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THE INFLUENCE OF TRAVEL DISTANCE AND TRANSPORT OPERATORS' OBJECTIVES ON FARES, TRANSPORT QUALITY AND GENERALISED TRANSPORT COSTS

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Abstract

A theoretical model is adopted in order to examine optimal fare and optimal quality of supply schemes for a transport operator. The analysis shows how fares and quality of supply are related to travel distance and to the transport operator's weight on profit versus consumer surplus. Under reasonable assumptions imposed on the actual functions, it is found that the more weight the operator gives to profits, the higher the fare level and the higher the generalised travel costs. How the operator's objectives influence the quality of transport and how travelling distance affects fares, quality of transport and generalised travel costs are ambiguous, and depend on the initial restrictions placed on the actual functions. The paper then investigates how different additional restrictions imposed upon the functions influence the results. The paper also examines the special case in which the quality of transport is exogenous to the transport operator. One important result then is that higher demands towards the transport operator regarding the quality of the transport supply do not necessarily reduce the transport users' generalised travel costs.

JEL Classification: L2, L9, R4

Keywords: Travel distance, operators' objectives, optimal fare, optimal quality, generalised costs.

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1. Introduction

Several economists have been engaged in searching for optimal fares and optimal supply within different areas of public transport. Two of the pioneering works within this field are Mohring (1972) and Turvey and Mohring (1975) who, by taking an example for urban bus transportation, are considering both direct costs from bus operations and the passengers' time costs in discussing optimal strategies in fare and supply management in transport route operations. In the spirit of Mohring's work several similar analyses have been carried out since, see for instance Jansson (1979), Larsen (1983) and Jansson (1993).

All the works mentioned above are searching for first best optimal policies regarding fares and services. Inspired by the seminal work of Baumol and Bradford (1970) on the theory of the second best, many authors have, however, been searching for optimal fare and supply in a context where financial constraints are to be fulfilled by the transport operator, see for instance Glaister and Lewis (1978), Hervik (1983), Jørgensen and Solvoll (1999) and Kolstad and Solvoll (2000). Additionally, Nash (1978), Glaister and Collings (1978) and Bøs (1978) are analysing optimal fare, supply schemes and financial results for the operators when they pursue other goals than social welfare maximisation; for example they aim to maximise profit, bus mileage or passenger mileage subject to a budget constrain. In order to reach the above objectives in practice, the operators offer more and more sophistic pricing and product differentiation schemes such that the capacity utilisation of the scheduled services will be as high as possible. Such yield management procedures are, in particular, well developed in scheduled flight services, see Button (1993) and Botimer (1996).

One specific problem that the operator faces when optimal management in transport route operations comes into practise is how should the fares and qualities of supply vary with the passengers' travel distance. Often transport companies design fare and supply schemes for all their operations, where the fares and sometimes also the quality of supply vary with the travel distance rather than choosing fare and supplies on single distances or routes. It is also seen that in cases where public authorities have the opportunity to regulate transport operations, they claim that such fare and supply schemes should be developed and encompassed for all routes operated by the company. Also if a transport company is operating under free market conditions, it may find it advantageously to design fare and supply schemes that are related to travel distance in a uniform way.

In Norway, some empirical works have analysed the actual relationships between fares and travelling distance for different modes of passenger transport; Ertkjern and Tausvik (1996) for bus transport, rail transport, regional air transport and fast–craft transport, Jørgensen and Solvoll (1999) for ferry transport and Kolstad and Solvoll (2000) for bus transport in different counties. Depending on mode of transport and range of travelling distance, all these works showed, as expected, that the relationships between fares and travelling distance were stepwise increasing or none convex continuously increasing. Broadly speaking, the analyses also showed, that travelling distance influenced fares more on fast modes (air transport) than on slow modes (bus transport). The same characteristics with the fare schemes are probably present for the majority of countries. Except for ferry transport where the Norwegian authorities control the fare schemes directly and where the fare level is the same all over the country, the authorities only regulate the operators' fare level indirectly, by claiming that the practised fares must be sanctioned in advanced.

Neither of these works show, however, how the quality of the transport service and thereby individual resources spent on travelling, measured be generalised travel costs, vary with travelling distance; is, for example, the quality of the buses, ferries and planes better on long routes than on short routes? Common sense tells us that this would be the case, but no empirical works have, as far as we know, been undertaken in any country analysing this issue thoroughly.

Although the travelling distance varies considerably among passengers on the same mode of transport and although the actual fares are highly related to the travelling distance, issues dealing with how fare and quality of transport services should be related to travelling distance are scarcely dealt with in both theoretical and empirical works. The bases for the latter assertion are the following: Firstly, no matter whether the operators aim to maximise profit, social welfare or pursue other objectives where cost effectiveness is important, finding such optimal relationships demands knowledge about the relationships between marginal costs and travelling distance for different levels of transport quality. Such relationships are, however, seldom estimated (Jørgensen and Preston, 2000); estimates on marginal costs are often average figures for all passengers and, thus, without the travel distance and quality dimensions. Secondly, when the operators have financial targets, designing optimal relationships between fare and transport quality on one hand and travelling distance on the other hand, also demands knowledge about how travelling distance influences the fare elasticity and transport quality elasticity with respect to demand. Very few studies have dealt with this problem too: see for example Button (1993) and Ippolito (1981) who concluded that the fare elasticity is likely to increase, in absolute terms, with the length of the journey.

Our main concern in this paper will be to deduce optimal fare and supply schemes for the operators where special focus will be placed on how fares and quality of supply should vary with the distance travelled by the passengers. These analyses will be carried out under different assumptions concerning the transport operators' objectives. The above discussions give also rise to analyse how generalised travel costs will vary with the travel distance when transport fares and qualities are designed optimally. Since the latter issue will be seen in the light of the operators' objectives or preference function, our work should be important when discussing welfare consequences for people living in different geographical areas. If, for example, privation of transport operators influences the preference function, the actual transport policy and thereby the relationship between generalised travel costs and travelling distance for their customers, the change in ownership structure will influence long distance travellers and short distance travellers differently.[1] The above discussion should, thus, be an interesting aspect when evaluating possible disadvantages under different regulatory regimes of the public transport sector for people living in rural areas versus urban areas of a region.

The rest of the paper is structured as follows: In section 2 we outline a model describing reasonable cost and demand conditions for transport operators. Furthermore, we deduce the first order conditions for optimal fares and quality of the transport services under different assumptions concerning the operators' objectives. Section 3 analyses the travel distance influence on fare, the quality of the transport service and generalised travel costs when the operators pursue different goals. In order to simplify the comparative analysis, section 4 analyses the travelling distance influence on fare in particular when the quality of the transport service is regarded as uncontrollable or exogenous for the transport operators. As far

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[1] How the ownership structure of a transport company influences its objectives is, for example, discussed in Jørgensen and Pedersen (1990).

as the Norwegian transport operators' are concerned, this is a reasonable assumption; at least in the short run. Finally, in section 5 we draw some conclusions and suggest how to go further with the issues.

2. A Model of a Transport Operator's Behaviour

2.1 The Cost Structure

The costs of operating a system of transport routes within a specific area, *C*, are assumed to be given by:

$$C = C(X, q, A). \tag{1}$$

X is the number of passengers served, *q* is a variable denoting the quality of transport supply, supposed to influence on the subjective time costs passengers experience by travelling and, finally, *A* measures the mean distance travelled by the passengers in the system of routes supplied. With regard to the operator's cost function, we suppose that:

$$C_X > 0, C_q > 0, C_A > 0, C_{XX} \geq 0, C_{qq} \geq 0, C_{AA} \geq 0, C_{XA} > 0, C_{qA} > 0, C_{Xq} > 0.$$ [2]

This means that costs are supposed to be strongly increasing and convex in the number of passengers served, the quality of services and the average distance travelled. Additionally, it is assumed that marginal costs in serving passengers and marginal costs in supplying higher quality are strongly increasing when the travel distance becomes longer. Finally, it is supposed that marginal costs in serving passengers are strongly increasing in the quality supplied.

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[2] Here and throughout the paper the notation Y_Z means the partial derivative of *Y* with regard to *Z*.

2.2 Generalised Travel Costs and Demand Structure

In order to simplify, let us suppose that the consumers are identical and that their generalised travel costs, *R*, are given by the sum of the fare, *p*, and the value of the time spent on the journey, *T*. The time value is supposed to be dependent on the quality of service and the distance travelled:

$$R = p + T(q, A) = R(p, q, A)\,. \tag{2}$$

Moreover, the *T* and *R* functions are assumed to satisfy the following conditions:

$$R_p = 1,\ R_q = T_q < 0\ ,\ R_A = T_A > 0,\ R_{qq} = T_{qq} > 0,\ R_{qA} = T_{qA} < 0,\ R_{AA} = T_{AA} \geq 0\,.$$

Firstly, this means that generalised costs increase equal to the fare level, they decrease when quality is improved and are higher for longer than for shorter travel distances. Secondly, it is supposed that the reductions in the time costs from improved quality of supply become less as the quality is increased and the reductions in generalised costs from improved quality of supply become stronger as the travel distance increases. Finally, we find it reasonable to assume that the marginal increase in time cost as the travel distance becomes longer does not decrease as the distance travelled increases.

Furthermore, the numbers of travellers are supposed to be conditional of the generalised costs, defined by an ordinary demand function:

$$X = X(R)\,. \tag{3}$$

We make the common assumptions concerning the demand function; i.e. strongly decreasing and convex, $X_R < 0, X_{RR} \geq 0$, telling us that increased generalised costs reduce the number of travellers at a decreasing rate. Additionally, it should be noticed from (3) that the travel distance has no direct effect on the demand but influences it indirectly through the generalised costs. From the restrictions placed on the *R* function follow that a partial increase in travelling distance decreases the travel activity. This means, all other things being constant, that the longer the distance between two places, the lower will be the number of travellers.

2.3 The Transport Operator's Objectives

By using the demand function in (3), we define the consumer surplus coming from the transport activity, *V*, by:

$$V = \int_{R(p,q,A)}^{\infty} X(r)dr = V(p,q,A)\,. \tag{4}$$

Additionally, we make two reasonable assumptions concerning the *V* function. Firstly, it is assumed that as the travel distance increases, the consumers' utility coming from a marginal increase in quality of the transport services will not be reduced, i.e. $V_{qA} \geq 0$. Secondly, it is reasonable to believe that the consumers' welfare of higher quality will not increase as the quality of the transport services improves, i.e. $V_{qq} \leq 0$.

Based on (1), (2) and (3), the transport operator's profit, π, from serving the passengers in the routes is defined by:

$$\pi = pX[R(p,q,A)] - C\{X[R(p,q,A)],q,A\} = \pi(p,q,A)\,. \tag{5}$$

The profit function is supposed to be strictly concave in *p* and *q*. In order to be able to discuss optimal fare and quality schemes for different kinds of objectives, however, we introduce the following utility function hold by the transport operator:

$$\begin{aligned} U &= (1-\alpha)V(p,q,A) + \alpha\pi(p,q,A) \\ &= (1-\alpha)\int_{R(p,q,A)}^{\infty} X(r)dr + \alpha\left\{pX\left[R(p,q,A)\right] - C\left[X\left(R(p,q,A),q,A\right)\right]\right\} \\ &= U(p,q;A,\alpha). \end{aligned} \tag{6}$$

The first term in the operator's utility function is the consumer surplus coming from the public transport activity multiplied by $(1-\alpha)$, while the second term is the operator's profit multiplied by α. This means that we can interpret α and $(1-\alpha)$ as the operator's weights placed on profits and consumer surplus, respectively. In our analyses we restrict ourselves to discuss cases where $\frac{1}{2} \leq \alpha \leq 1$.[3] In the special case where $\alpha = \frac{1}{2}$, the transport operator weights consumer surplus and producer surplus equally. If the relationship between λ and α in footnote 3 holds, the efficiency loss of rising public funds in this case is zero ($\lambda = 0$) and maximising (6) is equivalent to maximising the sum of profit and consumer surplus; that is the social surplus related to the transport activity. Furthermore, it is seen that when $\alpha = 1$, the operator is only concerned about profit. Providing that the relationship between λ and α holds such that λ is infinite, maximising U is still equivalent to maximising social surplus. In intermediate cases where $\frac{1}{2} < \alpha < 1$, the operator evaluates profit higher than consumer surplus.

2.4 The Operator's Choice of Fare and Quality

Maximising the utility function in (6) with regard to the fare level, *p*, and the quality of service, *q*, gives the following first order conditions for optimality:

$$U_p = (2\alpha - 1)X(\cdot) + \alpha[(p - C_X)X_R] = 0\,, \tag{7}$$

$$U_q = -(1-\alpha)X(\cdot)R_q + \alpha[(p - C_X)X_R R_q - C_q] = 0\,.^4 \tag{8}$$

[3] It can be shown that if there is an estimation of an exogenous shadow price of rising public funds (λ) through inefficiencies caused by ordinary taxation, this value will implicitly define α by the following fraction: $\lambda = \dfrac{2\alpha - 1}{1-\alpha}$, see for instance Lewis and Sappington (1988) and the discussion made in Pedersen (1995). Here λ must be interpreted as the extra costs which come when one is going to rise one unit of money for public spending given that one taxes in a way which causes the lowest efficiency loss in the economy. When the above relationship between λ and α holds, maximising U in (6) is, thus, equivalent to maximising social surplus when $\frac{1}{2} \leq \alpha \leq 1$.

[4] Given the assumptions made above, it can be shown that the second order conditions are satisfied.

In order to interpret the optimal fare and quality of supply scheme, the conditions in (7) and (8) may be rewritten as:

$$\frac{p - C_X}{p} = -\frac{(2\alpha - 1)X}{\alpha X_R p} = \frac{2\alpha - 1}{\alpha} \cdot \frac{1}{E_p}, \tag{9}$$

$$-XR_q = C_q \,. \tag{10}$$

It is easily seen that the first order conditions obtained give the well-known monopoly solution when $\alpha = 1$. From (9) it follows that in this case the relative fare difference from marginal costs of serving passengers is equal to the absolute value of the inverse fare elasticity (E_p). Moreover, it is also seen from (9) that we obtain the ordinary first-best welfare solution when $\alpha = \frac{1}{2}$, i.e. the fare paid by travellers, *p*, should be equal to the costs experienced by the operator of serving a marginal passenger, C_X. In all intermediate cases where $\frac{1}{2} < \alpha < 1$, the relative difference between fare and marginal costs will be positive and lower than in the monopoly case, and as the value of α increases, *ceteris paribus*, the relative fare difference from marginal costs becomes higher.

Additionally it is seen from (10) that the optimum of fare and quality is characterised by equality between the income stemming from the final quality unit supplied, $-XR_q$, and the costs experienced by supplying this unit, C_q. It should be noticed that this holds for all values of α. This is because our model, based on homogenous consumers, implies that the marginal income of improved quality is equal to the marginal increase in the consumers' surplus, see Spence (1975). The value of α has, thus, no direct effect upon the choice of optimal quality.

3. Comparative Statics Concerning Optimal Schemes

In order to characterise the optimal fare and supply scheme further we are now searching for what happens to fare, quality of transport, generalised travel costs and travel activity when the travelling distance (*A*) and the transport operator's weight on profit (α) change.

3.1 Travelling Distance Influence on Optimal Fare and Quality

By differentiation of (7) and (10) with respect to *A*, we obtain the following expressions for changed optimal values of *p* and *q*, respectively p^* and q^*:

$$\frac{\partial p^*}{\partial A} = \frac{-YU_{pA} - ZU_{pq}}{E}, \qquad (11)$$

$$\frac{\partial q^*}{\partial A} = \frac{ZU_{pp} + WU_{pA}}{E}, \qquad (12)$$

where:

$$U_{pp} = (3\alpha - 1)X_R - \alpha C_{XX}(X_R)^2 + \alpha(p - C_X)X_{RR} < 0,$$

$$Y = -X_R(R_q)^2 - XR_{qq} - C_{Xq}X_R R_q - C_{qq} = V_{qq} - C_{Xq}X_R R_q - C_{qq} < 0,$$

$$U_{pq} = (2\alpha - 1)R_q X_R - \alpha C_{XX}(X_R)^2 R_q - \alpha C_{Xq}X_R + \alpha(p - C_X)X_{RR}R_q \geq (<) 0$$

if $$(2\alpha - 1)R_q X_R - \alpha C_{XX}(X_R)^2 R_q - \alpha C_{Xq}X_R \geq (<) -\alpha(p - C_X)X_{RR}R_q,$$

$$U_{pA} = (2\alpha - 1)X_R R_A - \alpha C_{XX}(X_R)^2 R_A - \alpha C_{XA}X_R + \alpha(p - C_X)X_{RR}R_A \geq (<) 0$$

if $$\alpha[(p - C_X)X_{RR}R_A - C_{XA}X_R] \geq (<) \alpha C_{XX}(X_R)^2 R_A - (2\alpha - 1)X_R R_A,$$

$$W = -X_R(R_q + C_{Xq}) \geq (<) 0$$

if $$C_{Xq} \geq (<) -R_q,$$

$$Z = C_{qX}X_R R_A + C_{qA} + X_R R_A R_q + XR_{qA} \leq (>) 0$$

if $$V_{qA} = -X_R R_A R_q - XR_{qA} \geq (<) C_{qA} + C_{qX}X_R R_A,$$

and $$E = U_{pp}Y - U_{pq}W > 0.$$

The effects on optimal fare and optimal quality might be seen as a sum of a direct and an indirect effect. Taking a look at (11), the direct effect on p is $-YU_{pA}/E$, telling us what happens to the optimal fare for a predetermined value of q. The indirect effect is $-ZU_{pq}/E$, measuring what happens to the optimal fare as a consequence of possible changed value of optimal q. Analogously, as seen in (12), ZU_{pp}/E and WU_{PA}/E are the direct and the indirect effects on q, respectively. According to presumptions made and ensuring that the second order conditions are satisfied, we know that $U_{pp}<0$, $Y<0$ and $E>0$. Based on these assumptions only, the signs of the direct and indirect effects are, however, uncertain both in (11) and (12). This makes it, of course, impossible to draw unambiguous conclusions regarding the sums. With reasonable restrictions placed upon the actual functions it is, thus, impossible to draw unambiguously conclusions how travel distance influences fare and transport quality, no matter how the transport operator weights profit contra consumer surplus.

In order to come a bit further in our analysis, let us sort out situations where the signs of the changes in p and q are unambiguous. This will be the case when the direct and indirect effects have the same signs. From (11) and (12) we can identify the following cases:

If (a) $U_{pA}\geq 0, U_{pq}\geq 0$ and $Z\leq 0$ or (b) $U_{pA}\geq 0, U_{pq}\leq 0$ and $Z\geq 0$ then $\dfrac{\partial p^*}{\partial A}\geq 0$. (13)

If (a) $Z\leq 0, U_{pA}\geq 0$ and $W\geq 0$ or (b) $Z\leq 0, U_{pA}\leq 0$ and $W\leq 0$ then $\dfrac{\partial q^*}{\partial A}\geq 0$. (14)

If (a) $U_{pA}\leq 0, U_{pq}\leq 0$ and $Z\leq 0$ or (b) $U_{pA}\leq 0, U_{pq}\geq 0$ and $Z\geq 0$ then $\dfrac{\partial p^*}{\partial A}\leq 0$. (15)

If (a) $Z\geq 0, U_{pA}\geq 0$ and $W\leq 0$ or (b) $Z\geq 0, U_{pA}\leq 0$ and $W\geq 0$ then $\dfrac{\partial q^*}{\partial A}\leq 0$. (16)

In (13) we have found two sets of sufficient conditions giving us two cases where the optimal fare will be non-decreasing in the travel distance. In (13a) the direct effects on fare and quality are both non-negative ($U_{pA} \geq 0$ and $Z \leq 0$ respectively) and higher quality imposes the operator to choose at least as high fare as originally ($U_{pq} \geq 0$). In (13b) the direct effect on fare is non-negative, $U_{pA} \geq 0$, and the direct effect on quality is non-positive, implying $Z \geq 0$. Additionally, (13b) is characterised by a situation where lower quality induces the operator to choose a fare at least as high as originally ($U_{pq} \leq 0$). Analogously, from (14), we find two sets of conditions giving us two cases where the optimal quality is non-decreasing in the travel distance.

As mentioned previously, one would expect that fare and the quality of the transport services increase with travelling distance, no matter what weight the transport operator gives to profit (α). Let us, therefore, have a closer look at (13) and (14) and infer what restrictions upon the actual functions these inequalities in combination lead to. By combining the information in (13) and (14) we find the sufficient conditions for excluding any other possibilities than $\partial p^{*} / \partial A > 0$ and $\partial q^{*} / \partial A > 0$ are:

$$U_{pA} > 0, \; Z < 0, \; U_{pq} > 0, \; W > 0 . \tag{17}$$

When U_{pA} is positive and Z is negative, we know for sure that the direct effects on fare and quality respectively, caused by longer travel distance, are positive. Furthermore, if U_{pq} and W are positive, it is secured that also the indirect effects on fare and quality respectively are positive meaning that higher quality imposes increase in fare and higher fare imposes higher quality.[5] Therefore, let us take a closer look at the situation in (17).

The first inequality in (17) means that the marginal utility gain for the transport operator by increasing fare must be higher when the distance becomes longer, i.e. $U_{pA} > 0$. From the expression defining U_{pA} above, generally having an ambiguous sign, we see that this will surely hold in the welfare maximising case ($\alpha = \frac{1}{2}$) given that marginal costs of serving passengers are independent of the number of travellers, i.e. $C_{XX} = 0$. The second inequality in (17), restricting the sign of *Z*, concerns the effect on marginal utility of quality when the travel distance changes. Looking at the different terms defining *Z*, it should be noticed that V_{qA} measures the increase in the marginal consumer surplus with regard to quality when distance becomes longer, while $C_{qA} + C_{Xq} X_R R_A$ measures the change in the marginal costs in supplying higher quality when the travel distance increases. The first term in these marginal costs, C_{qA}, is the direct marginal cost increase, while the second term, $C_{Xq} X_R R_A$, measures the marginal cost reductions following from fewer travellers using the route system when the general costs increases as a consequence of longer distance. If the marginal growth in the consumers' welfare of quality dominates the net effects on the marginal costs of quality, *Z* will be negative. The third inequality in (17) states that fare and quality must be complements in the operator's utility function, i.e. $U_{pq} > 0$. This means that the operator's utility of increasing the fare marginally becomes higher as the quality is improved. From the expression

[5] It should be noticed that U_{pq} and *W* are different because we have chosen to use one of the first order conditions (equation (7)) and a combination of (7) and (8) (equation (10)) when doing the comparative statics. This means that *W* must be interpreted in the light of what happens to optimal *q* as a consequence of marginal changes in *p* given that *p* is optimally chosen, i.e. that equation (7) holds.

defining U_{pq} above, it is seen that this will always hold in the case of a welfare maximising operator because then fare is equal to marginal costs, i.e. $p = C_X$. It is also seen that this holds if the demand function is linear, i.e. $X_{RR} = 0$, without regard to the value of α. The fourth condition in (17) concerns the sign of *W*. It is seen that if $C_{Xq} > -R_q$, i.e. that the marginal costs in serving passengers increase more than the generalised costs decrease when quality of services increases, *W* is positive. If this holds, the operator will find it advantageously improving the quality, as the optimal fare becomes higher. The above discussion gives, thus, rise to the following conclusions:

Result 1: *The following sufficient conditions are identified securing that optimal fare and optimal quality will be higher as the travel distance becomes longer:*

(a) *The operator's marginal utility of fare must be increasing in the travel distance. This surely holds when the transport operator gives equal weight to profit and consumer surplus and when marginal costs of serving another passenger are independent on the number of travellers.*

(b) *The increase in the marginal consumer surplus of quality for longer travel distance must dominate the net increase in the operator's marginal costs of quality as the distance becomes longer.*

(c) *Price and quality must be complements in the operator's utility function.*

(d) *The growth in the marginal costs of serving passengers as quality increases must be higher than the reduction in the generalised costs caused by the same increase in quality.*

Before leaving the discussion of the impact on optimal fare and optimal quality of supply caused by marginal increase in travel distance, it might be useful to take a short look at the cases stated in (15) and (16) above. In (15) we find sufficient conditions identifying two cases which are contra-intuitive in the sense that the optimal scheme is characterised by fares being non-increasing in the travel distance. Analogously, (16) gives us sufficient conditions for two cases where optimal quality is non-increasing in the travel distance, which also might be seen as unlikely cases compared to what one should expect in practice. The situations described in (15) and (16) can, however, not be ruled out based on the original assumptions made in our theoretical model.

3.2 The Operator's Weight on Profit Influence on Optimal Fare and Quality

In order to study how the weight on profit influence on the optimal choices, we differentiate equations (7) and (10) with respect to α. It then follows:

$$\frac{\partial p^*}{\partial \alpha} = \frac{-Y(\pi_p + X)}{E}, \tag{18}$$

$$\frac{\partial q^*}{\partial \alpha} = \frac{W(\pi_p + X)}{E}, \tag{19}$$

where $\pi_p = (p - C_X)X_R + X$. If profit maximum fare is the highest possible one in our model, we know that for all relevant p's, $\pi_p \geq 0$. Then it follows directly from (18) that if the weight on profits becomes higher (and the weight on consumer surplus decreases), the optimal fare will increase. What happens with the optimal quality of services will depend on the sign of W. If $C_{Xq} > (\leq) - R_q$ such that W is positive (negative), the optimal quality of service increases (decreases) as the weight on profits becomes higher. Then the higher fares stimulate the transport operator to increase (decrease) quality. In summary, we can, thus, conclude:

Result 2: *If the operator weights profit more heavily in the utility function, it follows that:*

(a) *Optimal fare will increase.*

(b) *Optimal quality increases (decreases) if marginal costs of serving passengers increase more (less) than generalised costs are reduced as the quality is improved.*

(c) *If one of the sufficient conditions ensuring that optimal fare and optimal transport quality increases with travelling distance (*$W > 0$*), optimal quality will increase as the weight given to profit increases.*

3.3 Influence on Generalised Travel Cost and on the Number of Travellers

Let the values of generalised costs and numbers of travellers in optimum be denoted R^* and X^* respectively. Using (2), (3), (11), (12), (18) and (19) it is found that changes in these two variables for increased distance and increased weight on profit are given by:

$$\frac{dR^*}{dA} = \frac{\partial p^*}{\partial A} + R_q \frac{\partial q^*}{\partial A} + R_A = \frac{(XR_{qq} + C_{qq})U_{pA} - \alpha ZW + R_A E}{E}, \tag{20}$$

$$\frac{dX^*}{dA} = X_R \frac{dR^*}{dA} = X_R[R_A + \frac{(XR_{qq} + C_{qq})U_{pA} - \alpha ZW}{E}], \tag{21}$$

$$\frac{dR^*}{d\alpha} = \frac{\partial p^*}{\partial \alpha} + R_q \frac{\partial q^*}{\partial \alpha} = \frac{(XR_{qq} + C_{qq})(\pi_p + X)}{E} > 0, \tag{22}$$

$$\frac{dX^*}{d\alpha} = X_R \frac{dR^*}{d\alpha} < 0. \tag{23}$$

What happens to generalised costs when the travel distance becomes longer is according to (20) dependent on the sign and sizes of $\partial p^* / \partial A$, $R_q(\partial q^* / \partial A)$ and R_A. If (17) holds, however, R will increase more than with the value of the direct effect as the travel distance becomes longer, i.e. $(\partial R^* / \partial A) > R_A > 0$. It then follows from (21) the number of travellers will be reduced when the travel distance increases; that is $\partial X^* / \partial A < 0$. Furthermore, from (22) and (23) it follows that the generalised travel costs increase and the numbers of travellers decrease as α becomes higher for any travelling distances. The above discussion may, thus, be summarised as follows:

Result 3:

(a) When the transport operator gives higher weight to profit, generalised travel costs will increase and travel activity will be reduced.

(b) Under the original restrictions placed on the actual functions, the travelling distance influence on generalised travel costs and travel activity is ambiguous.

(c) When the conditions described under Result 1 hold, an increase in the travelling distance will increase generalised travel cost and decrease the numbers of travellers.

4. Comparative Statics for Predetermined Quality of Transport

Let us now suppose that the quality is predetermined (for instance by the superior public authorities) and, thus exogenous for the transport operator. In many countries, this is a reasonable assumption, at least in the short run. In order to simplify the discussion in this case, let us restrict ourselves to situations where the relationships between operator's costs and the number of passengers and between the number of passenger and generalised travel costs are linear; that is $C_{XX} = X_{RR} = 0$.

4.1 Influence on Optimal Fare of Changes in the Exogenous Variables

When $C_{XX} = X_{RR} = 0$, then (11) and (18) simplify to:

$$\frac{\partial p^*}{\partial A} = -\frac{U_{pA}}{U_{pp}} = -\frac{(2\alpha - 1)R_A - \alpha C_{XA}}{3\alpha - 1}, \qquad (24)$$

$$\frac{\partial p^*}{\partial \alpha} = -\frac{2X + (p - C_X)X_R}{(3\alpha - 1)X_R} > 0\,. \tag{25}$$

The sign of $\partial p^* / \partial A$ is now dependent on the sign of the direct effect only and in cases where $C_{XX} = X_{RR} = 0$, it follows that $U_{pA} \geq (<) 0$ if $\frac{2\alpha - 1}{\alpha} R_A \leq (>) C_{XA}$. This means that the optimal fare will be:

- Increasing in the travel distance if $\frac{2\alpha - 1}{\alpha} R_A < C_{XA}$;
- Decreasing in the travel distance if $\frac{2\alpha - 1}{\alpha} R_A > C_{XA}$.

The left hand side of these inequalities, measures the increase in the travellers' generalised costs stemming from longer distance, weighted by an index defining the relative more influence profit has to consumer surplus in the operator's utility function in (6). If profits and consumer surplus is equally valued, i.e. $\alpha = \frac{1}{2}$, this index is zero, while it is 1 when only profits count in this objective function, i.e. $\alpha = 1$. The right hand side of these inequalities, C_{XA}, is the increase in marginal costs of serving passengers when distance increases. This means that if "the weighted" generalised costs grow more slowly (faster) than the operator's marginal costs in serving passengers as the travel distance increases, the optimal fare will be higher (lower) for longer distances than for shorter ones. If we consider the case of pure welfare maximum, i.e. $\alpha = \frac{1}{2}$, it follows directly from (24) that the growth in the fare as the travel distance becomes longer should equalise the growth in the marginal costs of serving travellers as the distance increase, i.e. $\frac{\partial p^*}{\partial A} = C_{XA}$. In the profit maximising case, i.e. $\alpha = 1$, the fare will increase (decrease) with travel distance if the generalised costs increase more slowly (faster) than marginal costs in serving passengers as the travel distance becomes longer, i.e. $R_A \leq (>) C_{XA}$. Equation (25) tells us that optimal fare will increase when the operator gives higher weight to profit.

Equations (24) and (25) hold if *q* is supposed to be an exogenous variable. However, it is interesting to know how exogenous changes in the quality influence optimal fare. Given our simplifying assumptions that $C_{XX} = X_{RR} = 0$, it follows from (7) that:

$$\frac{\partial p^*}{\partial q} = -\frac{(2\alpha - 1)R_q - \alpha C_{Xq}}{3\alpha - 1} > 0 \,. \qquad (26)$$

No matter transport distance and how the transport operator weights profit versus consumer surplus, higher demands from the authorities regarding the quality of the transport supply, will always impose the transport operators to increase fares.

In order to elucidate more thoroughly how optimal fare is influenced by the exogenous variables above, let us have a closer look at (24), (25) and (26). By differentiation of (24) with regard to *A*, we obtain:

$$\frac{\partial^2 p^*}{\partial A^2} = -\frac{(2\alpha - 1)R_{AA} - \alpha C_{XAA}}{3\alpha - 1} \,. \qquad (27)$$

It seems reasonable to believe that $C_{XAA} \leq 0$, i.e. that the increase in marginal costs of serving passengers when the travel distance increases, is not increasing as the distance becomes longer. Having in mind that we have supposed that $R_{AA} \geq 0$, the expression in (27) will be non-positive. This means that the fare increase per distance unit should not to be higher as the travel distance becomes longer, i.e. $\frac{\partial^2 p^*}{\partial A^2} \leq 0$. In the case of pure welfare maximum ($\alpha = \frac{1}{2}$) and $C_{XAA} = 0$, the relationship between travel distance and fare is linear. Furthermore, by differentiation of (25) with regard to α, it is seen that:

$$\frac{\partial^2 p^*}{\partial \alpha^2} = \frac{3X_R[2X + (p - C_X)X_R]}{[(3\alpha - 1)X_R]^2} < 0. \tag{28}$$

This means that the influence on giving more weight on profits on optimal fare reduces as the value of α is increased. Moreover, differentiation of (26) with regard to *q* gives us that:

$$\frac{\partial^2 p^*}{\partial q^2} = -\frac{(2\alpha - 1)R_{qq} - \alpha C_{Xqq}}{3\alpha - 1}. \tag{29}$$

A sufficient condition for (29) to be negative, is that $C_{Xqq} \leq 0$, i.e. that the higher quality becomes, the lower is the growth in marginal costs of serving passengers as quality improves. In this case it is ensured that the optimal fare grows less with higher quality of transport as the quality is further improved. In the case where the transport operator aims to maximise social welfare and where $C_{Xqq} = 0$, the relationship between fare and transport quality is linear. The above discussion may be summarised as follows:

Result 4: *Given that the quality of transport supply is exogenous and that* $C_{XX} = R_{XX} = 0$*, we can conclude that:*

(a) *Optimal fare will be increasing and concave in the weight given to profit in the transport operator utility function.*

(b) *The less weight the transport operator gives to profit, the more likely it would be that optimal fare increases with the travelling distance. If* $C_{XAA} \leq 0$*, this relationship would be concave. In the case where the operator aims to maximise social welfare and where* $C_{XAA} = 0$*, optimal fare will increase linearly with the travelling distance.*

(c) *Optimal fare will increase when the transport authorities demand higher quality of transport from the operator and it will increase concavely if* $C_{Xqq} \leq 0$*. When the operator aims to maximise social welfare and* $C_{Xqq} = 0$*, the relationship is linear.*

4.2 Interdependence between the Effects of the Exogenous Variables

It is also interesting to know what happens with the marginal changes in optimal fare level, described by (24), (25) and (26), when there are changes in one of the other exogenous variables. Firstly, it is seen that:

$$\frac{\partial^2 p^*}{\partial A \partial \alpha} = -\frac{R_A + C_{XA}}{(3\alpha - 1)^2} < 0 . \qquad (30)$$

The interpretation of this is that as the transport operator gives higher weight to profit, the fare level will be less positively related to the travelling distance. This means, for example, that an operator maximising profits ($\alpha = 1$) will deduce an optimal fare system where the growth in fare when travel distance increases is lower than what an operator maximising welfare ($\alpha = \frac{1}{2}$) will calculate. Equation (30) may also be interpreted such that the longer the travel distance, the less influence have the operator's objectives on optimal fare. Secondly, it is found that:

$$\frac{\partial^2 p^*}{\partial q \partial A} = -\frac{(2\alpha - 1)R_{qA} - \alpha C_{XqA}}{3\alpha - 1} \geq 0 \quad \text{if} \quad C_{XqA} \geq 0 . \qquad (31)$$

This means that if the growth in marginal costs serving passengers when quality increases becomes higher when the travel distance increases, it is ensured that the optimal fare increases more with longer travel distances as the quality is improved. Finally, it is found that:

$$\frac{\partial^2 p^*}{\partial q \partial \alpha} = -\frac{(4\alpha - 1)R_q - (2\alpha - 1)C_{Xq}}{(3\alpha - 1)^2} > 0 . \qquad (32)$$

This inequality tells us that the growth in the optimal fare followed by improvements in quality increases with the weight given to profit in the objective function. This means that as we move from a social planner, maximising the sum of producer surplus and consumer surplus towards an operator concerned about profit only, fares becomes more strongly positively related to quality. In summary, we can conclude:

Result 5: *Given that quality of transport supply is exogenous, and that* $C_{XX} = X_{RR} = 0$ *we find that:*

(a) When the transport operator gives higher weight to profit, optimal fare will be less positively related to the travelling distance.

(b) The marginal impact on optimal fare from increased quality demands becomes higher as the travel distance and the weight put on profits in the operator's utility function increase.

4.3 Influence on Generalised Travel Costs and on the Number of Travellers

Also when the quality of transport supply is exogenous for the transport operator, it might be interesting to show what happens to the total individual resources spent on transport (R^*) and on the number of travellers (X^*) when A, α and q change. Using the expressions in (3), (24), (25) and (26) above give us that:

$$\frac{dR^*}{dA} = \frac{\partial p^*}{\partial A} + R_A = \frac{\alpha(R_A + C_{XA})}{3\alpha - 1} > 0, \tag{33}$$

$$\frac{dR^*}{d\alpha} = \frac{\partial p^*}{\partial \alpha} > 0, \tag{34}$$

$$\frac{dR^*}{dq} = \frac{\partial p^*}{\partial q} + R_q = \frac{\alpha(C_{Xq} + R_q)}{3\alpha - 1} \geq (<) 0 \quad \text{if} \quad C_{Xq} \geq (<) - R_q. \tag{35}$$

The generalised costs are, thus, increasing both in travel distance and in the weight given to profit in the transport operator's utility function. However, it is generally uncertain what happens with R when the transport quality is changed, see (35). The direct effect is of course that generalised costs are reduced when quality is improved, but the fare is shown to increase implying higher individual travelling costs. It is seen that this indirect effect through higher fare level dominates the direct effect if the marginal costs in serving passengers grow faster than the generalised costs are reduced as quality is improved. In summary, it is not sure that higher quality demands on transport supply from the authorities, will reduce generalised travel costs.

Using (3) it is easily seen that the number of passengers will be reduced when the travelling distance and the transport operator's weight on profit increase, while the quality of transport demands impacts on the number of passengers is ambiguously. Furthermore, using equations (2), (27), (28), (29) we obtain:

$$\frac{d^2R^*}{dA^2} = \frac{\partial^2 p^*}{\partial A^2} + R_{AA} = \frac{\alpha(R_{AA} + C_{XAA})}{3\alpha - 1} \geq (<) 0 \quad \text{if} \quad R_{AA} \geq (<) - C_{XAA}, \tag{36}$$

$$\frac{d^2R^*}{d\alpha^2} = \frac{\partial^2 p^*}{\partial \alpha^2} < 0, \tag{37}$$

$$\frac{d^2R^*}{dq^2} = \frac{\partial^2 p^*}{\partial q^2} + R_{qq} = \frac{\alpha(R_{qq} + C_{Xqq})}{3\alpha - 1} \geq (<) 0 \quad \text{if} \quad R_{qq} \geq (<) - C_{Xqq}, \tag{38}$$

$$\frac{d^2R^*}{dA d\alpha} = \frac{\partial^2 p^*}{\partial A \partial \alpha} < 0, \tag{39}$$

$$\frac{d^2R^*}{dq dA} = \frac{\partial^2 p^*}{\partial A \partial q} + R_{qA} = \frac{\alpha(R_{qA} + C_{XqA})}{3\alpha - 1} \leq (>) 0 \quad \text{if} \quad C_{XqA} \leq (>) - R_{qA}, \tag{40}$$

$$\frac{d^2R^*}{d\alpha dq} = \frac{\partial^2 p^*}{\partial \alpha dq} > 0. \tag{41}$$

It is seen from (36) and (38) that R can be both concave and convex in A and q, dependent on the size of R_{AA} compared to C_{XAA} and R_{qq} compared to C_{Xqq}, respectively. Equation (37) implies that generalised travel costs increase and are concave in α, meaning that as the weight on profit is increased, the slower grow the generalised costs. It is also found from (39) that $\frac{dR^*}{dA}$ is decreasing in α, implying that the marginal growth in individual resources spent on travelling with regard to distance becomes lower as the operator weights profits more heavily in his utility function. From (41) it is seen that $\frac{dR^*}{d\alpha}$ is increasing in *q*. This means that the marginal growth in generalised costs with regard to α becomes higher as the quality improves. Finally, equation (40) implies that the marginal impact on generalised costs from higher *A* can be both higher and lower as quality improves conditional on the size of R_{qA} compared to C_{XqA}. The above discussion can be summarised as follows:

Result 6: *Given that quality of transport supply is exogenous, and that* $C_{XX} = X_{RR} = 0$ *we have:*

(a) *Generalised travel costs are strictly increasing in the travelling distance and in the transport operator's weight on profit.*

(b) *The influence on generalised cost of increased transport quality demands from the regulators is ambiguous; they will increase (decrease) if* $C_{Xq} \geq (<) - R_q$.

(c) *The marginal impact on R from changed A becomes lower the higher weight the transport operator places on profits and lower (higher) as quality is improved if* $C_{XqA} \leq (>) - R_{qA}$.

(d) *The marginal impact on R from changed weight the transport operator gives to profit becomes higher the higher the demands from the authorities regarding the quality of the transport supply.*

5. Concluding Remarks

In this work we have developed a model aiming to discuss how travelling distance and the transport operator's weight on profit versus consumer surplus influence the fare levels, the quality of transport supply and generalised travel costs, measured as the sum of pecuniary costs and time costs for the traveller. As a starting point we assume that both fares and quality of transport supply are regarded as controllable variables for the transport operator. In order to simplify the analysis, we also, however, discuss thoroughly the specific case where the quality of transport supply is exogenous for the transport operator and controlled by the transport authorities. As emphasised previously, this may be a realistic case in many circumstances. Besides analysing how travelling distance and the operator's objectives influence fare and generalised travel costs in this particular case, we also discuss how changes in the transport authorities' demands regarding the quality of the transport supply, influence these variables.

In almost all practical examples within transportation, we believe, one will find that the fare levels and probably also the quality of supply are increasing as the travel distance for the passengers becomes longer. Common sense indicates too that travellers' generalised cost increase with the distance travelled; the eventual positive effects of increased transport quality are outweighed by longer travel distance and higher fares. Consequently, the number of travellers becomes less the longer the distance between two places. This is in line with the conclusions in the most common gravity models of trip distribution; see for example McDonald (1997). Our model, built on what seems to be reasonable assumptions regarding the transport operators' objectives and their costs and demand conditions, however, generally gives ambiguous answers to how fare, quality of supply and generalised travel costs vary with the distance travelled by the consumers. We have, therefore, inferred how different additional restrictions imposed on the actual functions influence the results.

Firstly, if fare and quality are complements in the operator's objective function and the operator's marginal utility with regard to fare and quality of supply is increasing in the travel distance, it is secured that both optimal fare and quality will be increasing in the travel distance. Under the above conditions will an increase in the travelling distance also increase generalised travel costs and reduce the number of travellers; the increase in fares as the distance becomes loner outweighs the effects of increased quality of transport supply. It is, however, worth noting that without imposing rather strict restrictions upon the transport operator's cost and demand conditions, it is ambiguous how travelling distance influence fares, quality of transport and generalised travel costs.

Secondly, if the transport operator weights profit more heavily in the utility function, optimal fare will increase. Furthermore, if an increase in quality of transport influences marginal costs of serving passengers more (less) than generalised costs are reduced, an increase in the operator's weight on profit will increase (decrease) optimal quality of supply. When having a closer look at the restrictions ensuring that fare and quality of transport increase with the travelling distance, it is seen that these conditions also ensure that quality of supply will increase with the operator's weight given to profit. Finally, generalised travel costs will always increase the more weight the transport operator gives to profit. If one accept the common hypothesis that privately owned companies give more weight to profit versus consumer surplus than the public ones, one should expect that transport users would prefer publicly owned transport companies.

Thirdly, in the version of the model where the quality of supply is predetermined, it is found, that the optimal fare is increasing and concave in the weight the transport operator gives to profit versus consumer surplus. In this case to, the travelling distance influence on optimal fare is according to the original restrictions on the actual functions ambiguous. The less weight the transport operator gives to profit versus consumer surplus and the less generalised travel costs are influenced by the travelling distance (i.e. the lower the travellers' time costs are and the higher the speed of the mode), the more likely it is that fare is increasing and concave in the travelling distance. Such relationship between fare and travel distance is not unusual; in Norway, concave positive relationships between fares and travelling distance are, for example, estimated for trains and plains, see Ertkjern and Tausvik (1996). Furthermore, when the regulators demand higher quality of supply from the operators, optimal fares will increase. When the quality of transport supply is exogenous for the transport operators, generalised travel costs will increase when the travel distance becomes longer and when the operator gives higher weight to profit. The effect on generalised costs of increased demands for the quality of transport supply is, however, ambiguous; these cost increase (decrease) if the marginal costs increase more (less) than the generalised costs are reduced as the quality increases. It is, thus, not sure that the travellers will benefit from increased quality demand of the transport supply. As far as the interdependence between the effects of the exogenous variables is concerned, it is seen that the optimal fare and generalised travel costs are less related to travel distance as the weight on profits versus consumer surplus in the objective function is increased. It also seems most likely that optimal fare and generalised travel costs are stronger positively related to travel distance as the quality is improved. Higher quality demands and more weight on consumer surplus, make, thus, generalised travel cost more sensitive to the travel distance.

The most important conclusions of the analysis above are that it is not as clear as one should expect how travel distance influences fare, quality of transport supply and generalised travel costs. Nevertheless, our model analysis has made us aware of some important mechanisms influencing on the optimal fare and quality of supply schemes which one should have in mind before turning over to empirical studies of the fare system and of the quality schemes for different modes of transport. In order to operationalise the model further and in that way develop econometric models for estimating the actual relationships and testing relevant hypotheses, it may be sensible to impose special functional forms as far as the cost structure and the demand conditions are concerned.

The data needed for such econometric analyses are probably attainable in many countries. Firstly, it is probably easy to obtain information about how fares vary with the travelling distance for all modes of transport and for different transport operators. Secondly, different aspects of the ownership structure of a transport company may give a good indication on how the company weights profit versus consumer surplus, (see, for example, Jørgensen and Pedersen, 1990 and Jørgensen *et al*, 1995, for a discussion on this issue) and such information is also attainable; at least in Norway. Thirdly, it is possible to give indicators of the quality of the transport supply for routes with different average travelling distance; for example by finding the average age of the modes serving the routes. Finally, even though the travel distance dimension is scarcely dealt with in empirical studies analysing the demand and the costs conditions for different modes of transport, a vast amount of data are available for such analyses. By taking our theoretical model as a basis, it should, therefore, be possible to come up with fruitful empirical analyses.

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