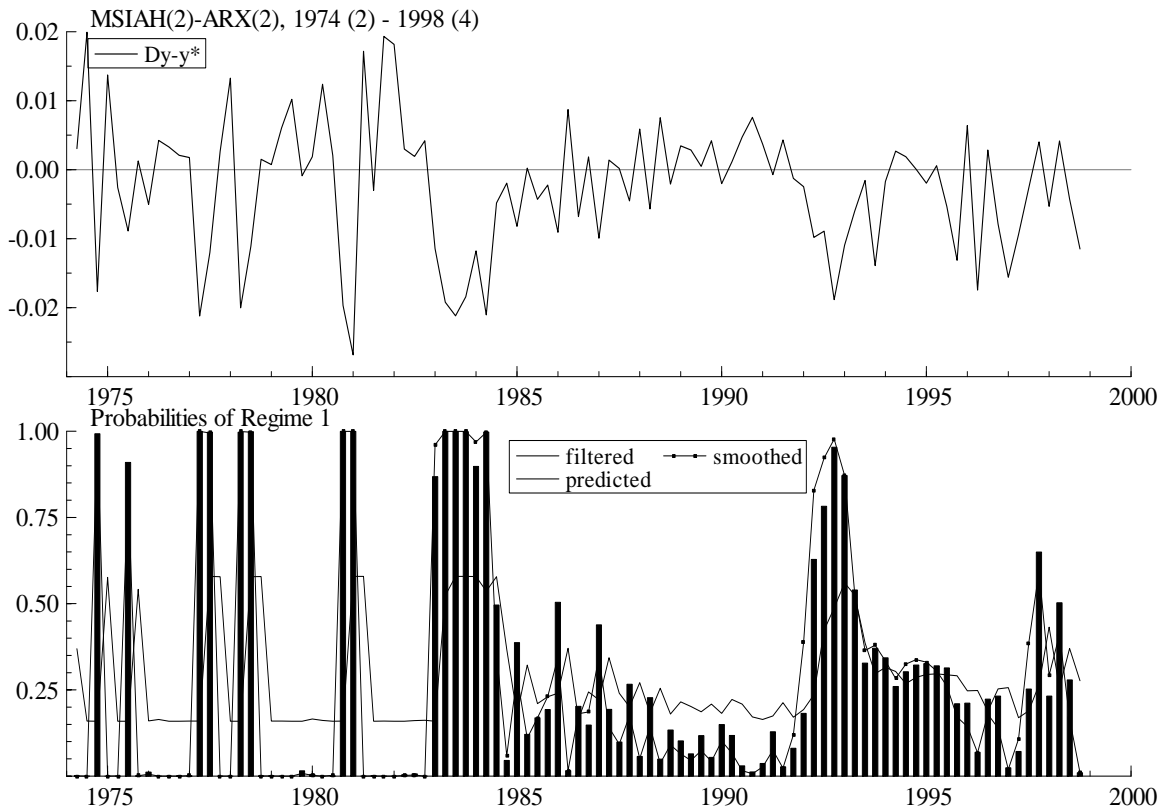




The analysis for France is clear cut as the negative estimates of ϕ_s found in both regimes ($s_t = 0$ and $s_t = 1$) for the exchange rate do not support the presence of bubbles (see Table 1). The significant negative estimate of ϕ_0 for the exchange rate and for the money supply shows stationarity for both variables in regime $s_t = 0$. The regime $s_t = 1$ produced a non-significant negative estimate of ϕ_1 for the exchange rate and for the money supply, that is, a likely non-stationarity. Hence, these results may reveal that the exchange rate behaviour is due to the money supply behaviour and so the hypothesis of the presence of bubbles is again rejected. Figure 2–France shows that the stationarity observed in the exchange rate for the 1981-1985 period and for the 1991-1993 period described in regime $s_t = 0$, in particular, are associated to a mix of non-stationary and stationary behaviours generated by the money supply in regime $s_t = 0$ and $s_t = 1$, respectively. Once again, the graphical analysis reinforces the hypothesis of no bubble driving the exchange rate. Figure 2-France displays the graphs of regime probabilities.

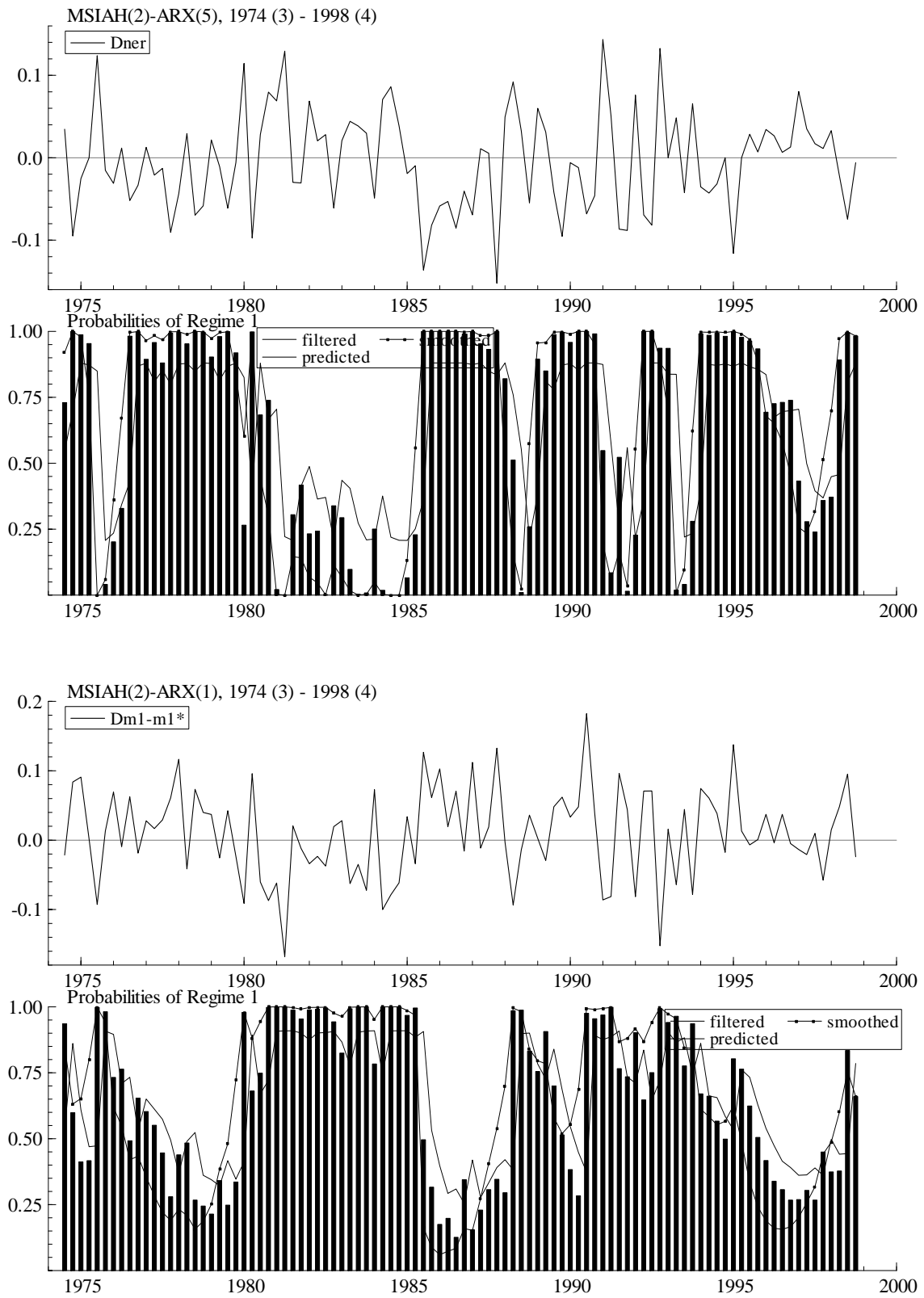
Figure 2-France

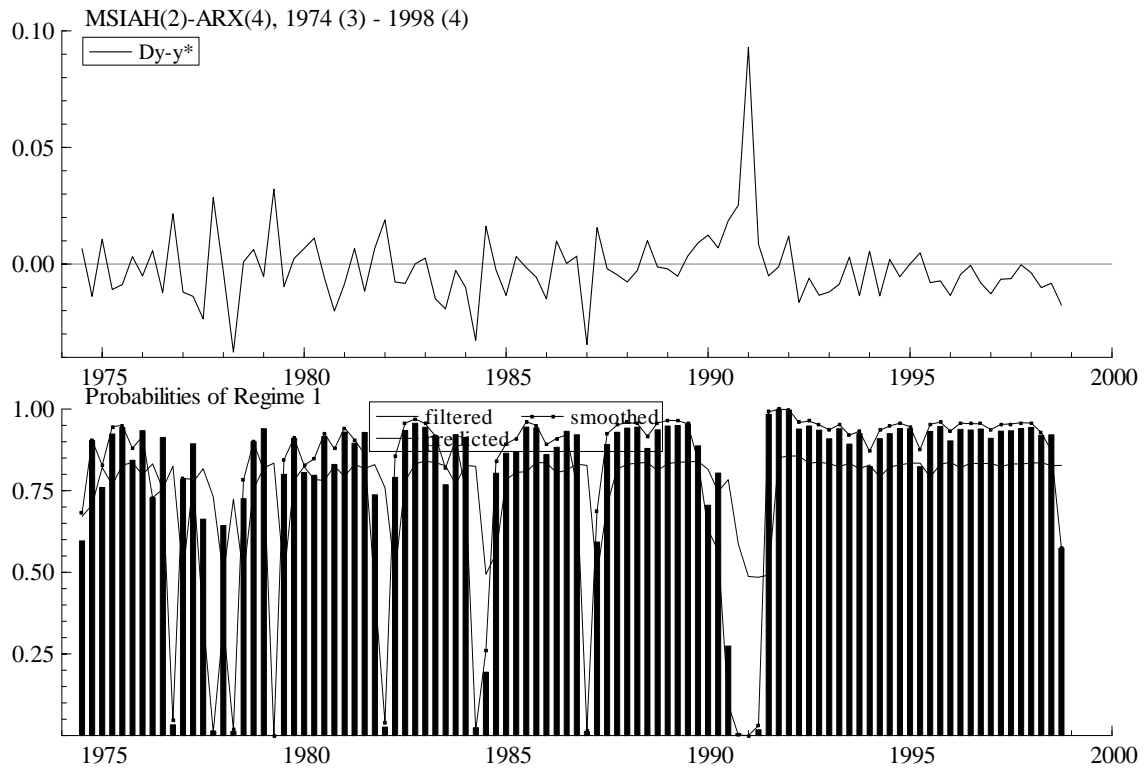




For Germany a positive estimate of ϕ_s is not computed in both regimes for the exchange rate (see Table 1). The significant negative estimate of ϕ_s found in regime $s_t = 0$ and non-significant estimate found in regime $s_t = 1$ leads to the conclusion that the exchange rate series is a stationary and non-stationary process, respectively, in different regimes. The estimate of ϕ_s for the money supply is found positive in regime $s_t = 0$, though statistically non-significant, and non-stationary in regime $s_t = 1$. As such, the hypothesis of bubbles in the German exchange rate is rejected for the sample period. This conclusion is supported by analysing graphically the regime probabilities in which the non-stationarity for the exchange rate and for the money supply observed in regime $s_t = 1$ prevails for the sample period as whole (see Figure 3-Germany).

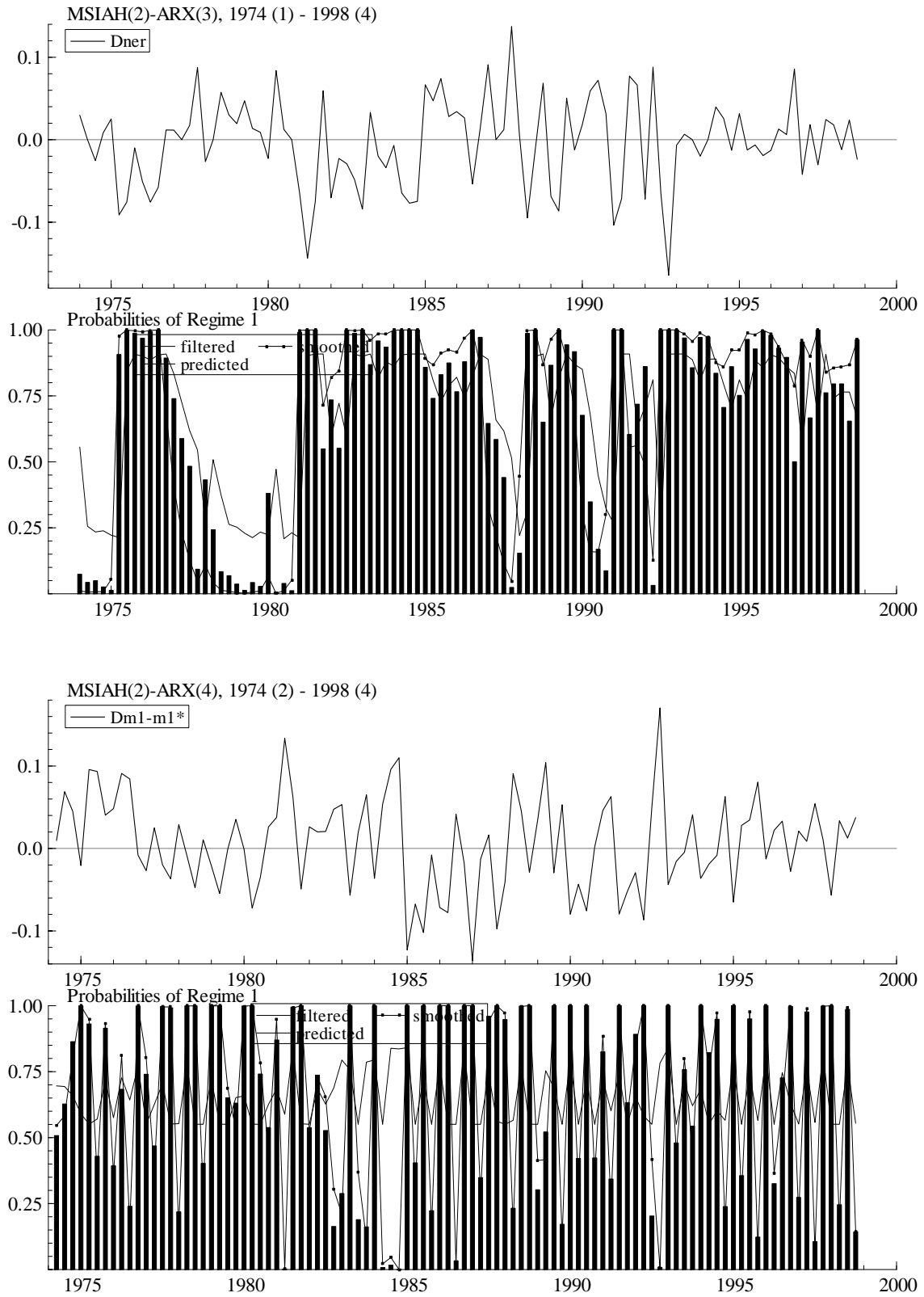
Figure 3-Germany

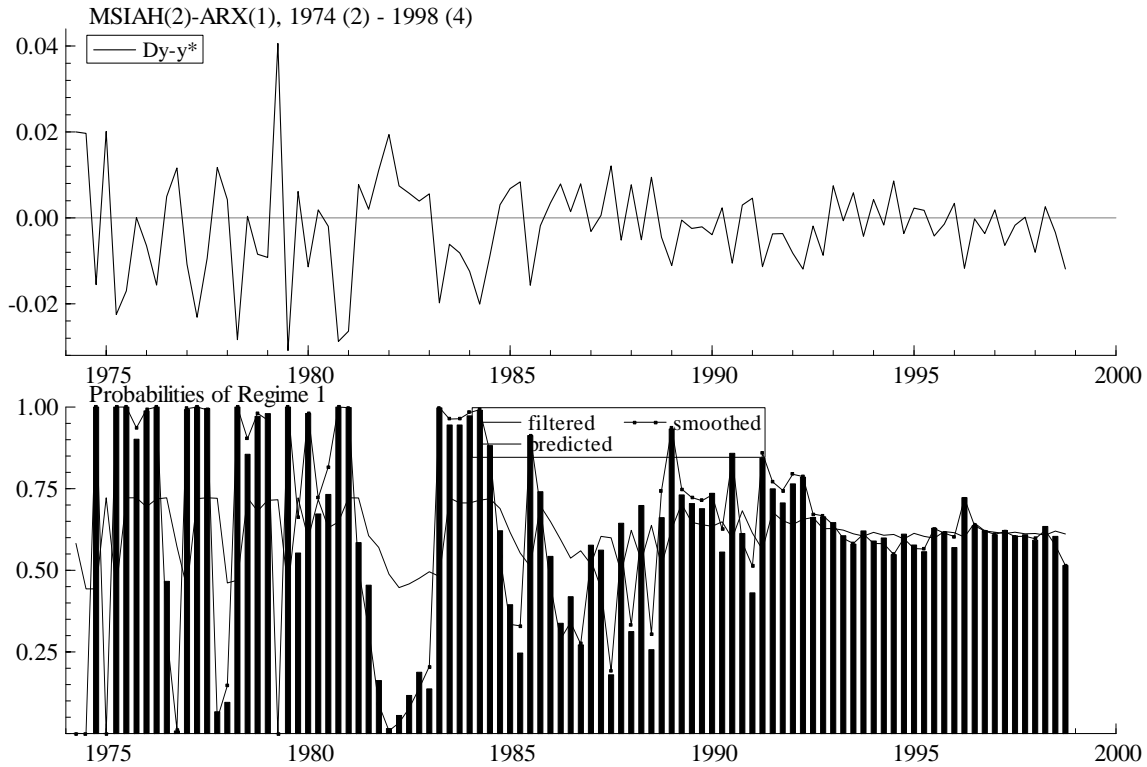




Finally, the United Kingdom case demonstrates a similar conclusion as for Germany about the presence of bubbles. The significant negative estimates of ϕ_s in regime $s_t = 0$ confirm that the exchange rate is a stationary process reinforced by a similar result for the money supply (see Table 1). The regime $s_t = 1$ demonstrates that the exchange rate is non-stationary associated to a similar behaviour for the money supply. The results for both regimes do not allow for the conclusion of bubbles driving the exchange rate. Figure 4-UK that displays the probabilities of regimes confirm this finding as the graphical behaviour for the exchange rate and the money supply follows the non-stationary regime $s_t = 1$.

Figure 4-UK





Although it is not possible to identify robustly different regimes (non-bubble and bubble regimes) based on only regime probabilities, it is possible to state the prevailing regimes for each variable. Table 2 displays the regime probability for each variable based on the number of observations by regimes.

Table 2 – Regime Probabilities (%)*

Country	State Variable ⁷	ϕ_s	
		$s_t = 0$	$s_t = 1$
Canada	Exchange Rate [4]	13.6	86.4
	Money Supply [4]	19.5	80.5
	Output[4]	15.0	85.0
France	Exchange Rate [3]	51.7	48.3
	Money Supply [1]	83.8	16.2
	Output [2]	72.2	27.8
Germany	Exchange Rate [5]	63.4	36.6
	Money Supply [1]	64.6	35.4
	Output [4]	81.9	18.1
UK	Exchange Rate [3]	69.1	30.9
	Money Supply [4]	65.1	34.9
	Output [1]	61.4	38.6

* Figures in square bracket are the lag order in the ADF regression as in Table 1.

⁷ Note that all variables are expressed in logarithmic form.

By comparing results in Table 1 and Table 2 for Canada, for instance, it may be interpreted that the regime probability of the explosive behaviour running the exchange rate and the money supply in $s_t = 0$ is much less than the probability for the non-explosive regime $s_t = 1$. It means that a fundamental solution prevails for the explosive case⁸. France presented the regime probability of a stationary behaviour in regime $s_t = 0$ for the exchange rate and the money supply slightly greater than the regime probability for a non-stationary regime $s_t = 1$. Once again, the fundamental solution for France prevailed. Germany and UK did not exhibit an explosive behaviour for either the exchange rate or the money supply and the prevailing regime probability is stationary. The unexpected regime probabilities found for Germany and UK supporting the stationarity for the exchange rate may require further investigations.

Finally tests for linearity based on MS-likelihood ratios rejected the null hypothesis of one regime and revealed non-linear behaviours for the exchange rate. Table 3 displays linearity statistics:

Table 3 - Likelihood Ratio (LR) Linearity Tests

Country	LR	χ^2
Canada	30.804	(7)[0.0001]**
France	13.5779	(6)[0.0347]*
Germany	22.459	(8)[0.0041]**
UK	12.0129	(6)[0.0417]*

** = 1% and * = 5% level of significance.

Figures in parentheses denote degrees of freedom and in squared brackets are p -values.

To summarise, the analysis to detect the presence of bubbles based on the MS-unit root procedure does not reveal clear evidence of bubbles driving the stochastic process of the exchange rate. The result for Canada was the only one that produced significant positive estimate of ϕ_s in the exchange rate followed by similar results in at least one exchange rate fundamental. The graphical analysis for Canada also presented an explosive regime in the exchange rate associated to a non-explosive regime in fundamentals which may suggest evidence of bubbles. However, this conclusion is not robust and requires a further investigation. Moreover, as tests detected non-linear behaviours there may be periods in graphs demonstrating some evidence of regime

⁸ An exception must be made for some quarters before and after 1980 and 1990 as already aforementioned.

mismatches between the exchange rate and its fundamentals. These periods may require a further investigation. Next section presents an additional approach for bubble tests based on MS-VECM.

4.2 - The Markov-Switching Vector Autoregression of Nonstationary TS

The Hamilton (1988, 1989) approach models regime switching behaviour of univariate time series. In particular, the underlying idea of this class of regime-switching models consists of examining the effect of an unobservable regime variable s_t , denoting different states of the world, on the parameters of a stochastic process. The increasing interest in analysing multivariate systems with regime switching encouraged the development of a new technique based on vector autoregression (VAR) modelling. This new approach was proposed by Krolzig (1996) who developed the idea of cointegrated Markov-Switching vector autoregression (MS-VAR). In particular, it is designed to investigate the statistical properties of multivariate time series subject to regimes shifts. Note that although the parameters are time-varying, they are constant conditional on s_t . Essentially, Krolzig extends the univariate case to the multivariate case by generating the finite order vector autoregressive (VAR) model with regime switching which is expressed generically by the following equation:

$$x_t = v(s_t) + A_1(s_t)x_{t-1} + \dots + A_p(s_t)x_{t-p} + \mu_t \quad (11)$$

where the sample values x_0, \dots, x_{1-p} are fixed and constitute elements of a vector matrix. The dependence of the parameters on the realised regime s_t is denoted by the parameter shift functions $v(s_t), A_1(s_t), \dots, A_p(s_t)$. For instance, the state-space representation for the intercept is expressed as:

$$v(s_t) = \begin{cases} v_1 & \text{if } s_t = 0 \\ \vdots & \\ v_M & \text{if } s_t = M - 1 \end{cases} \quad (12)$$

The parameters in equation (11) are subject to a prevailing regime which is stochastic and unobservable. Thus, a complete description of the data generating process requires formulation of a rule, a regime generating process, and then the evolution of regimes can be inferred from the data. The regime generating process in

Markov-switching models constitutes an ergotic Markov chain based on a finite number of states $s_t = 1, \dots, M$ and defined by transition probabilities:

$$p_{ij} = P_r(s_{t+1} = j | s_t = i), \quad \sum_{j=1}^M p_{ij} = 1 \quad \forall_{i,j} \in \{1, \dots, M\} \quad (13)$$

The idea of a MS-VAR process is based on the existence of a finite-order vector moving average (VARMA) representation⁹. Such a VARMA structure may be approximated by a finite-order linear VAR model, and estimated by Johansen's maximum likelihood procedure (see Johansen, 1995).

The cointegration relationships are referred to as the long-run equilibrium of the system. Like the standard vector equilibrium correction model (VECM) proposed by Davidson *et al.* (1978) and Engle and Granger (1987), a Markov-switching vector equilibrium correction model (MS-VECM) can also be implemented. The basic idea is that the error correction mechanism contained within a MS-VECM also allows for the errors arising from regime shifts to be corrected towards the stationary distribution of the regimes (see Psaradakis *et al.*, 2004). A generalisation of this model is as follows:

$$\Delta x_t = v(s_t) + \sum_{i=1}^{p-1} \Gamma_i(s_t) \Delta x_{t-i} + \Pi(s_t) x_{t-p} + u_t \quad (14)$$

where $\Gamma_i(s_t) = -\left(\mathbf{I}_k - \sum_{j=1}^i \mathbf{A}_j \right)$ is the coefficient matrix on the differenced variables

subject to the regime s_t , and $\Pi(s_t) = \alpha \beta' = \mathbf{I}_k - \sum_{j=1}^p \mathbf{A}_j$ is the coefficient matrix subject

to the regime s_t , which is composed of the adjustment velocity matrix α and the cointegration matrix β' . The matrix $\Pi(s_t)$ corresponds to the error correction mechanism¹⁰. The rank r of the matrix $\Pi(\alpha \beta')$ defines general conditions for co-breaking¹¹. The idea behind the concept of co-breaking consists of removing the effects of regime switching by taking linear combinations of variables.

⁹ The intercept term is composed of an unconditional mean \bar{v} plus a moving average MA(∞) representation.

¹⁰ Note that this correction mechanism is closely related to the concept of a multiple dynamic equilibrium in economics, defined by the equilibrium value of the cointegration vector and the drift.

¹¹ In fact, this concept of co-breaking is closely related to the concept of cointegration for multiple time series subject to regime switching introduced by Hendry (1996).

Similar to the unit root test approach for regime switching, the investigation of the presence of a bubble by a MS(s_t)-VECM(p) requires the analysis of the sign on the adjustment coefficient $\alpha(s_t)$ in equation (14). Once again, assuming that the stochastic process is characterised by two regimes $s_t \in [0,1]$, one corresponds to the fundamental solution and the other is the bubble solution. The hypothesis of a bubble driving the stochastic process is accepted if $\alpha_0 > 0$ or $\alpha_1 > 0$ are found.

To analyse the presence of bubbles in the exchange rates, a MS-VECM was applied to each country individually according to equation (14). The econometric approach consists, firstly, of estimating a linear VAR with finite order¹². Next, based on the estimated cointegration matrix, the EM (Expectation-Maximisation) algorithm is used to estimate the remaining coefficients of a MS-VECM¹³. In practice, the analysis of a bubble driving the stochastic process consists of examining the significance of the coefficient on $VECM_{t-1}$ in equation (14) which is subject to regime shifts. If it is found to be negative and statistically significant then the process converges towards the fundamental equilibrium, but if it is found to be positive and statistically significant, then it may be interpreted as an explosive process indicating the presence of bubbles¹⁴.

For the case of Canada, a MS(2)-VECM(2) is estimated and the lag order is based on the minimum Akaike information criterion (AIC). The log-likelihood statistic 261.382(240.281), AIC -4.795(-4.632), likelihood ratio test 42.199 all reject the linearity hypothesis of the stochastic process. Even though the Canadian data have produced a positive coefficient on $VECM_{t-1}$ in the regime $s_t=0$ (see Table 4) it is not statistically significant. This may suggest that there is a non-stationary component in the joint exchange rate-fundamentals process, but its statistical insignificance does enable a bubble conclusion. Regime 1 is characterised as non-stationary since the negative coefficient on the ECM is statistically not significant. Also, based on the duration estimates it is possible to observe that the time path for the Canadian exchange rate is remarkably turbulent as regime 0 (duration=11.7) prevails over regime 1 (duration=7.65). The transition probability reinforces this conclusion since the transition

¹² This procedure aims at estimating the parameters of a linear VAR which comprises the ECM. Once again, a linear VAR is required as there is only one fundamental solution for the exchange rate. The other ones are assumed to be bubble solutions.

¹³ Note that the VAR estimation procedure has only been applied to the ECM. The other coefficients are estimated in accordance with the Engle and Granger (1987) procedure.

¹⁴ For simplicity, the critical values are taken from the Dickey-Fuller distribution as the distribution for this approach requires a complex simulation process not available for this thesis.

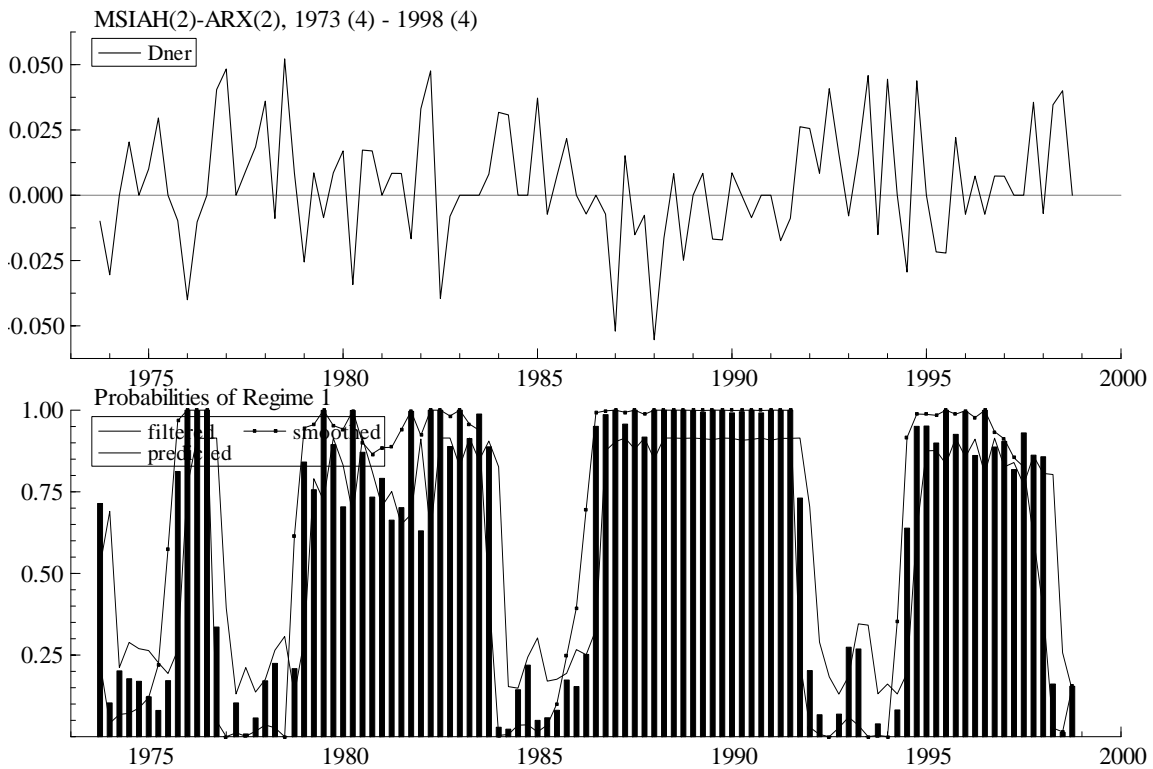
probability of leaving the turbulent regime 0 to non-stationary regime 1 is just 8%. Table 4 and Figure 5-Canada display the main estimated coefficients and the regime probabilities, respectively.

Table 4 – Main ML Estimation Results (Canada)¹

MSIAH(2)-VECM(2)	Regime $s_t = 0$ *	Regime $s_t = 1$ *
Intercept	-0.0306(-1.1192)	-0.0355(5.662)
VECM_{t-1} $\alpha(s_t)$	0.0104(0.9592)	-0.0181(-1.6215)
Standard Error	0.01592	0.01358
Duration	11.7 (61 obs.)	7.65(39 obs.)
Log-likelihood	261.976 (243.532)	
LR linearity test	36.887, $\chi^2(9) = [0.0000]$ **	
Transition Matrix	Regime $s_t = 0$	Regime $s_t = 1$
Regime $s_t=0$	0.92	0.08
Regime $s_t=1$	0.13	0.87

* Figures in parentheses denote *t*-statistics.

Figure 5-Canada



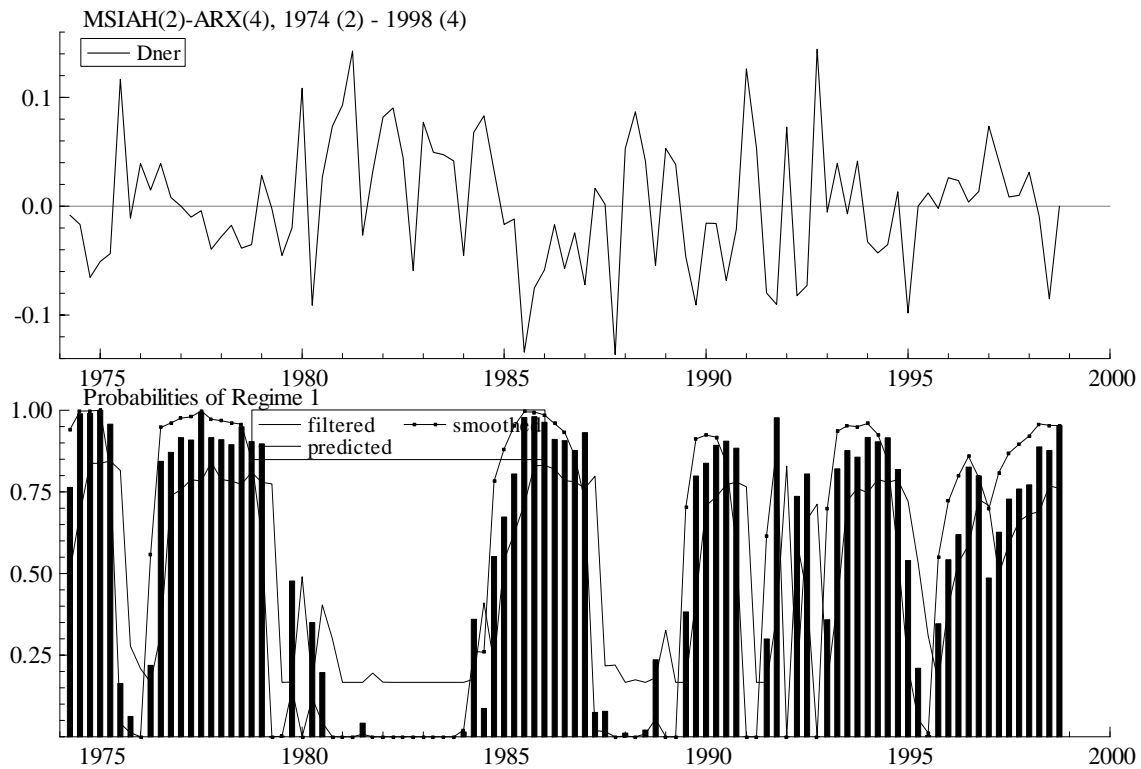
For France a MS(2)-VECM(4) is estimated in accordance with the minimum AIC information criterion. Once again, the log-likelihood statistic 171.5905(150.2075), AIC criterion -2.82(-2.7315), and the likelihood ratio test 42.7659 reject the linearity hypothesis of the stochastic process. The model reveals a significantly negative estimated coefficient on $VECM_{t-1}$ in the regime $s_t=0$ (see Table 4) which may be interpreted as evidence of cointegration between the exchange rate and its fundamentals. The regime 1 reveals non-stationary behaviour denoted by a statistically insignificant negative coefficient on the $VECM_{t-1}$. The transition probabilities of the regimes are close, leading to durations that are quite similar between regimes. The lengths of deviation periods of the exchange rate from its fundamentals are quite similar to the length of long-run equilibrium periods. Moreover, the presence of explosive behaviour of the exchange rate is not observed. Hence, the data for France do not present evidence of bubbles driving exchange rate movements. Table 5 and Figure 6-France demonstrate the main computations and regime probabilities, respectively.

Table 5 – Main ML Estimation Results (France)

MSIAH(2)-VECM(4)*	Regime $s_t = 0$	Regime $s_t = 1$
Intercept	0.1671(5.3735)	0.1678(2.1981)
VECM_{t-1}	-0.049(-6.2178)	-0.035(-1.8672)
Standard Error	0.02191	0.05228
Duration	6.44 (50 obs.)	6(48 obs.)
Log-likelihood	171.5905 (150.2075)	
LR linearity test	42.765, $\chi^2(15) = [0.0002]**$	
Transition Matrix	Regime $s_t = 0$	Regime $s_t = 1$
Regime $s_t=0$	0.84	0.16
Regime $s_t=1$	0.16	0.84

* Figures in parentheses denote *t*-statistics.

Figure 6-France



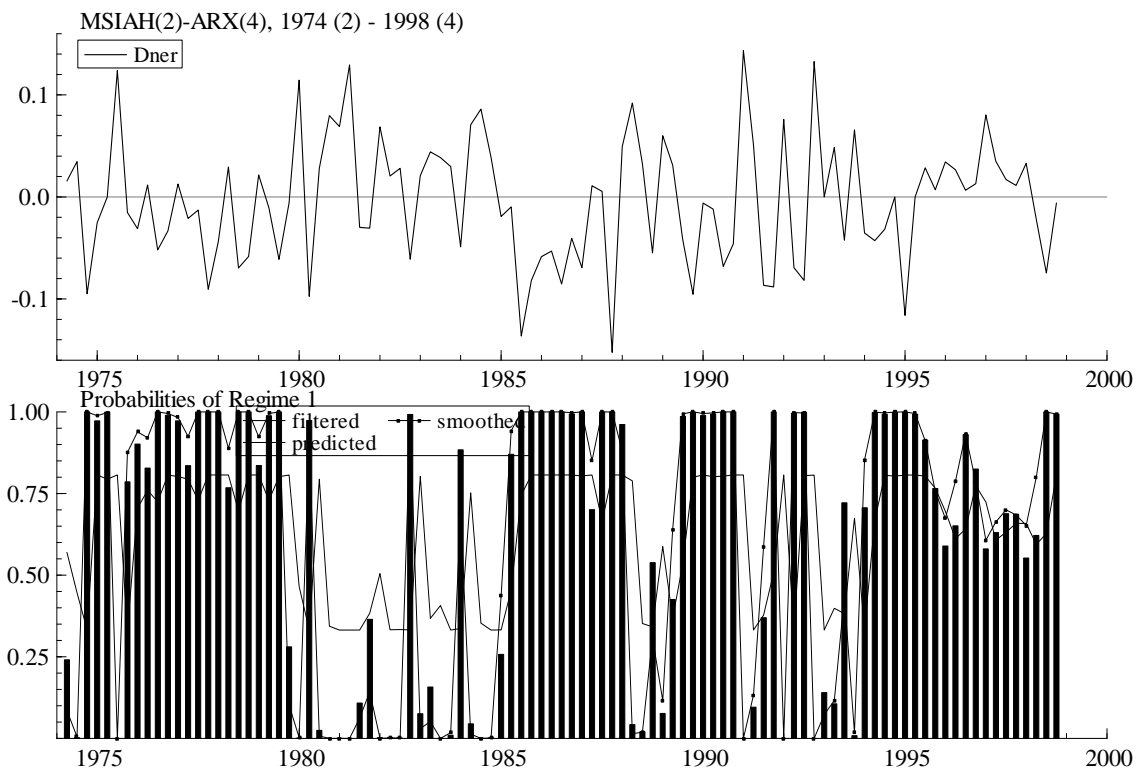
A MS(2)-VECM(4) is also selected for Germany based on the AIC information criterion. The log-likelihood statistic 161.0411(146.2912), AIC criterion -2.6069(-2.652) and likelihood ratio test 29.499 reject the linearity hypothesis of the system. The computations for coefficients on $VECM_{t-1}$ are found to be statistically insignificant which means that cointegration between the exchange rate and its fundamentals is not supported by the data. Deviations from the fundamentals solution are not corrected in the long run. Despite the coefficient on $VECM_{t-1}$ being positive in regime 1, indicating an explosive component in exchange rate behaviour, it is not significant. Furthermore, taking into account the duration for both regimes shows that the stochastic process defined by the non-stationary regime 0 prevails over the weak explosive path generated by the regime 1, which is reinforced by the higher transition probability of leaving regime 1 to regime 0, that is, $0.33 > 0.19$ (see Table 6). Thus, it is possible to conclude that bubbles are not driving the exchange rate movements in Germany for the sample period. Table 6 and Figure 7-Germany display the main estimates and regime probabilities, respectively.

Table 6 – Main ML Estimation Results (Germany)

MSIAH(2)-VECM(4)*	Regime $s_t = 0$	Regime $s_t = 1$
Intercept	-0.0181(-0.5722)	0.0422(0.427)
VECM $_{t-1}$	-0.0269(-1.4633)	0.0089(0.0819)
Standard Error	0.03195	0.03376
Duration	5.18 (62 obs.)	3.01(37 obs.)
Log-likelihood	161.0411 (146.291)	
LR linearity test	29.499, $\chi^2(15) = [0.0139]^*$	
Transition Matrix	Regime $s_t = 0$	Regime $s_t = 1$
Regime $s_t=0$	0.81	0.19
Regime $s_t=1$	0.33	0.67

* Figures in parentheses denote t -statistics.

Figure 7-Germany



Finally, the results for the United Kingdom are estimated from a MS(2)-VECM(4) supported by the AIC information criterion. The log-likelihood statistic 178.239(157.811), AIC criterion -2.954(-2.885), likelihood ratio test statistic 40.855 all reject the linearity of the system. The estimates of the VECM $_{t-1}$ are also not significant, but both are negative, so that it is possible to infer that there is no cointegration between the exchange rate and its fundamentals (see Table 7). Furthermore, based on these

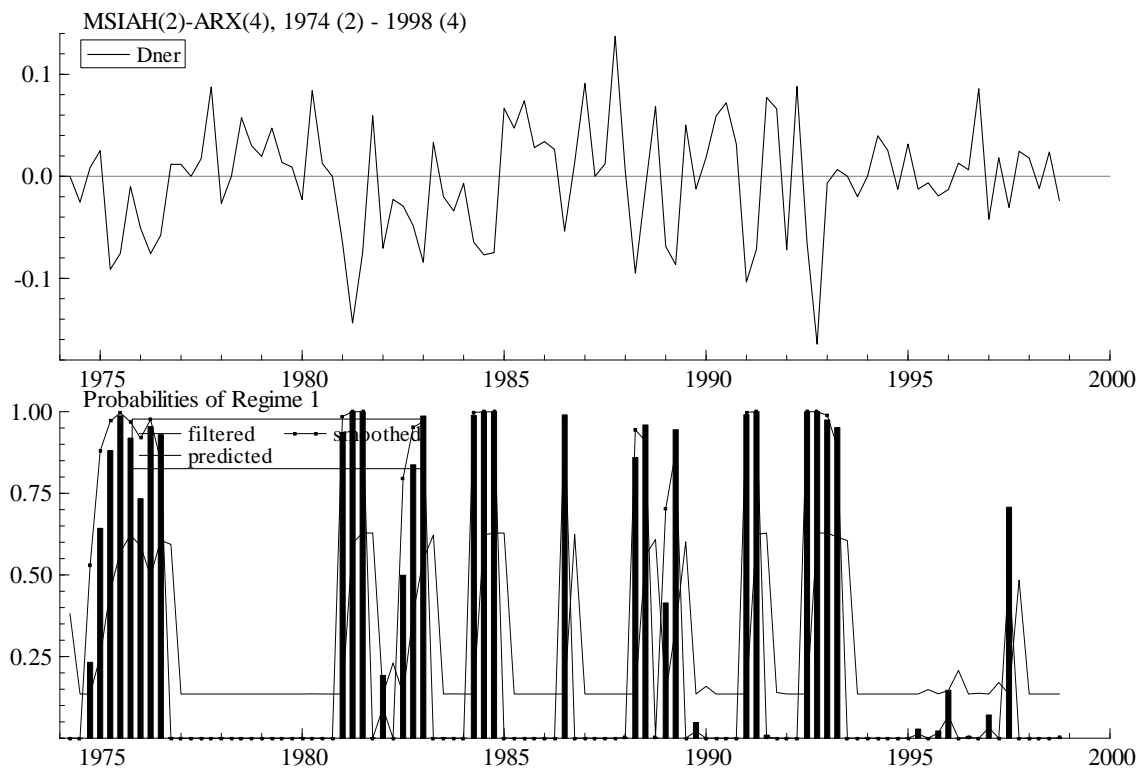
results the hypothesis of bubbles driving the exchange rate in the UK can be rejected. Table 7 and Figure 8-UK display the main calculations.

Table 7 – Main ML Estimation Results (UK)

MSIAH(2)-VECM(4)*	Regime $s_t = 0$	Regime $s_t = 1$
Intercept	-0.2805(-2.433)	-0.0021(-0.0143)
VECM $_{t-1}$	-0.0517(-1.8213)	-0.006(-0.1643)
Standard Error	0.01474	0.03334
Duration	2.69 (26.9 obs.)	7.39(72 obs.)
Log-likelihood	178.2394 (157.8114)	
LR linearity test	40.8559, $\chi^2(15) = [0.0003]**$	
Transition Matrix	Regime $s_t = 0$	Regime $s_t = 1$
Regime $s_t=0$	0.63	0.37
Regime $s_t=1$	0.13	0.87

* Figures in parentheses denote *t*-statistics.

Figure 8-UK



According to Psaradakis *et al.* (2004) the conventional tests of linear cointegration are reasonable in detecting long-run relationships. Even if occasionally the error process follows a non-stationary path due to different prevailing regimes, the tests are powerful enough to detect evidence of cointegration when the state indicators $\{s_t\}$

are uncorrelated. The conclusions found for cointegration in this section reinforce the results reached in the previous section using MS-unit root tests in rejecting the presence of bubbles in the exchange rates for these four industrialised market economies.

5 - Conclusion

The standard tests for unit roots and cointegration are unable to detect periodically collapsing bubbles. This type of stochastic process does not model regime shifts suitably and this affects test performances. Thus, a more flexible econometric technique was used, based on Markov switching (MS) regimes, where the parameters are defined conditional on an unobservable regime variable, and the transition between different regimes is determined by an ergodic Markov chain.

The MS-regime unit root tests applied to each country did not reveal robustly the presence of bubbles governing the exchange rate, as the explosive roots detected in this variable were also detected in at least one of its fundamentals. The bubble process is an independent stochastic process. Tests, nonetheless, detected non-linear behaviours for exchange rates. Due to the potential weakness of MS-unit root tests a MS-ECM test was investigated based on a MS(M)-VECM(p) model with two distinct regimes ($M=2$), and different lag orders (p) for the VAR representation. The empirical evidence found in this analysis of the exchange rate in all four countries did confirm the results reached by the MS-regime unit root tests; namely that the interpretation of the estimates did not support the presence of bubbles. Although some countries have significant intercepts in different regimes, the estimated coefficients on the $VECM_{t-1}$ when positive, were not found to be significant. Tests also revealed non-linear behaviours for exchange rates.

Summarising, the hypothesis of periodically collapsing bubbles driving the exchange rate away from the fundamentals solution cannot be accepted for these four countries. Moreover, the Markov switching regime approach revealed significant non-linearities and different regimes. The existence of different regimes in the exchange rate is a finding that confirms previous results on non-linear exchange rate models [see Meese and Rose (1991), Taylor and Peel (2000), Yue and Kana (2000), Taylor and Peel and Sarno (2001), Kilian and Taylor (2003)]. However, these non-linearities do not appear to be linked to the explosive behaviour characteristic of processes driven by bubbles.

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